Toward a Quantitative Theory of Food Consumption Choices and Body Weight

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Toward a quantitative theory of food consumption choices and body weight

Sebastien Buttet* and Veronika Dolar†

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Abstract

We propose a calibrated dynamic model of food consumption choices and body weight to study changes in daily caloric intake, weight, and the away-from-home share of calories consumed by adult men and women in the U.S. during the period between 1971 and 2006. Calibration reveals substantial preference heterogeneity between men and women. For example, utility losses stemming from weight gains are ten times greater for women compared to men. Counterfactual experiments show that changes in food prices and household income account for half of the increase in weight of adult men, but only a small fraction of women’s weight. We argue that quantitative models of food consumption choices and body weight have a unique role to play in obesity economics future research.

JEL Classification: I10, D91

Keywords: Obesity, body weight, calories, food away from home, price per calorie.

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1 Introduction

The goal of this paper is to offer a quantitative analysis of changes in food consumption choices and body weight for adult men and women in the United States during the period between 1971 and 2006. While earlier research in obesity economics (e.g., Cutler et al. (2003)) conclusively demonstrated that changes in eating habits, especially the continued increase in daily caloric intake when calories expended remained constant, are responsible for the recent body weight gain in the US, there is much less agreement over what accounts for changes in eating habits of Americans and thus weight (Cawley, 2011)?

Controlled experiments, on the one hand, show that lowering prices on healthy foods and hiking prices of unhealthy foods usually induce people to switch to healthier food choices (French et al., 1997, 2001; Epstein et al., 2007) but these changes in food choices do not necessarily translate to lower body weight (Schroeter et al., 2008; Fletcher et al., 2014). Statistical analysis of observational data, on the other hand, suggest that broad-based reductions in food prices tend to lead to body weight increase (Lakdawalla and Philipson, 2009; Lakdawalla and Zheng, 2011). However, when one looks at the effect of price changes for more narrowly defined food categories, such as food away from home versus food at home, results are more mixed. For example, Chou et al. (2004) show that prices at full-service or fast-food restaurants are negatively correlated with adult body-mass index, while Anderson and Matsa (2011) and Beydoun et al. (2008) find no causal relationship between restaurant prices and obesity.

This paper offers an alternative research option to statistical analysis of observational data and controlled experiments. We propose a calibrated dynamic model rooted in micro-economic foundations to analyze the quantitative impact of rising household income

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1 Cutler et al. (2003) show that declines in energy expenditures in the U.S. are too small to account for the observed changes in weight between 1965 and 1995. Most of the switch to a sedentary lifestyle ended by the 1970s while daily calorie intake and obesity rates continued to increase after the 1970s. Understanding changes in total calories consumed is thus an important issue to consider when analyzing the obesity epidemic.

2 Schroeter et al. (2008) and Fletcher et al. (2014) show that extra sales taxes on soda or food sold at restaurants, while discouraging consumption of this type of food, do nothing to improve people’s health or reduce obesity.
and declining food prices on body weight, daily caloric intake, and the away-from-home fraction of calories consumed by American men and women since the 1970s.

We view our work as being complementary to traditional applied micro-economic research since using a structural model addresses certain shortcomings of controlled experiments or observational data studies. First, no endogeneity issues such as reverse causality arise in structural models because the underlying theory imposes explicit restrictions on the economic mechanisms through which household income and food prices affect food consumption choices and body weight. Second, structural models have an advantage over controlled experiments. They provide a single and consistent framework which allows assessing linkages between food consumption choices and body weight. Controlled experiments, in contrast, study consumers’ decisions over a handful of food items only and thus are not helpful in connecting eating decisions to body weight.

In two important contributions to the obesity economics literature, Lakdawalla et al. (2005) and Lakdawalla and Philipson (2009) show that food consumption choices can be formulated as a dynamic program where body weight, the state variable, also enters the utility function. We extend their work by nesting two types of food: food consumed away from home (FAFH) and food at home (FAH) with a constant elasticity of substitution function. Given our functional form assumptions, a rule of thumb for food consumption choices is that the relative price of food affects what type of food people eat while real

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3For example, in our model, the weight law of motion links calorie consumption and weight in the current period to next period body weight, while first-order conditions determine the optimal allocation of resources toward food and non-food consumption given household income and relative food prices.

4The assumptions in Lakdawalla et al. (2005) and Lakdawalla and Philipson (2009) are new and interesting because state variables do not usually enter agents’ utility in dynamic economic models. For example, in the one-sector growth model of macroeconomics, market goods are produced using physical or human capital (the state variables) as inputs. However, only the stream of market good consumption enters agent’s utility, not physical or human capital (Lucas et al., 1989; Ljungqvist and Sargent, 2012). In the field of obesity economics, however, there are good reasons to believe that weight in the utility function makes sense. First, weight is a proxy for health. Today, Americans are heavier than what the medical field recommends and the obesity epidemic is associated with many of the leading causes of preventable death such as heart disease, stroke, type-II diabetes and certain types of cancer (National Institute of Health, 2005). Second, people care about the way they look and being too skinny or too fat can affect people’s self-esteem above and beyond medical considerations.
income determines total number of calories consumed and thus weight. As a result, households respond optimally to a decline in the relative price of FAFH by reducing the share of calorie intake from FAH, while rising household income leads to an increase in total calories consumed and weight.\textsuperscript{5}

We use data moments for total calories, food shares, and weight for men and women from the 1971 National Health and Nutrition Examination Survey (NHANES I) to calibrate preferences parameters. We also derive an analytical expression linking preference parameters, including the elasticity of substitution, to empirical estimates of price and cross-price elasticity of demand for FAFH. One interesting result from the calibration is that the elasticity of substitution between FAFH and FAH is negative for both men and women. The main takeaway from the calibration, however, is that there is substantial preference heterogeneity between men and women. For example, utility losses stemming from weight gain are ten times greater for women compared to men.\textsuperscript{6}

Once preference parameters are calibrated, our task is to quantify how much of the observed changes for men and women in total daily calories consumed, weight, and the fraction of calories consumed away from home between 1971 and 2006 can be accounted for by changes in relative food prices and household income? We conduct four different counterfactual experiments changing food prices and household income one at a time and then all together. As one would predict, for men, decline in the relative price of FAFH leads to a reallocation of resources away from FAH as well as an increase in total calorie

\textsuperscript{5}The assumption of perfect rationality might be too strong when it comes to food choices. People with self-control problems or time-inconsistent preferences would find it optimal to discount future health costs and choose immediate gratification from food consumption. Adapting the bounded rationality modeling strategies in Gruber and Koszegi (2004), Gruber and Koszegi (2001), or O’Donoghue and Rabin (1999) to study the impact of declining food prices on weight would be a valuable contribution to the economics of obesity literature.

\textsuperscript{6}We do not seek to explain preference heterogeneity in this paper. However, we hypothesize that large utility costs of being obese for women could come from discrimination in the workforce toward obese women. For example, Cawley (1999) who studies the impact of obesity on wages, finds that for white females a difference in weight of two standard deviations (roughly 65 pounds) is associated with a difference in wages of 9 percent. In contrast, the impact of being overweight or obese for men is more muted. For white men, being obese does not influence wages in a statistically significant way, while for back men, being obese possibly leads to higher wages.
consumed and weight. Interestingly, the weight gain resulting from a simultaneous change in food prices and income is lower than weight gain when only food prices change which can be explained as follows. When weight is greater than the best physiologically determined weight, consuming one more unit of food yields a lower marginal utility compared to consuming one more unit of the market good. As a result, men find it optimal to allocate the extra income to market good rather than food consumption when income goes up. For women, changes in food prices and income, while affecting the composition of what food type women eat, have almost no impact on total calories consumed, and thus body weight, because of high utility cost of weight gain.

Our numerical experiments highlight the importance of carefully modeling how body weight changes affect utility, as first pointed out by Philipson and Posner (2003) and Lakdawalla et al. (2005). In fact, we believe that quantitative models of food consumption choices and body weight have a unique role to play in obesity economics future research. Thirty years ago, in an otherwise unrelated context, Mehra and Prescott (1985) used a quantitative model to uncover a puzzle of asset prices and ended up writing one of the most influential paper at the intersection of macroeconomics and finance. They showed that, for a class of general equilibrium models where agents have constant relative risk aversion utility, the premium that investors pay to hold Treasury bills over equities implies a level of risk aversion which is much greater than what most direct evidence regarding individual behavior toward risk suggests. The equity premium paper generated thousands of citations and hundreds of related papers, many of them aimed squarely at trying to resolve the puzzle. It also led to the examination of new assumptions about individual behavior and preferences, including the development and introduction of habit-forming utility by Abel (1990), Epstein-Zin preferences (Epstein and Zin, 1989), and prospect theory based on loss aversion (Benartzi and Thaler, 1995).

Similar to what the equity premium puzzle paper accomplished for the fields of macroeconomics and finance, we argue that further studying the interplay between theory and data could unleash a very productive research agenda in obesity economics. For example, if one takes our model seriously, it remains unclear why are women more affected by changes in body weight compared to men? Why is the elasticity of substitution between
food type greater for women compared to men? Is the rational paradigm best suited to address how changes in household income and food prices affect eating decisions and body weight, as opposed to models where agents have commitment issues and seek short-term gratification from food consumption at the expense of long-term health? Answering these empirical questions becomes much easier once a quantitative theory of food consumption choices and body weight is up and running.

The remainder of the paper is organized as follows. In Section 2, we present a dynamic model of eating decisions and body weight. In Section 3, we review data about changes in weight, total calories consumed, and the away-from-home share of caloric intake between 1971 and 2006. We also explain how to construct the price per calorie for FAFH and FAH. In Section 4, we show how to calibrate model parameters. In Section 5, we perform numerical experiments, including a quantitative assessment of welfare changes. We offer concluding remarks in Section 6.

2 Theory

2.1 An Infinite-Horizon Model of Eating Decisions and Weight

We present a discrete time infinite horizon dynamic model of food choices and weight. We denote by $f_{1t}$ daily calories eaten away from home and $f_{2t}$ calories consumed at home. We also let $\theta_t \in [0, 1]$ the share of calories eaten away from home with $\theta_t = \frac{f_{1t}}{f_{1t} + f_{2t}}$ where the denominator $f_{1t} + f_{2t}$ denotes total calories consumed in period $t$. By construction, the share of calories eaten at home is equal to $1 - \theta_t = \frac{f_{2t}}{f_{1t} + f_{2t}}$.

Calorie consumption of both types of food is nested into food consumption $c_{ft}$ with a constant elasticity of substitution function:

$$c_{ft} = (\eta f_{1t}^p + (1 - \eta) f_{2t}^p)^\frac{1}{\rho}$$  \hspace{1cm} (1)

with $\eta \in [0, 1]$ and $\rho \leq 1$. Note that because the food consumption function is homogenous of degree one, food consumption can be rewritten in terms of food share and total daily calories consumed with $c_{ft} = (f_{1t} + f_{2t})(\eta \theta_t^p + (1 - \eta)(1 - \theta_t)^p)^\frac{1}{\rho}$.

Agents derive utility from consumption of a market good composite, $c_t$, food consumption, $c_{ft}$, and weight $W_t$. As in Lakdawalla and Philipson (2009), weight, the model state
variable, enters utility. The period-\(t\) utility function is equal to:

\[
U(c_t, c_{ft}, W_t) = \nu c_{ft} + \frac{c_t}{1 + \kappa(W_t - \bar{W})^2}
\]  
(2)

where \(\nu\) and \(\kappa\) are two positive constants and \(\bar{W}\) denotes agent’s best physiological weight. The intertemporal objective function is equal to the sum of period-\(t\) utility discounted back to the first period:

\[
\sum_{t=1}^{+\infty} \delta^{t-1} U(c_t, c_{ft}, W_t)
\]  
(3)

where \(\delta \in (0, 1)\) is the pure time discount factor.

The budget constraint is given by:

\[
c_t + p_{1t} f_{1t} + p_{2t} f_{2t} = I_t
\]  
(4)

where \(p_{1t}\) is the price per calorie of FAFH, \(p_{2t}\) the price per calorie of FAH, \(I_t\) denotes real income, and we normalized the price of non-food to one.\(^7\)

Finally, the inter-temporal weight law of motion links weight in the next period to current weight and total daily calorie consumption:

\[
W_{t+1} = W_t + \zeta(f_{1t} + f_{2t} - \mu(W_t))
\]  
(5)

where \(\zeta > 0\) is a parameter that converts calorie consumption into weight gain and \(\mu(W_t)\) is the number of calories needed to maintain a constant weight. We consider the linear case where \(\mu(W_t) = \beta_0 + \beta_1 W_t\) with \(0 < \beta_1 < \frac{1}{\zeta}\) in which case the weight law of motion becomes:

\[
W_{t+1} = (1 - \zeta\beta_1)W_t + \zeta(f_{1t} + f_{2t} - \beta_0)
\]  
(6)

\(^7\)Our model is partial equilibrium and thus food prices and household income are determined outside of the model. Cawley (2004) shows that body weight affects wages for white females but we do not capture this mechanism in our model. Alternatively, Gomis-Porqueras and Peralta-Alva (2008) propose two explanations for the decline in the cost of food away from home: a) productivity improvements in the production of food prepared away from home and b) declines in income taxes and in the gender wage gap. While both changes increase the relative cost of preparing food at home from scratch, the authors find that changes in income taxes and the gender wage gap have the largest impact and account for about three quarter of the increase in calorie consumption of the American population during the last 40 years. One important difference with Gomis-Porqueras and Peralta-Alva (2008) is that, in our paper, weight enters utility.
We are now ready to formulate agents’ optimization problem. For any given sequence of prices and income, \( \{p_{1t}, p_{2t}, I_t\}_{t \geq 1} \), and an initial weight, \( W_1 \), the representative agent chooses an optimal sequence of market and food consumption \( \{c_t, c_{ft}, f_{1t}, f_{2t}\}_{t \geq 1} \) to maximize the objective function in equation (3) subject to the food aggregation equation (1), the budget constraint (4), the weight law of motion (6), and non-negativity constraints for calories, food, and non-food consumption.

Before presenting the solution to households’ maximization problem in the next section, we make two important comments. First, note that, as far as weight is concerned in equation (6), it does not matter whether calories are consumed at home or away from home. Research by Buchholz and Schoeller (2004) supports the view that a “calorie is a calorie” regardless of macronutrient composition implying that daily caloric intake, not what people eat, determines body weight (see also Nestle (2012)). On the other hand, Feinman and Fine (2004), stating the second law of thermodynamics, show that calories from with different macronutrient compositions could potentially affect weight differently. Our analysis does not consider macronutrient composition of what people eat but rather whether food is consumed at home or away from home. In addition, it is clear that calories from different types of food (for examples transfat vs complex sugars) affect health differently. Since we do not model health directly, we chose the simplest framework allowing us to answer our question which is why a calorie is a calorie in the weight law of motion.\(^8\) Including differences in nutritional values into the weight law of motion and agents’ preferences would greatly complicate the analysis without helping much to answer our quantitative question about the impact of food prices and household income on body weight.

Second, as highlighted in the Introduction section, our model shares many similarities with the frameworks proposed by Philipson and Posner (2003) and Lakdawalla and Philip-

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\(^8\)Evidence from the medical literature increasingly suggests that the type of food people eat as well as how much they eat affect health, especially longevity. For example, Johnson et al. (2013) find that reductions in nutrient intake in the absence of malnutrition, in particular reduce intake of insulin, extends lifespan in many different species. Our theoretical framework, however, does not directly consider the impact of weight on health which explains why we choose not to model nutritional value of what people eat.
son (2009), in particular food consumption choices are dynamic and weight appears in the utility function. There are two important differences, however. First, we do not model calories expenditures. Cutler et al. (2003) show that declines in energy expenditures in the U.S. are too small to account for the observed changes in body weight after 1965. Since we study body weight changes after 1970, not modeling energy expenditures seems reasonable enough. Second, agents can consume different food types, in this paper FAFH and FAH. Since the obesity economics literature emphasizes how important substitution between food items is when it comes to body weight analysis, we believe that studying the properties of a theoretical framework where agents choose among several food choices is a nice addition to the literature. In the next section, we solve the agent’s maximization problem.

2.2 Dynamic Programming

Following the insight of Lakdawalla and Philipson (2009), we formulate the previous inter-temporal optimization problem as a dynamic program where weight is the state variable. We denote by \( V(W) \) the value function which is determined by following the Bellman equation:

\[
V(W) = \max_{(c,c_f,f_1,f_2)} \left[ \nu c_f + \frac{c}{1 + \kappa(W - \bar{W})^2} + \delta V(W') \right] \tag{7}
\]

s.t.

\[
\begin{align*}
W' &= (1 - \zeta \beta_1)W + \zeta (f_1 + f_2 - \beta_0) \\
c_f &= (\eta f_1^\rho + (1 - \eta) f_2^\rho) \frac{1}{\rho} \\
c + p_1 f_1 + p_2 f_2 &= I
\end{align*} \tag{8}
\]

where \( W' \) denotes weight next period.

Using the budget constraint (4), the food aggregation function (1), and the weight law of motion (6), we re-write the Bellman equation as:

\[
V(W) = \max_{(f_1,f_2)} \left[ \nu (\eta f_1^\rho + (1 - \eta) f_2^\rho) \frac{1}{\rho} + \frac{I - p_1 f_1 - p_2 f_2}{1 + \kappa(W - \bar{W})^2} \right] + \delta V((1 - \zeta \beta_1)W + \zeta (f_1 + f_2 - \beta_0)) \tag{9}
\]

Assuming that the value function \( V \) is differentiable, the first-order optimality condi-
tion with respect to $f_1$ yields:

$$\nu \eta f_1^{p-1}(\eta f_1^p + (1 - \eta)f_2^p)^{1-p} - \frac{p_1}{1 + \kappa(W - \bar{W})^2} = 0$$

(10)

$$\delta \zeta V'((1 - \zeta \beta_1)W + \zeta (f_1 + f_2 - \beta_0)) = 0$$

where $V'$ is the first-order derivative of the value function. Similarly, the first-order optimality condition with respect to $f_2$ is:

$$\nu (1 - \eta)f_2^{p-1}(\eta f_1^p + (1 - \eta)f_2^p)^{1-p} - \frac{p_2}{1 + \kappa(W - \bar{W})^2} = 0$$

(11)

$$\delta \zeta V'((1 - \zeta \beta_1)W + \zeta (f_1 + f_2 - \beta_0)) = 0$$

In addition, the envelope theorem yields:

$$V'(W) = \frac{-2\kappa(W - \bar{W})(I - p_1 f_1 - p_2 f_2)}{(1 + \kappa(W - \bar{W})^2)^2} + \delta(1 - \zeta \beta_1)V'((1 - \zeta \beta_1)W + \zeta (f_1 + f_2 - \beta_0))$$

(12)

We solve the above dynamic program when the weight is in steady state defined as $W' = W = W^*$. We denote by $f_1^*(f_2^*)$ steady-state calories eaten away from (at) home, respectively. According to the weight law of motion in equation (6), the following relationship between steady-state weight and calories consumption holds:

$$W^* = \frac{f_1^* + f_2^* - \beta_0}{\beta_1}$$

(13)

In steady-state, equations (10) and (11) yield:

$$\frac{p_2 - p_1}{1 + \kappa(\frac{f_1 + f_2 - \beta_0}{\beta_1} - W)} = \nu((1 - \eta)f_2^{p-1} - \eta f_1^{p-1})(\eta f_1^p + (1 - \eta)f_2^p)^{1-p}$$

(14)

where we dropped the star notation for convenience.

In the steady-state, the envelope condition (12) is equal to:

$$V'(W) = -\frac{2\kappa(W - \bar{W})(I - p_1 f_1 - p_2 f_2)}{(1 + \kappa(W - \bar{W})^2)^2}$$

(15)

Substituting equation (15) into the first-order condition (10) yields:

$$\frac{p_1}{1 + \kappa(\frac{f_1 + f_2 - \beta_0}{\beta_1} - W)^2} = \nu \eta f_1^{p-1}(\eta f_1^p + (1 - \eta)f_2^p)^{1-p}$$

$$- \frac{2\delta \zeta \kappa(\frac{f_1 + f_2 - \beta_0}{\beta_1} - \bar{W})(I - p_1 f_1 - p_2 f_2)}{(1 + \kappa(\frac{f_1 + f_2 - \beta_0}{\beta_1} - W)^2)^2}$$

(16)

Together equations (14) and (16) are necessary and sufficient to calculate the steady state calorie consumption of food eaten away from and at home, $f_1^*$ and $f_2^*$. In turn, the steady-state share of food eaten away from home is equal to $\theta^* = \frac{f_1}{f_1^* + f_2^*}$ and the steady state weight is $W^* = \frac{f_1^* + f_2^* - \beta_0}{\beta_1}$.
3 Nutritional and Economic Data

3.1 Total calories, food shares, and weight

We analyze changes in total calories consumed, fraction of calories eaten away from and at home, weight, and body-mass index for the period between 1971 and 2006 using NHANES I and NHANES 2005-06 survey data. The choice of the time period is purposeful. On the one hand, weight growth is not a recent phenomenon. For example, Costa and Steckel (1995) document that average body weight of Americans has increased continuously in the past 150 years, especially after WWII. In addition, the transition from labor intensive to more sedentary lifestyle has been mostly completed by 1970 so that the calories expended remain fairly constant since 1970 (Cutler et al. (2003)). On the other hand, there have been no significant changes in obesity prevalence in youth or adults after 2003 (Ogden et al., 2014).

What makes weight gains for the period after 1970 different, however, is that for the first time weight gains are associated with a decrease in health, not an increase. Weight gains in the first part of the twentieth century represented improvements in health, notably increased longevity (Fogel, 1994). Today, Americans are heavier than what the medical field recommends and the obesity epidemic is associated with many of the leading causes of preventable death such as heart disease, stroke, type-II diabetes and certain types of cancer (National Institute of Health, 2005).

Both NHANES I and NHANES are survey data so appropriate weights must be used to calculate nationally representative averages and standard deviation for total calories consumed, food shares, and weight. Data about body weight comes from NHANES examination component and is measured by trained medical personnel. Information about total daily calories and the fraction of calories consumed away from home on the other hand is included in the interview part and thus is self-reported by individuals which can lead to underreporting (Cawley, 2004).

Obesity prevalence among adults and youth remains high with more than one-third of U.S. adults and 17 percent of youth being obese in 2012 (Ogden et al., 2014). In addition, obesity affects some groups more than others. For example, non-Hispanic blacks have the highest age-adjusted rates of obesity (47.8 percent) followed by Hispanics (42.5 percent), non-Hispanic whites (32.6 percent), and non-Hispanic Asians (10.8 percent).

Finkelstein et al. (2009) estimate medical costs associated with being overweight or obese to be as high as $147 billion, or 10 percent of all medical costs in 2008 (see also Tsai et al. (2011)). In addition,
We present our results for men and women age 20 and older in Tables 1 and 2, respectively. Total daily calories and the share of calories eaten away from home increased for both men and women, and in percentage terms, changes are more pronounced for women.\textsuperscript{12} For example, calories consumed away from home increased from 30 percent to 41 percent for men versus 20 percent to 36 percent for women. These changes in eating habits led to weight gain of 23 pounds (13 percent) for men and 20 pounds (16 percent) for women.\textsuperscript{13} Finkelstein et al. (2009) show that obesity-related costs now exceed health-care costs associated with smoking or problem drinking and that yearly medical costs for people who are obese are $1,429 higher than those of normal weight.

\textsuperscript{12}The question about where do people eat their meal has changed over time. In NHANES I, individuals can choose among the following four locations: at home, in school, in restaurants, and other, while in NHANES 2005-06, the location question is: “Did you eat this food at home?” and the possible answers are yes, no, refused to answer, do not know, and missing information. To maintain consistency across the two data sets, we define food eaten at home as any food item for which individuals answered at home in NHANES I and yes in NHANES 2005-06. We define food eaten away from home as all food items not eaten at home. We calculate the fraction of calories eaten away from home as one minus the fraction of calories eaten at home.

\textsuperscript{13}Simple t-tests show that differences in mean weight and mean body-mass index over time are statistically significant for both men and women.
Table 1: Changes in total calories consumed, food shares, weight, and body-mass index for men age 20 and older (standard error in parenthesis)

<table>
<thead>
<tr>
<th></th>
<th>1971-75</th>
<th>2005-06</th>
<th>%Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Calories</td>
<td>2433 (27.4)</td>
<td>2543 (39.1)</td>
<td>4.5</td>
</tr>
<tr>
<td>Calories share - FAFH</td>
<td>.30 (.002)</td>
<td>.41 (.003)</td>
<td>12.0</td>
</tr>
<tr>
<td>Calories share - FAH</td>
<td>.70 (.003)</td>
<td>.59 (.003)</td>
<td>-12.0</td>
</tr>
<tr>
<td>Weight (lbs.)</td>
<td>175.7 (.58)</td>
<td>198.2 (1.1)</td>
<td>12.8</td>
</tr>
<tr>
<td>Body-mass Index(a)</td>
<td>25.9 (.08)</td>
<td>29.0 (.27)</td>
<td>12.0</td>
</tr>
</tbody>
</table>

\(a\) Body-mass Index = \(703 \times \frac{Weight}{Height^2}\).

Table 2: Changes in total calories consumed, food shares, weight, and body-mass index for women age 20 and older (standard error in parenthesis)

<table>
<thead>
<tr>
<th></th>
<th>1971-75</th>
<th>2005-06</th>
<th>%Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Calories</td>
<td>1538 (14.1)</td>
<td>1802 (14.6)</td>
<td>17.2</td>
</tr>
<tr>
<td>Calories share - FAFH</td>
<td>.20 (.002)</td>
<td>.36 (.003)</td>
<td>17.0</td>
</tr>
<tr>
<td>Calories share - FAH</td>
<td>.80 (.003)</td>
<td>.64 (.003)</td>
<td>-17.0</td>
</tr>
<tr>
<td>Weight (lbs.)</td>
<td>145.8 (.54)</td>
<td>168.9 (.98)</td>
<td>15.8</td>
</tr>
<tr>
<td>Body-mass Index(a)</td>
<td>25.2 (.10)</td>
<td>28.8 (.30)</td>
<td>14.3</td>
</tr>
</tbody>
</table>

\(a\) Body-mass Index= \(703 \times \frac{Weight}{Height^2}\)
3.2 Price per calorie

We explain how to use household expenditures data on FAFH and FAH to construct the price per calorie for both of these food types. Using price per calorie measure ensures that food prices reflect changes for all foods, rather than specific items as is usually the case in price indices (Dolar, 2014a).\(^{14}\)

We define price per calorie for food consumed away from home, \(p_{1t}\), and food consumed at home, \(p_{2t}\), by the following expression:

\[
p_{jt} = \frac{\text{Per capita daily expenditures (2006 dollars) on food } j \text{ in year } t}{\text{Per capita daily calories produced of food } j \text{ in year } t}
\]  

(17)

for \(j = \{1, 2\}\) and \(t = \{1971, 2006\}\).

U.S. Department of Agriculture data on household expenditures shows that families spent 9.9 percent of their disposable income on food at home and 3.6 percent on food away from home for the period between 1971 and 1975, while in 2006 expenditures shares for food at and away from home are equal to 5.7 and 4.1 percent. As a result, total expenditure on food declined from 13.5 percent of disposable income in 1973 to only 9.8 in 2006 (see Table 3).\(^{15}\)

\(^{14}\)As pointed out by Goldman et al. (2011) and Christian and Rashad (2009), the use of price per calorie is superior to using standard price index since the index does not take into account differential impacts on body weight of consuming various foods. In the construction of the price per calorie Goldman et al. (2011) use 59 food items from American Chamber of Commerce Researchers Association (ACCRA), now known as Council of Community and Economic Research (C2ER). A slightly different prices per calorie are also used by Grossman et al. (2013) where they use 21 food items from C2ER. Given the fact that about 320,000 foods and beverage products are available it the United States, and that an average supermarket carries 30,000 to 40,000 of them, using of 20 to 60 food items is very limited (Nestle (2006)).

\(^{15}\)An alternative definition for expenditures shares on food away from home and food at home uses relative importance from Consumer Price Index for All Urban Consumers, Bureau of Labor Statistics. Relative importance for food away from home and food at home are equal to 5.1 percent and 19.6 percent in 1971-75, respectively, and 5.9 percent and 7.9 percent in 2005-06, respectively. As a result, food expenditures share from relative importance declined from 24.7 percent in 1971-75 to 13.8 percent in 2005-06. Note that food expenditure share from relative importance are much higher compared to food expenditure shares in Table 7 of USDA Food Expenditure Series which are equal to 13.5 percent in 1973 and 9.8 in 2006.
Table 3: Food expenditures shares and disposable income

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Expenditures share, FAFH&lt;sup&gt;a&lt;/sup&gt;</td>
<td>.036</td>
<td>.057</td>
</tr>
<tr>
<td>Expenditure share, FAH&lt;sup&gt;a&lt;/sup&gt;</td>
<td>.099</td>
<td>.041</td>
</tr>
<tr>
<td>Food expenditure share</td>
<td>.135</td>
<td>.098</td>
</tr>
<tr>
<td>Daily real disposable income, (2006 dollars)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>$57.23</td>
<td>$89.53</td>
</tr>
</tbody>
</table>

<sup>a</sup> Source: USDA Food Expenditure Series Table 7.

<sup>b</sup> Source: Bureau of Economic Analysis - NIPA Table 2.1 Personal Income and its Disposition (Line 38).

We present changes in price per calories data for food away from home and food at home in Table 4. The numerator in equation (17) is equal to daily real disposable income times food expenditure shares. National Income and Product Accounts data shows that daily real disposable income expressed in 2006 dollars increased from $57.23 in 1971-75 to $89.53 in 2006 (see Table 3). On the other hand, data for daily calories produced (the denominator in equation (17)) comes from Food Availability Data System of the U.S. Department of Agriculture and Lin and Guthrie (2012).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Price per thousand calories - FAFH</td>
<td>$5.56</td>
<td>$4.04</td>
</tr>
<tr>
<td>Price per thousand calories - FAH</td>
<td>$3.36</td>
<td>$3.03</td>
</tr>
<tr>
<td>Relative price (home vs. away)</td>
<td>.60</td>
<td>.75</td>
</tr>
</tbody>
</table>

Table 4: Changes in per calorie food prices

The main takeaway from Table 4 is that in absolute terms, calories of both food away and at home have become cheaper over time, while in relative terms, the price of one calorie of food at home increased by 25 percent.
4 Calibration

In this section, we explain how to use data presented in the previous section to calibrate model parameters appearing in the weight law of motion in equation (6) as well as preferences parameters in first-order conditions (14) and (16). We discuss what calibrated parameter values imply for food consumption choices and body weight. Finally, we compare our results to the obesity economics empirical literature, in particular estimates of food price elasticity.

4.1 Weight law of motion

The weight law of motion (6) contains three parameters ($\zeta, \beta_0, \beta_1$) that must be calibrated. The parameter $\zeta$ relates changes in body weight to total calorie consumed above and beyond what is needed to maintain a constant weight. It is well established in the nutrition literature that people gain ten pounds per year if they eat an extra one hundred calories every day above and beyond the recommended daily calorie intake (Shils et al. (1998)). Accordingly, we set $\zeta = \frac{10}{100 \times 365} = 2.7397 \times 10^{-4}$.

Second, the minimum number of calories required to maintain a constant weight is equal to $\mu(W) = \beta_0 + \beta_1 W$ where $W$ denotes body weight measured in pounds. One hundred years ago, Harris and Benedict (1918) proposed an equation to estimate men’s and women’s basal metabolic rate (BMR) and daily kilocalorie requirements. The estimated BMR value is multiplied by a number that corresponds to the individual’s activity level and the resulting number is the recommended daily kilocalorie intake to maintain current body weight. They found that daily calorie requirements differ for men and women and the heavier an individual is the more calories need to be consumed to maintain a constant weight. We use a recent technical report on dieting and energy intake by the Food and Nutrition Board (2002, p.185) which finds that, assuming a moderate level of physical activity, men need to consume an addition 8.09 calories per day for each extra pound to maintain a constant weight, while calorie requirements for women are equal to an additional 4.76 calories per day for each extra pound. As a result, we set $\beta_1^m = 8.09$ and $\beta_1^f = 4.76$.

See also Roza and Shizgal (1984) for a more recent estimation of the Harris-Benedict equation.
Third, given the parameter $\beta_1$, we choose $\beta_0$ so that steady-state weight of men and women is equal to best weight $\bar{W}$. According to equation (13), we have $\beta_0 = f_1 + f_2 - \beta_1 \bar{W}$ where $f_1 + f_2$ represents steady state daily caloric intake shown in Table 5. As a result, we have $\beta_0^m = 2433 - 8.09 \times 175.5 = 1011.59$, while for women $\beta_0^f = 1538 - 4.76 \times 145.8 = 843.99$.\(^{17}\)

4.2 Preference parameters

Finally, we calibrate four preference parameters ($\rho, \eta, \nu, \kappa$). First-order optimality conditions in (14) and (16) provide two equations. To obtain two more conditions, we derive an analytical formula in the Appendix for the price and cross-price elasticity of FAFH when the steady-state weight is equal to best weight $\bar{W}$. Using empirical estimates of Reed et al. (2005) for price and cross-price elasticity of FAFH, we have an exactly identified system of four equations and four unknowns which can be solved to obtain calibrated preference parameters.

**Proposition 1** (Identification). Let $(f_1, f_2)$ such that $\frac{f_1 + f_2 - \beta_0}{\beta_1} = \bar{W}$. Then $(f_1, f_2)$ is a solution to the system of first-order conditions in (14) and (16) if and only if the parameters $(\rho, \eta, \nu, \kappa)$ are determined by equations (18)-(21):

$$\rho = 1 + \frac{\pi(1 - \theta) + \theta}{\pi(\epsilon_{11} - \epsilon_{21})}$$  \hspace{1cm} (18)

where $\epsilon_{11}$ and $\epsilon_{21}$ are the price and cross-price elasticity of demand for FAFH.

$$\eta = \frac{1}{1 + \pi\left(\frac{1-\theta}{\theta}\right)^{1-\rho}}$$  \hspace{1cm} (19)

$$\nu = \frac{p_1^\theta \frac{1-\theta}{\rho}}{\eta^\frac{1}{\theta} (\theta + \pi(1 - \theta))^{\frac{1-\theta}{\rho}}}$$  \hspace{1cm} (20)

\(^{17}\)Note that if individuals underreport total calories consumed daily in NHANES, then the calibrated value for $\beta_0$ is lower than what true calorie consumption would imply. Dolar (2014b) discusses reasons leading individuals to under-report their daily calorie intake. Following methodologies proposed by Courtemanche et al. (2014) or Cawley (2004), she explains how to adjust self-reported calories in NHANES and studies the relationship between the adjusted measure for calories consumed and body weight.
\[ \kappa = -\frac{\beta_1 p_1 \theta (1 - \delta (1 - \zeta \beta_1))}{2 \delta (I - p_1 (f_1 + f_2)) (\theta + \pi (1 - \theta)) (f_1 + f_2) (\epsilon_{11} \theta + \epsilon_{21} (1 - \theta))} \]  

(21)

Proof. See the Appendix.

In Table 5, we present a summary of data for men and women during the period between 1971 and 1975, which is needed in equations (18) to (21) to calculate preferences parameters numerical values.

<table>
<thead>
<tr>
<th></th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total daily calories, ( f_1 + f_2 )</td>
<td>2433</td>
<td>1538</td>
</tr>
<tr>
<td>Fraction of food eaten away from home, ( \theta )</td>
<td>.30</td>
<td>.20</td>
</tr>
</tbody>
</table>

Table 5: Calibrated total calories and food shares for men and women, 1971-75

In addition, we know from the data section that the price of 1000 calories for FAFH in 1971 is equal to \( p_1 = $5.56 \), the relative food price is \( \pi = .6 \), and household daily real income is equal to \( I = $57.23 \). Finally, we set the pure time discount factor \( \delta = .98^{\frac{1}{365}} \) and use empirical estimates for price and cross-price elasticity for FAFH from Reed et al. (2005) with \( \epsilon_{11} = -.692 \) and \( \epsilon_{12} = .168 \). Preferences parameters values for men and women are shown in Table 6.

<table>
<thead>
<tr>
<th></th>
<th>( \rho )</th>
<th>( \eta )</th>
<th>( \nu )</th>
<th>( \kappa )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>-.3920</td>
<td>.3362</td>
<td>.0079</td>
<td>( 5.41 \times 10^{-6} )</td>
</tr>
<tr>
<td>Females</td>
<td>-.3162</td>
<td>.2210</td>
<td>.0067</td>
<td>( 4.02 \times 10^{-5} )</td>
</tr>
</tbody>
</table>

Table 6: Calibrated preference parameters

\(^{18}\)Reed et al. (2005) estimate cross-price elasticity of food away from home with five different categories of food at home: fruits and vegetables, dairy, meats, cereals and bakery, and fat and sugar. Cross-price elasticity for the five food categories are equal to \((.50, 1.32, - .74, .18, .66)\). We aggregate the five food categories into food at home using food expenditures weights. According to the Bureau of Labor Statistics, food expenditures for the five food groups as a fraction of disposable income are equal to \((.03, .03, .07, .03, .04)\). Normalizing expenditures to one, the cross-price elasticity \( \epsilon_{21} \) is equal to \( \epsilon_{12} = .16 \times .5 + .15 \times 1.32 + .36 \times (- .74) + .13 \times .18 + .20 \times .66 = .168 \).
One interesting result from the calibration is that the parameter $\rho$ is negative for both men and women. The main takeaway from the calibration however is that there is substantial preference heterogeneity between men and women. For example, the parameter $\nu$ which measures marginal utility from food consumption is fifty percent greater for men compared to women. Second, the parameter $\kappa$ which measures utility losses when steady-state weight deviates from best weight $\bar{W}$ is ten times greater for women compared to men. In section 5 where we conduct numerical experiments, we will show that the higher the value for the parameter $\kappa$, the lower the impact of changes in household income and food prices on food decisions and body weight.

4.3 Discussion

Parameters’ calibration was designed to ensure that steady-state values for men and women for body weight, daily caloric intake, and calorie share for FAFH and FAH are equal to their data counterparts for the period between 1971 and 1975.

To gain confidence that our model is a good framework to study food consumption choices, we analyze model predictions for data moments that we did not use directly in the calibration – what an econometrician could refer to as an over-identified model.

Food expenditure on FAFH is equal to $5.56 \times 10^{-3} \times 517.5 = .050$ where 517.5 is the average calories consumed away from home by men and women from Table 5. In comparison, household expenditures on FAFH in the data in 1973 is equal to 3.6 percent (see Table 3). Similarly, food expenditure on FAH is equal to $3.36 \times 10^{-3} \times 1469 = .086$ where 1469 is the average calories consumed away from home by men and women from Table 5. In contrast, household expenditures on FAH in the data in 1973 is equal to 9.9 percent (see Table 3). As a result, total expenditures on food is equal to 13.6 percent compared to 13.5 percent in the data.

Second, we calculate price and cross-price elasticity for food at home. We used data from Reed et al. (2005) to link the price elasticity for food away from home to the elasticity of substitution in the calibration of preference parameters. In the Appendix, we show that the price elasticity for food at home $\epsilon_{22}$, and the cross-price elasticity between calories consumed away from home and the price of food at home, $\epsilon_{12}$, are determined by the
following system of equations:

\[ \epsilon_{12} - \epsilon_{22} = \pi(\epsilon_{21} - \epsilon_{11}) \] (22)

\[ \epsilon_{12}(\frac{\theta^2}{\epsilon_{11}\theta + \epsilon_{21}(1 - \theta)} + \frac{1 - \theta}{\epsilon_{11} - \epsilon_{21}}) + \epsilon_{22}(\frac{(1 - \theta)^2}{\epsilon_{11}\theta + \epsilon_{21}(1 - \theta)} - \frac{1 - \theta}{\epsilon_{11} - \epsilon_{21}}) = 0 \] (23)

Using the average away-from-home calorie share, the solution to the previous equations is equal to \((\epsilon_{12}, \epsilon_{22}) = (0.433, -0.086)\). As a result, the food price elasticity matrix \(\epsilon\) is equal to:

\[ \epsilon = \begin{bmatrix} -0.692 & 0.433 \\ 0.168 & -0.086 \end{bmatrix} \] (24)

Negative coefficients on the diagonal of the elasticity matrix imply downward-sloping demand functions for both food types. In addition, as in Reed et al. (2005), we find that food at home and food away from home are gross substitutes as indicated by the positive values for the cross-price elasticity. The fact that the model matches food expenditure shares in 1973 and that price and cross-price elasticity are close to values estimated in the obesity economics empirical literature gives us confidence that the model is a good framework to study food consumption choices and resulting body weight. We proceed with counterfactual experiments in the next section.

5 Numerical Experiments

We conduct four different experiments. We change food prices away from and at home one at a time, then together, and finally food prices and household income all together. We present results in Table 7.

In experiment I, we change price per thousand calories for FAFH from its 1973 value \(p_1 = 5.56\) to its 2006 value \(p_1 = 4.04\) leaving all other parameters constant. Qualitative results are as predicted by the theory. The share of FAFH increase, and so do total caloric intake and weight. Quantitatively however, changes are much more pronounced for men compared to women. For men, the fraction of FAFH increases by 6 percentage points from 29 percent to 35 percent. Daily caloric intake increases by 52 calories which translates into weight gain of 6.4 pounds. For women, the fraction of FAFH increases by
4 percentage points. However, changes in daily caloric intake and weight are very small due to large value of parameter $\kappa$.

Changing price per calorie for FAH from its 1973 value $p_2 = 3.36 \times 10^{-3}$ to its 2006 value $p_1 = 3.03 \times 10^{-3}$ in experiment II leads to a calories reallocation toward FAH and small weight gains for men and women. In experiment III, where we change both prices simultaneously, weight gains are the largest for men and women. For men, changes in food prices account for slightly less than half of body weight gain, daily caloric intake, and share of FAFH.

In experiment IV, we change food prices and household income all together. Note that due to our choice of utility functional form, it is easy to show that changes in household income alone have no impact on steady-state values when weight is equal to best weight $\bar{W}$. For this reason, we do not consider changes in income alone. Changes in household income however do have an impact on food consumption choices away from the calibrated steady state as can be seen in the last column of Table 7. Interestingly, when food prices and income are altered together, the resulting weight for men is lower than when food prices are changed alone which can be explained as follows. Because of utility loss

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19Reed et al. (2005) however estimate large positive values for income elasticity of food. We leave it for future research to reconcile Reed’s empirical with theory.
stemming from deviation from best weight, consuming one more unit of food yields a lower marginal utility compared to consuming one more unit of the market good. As a result, men find it optimal to allocate the extra income to market good rather than food consumption.

Finally we calculate changes in steady-state welfare for men and women using equation (7). An important advantage of using structural models over direct statistical analysis of food choices and body weight is that changes in agents’ welfare can be quantified. Because food prices declined while real household income increased (as in our experiment IV), welfare is expected to go up between 1971 and 2006. By how much can be seen in Table 8. An interesting result is that for both men and women, the percentage change in welfare is less than the percentage in household income because weight gains reduce utility.

<table>
<thead>
<tr>
<th></th>
<th>1971-75</th>
<th>2005-06</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>206.5</td>
<td>249.6</td>
<td>20.9%</td>
</tr>
<tr>
<td>Women</td>
<td>100.9</td>
<td>135.8</td>
<td>34.5%</td>
</tr>
</tbody>
</table>

Table 8: Changes in welfare for men and women

6 Conclusion

The field of obesity economics stands at a crossroads, partially because statistical analysis of observational studies and controlled experiments give somewhat conflicting results about the impact of food prices and income on food consumption choices and weight. An obvious research agenda to address this issue is to refine or even introduce new estimation strategies like Rashad (2006), who searches for instrumental variables for calories consumption, or Anderson and Matsa (2011) who use longitudinal data to examine the relationship between food prices and body weight.

In this paper, we proposed a different approach and introduced a calibrated dynamic model of eating decisions and weight rooted in microeconomic foundations. In particular,
we explained how to use data moments to pin down preferences parameters and used the model to run counter-factual experiments. The overarching message from our analysis is that future work in obesity economics should seek to develop and incorporate quantitative models as part of its research toolkit to explain stylized facts of the obesity epidemic in the US. A pressing question is what are the underlying factors causing preference heterogeneity among different subgroups of the population? As far as men and women are concerned, we hypothesized that high levels of discrimination in the workplace towards obese women compared to obese men could explain higher utility costs of weight gains for women. Another open question is whether models based on the rational expectations paradigm, as opposed to models where agents have commitment issues and seek short-term gratification from food consumption at the expense of long-term health, provide the best framework to analyze food consumption choices. We leave answering questions about these interesting topics for future research.
7 Appendix

7.1 Proof of Proposition 1

We first prove the sufficiency part of the proposition. Consider the pair \((f_1, f_2)\) which is a solution for first-order conditions (14) and (16) and satisfy \(\frac{f_1 + f_2 - \beta_0}{\beta_1} = \bar{W}\) as well. We must prove that parameters \((\rho, \eta, \nu, \kappa)\) are determined by equations (18) to (21).

7.1.1 Equation (18), \(\rho\)

Before we derive an expression for parameter \(\rho\), we must differentiate first-order equation (14) with respect to food prices, \(p_1\) and \(p_2\). Equation (14) reads:

\[ p_2 - p_1 = (1 + \kappa(\frac{f_1 + f_2 - \beta_0}{\beta_1} - \bar{W})^2)\nu((1 - \eta)f_2^{\rho - 1} - \eta f_1^{\rho - 1})(\eta f_1^\rho + (1 - \eta)f_2^\rho)^{\frac{1+\rho}{\rho}} \]  

(25)

Take logarithm of the previous expression.

\[
\ln(p_2 - p_1) = \ln(1 + \kappa(\sum_s f_s - \beta_0 - \bar{W})^2) + \ln(\nu) \\
+ \frac{1 - \rho}{\rho} \ln(\eta f_1^\rho + (1 - \eta)f_2^\rho) + \ln((1 - \eta)f_2^{\rho - 1} - \eta f_1^{\rho - 1})
\]

(26)

Differentiate with respect to \(p_k\):

\[
\forall k = 1, 2 \\
\frac{d}{dp_k}(\ln(p_2 - p_1)) = \frac{2\kappa(W - \bar{W})}{\beta_1(1 + \kappa(W - \bar{W})^2)} + \\
\frac{1 - \rho}{\eta f_1^\rho + (1 - \eta)f_2^\rho}(\eta f_1^{\rho - 1}\frac{\partial f_1}{\partial p_k} + (1 - \eta)f_2^{\rho - 1}\frac{\partial f_2}{\partial p_k}) - (1 - \rho)(1 - \eta)f_2^{\rho - 2}\frac{\partial f_2}{\partial p_k} - \eta f_1^{\rho - 2}\frac{\partial f_1}{\partial p_k}
\]

(27)

Regroup terms \(\frac{\partial f_1}{\partial p_k}\) and evaluate the previous expression when \(W = \bar{W}\) gives:

\[
\forall k = 1, 2 \\
\frac{d}{dp_k}(\ln(p_2 - p_1)) = \frac{\partial f_1}{\partial p_k}(\frac{(1 - \rho)\eta f_1^{\rho - 1}}{\eta f_1^\rho + (1 - \eta)f_2^\rho} + \frac{(1 - \rho)\eta f_1^{\rho - 2}}{(1 - \eta)f_2^{\rho - 1} - \eta f_1^{\rho - 1}}) + \\
\frac{\partial f_2}{\partial p_k}(\frac{(1 - \rho)(1 - \eta)f_2^{\rho - 1}}{\eta f_1^\rho + (1 - \eta)f_2^\rho} - \frac{(1 - \rho)(1 - \eta)f_2^{\rho - 2}}{(1 - \eta)f_2^{\rho - 1} - \eta f_1^{\rho - 1}})
\]

(28)
Define \( \epsilon_{jk}^p = \epsilon_{f_j,p_k} = \frac{p_k}{f_j} \frac{\partial f_j}{\partial p_k} \) for \( j \in \{1, 2\} \) and \( k \in \{1, 2\} \). The previous expression becomes:

\[
\forall k = 1, 2
\]

\[
p_k \frac{d}{dp_k} (\ln(p_2 - p_1)) = \epsilon_{1k}^p \left( \frac{1 - \rho}{\eta f_1^p} + (1 - \eta) f_2^p \right) + \frac{(1 - \rho) \eta f_1^p - 1}{(1 - \eta) f_2^p - 1 - \eta f_1^p - 1}
\]

\[
\epsilon_{2k}^p \left( \frac{1 - \rho}{\eta f_1^p} + (1 - \eta) f_2^p \right) - \frac{(1 - \rho)(1 - \eta) f_2^p - 1}{(1 - \eta) f_2^p - 1 - \eta f_1^p - 1}
\]

By construction, we have \( \frac{\theta}{1 - \theta} = \frac{f_2}{f_2} \). For \( k = 1 \), the previous expression can be written as:

\[
\frac{1}{1 - \pi} = \epsilon_{11}^p \left( \frac{1 - \rho}{1 + \frac{\rho}{(1 - \theta)^{\rho - 1}}} + \frac{1 - \rho}{1 - \theta} \right) + \frac{1 - \rho}{1 - \theta} \left( \frac{\rho}{(1 - \theta)^{\rho - 1}} \right)
\]

\[
\epsilon_{21}^p \left( \frac{\rho}{1 - \theta} \right) + 1 + \frac{1 - \rho}{1 - \theta} \left( \frac{\rho}{(1 - \theta)^{\rho - 1}} \right)
\]

where \( \pi = \frac{\rho}{\theta} \).

When \( \frac{\sum_j f_j - \theta \bar{B}_j}{\bar{B}_j} = \bar{W} \), first-order conditions (14) and (16) take the following form:

\[
p_2 - p_1 = \nu(\eta f_1^p + (1 - \eta) f_2^p) \frac{1 - \rho}{\theta} ((1 - \eta) f_2^p - 1 - \eta f_1^p - 1)
\]

and

\[
p_1 = \nu \eta f_1^p + (1 - \eta) f_2^p \frac{1 - \rho}{\theta}
\]

Divide equation (31) by (32) yields:

\[
\pi = \frac{1 - \eta}{\eta} \left( \frac{f_2}{f_1} \right)^{\rho - 1} = \frac{1 - \eta}{\eta} \left( \frac{1 - \theta}{\theta} \right)^{\rho - 1}
\]

Using the previous expression and for \( k = 1 \), equation (30) becomes:

\[
\frac{1}{1 - \pi} = (1 - \rho) \left( \epsilon_{11}^p \left( \frac{1}{1 + \frac{\rho}{(1 - \theta)^{\rho - 1}}} + \frac{1}{\pi - 1} \right) + \epsilon_{21}^p \left( \frac{1}{1 + \frac{\rho}{\eta(1 - \theta)^{\rho - 1}}} + \frac{1}{\pi - 1} \right) \right)
\]

We have:

\[
\frac{1}{1 + \frac{\rho}{\eta(1 - \theta)^{\rho - 1}}} + \frac{1}{\pi - 1} = \frac{\theta(\pi - 1) + \theta + \pi(1 - \theta)}{\theta + \pi(1 - \theta)}(\pi - 1)
\]

Using the previous expression and for \( k = 1 \), equation (30) becomes:

\[
\frac{1}{1 - \pi} = (1 - \rho) \left( \epsilon_{11}^p \left( \frac{1}{1 + \frac{\rho}{(1 - \theta)^{\rho - 1}}} + \frac{1}{\pi - 1} \right) + \epsilon_{21}^p \left( \frac{1}{1 + \frac{\rho}{\eta(1 - \theta)^{\rho - 1}}} + \frac{1}{\pi - 1} \right) \right)
\]

\[
\frac{1}{1 + \frac{\rho}{\eta(1 - \theta)^{\rho - 1}}} + \frac{1}{\pi - 1} = \frac{\theta(\pi - 1) + \theta + \pi(1 - \theta)}{(\theta + \pi(1 - \theta))(\pi - 1)}
\]

In addition,

\[
\frac{1}{1 + \frac{\rho}{\eta(1 - \theta)^{\rho - 1}}} + \frac{1}{\pi - 1} = \frac{\pi(1 - \theta)}{\pi(1 - \theta) + \theta} + \frac{\pi}{1 - \pi} = \frac{(1 - \theta)(1 - \pi) + \pi(1 - \theta) + \theta}{(\pi(1 - \theta) + \theta)(1 - \pi)}
\]

\[
= \frac{\pi(1 - \theta) + \theta)(1 - \pi)}{(\pi(1 - \theta) + \theta)(1 - \pi)}
\]

25
As a result, equation (34) can be written as:

\[
1 = \frac{(1 - \rho)\pi}{\pi(1 - \theta) + \theta (e_{21}^P - e_{11}^P)} (37)
\]

Solving for \( \rho \) gives:

\[
\rho = 1 + \frac{\pi(1 - \theta) + \theta}{\pi(e_{11}^P - e_{21}^P)} (38)
\]

which is equation (18).

7.1.2 Equation (19), \( \eta \)

As noted in equation (33), we have:

\[
\pi = \frac{1 - \eta f_2}{\eta f_1} (\rho)^{-1} = \frac{1 - \eta}{\eta} \left( \frac{1 - \theta}{\theta} \right)^{-1} (39)
\]

Solving for \( \eta \) yields:

\[
\eta = \frac{1}{1 + \pi (\frac{1 - \theta}{\theta})} (40)
\]

which is equation (19).

7.1.3 Equation (20), \( \nu \)

When \( \frac{f_1 + f_2 - \beta_0}{\beta_1} = \bar{W} \), first-order condition (16) takes the following form:

\[
p_1 = \nu \eta f_1^p (\eta f_1^p + (1 - \eta) f_2^p)^{\frac{1 - \rho}{\rho}} (41)
\]

We can write the previous expression as:

\[
p_1 = \nu \eta \frac{1}{\eta} (\frac{1 - \eta}{\eta} \left( \frac{f_2}{f_1} \right)^p)^{\frac{1 - \rho}{\rho}} = \nu \eta \left( 1 + \frac{\pi(1 - \theta)}{\theta} \right)^{\frac{1 - \rho}{\rho}}
\]

\[
= \nu \eta \frac{1}{\theta} \left( (\theta + \pi(1 - \theta)) \frac{1 - \rho}{\rho} \right) (42)
\]

As a result, we obtain:

\[
\nu = \frac{p_1 \theta^{\frac{1 - \rho}{\rho}}}{\eta \left( \theta + \pi(1 - \theta) \right)^{\frac{1 - \rho}{\rho}} (43)
\]

which is equation (20).
7.1.4 Equation (21), κ

Before we derive an expression for κ, we must differentiate first-order condition (16) with respect to food prices, \( p_1 \) and \( p_2 \). First equation in equation (16) reads:

\[
\frac{p_1}{1 + \kappa \left( \frac{f_1 + f_2 - \beta_0}{\beta_1} - \bar{W} \right)^2} = \nu \eta f_1^{\rho - 1} \left( \eta f_1^{\rho} + (1 - \eta) f_2^{\rho} \right) ^{\frac{1}{\rho}} - \frac{2 \delta \kappa \left( \frac{f_1 + f_2 - \beta_0}{\beta_1} - \bar{W} \right)(I - p_1 f_1 - p_2 f_2)}{(1 - \delta(1 - \zeta \beta_1))(1 + \kappa \left( \frac{f_1 + f_2 - \beta_0}{\beta_1} - \bar{W} \right)^2)}
\]

(44)

First rewrite a bit:

\[
p_1(1 - \delta(1 - \zeta \beta_1))(1 + \kappa \left( \frac{f_1 + f_2 - \beta_0}{\beta_1} - \bar{W} \right)^2) + 2 \delta \kappa \left( \frac{f_1 + f_2 - \beta_0}{\beta_1} - \bar{W} \right)(I - p_1 f_1 - p_2 f_2) = \frac{2 \delta \kappa \left( \frac{f_1 + f_2 - \beta_0}{\beta_1} - \bar{W} \right)(I - p_1 f_1 - p_2 f_2)}{(1 - \delta(1 - \zeta \beta_1))(1 + \kappa \left( \frac{f_1 + f_2 - \beta_0}{\beta_1} - \bar{W} \right)^2)}
\]

\[
\nu \eta f_1^{\rho - 1} \left( \eta f_1^{\rho} + (1 - \eta) f_2^{\rho} \right) ^{\frac{1}{\rho}}
\]

(45)

Take logarithm:

\[
\ln \left( p_1(1 - \delta(1 - \zeta \beta_1))(1 + \kappa \left( \frac{f_1 + f_2 - \beta_0}{\beta_1} - \bar{W} \right)^2) + 2 \delta \kappa \left( \frac{f_1 + f_2 - \beta_0}{\beta_1} - \bar{W} \right)(I - p_1 f_1 - p_2 f_2) \right) - \ln(1 - \delta(1 - \zeta \beta_1)) - 2 \ln(1 + \kappa \left( \frac{f_1 + f_2 - \beta_0}{\beta_1} - \bar{W} \right)^2) = \ln(\nu) + \ln(\eta) + (\rho - 1) \ln(f_1) + \frac{1 - \rho}{\rho} \ln(\eta f_1^{\rho} + (1 - \eta) f_2^{\rho})
\]

(46)

Differentiate with respect to \( p_k \):

\[
\forall k = 1, 2,
\]

\[
\frac{(1 - \delta(1 - \zeta \beta_1))(1 + \kappa(W - \bar{W})^2) + p_1 + 2 \kappa(W - \bar{W}) \sum_s \frac{\partial f_2}{\partial p_k}}{p_1 - \delta(1 - \zeta \beta_1))(1 + \kappa(W - \bar{W})^2) + 2 \delta \kappa(I - \sum_s p_s f_s)(W - \bar{W})} + \frac{2 \delta \kappa}{\beta_1} \frac{(I - p_1 f_1 - p_2 f_2) \sum_s \frac{\partial f_2}{\partial p_k} - \beta_1(f_k + \sum_s p_s \frac{\partial f_s}{\partial p_k})(W - \bar{W})}{p_1(1 - \delta(1 - \zeta \beta_1))(1 + \kappa(W - \bar{W})^2) + 2 \delta \kappa(I - \sum_s p_s f_s)(W - \bar{W})} - \frac{4 \kappa(W - \bar{W}) \sum_s \frac{\partial f_2}{\partial p_k}}{\beta_1(1 + \kappa(W - \bar{W})^2)} = (1 - \rho) \frac{\eta f_1^{\rho - 1} \frac{\partial f_1}{\partial p_k} + (1 - \eta) f_2^{\rho - 1} \frac{\partial f_2}{\partial p_k}}{\eta f_1^{\rho} + (1 - \eta) f_2^{\rho}} - \frac{1 - \rho}{\rho} \frac{\partial f_1}{\partial p_k}
\]

(47)
Regroup terms in $\frac{\partial f}{\partial p_k}$ and evaluate at $W = \bar{W}$:

$$\forall k = 1, 2$$

$$\frac{p_k^2}{p_1} \sum_{k=1}^{2} = \frac{\partial f_1}{\partial p_k}(- \frac{2\kappa \delta \zeta(I - p_1 f_1 - p_2 f_2)}{\beta_1 p_1 (1 - \delta(1 - \zeta_1))} + \frac{(1 - \rho)\eta f_1^{\rho-1}}{\rho f_1} - \frac{1 - \rho}{f_1})$$  \hspace{1cm} (48)

$$+ \frac{\partial f_2}{\partial p_k}(- \frac{2\kappa \delta \zeta(I - p_1 f_1 - p_2 f_2)}{\beta_1 p_1 (1 - \delta(1 - \zeta_1))} + \frac{(1 - \rho)(1 - \eta)f_2^{\rho-1}}{\eta f_2^{\rho}})$$

Rewriting in terms of elasticity and using food shares, we get:

$$\forall k = 1, 2$$

$$\frac{p_k^2}{p_1} \sum_{k=1}^{2} = \epsilon_{1k}(- \frac{2\kappa f_1 \delta \zeta(I - p_1 f_1 - p_2 f_2)}{\beta_1 p_1 (1 - \delta(1 - \zeta_1))} - \frac{1 - \rho}{1 + \frac{\eta}{\rho(1 - \theta)^\rho}})$$  \hspace{1cm} (49)

$$+ \epsilon_{2k}(- \frac{2\kappa f_2 \delta \zeta(I - p_1 f_1 - p_2 f_2)}{\beta_1 p_1 (1 - \delta(1 - \zeta_1))} + \frac{1 - \rho}{1 + \frac{\eta}{\rho(1 - \theta)^\rho}})$$

Using equation (33) and for $k = 1$, equation (49) becomes:

$$1 = \epsilon_{11}(- \frac{2\kappa f_1 \delta \zeta(I - p_1 f_1 - p_2 f_2)}{\beta_1 p_1 (1 - \delta(1 - \zeta_1))} - \frac{1 - \rho}{1 + \frac{\theta}{\pi(1 - \theta)^\theta}})$$

$$+ \epsilon_{21}(- \frac{2\kappa f_2 \delta \zeta(I - p_1 f_1 - p_2 f_2)}{\beta_1 p_1 (1 - \delta(1 - \zeta_1))} + \frac{1 - \rho}{1 + \frac{\theta}{\pi(1 - \theta)^\theta}})$$  \hspace{1cm} (50)

Rewriting slightly, we have:

$$\frac{2\kappa \delta \zeta(I - p_1 f_1 - p_2 f_2)}{\beta_1 p_1 (1 - \delta(1 - \zeta_1))} (\epsilon_{11} f_1 + \epsilon_{21} f_2) = \frac{(1 - \rho)\pi(1 - \theta)(\epsilon_{21} - \epsilon_{11})}{\pi(1 - \theta) + \theta} - 1$$  \hspace{1cm} (51)

Note that given equation (18), the right-hand side of the previous expression is equal to $-\theta$. Solving for $\kappa$, we get:

$$\kappa = -\frac{\beta_1 p_1(1 - \delta(1 - \zeta_1))}{2\delta \zeta(I - p_1 f_1 - p_2 f_2)(\epsilon_{11} f_1 + \epsilon_{21} f_2)}$$  \hspace{1cm} (52)

We rewrite the previous equation as:

$$\kappa = -\frac{\beta_1 p_1(1 - \delta(1 - \zeta_1))}{2\delta \zeta(I - p_1(f_1 + f_2)(\theta + \pi(1 - \theta)))(f_1 + f_2)(\epsilon_{11} \theta + \epsilon_{21}(1 - \theta))}$$  \hspace{1cm} (53)

which is equation (21).

To complete the proof we must show the necessity part. If total calories consumed satisfy $\frac{f_1 + f_2 - \beta_1}{\beta_1} = \bar{W}$ and preference parameters are given by equations (18)-(21), we must show that the pair $(f_1, f_2)$ satisfy first-order conditions (14) and (16).
Start with equation (20):

\[ \nu = \frac{p_1 \theta^{1-\rho}}{\eta^2 (\theta + \pi (1 - \theta))^{\frac{1-\rho}{\theta}}} \]  

(54)

Rearranging, we obtain:

\[ p_1 = \nu \eta^\frac{1}{\theta} (\theta + \pi (1 - \theta))^{\frac{1}{1-\theta}} = \nu \eta^\frac{1}{\theta} (1 + \frac{\pi (1 - \theta)}{\theta})^{\frac{1}{1-\theta}} \]  

(55)

From equation (33), we know that:

\[ \frac{\pi (1 - \theta)}{\theta} = \frac{1 - \eta}{\eta} \left( \frac{f_2}{f_1} \right)^{\rho} \]  

(56)

As a result, equation (55) can be written as:

\[ p_1 = \nu \eta^\frac{1}{\theta} (1 + \frac{1 - \eta}{\eta} \left( \frac{f_2}{f_1} \right)^{\rho})^{\frac{1}{1-\theta}} \]  

(57)

Rearranging once more, we get:

\[ p_1 = \nu \eta f_1^{\rho-1} (\eta f_1^{\rho} + (1 - \eta) f_2^{\rho})^{\frac{1}{1-\theta}} \]  

(58)

which is first-order condition (16) when \( \frac{f_1 + f_2 - \beta_0}{\beta_1} = \bar{W} \).

Next we must show that first-order condition (14) holds. That is:

\[ p_2 - p_1 = \nu (\eta f_1^{\rho} + (1 - \eta) f_2^{\rho})^{\frac{1}{1-\theta}} ((1 - \eta) f_2^{\rho-1} - \eta f_1^{\rho-1}) \]  

(59)

Starting with equation (33) again, we have:

\[ \pi = \frac{1 - \eta}{\eta} \left( \frac{f_2}{f_1} \right)^{\rho-1} \]  

(60)

Subtracting one on both sides and multiplying by \( p_1 \) we get:

\[ p_2 - p_1 = p_1 \left( \frac{1 - \eta}{\eta} \left( \frac{f_2}{f_1} \right)^{\rho-1} - 1 \right) \]  

(61)

We substitute \( p_1 \) from equation (58):

\[ p_2 - p_1 = \nu \eta f_1^{\rho-1} (\eta f_1^{\rho} + (1 - \eta) f_2^{\rho})^{\frac{1}{1-\theta}} \left( \frac{1 - \eta}{\eta} \left( \frac{f_2}{f_1} \right)^{\rho-1} - 1 \right) \]  

(62)

Rearranging, we get

\[ p_2 - p_1 = \nu (\eta f_1^{\rho} + (1 - \eta) f_2^{\rho})^{\frac{1}{1-\theta}} ((1 - \eta) f_2^{\rho-1} - \eta f_1^{\rho-1}) \]  

(63)

which is first-order condition (14) when \( \sum_{j} \beta_j = \bar{W} \).
7.2 Equations (22) and (23)

Rewriting equations (29) and (49) for \( k = 2 \) yields:

\[
\frac{\pi}{\pi - 1} = \epsilon_{12}\left(1 + \frac{1 - \rho}{1 - \eta (1 - \theta)} + \frac{1 - \rho}{\eta (1 - \theta)^{\rho - 1} - 1}\right) + \epsilon_{22}\left(\frac{1 - \rho}{\eta (1 - \theta)^{\rho - 1} - 1} + \frac{1 - \rho}{\eta (1 - \theta)^{\rho - 1} - 1}\right)
\]

(64)

and

\[
0 = \epsilon_{12}\left(-\frac{2\kappa_1 \delta \zeta (I - p_1 f_1 - p_2 f_2)}{\beta_1 p_1 (1 - \delta (1 - \zeta \beta_1))} - \frac{1 - \rho}{1 + \frac{\eta (1 - \theta)}{1 - \eta (1 - \theta)^{\rho - 1} - 1}}\right) + \epsilon_{22}\left(-\frac{2\kappa_2 \delta \zeta (I - p_1 f_1 - p_2 f_2)}{\beta_1 p_1 (1 - \delta (1 - \zeta \beta_1))} + \frac{1 - \rho}{1 + \frac{\eta (1 - \theta)}{1 - \eta (1 - \theta)^{\rho - 1} - 1}}\right)
\]

(65)

Note from equation (19) that:

\[
\frac{1 - \eta}{\eta} = \pi (\frac{1 - \theta}{\theta})^{1 - \rho}
\]

(66)

As a result, we have:

\[
1 + \frac{1 - \eta}{\eta} (\frac{1 - \theta}{\theta})^\rho = 1 + \frac{\pi (1 - \theta)}{\theta} = \frac{\pi (1 - \theta) + \theta}{\theta}
\]

(67)

\[
\frac{1 - \eta}{\eta} (\frac{1 - \theta}{\theta})^{\rho - 1} - 1 = \pi - 1
\]

(68)

\[
1 + \frac{\eta}{1 - \eta} (\frac{\theta}{1 - \theta})^\rho = 1 + \frac{\theta}{\pi (1 - \theta)} = \frac{\pi (1 - \theta) + \theta}{\pi (1 - \theta)}
\]

(69)

\[
\frac{\eta}{1 - \eta} (\frac{\theta}{1 - \theta})^{\rho - 1} - 1 = \frac{1}{\pi} - 1 = \frac{1 - \pi}{\pi}
\]

(70)

Equations (64) can be written as:

\[
\frac{1}{1 - \rho} \frac{\pi}{\pi - 1} = \epsilon_{12}\left(\frac{\theta}{\theta + \pi (1 - \theta)} + \frac{1}{\pi - 1}\right) + \epsilon_{22}\left(\frac{\pi (1 - \theta)}{\theta + \pi (1 - \theta)} + \frac{\pi}{1 - \pi}\right)
\]

(71)

We have:

\[
\frac{\theta}{\theta + \pi (1 - \theta)} + \frac{1}{\pi - 1} = \frac{\theta (\pi - 1) + \theta + \pi (1 - \theta)}{(\theta + \pi (1 - \theta))(\pi - 1)} = \frac{\pi}{(\theta + \pi (1 - \theta))(\pi - 1)}
\]

(72)
\[
\frac{1 - \theta}{\theta + \pi(1 - \theta)} + \frac{1}{1 - \pi} = \frac{(1 - \theta)(1 - \pi) + \theta + \pi(1 - \theta)}{(\theta + \pi(1 - \theta))(1 - \pi)} = \frac{1}{(\theta + \pi(1 - \theta))(1 - \pi)}
\] (73)

As a result,
\[
\frac{\theta + \pi(1 - \theta)}{1 - \rho} = \epsilon_{12} - \epsilon_{22}
\] (74)

Using the definition for the parameter \(\rho\) in equation (18), we rewrite the previous expression as:
\[
\epsilon_{12} - \epsilon_{22} = \pi(\epsilon_{21} - \epsilon_{11})
\] (75)

which is equation (22).

Equations (65) can be written as:
\[
0 = \epsilon_{12}\left(\frac{2\kappa f_1 \delta \zeta (I - p_1 f_1 - p_2 f_2)}{\beta_1 p_1(1 - \delta(1 - \zeta \beta_1))} + \frac{(1 - \rho)\pi(1 - \theta)}{\theta + \pi(1 - \theta)}\right) + \epsilon_{22}\left(\frac{2\kappa f_2 \delta \zeta (I - p_1 f_1 - p_2 f_2)}{\beta_1 p_1(1 - \delta(1 - \zeta \beta_1))} - \frac{(1 - \rho)\pi(1 - \theta)}{\theta + \pi(1 - \theta)}\right)
\] (76)

From the definition of the parameter \(\kappa\) in equation (21), we have:
\[
\frac{2\kappa f_1 \delta \zeta (I - p_1 f_1 - p_2 f_2)}{\beta_1 p_1(1 - \delta(1 - \zeta \beta_1))} = \frac{\theta^2}{\epsilon_{11}\theta + \epsilon_{21}(1 - \theta)}
\] (77)

and
\[
\frac{2\kappa f_2 \delta \zeta (I - p_1 f_1 - p_2 f_2)}{\beta_1 p_1(1 - \delta(1 - \zeta \beta_1))} = \frac{(1 - \theta)^2}{\epsilon_{11}\theta + \epsilon_{21}(1 - \theta)}
\] (78)

In addition, we have:
\[
\frac{(1 - \rho)\pi(1 - \theta)}{\theta + \pi(1 - \theta)} = \frac{1 - \theta}{\epsilon_{11} - \epsilon_{21}}
\] (79)

We can therefore write equation (76) as:
\[
0 = \epsilon_{12}\left(\frac{\theta^2}{\epsilon_{11}\theta + \epsilon_{21}(1 - \theta)} + \frac{1 - \theta}{\epsilon_{11} - \epsilon_{21}}\right) + \epsilon_{22}\left(\frac{(1 - \theta)^2}{\epsilon_{11}\theta + \epsilon_{21}(1 - \theta)} - \frac{1 - \theta}{\epsilon_{11} - \epsilon_{21}}\right)
\] (80)

which is equation (23).
References


Food and Nutrition Board (2002). Dietary reference intakes for energy, carbohydrate, fiber, fat, fatty acids, cholesterol, protein and amino acids. Technical report, Food and


