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Mathematics in Contemporary Society Chapter 10

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Chapter 10

More on the Metric System

How does the metric system compare to the US Customary System? We can look at a chart of conversions:

USCS – Metric Conversions

Conversion Factors for U.S. Customary and Metric System

USCS to Metric	**Metric to USCS**
Distance Measures	**Distance Measures**
1 inch = 2.54 centimeters	1 centimeter = 0.394 inches
1 foot = 30.5 centimeters	1 meter = 3.28 feet
1 foot = 0.305 meters	1 meter = 1.09 yards
1 yard = 0.914 meters	1 kilometer = 3281 feet
1 mile = 1.61 kilometers	1 kilometer = 0.621 miles
Weight (Mass) Measures	**Weight (Mass) Measures**
1 ounce = 28.4 grams	1 kilogram = 35.3 ounces
1 ounce = 0.028 kilograms	1 kilograms = 2.2 pounds
1 pound = 454 grams	
1 pound = 0.454 kilograms	
1 ton = 908 kilograms	
Volume Measures	**Volume Measures**
1 quart = 0.946 liters	1 liter = 1.06 quarts
1 gallon = 3.79 liters	1 liter = 0.264 gallons

We notice from the table that a meter and yard are about the same. A liter and a quart are about the same. A kilogram is a little more than 2 pounds. A kilometer is $6/10^{th}$ of a mile.

We can use the chart to easily convert between both systems.

Example 1 Convert 25 kilogram to pounds.

All we need to do is multiply by the conversion factor:

$$\frac{25 \text{ \sout{kilograms}}}{1} \cdot \frac{2.2 \text{ pounds}}{1 \text{ \sout{kilogram}}} \cdot = 25 \cdot 2.2 \text{ pounds} = 55 \text{ pounds (lb)}$$

We notice that we could have also used the other conversion factor:

$$\frac{25 \text{ \sout{kilograms}}}{1} \cdot \frac{1 \quad \text{pounds}}{0.454 \text{ \sout{kilogram}}} \cdot = \frac{25}{0.454} \text{ pounds} = 55.07 \text{ pounds (lb)}$$

Question 1: **Convert 15 miles to kilometers.**
Question 2: **Convert 150 centimeters to inches.**
Question 3: **Convert 20 quarts to liters.**

A little note about mass: Grams and kilograms are measures of mass; pounds and kilograms are measures of weight. What the difference?

Weight is dependent upon gravity. I'm sure you've heard that your weight on the moon is not the same as it is on the earth. A 120-pound person weighs only about 20 pounds on the moon, because the moon's gravitational pull is so much less than Earth's gravity.

Mass is different; mass is a measure of the amount of matter contained in an object. This amount doesn't change because of gravity. A 55-kilogram person has the same mass on the earth and the moon.

We don't have to worry about the difference here on Earth because your mass and weight are not going to change. (As long as you stay on the Earth!)

Units of Temperature

The temperature measure we are probably most familiar with is Fahrenheit. On Thursday (4/7/11) at 2 pm the temperature is about 46º Fahrenheit (or 46º F). We should know that water freezes at 32º F and boils at 212º F. Our body temperature is about 98.6º F

Celsius temperatures are used most often internationally (as part of the metric system), although we see them in the United States (often both are reported). Today's 2 pm temperature is about 8º Celsius (or 8º C). We should know that water freezes at 0º C and boils at 100º C. Our body temperature is about 37º C.

Conversions

We should know how to convert from one temperature to the other.

To convert any Celsius temperature to Fahrenheit, multiply the temperature by 1.8 and add 32, using the formula:

$$F=(C\bullet 1.8) + 32^\circ$$

Example: Convert 25º C to Fahrenheit temperature.

Answer:
$F = (25^\circ \bullet 1.8) + 32^\circ$
$F = 45^\circ + 32^\circ$
$F = 77^\circ$

To convert any Fahrenheit temperature to Celsius, subtract 32 first and divide by 1.8, using the formula:

$$C = (F-32) \div 1.8$$

Example: Convert 60° F to Celsius temperature.

Answer:
$C = (60^\circ - 32^\circ) \div 1.8$
$C = 28^\circ \div 1.8$
$C = 15.6^\circ$

Question 4: Convert 40° Celsius to Fahrenheit.
Question 5: Convert 88° Fahrenheit to Celsius.

*Note: Kelvin temperature is often used by scientists. The interesting feature of Kelvin temperature is that 0° Kelvin represents **absolute zero**, the theoretically coldest temperature possible. The scale (by degree) is otherwise the same as Celsius.*

Units of Energy and Power

Energy is what makes matter move or heat up. We are probably very familiar with the Calorie, which is a measure of the energy obtained from different foods. The food Calorie is actually equal to 1,000 calories. One calorie is enough energy to raise 1 gram of water by 1° C.

In the metric system, the unit of energy is the **joule**, which is about 0.24 calories.

Power is a rate of energy usage. In the metric system, the unit of power is the **watt**, which is 1 joule of energy for one second:

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ second}}$$

One watt is a very small amount of power. Even the dimmest bulbs use at least 40 watts of power, which is 40 joules per second of usage.

If a 40 watt bulb (40 joules per second) is used for an hour, that would be:

$$\frac{40 \text{ joules}}{1 \ \cancel{\text{second}}} \times \frac{1 \ \cancel{\text{hour}}}{1} \times \frac{60 \ \cancel{\text{min}}}{1 \ \cancel{\text{hr}}} \times \frac{60 \ \cancel{\text{sec}}}{1 \ \cancel{\text{min}}} = 144{,}000 \text{ joules of energy used}$$

Question 6: How much energy does a 100 watt bulb use in 2 hours?

A special unit of energy usage is the kilowatt-hour, often seen on utility bills (check one out). A kilowatt-hour is a kilowatt (1,000 joules per second) of usage for one hour. How much energy is this? It's:

$$\frac{1000 \text{ joules}}{1 \ \cancel{\text{second}}} \times \frac{1 \ \cancel{\text{hour}}}{1} \times \frac{60 \ \cancel{\text{min}}}{1 \ \cancel{\text{hr}}} \times \frac{60 \ \cancel{\text{sec}}}{1 \ \cancel{\text{min}}} = 3{,}600{,}000 \text{ joules of energy used}$$

We can now use 1 kilowatt-hour = 3,600,000 joules for future calculations.

With this information, we can solve a problem relating to energy usage:

Example 1 A 100-watt bulb is left on for the month of November (30 days). How much energy is used? If a kilowatt-hour of usage costs 20 cents, how much will it cost?

First we convert the usage to joules:

$$\frac{100 \text{ joules}}{1 \text{ ~~second~~}} \times \frac{30 \text{ ~~days~~}}{1} \times \frac{24 \text{ hrs}}{1 \text{ ~~day~~}} \times \frac{60 \text{ ~~min~~}}{1 \text{ ~~hr~~}} \times \frac{60 \text{ ~~sec~~}}{1 \text{ ~~min~~}} = 259{,}200{,}000 \text{ joules}$$

This seems like a lot, but 1 kilowatt-hour = 3,600,000 joules. Therefore:

$$\frac{259{,}200{,}000 \text{ joules}}{1 \text{ kilowatt-hour}} = \frac{259{,}200{,}000 \text{ joules}}{3{,}600{,}000 \text{ joules}} = 72 \text{ kilowatt-hours}$$

At $0.20 per kilowatt-hour, this is:

72 x $0.20 = $14.40

Density

The density of a substance or object is the ratio of the mass of the object to its volume. Certain objects take up a lot of space, but don't have very much mass in comparison (or weight either). For example, a 1 kilogram block of Styrofoam has a volume of about 1,400 cubic centimeters (cm^3). Another object may have the same mass but much less volume. A 1 kilogram block of iron has a volume of about 130 cm^3. Therefore, the density of iron (or anything made of iron) is much larger than the density of Styrofoam.

How do we calculate density? We use the formula:

$$\text{Density} = \frac{\text{mass (usually in grams)}}{\text{volume (usually in cm}^3\text{)}}$$

If we know the mass and volume of any object, we can calculate its density.

Example 2 If a rock has a mass of 200 grams and a volume of 40 cm^3, then we can easily calculate the density:

$$\text{Density} = \frac{200 \text{ g}}{40 \text{ cm}^3} = 5 \text{ g/cm}^3$$ (5 grams for each cubic centimeter.)

We can compare the density of one object to any other object. If another rock had a density of 3 g/cm^3, it would less dense.

<u>Question 7:</u> **Name three objects with a low density. Name three objects with a high density.**

Note: The density of water is 1 g/cm^3. Anything with a smaller density than 1 (feathers, Styrofoam, ice) will float in water. Anything with a density greater than 1 (rocks, calculators, coins) will sink in water.

Other Densities

We can calculate the density for other things besides mass and volume. For instance, **population density** is calculated using the formula:

$$\text{Population Density} = \frac{\text{population of people in some area}}{\text{area (in } ft^2 \text{ or } mi^2 \text{ or } km^2)}$$

For example: If there are 30,000 people living in the town of Appleton, which has an area of 25 mi^2, we can calculate the population density:

$$\text{Population Density} = \frac{30000}{25\ mi^2} = \text{(1200 people per square mile)}$$

We can compare the population density of one area to another. If the town of Big Rock has a population density of 2000 people/mi^2, it is more densely populated (even though the population of the town may be less than 30000 people) per square mile.

<u>Question 8:</u> **Name a place that has a high population density. Name a place with a low population density.**

Blood-Alcohol Concentration

Concentration measures the amount of one substance within another substance. Blood-Alcohol Concentration measures how much alcohol is contained in the blood.

For example: If your Blood-Alcohol Concentration was 0.1%, then 0.1% of your blood contains alcohol. If a person has 5 liters of blood, then the total amount of alcohol in their system is:

$$0.1\% \bullet 5 \text{ liters} = 0.001 \bullet 5 \text{ L} = 0.005 \text{ L} = 5 \text{ milliliters (ml)}$$

This might not seem like very much, but a Blood-Alcohol Concentration of only 0.005% impairs brain functions, and 0.5% is usually deadly.

<u>Question 9:</u> **If a person's Blood-Alcohol Concentration is 0.2%, calculate how much alcohol is in the person's blood if he/she has 5.8 liters of blood. Would you consider this person to be intoxicated?**

Note:

1) Blood Alcohol Content (BAC) is measured by dividing the weight of alcohol (in grams) per 100 milliliters of blood. This more common legal definition gives slightly different measures.

2) Although a can of beer may have about 400 ml of alcohol, we would have to take into account the length of time it takes to drink it, the length of time it takes for alcohol to enter the bloodstream, and the body's metabolic rate of eliminating alcohol (roughly 10-15 ml per hour).

Problem Solving Techniques for the Real World

In the real world, problems aren't as simple as what we find in a textbook. We need to figure how to get to work, find time for school, have a social life, pay our bills, do our homework, etc., etc., etc. We may think there's no solution that makes all things possible.

Even so, there are problem solving techniques that can be applied to any problem to make it simpler to (maybe) solve.

Example: Suppose we (you and I) are presented with the problem:

How can everyone get a B or better in MA-321?

Here is some advice for this problem (and many others):

1) **Consider this four-step process:**

   a. **Understand the problem:**

   Be clear about what we mean by MA-321. Is it just one class or all of the MA-321 classes? (To make it easier, let's say it's just our class.)

   Who does this apply to? How many students actually have an average below B?

   What is the current class average?

   Which part of the class (exams, labs, homework, etc.) is causing students the most trouble?

   Are we fixing this problem for this semester or for future semesters?

   b. **Devise a strategy for solving it:**

   Try something—create study groups of three or four students.

   c. **Carry out the strategy; revise if necessary:**

   Are grades improving? If not, try something else.

   d. **Look back to check, interpret and explain your result:**

   Did the strategy improve grades? Do we know by how much?

With these in mind, we can consider other hints:

2) **There may be more than one answer to the problem.**

Should we examine what the students are doing? Do they need to study more often? How is their attendance? Are they handing in the work on time?

Should we consider what the professor is doing? Are the tests too difficult? Are the notes confusing? Is there too much work? Are the lab assignments difficult to understand?

Should we ask the math department why this course was designed to include the topics that it has? Would less material be better?

What about the administration? Maybe the classes are too large, or the computers in the labs are antiquated. Perhaps Blackboard is not accessible on a regular basis.

3) **Since there may be many answers, there are probably many strategies.**

We could ask the students what they need to create a better environment for learning. We could ask the professor what he wants the students to do that will help them get better grades. We could look at the exam scores to see if students are doing poorly on exams. We could examine homework scores; perhaps not all students are handing in assignments. We could look at the GPA of students taking MA-321; perhaps the average GPA of an MA-321 student is below a B average. We could assign tutors to any student who has MA-321 classes; how is this class doing in comparison to others?

4) **Use appropriate tools.**

Make sure that you have what you need to measure what you're examining. Do we have access to the students' grades in this course? If needed, do we have access to their school records? Do we need to obtain access to the records of other MA-321 classes?

We must have access to whatever we need to perform our task successfully—computers, records, surveys, financing, time, etc.

5) **Consider smaller problems.**

If resources are limited, perhaps a smaller focus group of students (those doing poorly in class) could be examined. By understanding the background of a smaller set of students, and attempting methods on these students, we may better understand how to approach the larger problem.

Or we may focus on one element that is causing students problems, such as lab assignments. We could then address ways to improve scores on lab assignments only, to better understand students' reactions.

6) **Approximate what the needs are.**

If your class average is 75 right now, you should have a sense of what it's going to take to bring your average up to an 85.

On a larger scale, if we know the average of any class, we can approximate what level of performance increase is required.

7) **Try something completely different.**

Work backwards. Consider what topics are going to be on the final exam and determine what students need to learn in the next six weeks to do well on the final.

8) **Have access to the appropriate sources and resources.**
Has anyone else tried to address a similar problem in MA-321 or another class? What did they do?
What resources are already available from the Math Center, or the Math Department, or the Academic Computing Center, or Instructional Support Services?

9) **If you're tired or aggravated, put the problem down and look at it again when you're feeling refreshed.**

You all know the feeling of trying to finish a task when you don't have the energy for it. Sometimes, a break away from it all is what you need.

Writing Assignment: Consider the following real world problems/situations. Using the suggestions in the notes, <u>briefly</u> identify four things you could do to address and solve each problem. (If you wish, you can apply the four-step process from the notes.)

a) Queensborough Community College students have been complaining that there's not enough parking on or around campus.
b) Your family would like to plan a trip to the Caribbean this summer.
c) Your company is trying to obtain financing for new computer equipment for the company headquarters.
d) Textbooks are becoming so expensive that many students do not bother purchasing them.
e) The New York Mets haven't won a World Series in almost thirty years.

Lab Assignment #8—Problem Solving

Due ____________________

Here are some problems to solve. Use Excel (and each other) for help.
With each problem, include a short written explanation of how you solved the problem. Answer any additional questions.

1) A sheet of paper has an area of 200 square inches. What dimensions will give the sheet the smallest perimeter?

2) A room is 24 feet long, 15 feet wide and 12 feet high. How much will it cost to remodel the room if carpeting costs $28 per square yard, a $35 can of blue wall paint covers 250 square feet and a $28 can of white ceiling paint covers 175 square feet?

3) You start with $7,000. You convert all of your money to British currency. In London, you spend 2,647 British pounds. When you return home, you convert the British currency back to US currency. How much money do you have left? (Use the conversion rates in the notes.)

4) a) A business associate, who owes you $50,000, makes the following proposal: Pay 1 cent on November 1, 2 cents on November 2, 4 cents on November 3, 8 cents on November 4, etc., until November 30. Use Excel to determine each individual payment and the sum of all payments day by day (make a column for each).
b) Is it a fair deal? (If no, when does it stop being fair?)

   c) Is there a shortcut formula?

   d) What if the deal was extended until December 31?

5) The original price of a stereo was reduced by 25%. The sales price was then reduced by 20%. This price was then reduced by 35%. If the stereo is now selling for $351.00, what was the original price?
6) Create a set of 15 numbers that have a mean of 75, a median of 70, a mode of 81 and a standard deviation of 24.1
7) If a kilowatt-hour of energy costs 45 cents, how much does it cost to leave a 40-watt light bulb on for the entire month of December?