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Cooperation and Reciprocity in Anonymous Interactions: Other-Regarding Preferences and Quasi-Magical Thinking

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by

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Dedication

This thesis is dedicated to the memory of my grandfather, Richard Lindbloom; of my nephew, Brian Klevans, Jr.; and of my mother, Jo Ann Klevans.

Contents

It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience.

—Albert Einstein

1 Introduction

People often cooperate with each other, even when one of the parties has every incentive to take advantage of another party. Such behavior appears to contradict the assumption of individual rationality that underlies much of economic theory. Game theory, the field that studies decision-making when outcomes depend on the choices made by others as well as by oneself, is traditionally predicated on individual rationality. This principle implies that outcomes necessarily occur in equilibrium: nobody could have gained from having made some other choice, given the choices made by the others. Instead, in experiments we observe otherwise: people often cooperate in prisoner's dilemmas, reward trust in trust games, and punish unfairness in ultimatum games, despite every attempt having been made by the experimenters to ensure subjects are confident they will remain anonymous to each other (and often to the experimenter as well).

Since individual rationality does not always hold, at least one of the following must be true: some people are not solely individualistic, or some people are not completely rational (in the game-theoretic sense). This does not vitiate the methods of equilibrium analysis in game theory; on the contrary, they are likely to be of use provided one accounts for other-regarding preferences, or for bounded rationality. However, the goal of a tractable model incorporating these principles has been elusive. In this context, Einstein's quote is arguably no less applicable to economics than it is to physics: we ought to strive to explain as much of observed behavior as we can given a minimum of complexity in the foundations of the theory.

This prepare presents a model that is intended to open a new path toward this goal. Guided by the results of the experimental literature within the field of behavioral game theory, I incorporate other-regarding preferences as well as a specific manifestation of bounded rationality that has been given the term "quasi-magical thinking" (Shafir and Tversky 1992). Together, they suggest four emotions at play in the decision-making process. Envy and guilt are described by other-regarding preferences, with decisions made to avoid experiencing these feelings. Quasi-magical thinking, on the other hand, is influenced by the immediate emotions of hope and fear. The distinction between immediate and anticipated emotions is relevant, as it separates nonconsequential decision-making (of which quasi-magical thinking is an example) from the consequential decision-making assumed in standard economic theory (Rick and Loewenstein 2010).

These are not modelled as dichotomous phenomena, but rather interact closely, resulting in interesting predictions. Other-regarding preferences are defined similarly as in the seminal model of Fehr and Schmidt 1999, but with three innovations: they are given a nonlinear specification allowing for interior solutions (so that players do not necessarily jump from rational behavior to cooperative behavior as parameters vary); the preferences are not defined over realized outcomes but rather on expected outcomes, allowing for probabilistic decision-making; and the potential of a uneven split serving as the benchmark for a fair outcome is accounted for, with a bargaining model offered to suggest how such a situation might come about. Quasi-magical thinking, proposed in Shafir and Tversky 1992 as decision-making that follows the erroneous belief (which may be understood to be false) that one's actions affect others' actions, is defined for asymmetric games as well as symmetric ones. In symmetric games, which present identical choices to each player, the natural definition for such belief is that choosing some action induces the opponent to choose the same action, but it is not clear how to define magical beliefs in asymmetric games. I tackle this problem by assuming that a quasi-magical thinker reveals how much they care about the opponent's payoff when they choose an action, and the worst possible interpretation of the signal is made and reciprocated by the opponent.

While I do not purport to have an all-encompassing model that is ready to apply to all possible situations, I believe that this model offers some novel and useful methods to analyze decision-making. The literature review will discuss experimental results and theoretical models that are most salient to the topic at hand. Then the model will be laid out, with illustrative examples provided as appropriate. A summary of the model, discussion of its limitations, and future avenues of research concludes the paper.

2 Literature Review

Four games with distinct characteristics and strong interest among experimenters are given particular attention. The prisoner's dilemma is the classic illustration of the conflict between individual and collective interest, while the traveler's dilemma shows how disregarding an equilibrium strategy can greatly improve one's outcome provided the opponent does the same. The ultimatum game and trust game introduce the potential to engage in reciprocity, i.e. forfeiting money in order to reward or punish an opponent who is either kind or mean, with the former likely to result in players exhibiting negative reciprocity, and the latter allowing players the choice to display positive reciprocity.

In the standard allegory justifying the name "prisoner's dilemma," two partners in crime are in jail and facing three criminal charges, each of which carries a sentence of, say, three years. The prosecutor has sufficient evidence to prove their guilt on two of the charges, but not the third. However, each defendant is able to provide evidence implicating the other on the third count, so the defendant offers each a deal in order to elicit the desired evidence: provide the evidence, and one of his charges will be dropped. The prisoners are separated and unable to communicate when the deal is offered.

The traveler's dilemma, crafted as a paradox of rationality (Basu 1994), is played by two airline customers whose luggage has been lost by an airline. Each traveler's luggage contains identical copies of an antique, and they ask to be compensated for its value but cannot prove what the value is. The airline manager with whom they are discussing the matter has a plan; he will separate the travelers and ask them to independently provide a value for the luggage. If different values are provided, then the lower value is taken to be the true value and each is compensated that amount, plus a \$2 reward to the one providing the lower and presumably honest value, and minus a \$2 penalty to the one the higher and presumably dishonest value. The airline is not willing to reimburse more than \$100 per person, but does not wish to take money from either customer, so claims may not be less than \$2 or more than \$100. However, the travelers do not simply provide their best appraisals of the value, as a normal person might. Instead, they surprise the manager by making claims of \$2. This is because they are rational payoff-maximizers, and this is the equilibrium strategy. Any higher claim gives the other traveler an opportunity to undercut the claim and thereby get a higher payment.

The ultimatum game (Güth, Schmittberger, and Schwarze 1982) and trust game (Berg, Dickhaut, and McCabe 1995) are sequential games in which a player is given an endowment and has an option of sending some of it to the opponent, who can take an action to reciprocate towards the first-stage player. In the ultimatum game, the second-stage player can reject the offer, which results in both players leaving with nothing. In the trust game, the endowment is multiplied by some factor, which leaves the second-stage player the ability to repay the first-stage player. The ultimatum game is a test of negative reciprocity, while the trust game is a test of positive reciprocity.

Relevant experimental findings are discussed in the following section, proceeded by a summary of the theoretical literature developed to account for many of the findings. The most salient literature is addressed; repeated games are mostly omitted, and some topics outside of the scope of the model, such as framing effects, are not covered.

2.1 Experimental Literature

Evidence that people engage in quasi-magical thinking was presented in an experiment reported in Shafir and Tversky 1992, which presented three groups of subjects with a prisoner's dilemma. One group was told the opponent had cooperated, and another was told the opponent had defected. The third was told nothing about the opponent's decision. More participants cooperated when told the opponent cooperated than when told the opponent defected, as expected, but still more cooperated when they were not informed either way. Presumably, many people have a predisposition to try to do their part to bring about a favorable outcome. An eye-tracking study (Hristova and Grinberg 2008) corroborated these findings, finding that players paid more attention to all possible outcomes when uninformed of the opponent's choice, and otherwise were more likely to compare only the two possible outcomes given the other's choice.

A survey of ultimatum bargaining games is given in C. Camerer 2003. Most offers range from 40% to 50% of the endowment, and such offers are usually accepted, but offers of less than 20% are likely to be rejected. The motives for making fair offers are ambiguous, since either altruism or fear of rejection can cause a proposer to behave fairly. Dictator game experiments, which remove the ability of the responder to reject an offer, eliminate this ambiguity. Both motives are likely to be relevant, since dictator game offers are lower (suggesting that high offers in ultimatum games are strategic) but positive (indicating altruism is also a factor). While results are robust within the typical experimental population, strikingly different outcomes arise in less market-integrated cultures that follow very different fairness norms. Also, when the fair solution is not readily apparent, disagreement of what constitutes a fair outcome, and hence a higher rejection rate, is more likely. This occurs, for example, when outside options are adjusted so that one or both players leave with some positive amount in the event of a rejection, leaving one with a strategic advantage over the other.

An early experiment of the traveler's dilemma was conducted for Capra et al. 1999, which had subjects play ten rounds of a traveler's dilemma for a variety of bonus/penalty amounts ranging from 5¢ to 80¢, with claims limited to amounts between 80¢ and 200¢ inclusive. For low bonus/penalty parameters, average claims were consistently close to 200¢, while for high parameters initial claims were somewhat lower but fell to near-equilibrium levels by the final period. Intermediate values of the bonus-penalty resulted in average claims stabilizing near the middle of the choice set at 120¢ to 150¢. In another experiment, 51 members of the Game Theory Society were asked to submit claims for the standard claim and bonus/penalty amounts, in order to text whether experts play rationality. Ten participants did not, as they submitted the maximum claim in spite of the fact that this strategy is dominated: claiming 99 will always yield a higher payoff than claiming 100. Only ten others claimed less than 94, with three playing the equilibrium strategy; the remaining claims ranged from 94 to 99 (C. F. Camerer, Ho, and Chong 2004).

An informative experiment of the trust game is given in Burks, Carpenter, and Verhoogen 2003, and a meta-analysis is given in Johnson and Mislin 2011. Patterns uncovered by the meta-analysis include the following: on average, 50% of the amount available to the senders is sent, and 37% of the amount available to the receivers is returned. Playing against a human significantly increases trust, subjects in Africa are less trusting, providing the receiver with a guaranteed endowment possibly reduces the amount sent, and students are less trustworthy than others. Burks, Carpenter, and Verhoogen 2003 found that having subjects play both roles reduced both trust and trustworthiness provided they were informed of this beforehand, and also find that a measure of Machiavellian personality traits obtained by questionnaire is associated with less trust but not with less trustworthiness. Another experiment of the trust game found that subjects often sent money despite reporting that they expected to receive less than the amount sent, although they sent less on average then those who expected to at least break even (Ashraf, Bohnet, and Piankov 2006), a finding which I am inclined to interpret as evidence of quasi-magical thinking. Such players send money for the sake of sending money, out of hope if not expectation that the opponent will return a fair share of the investment.

2.2 Theoretical Literature

The seminal model that formalized the notion that people want to help those who help them, and hurt those who hurt them, was introduced in Rabin 1993, while Fehr and Schmidt 1999 explored the implications when people derive utility from their relative payoffs compared to another in addition to their own payoff. Rabin 1993 transformed games into psychological games for which utilities are subjected to beliefs about the opponent's intentions, as well as beliefs about the opponent's beliefs about the player's intentions, and applied a "kindness" function to actions and beliefs. "Fairness equilibria" occur when the beliefs are correct. Fehr and Schmidt 1999 introduced a utility function of the form

$$U(x_i) = x_i - \alpha_i \max\{0, x_j - x_i\} - \beta_i \max\{0, x_i - x_j\},$$

with utility adversely affected when the opponent receives a higher payoff, and also (perhaps less so) when the opponent receives less. These dual concepts of reciprocity and inequity-aversion were combined into a single model by Bolton and Ockenfels 2000, while Dufwenberg and Kirchsteiger 2004 extended the theory to the class of sequential games, which includes the ultimatum game and trust games. The role of emotions, such as kindness and vengeance, that depend on experience is modelled in Cox, Friedman, and Gjerstad 2007.

Explicit models of bounded rationality are proposed in McKelvey and Palfrey 1995 and C. F. Camerer, Ho, and Chong 2004. The former assumes that players systematically make random mistakes in choosing best-responses, but better outcomes remain more likely than worse ones; the propensity to make a mistake is given by a single parameter, and "quantal response equilibrium" is achieved when players determine the opponent's probability distribution over strategies. The cognitive hierarchy model proposes that players perform backward inductive reasoning to varying depths of iteration, and that each players believes that the other players perform one less level of iteration than they do. Magical thinking is explored for the class of symmetric games in Daley and Sadowski 2017, while an essentially equivalent concept called "superrationality" was proposed by Hofstader 1983. The differing terminology reflects different interpretations of the idea: magical thinking is generally regarded as a cognitive bias, while superrationality is considered a deliberate form of reasoning that is believed to be common knowledge.

3 Model

Consider a two-person non-cooperative game in which each of Players 1 and 2 must choose one of two actions; call them a "high" action H and a "low" action L. The payoff to Player 1 is a_{jk}, where $j \in \{H, L\}$ is Player 1's action and $k \in \{H, L\}$ is Player 2's actions; the payoff to Player 2 given these actions is b_{jk}. This game is expressed in normal form by

	H	L
H	(a_{HH}, b_{HH})	(a_{HL}, b_{HL})
L	(a_{LH}, b_{LH})	(a_{LL}, b_{LL})

For now, I do not apply a utility function to the payoffs. Suppose Player 1 plays H with probability x_1 and Player 2 plays H with probability x_2; the quantities x_i are referred to as "strategies." Let $\pi_i(x_i, x_{-i})$ denote Player i's expected payoff given the strategies x_i and x_{-i} (where $-i$ refers to the opponent). For brevity, where there is believed to be no potential confusion, $\pi_i(x_i, x_{-i})$ will simply be expressed by π_i.

These games will be treated as continuous games. Depending on the context, strategies may either represent probabilities of playing one of two possible strategies, or represent selections from a continuous range of possible outcomes. The latter interpretation is only possible because utility functions are not yet invoked.

Example 1. *Prisoner's Dilemma*

Suppose players choose whether to cooperate or to defect. Choosing to defect grants a payment of 1, and an additional payment of 2 is made to a player if the opponent cooperates. This game has the following normal form, with x_i representing the probability that Player i cooperates:

	Cooperate	Defect
Cooperate	$(2, 2)$	$(0, 3)$
Defect	$(3, 0)$	$(1, 1)$

Example 2. *Trust Game*

Two players, an sender (Player 1) and a receiver (Player 2), are each given an endowment of 1. The send may invest some of his or her endowment by sending it to the receiver. The amount is multiplied by three and delivered to the receiver, who may return some fraction of the amount received. The matrix form of this game, with x_1 representing the amount sent and x_2 the fraction returned, is

	Return All	Return None
Send All	$(3, 1)$	$(0, 4)$
Send None	$(1, 1)$	$(1, 1)$

Note that this is not the normal form of the sequential game, which would take into account distinct responses to each investment amount. This matrix representation simply serves to define the payoffs.

Example 3. *Ultimatum Game*

A proposer (Player 1) is given an endowment of 1 and chooses to offer some amount to a responder (Player 2), who chooses whether to accept or reject, with a rejection resulting in payoffs of 0 to both players. The strategy x_1 represents the offer amount, and the strategy x_2 represents the probability of accepting the amount offered.

	Accept	Reject
Offer All	$(0, 1)$	$(0, 0)$
Offer None	$(1, 0)$	$(0, 0)$

Example 4. *Mini-Ultimatum Game*

This is a discrete version of the standard ultimatum game in which the offer amount must be selected from two choices: in this case, a high offer of 2 out of 4, or a low offer of 1 out of 4. The responder only chooses whether to accept or reject a low offer. The strategy x_1 represents the probability of making a high offer, and x_2 represents the probability of accepting a low offer if it is made.

	Accept Low	Demand High
High Offer	$(2, 2)$	$(2, 2)$
Low Offer	$(3, 1)$	$(0, 0)$

3.1 Cooperation

In order to define the players' preferences for fairness, it is necessary to establish what constitutes a "fair" outcome. The simplest such notion presumes that an outcome is fair if it grants both players equal payoffs. It is plausible that asymmetries in payoffs may lead a player to feel entitled to a higher payoff than the other. For that reason I provide an alternative method to find the benchmark that an other-regarding player might consider to be most fair, using a bargaining model similar to one outlined in (Luce and Raiffa 2012).

Suppose a solution (x_1, x_2) is proposed by one of the players or by some third-party, and the players were to discuss whether this solution was appropriate. Assume that the players are particularly spiteful negotiators, and one of the players notices that by playing $\hat{x}_i \neq \hat{x}_{-i}$, while he might be worse off than under the proposed solution, the opponent would incur a larger loss, i.e.

$$\pi_i(x_i, x_{-i}) - \pi_i(\hat{x}_i, x_{-i}) < \pi_i(x_i, x_{-i}) - \pi_{-i}(\hat{x}_i, x_{-i}).$$

The opponent in turn looks for a counter-threat strategy $\hat{x}_{-i} \neq x_{-i}$ satisfying

$$\pi_i(x_i, x_{-i}) - \pi_i(\hat{x}_i, \hat{x}_{-i}) > \pi_{-i}(x_i, x_{-i}) - \pi_{-i}(\hat{x}_i, x_{-i}),$$

turning the tables on Player i and rendering the threat ineffective. However, if such a counter-threat does not exist, then Player $-i$ agrees that the proposed solution is not acceptable, and they go back to the drawing board.

This bargaining process can be analyzed using the theory of zero-sum games, as by making the threat $\hat{x}_i$, Player i is seeking to increase the relative payoff $\pi_i - \pi_{-i}$, even though the individual payoffs are both decreased. Likewise, Player $-i$'s counter-threat seeks to reduce, and hopefully negate, the effectiveness of the threat by decreasing $\pi_i - \pi_{-i}$, which of course is the same as increasing $\pi_{-i} - \pi_i$. These differences may be regarding as the payoffs of a new game, which may be called a "difference game" (and is also known as a "relative payoff game"). This new game is zero-sum (meaning that the sum of each player's payoffs in any outcome is zero), hence as each player makes threats and counter-threats increasing their relative payoffs, the minimax theorem guarantees that when an equilibrium solution is reached, $\pi_i - \pi_{-i}$ must equal some unique constant λ_i. This quantity is known as the value of the zero-sum difference game, and it may be called "leverage" in the context of this bargaining model. Thus, Player 1 must accept any solution for which $\pi_1 - \pi_2 \leq \lambda_1$, and similarly Player 2 must accept any solution satisfying $\pi_2 - \pi_1 \leq \lambda_2 = -\lambda_1$. Given these constraints, each player would look for an acceptable solution granting the highest possible payoff. Call such a solution an "egalitarian solution," defined as a solution $(\overline{x}_1, \overline{x}_2)$ that maximizes π_i subject to $\pi_i - \pi_{-i} \leq \lambda_i$ for one of the players; the name reflects the fact that it is essentially equivalent to its namesake in cooperative game theory, as will be discussed further.[1]

Contrasting with the egalitarian solution is the outcome $(\underline{x}_1, \underline{x}_2)$ that would arise were the bargaining players to fail to reach an acceptable solution, unleashing a threat and a counter-threat. More precisely, define this outcome as that which minimizes π_i subject to $\pi_i - \pi_{-i} = \lambda_i$. This may be identified as the disagreement point of the Nash bargaining problem from cooperative game theory, which seeks to identify a fair division of possible payoffs given a disagreement point that grants both players some suboptimal amount if an agreement is not reached. The cooperative game theoretic approach is axiomatic, as it proposes the properties a fair solution should satisfies and determines the solution satisfying such properties. It turns out that not all reasonable properties can be fulfilled simultaneously. For instance, the Nash bargaining solution (which maximizes the product of surplus payoffs) is unaffected by the addition of irrelevant alternatives; however, increasing the available resources might leave one agent worse off (Nash 1950). On the other hand, the Kalai-

[1] In some coordination games it would be unclear to the players how to achieve an egalitarian solution without communicating. This complication can be ruled out by restricting the model to games such that if $(\overline{x}_1, \overline{x}_2)$ and $(\overline{y}_1, \overline{y}_2)$ are egalitarian solutions, so are $(\overline{x}_1, \overline{y}_2)$ and $(\overline{y}_1, \overline{x}_2)$.

Smorodinsky solution (which maximizes payoffs under the requirement that the ratio of each agent's payoff to the disagreement payoff is equal to the ratio of that agent's best possible payoff to the disagreement payoff) satisfies the latter property but not the former (Kalai and Smorodinsky 1975). If the solution that grants equal surplus payoffs to each player is chosen, then both of the above properties hold, but scale-invariance does not hold; in the context of the bargaining problem, payoffs represent utilities and therefore should be unique up to a positive linear transformation, but the egalitarian solution requires that the agents' utilities be compared with one another (Kalai 1977).

The equivalence between my egalitarian solution and Kalai's is explained as follows: if the condition is not met, then the player with the lower surplus payoff can play a threat, and expect a counter-threat leading to the outcome $(\underline{x}_1, \underline{x}_2)$, which is more costly to the opponent than to the player. The problem of interpersonal utility comparison is dealt with by assuming the players base their decisions on monetary payoffs rather than utilities. Changing this assumption would require normalizing the utility functions by imposing common knowledge of relative marginal utilities at some outcome, say, the disagreement outcome.

I apply the concept to three of the above examples to demonstrate the egalitarian solution and how it comes about in the bargaining process:

1. Consider the mini-ultimatum game in Example 4, and suppose a split of 3 for the proposer and 1 for the responder is proposed. The responder's threat is to reject the low offer; carrying out the threat lowers the proposer's payoff by 3, but the responder's by only 1. If the responder demands 3, then the proposer's threat is to offer nothing, which is more costly to the responder than the proposer. No such threat is available to either player if 2 is offered and accepted, and since this outcome is Pareto-optimal, it is the egalitarian solution. The threat solution is reached when an offer is rejected, leaving both players with 0.

2. In the prisoner's dilemma of Example 1, mutual cooperation is the egalitarian solution. If either player defects with any positive probability, the opponent's threat is to defect. The threat solution is the Nash equilibrium of mutual defection.

3. Consider the trust game of Example 2. The egalitarian solution is for the entire endowment to be sent (as required for Pareto-optimality), and for two-thirds of the amount received to be returned (which is required for fairness, as then both players end up with the

same payoff). Note that whether the receiver is provided a show-up fee equal to the endowment, which would ensure both have the same payoff in the Nash equilibrium/disagreement outcome, or not has no effect on the egalitarian solution.

Note that the threat solution is a Nash equilibrium in the prisoner's dilemma but not in the ultimatum game. However, in the associated difference game, it is an equilibrium in both examples, as it will be in any game. Also note that the egalitarian solution is not unique, as can be demonstrated by a simple coordination game with two distinct equilibria granting identical payoffs.

A natural question one might ask is whether players actually employ this reasoning in evaluating the fairness of an outcome. Experiments employing a variant of the ultimatum game, in which one or both players keep all or part of their proposed shares in the event of a rejected offer, have shown that many proposers and responders continue to use the focal point of an even split as the criterion for fairness (such as Güth and Huck 1997). For example, if proposers always get to keep at least one-fourth of the amount they propose for themselves, in the egalitarian solution, the proposer offers three-eights of the endowment and the responder accepts, since then the difference in payoffs (1/4) is the same as in the threat outcome of a rejected offer. However, in one such experiment (Fellner and Güth 2003), many even splits were proposed, and many low offers were rejected.

In addition, I have constructed a game for which the egalitarian solution is rather unintuitive. Take the prisoner's dilemma scenario, but alter it so that one of the prisoners has some sinister plot against the other prisoner, to be carried out if the latter is believed to defect. However, the plot must be carried out immediately, before the other prisoner's decision can be known. Suppose also that the other prisoner is aware of the plot, but has no defense or counter-threat against it. Carrying out the plot is costly for the threatening prisoner, as he will be the principal suspect for this new crime and likely end up with a worse sentence, but suppose the outcome is far more dreadful for the victim. Payoffs capturing this scenario are given by

	Cooperate	Defect
Cooperate	$(7, 7)$	$(5, 8)$
Defect	$(8, 5)$	$(6, 6)$
Threaten	$(4, 1)$	$(4, 1)$

The egalitarian solution is for the threatening prisoner to defect, and for the other to cooperate. Since it seems unreasonable that the threat would be carried out without knowledge of the opponent's action, this probably is not reasonable to consider as an optimal and equitable solution no matter how spiteful the prisoners are.

Having discussed the problem of determining the cooperative outcome, it remains to define how players respond to deviations from cooperative behavior.

3.2 Other-Regarding Preferences

Throughout the model, players are assumed to exhibit these preferences.

Definition 1. *An other-regarding player's preferences are defined by the utility function*

$$u_i(\pi_i, \pi_{-i}, \beta_i) = \pi_i - \frac{\beta_i}{\sigma_i} (\pi_i - \pi_{-i} - \rho_i)^2 ,$$

where $\sigma_i = \pi_i(\overline{x}_i, \overline{x}_{-i}) - \pi_i(\underline{x}_i, \underline{x}_{-i})$*, and* ρ_i *is the difference between Player i's payoff and Player* $-i$*'s in whatever outcome they deem fair. The reference point* ρ_i *is assumed to be common knowledge. The free parameter* β_i *satisfies* $\beta_i \in (0, \infty)$*, and the opponent's preferences are assumed to have the same functional form with a known, and possibly distinct, parameter* $\beta_{-i} \in (0, \infty)$*. Both* β_i *and* β_{-i} *are common knowledge.*

The reference point ρ_i is assumed to be common knowledge, even though this may not be the case in practice. It could be that simply $\rho_i = 0$; I also suggest $\rho_i = \overline{\lambda_i} = \pi_i(\overline{x}_i, \overline{x}_{-i}) - \pi_{-i}(\overline{x}_i, \overline{x}_{-i})$. Typically it is assumed that $\beta_1 = \beta_2$, to simplify the mathematics and because this is a reasonable belief for the players to hold.

The quantity σ_i ensures that decisions are scale-invariant, i.e. multiplying both players' payoff matrices by the same positive number does not affect the predicted outcome. An alternative definition is $\sigma_i = \sigma_{-i} = \max_{j,k \in \{H,L\}} |a_{jk} - b_{jk}|$, or the difference in payoffs for the most unequal possible outcome. The definition used is deliberately inapplicable to degenerate games, like the dictator game, which have $\sigma_i = \sigma_{-i} = 0$. For such games the utility function is undefined. This reflects the possibility that other-regarding preferences might reflect strategic considerations instead of distributional preferences over outcomes.

Definition 2. *Given $\beta_i \in (0, \infty)$, if x_{-i} is known, a rational other-regarding (ROR) player selects a best response from the set*

$$x_i^*(x_{-i}; \beta_i) = \underset{x \in [0,1]}{\operatorname{argmax}}\, u_i(x, x_{-i}; \beta_i),$$

where $u_i(x_i, x_{-i}; \beta_i) = u_i\left(\pi_i(x_i, x_{-i}), \pi_{-i}(x_i, x_{-i}); \beta_i\right)$. For $\beta_i \in \{0, \infty\}$

$$x_i^*(x_{-i}; 0) = \lim_{\beta \to 0^+} x_i^*(x_{-i}; \beta)$$

$$x_i^*(x_{-i}; \infty) = \lim_{\beta \to \infty} x_i^*(x_{-i}; \beta)$$

Treating $\beta = 0$ as a limiting case removes some equilibria that are not strict, such as the standard solution to the ultimatum game (which is often resolved by taking an infinitesimally small offer to be the rational offer); in effect, $\beta = 0$ can be thought of as a very small but positive β, so that there is always some utility gain from rejecting an offer of zero. The case of $\beta = \infty$ is hypothetical, but will prove to be useful in defining QMT behavior.

It will also be interesting to study the Nash equilibria that exist given these preferences, for in the simultaneous (or static) case, and in the sequential case in which subgame perfect equilibria are selected by rational players. (Subgame perfect equilibria are those under which the player moving second plays the best response to the first player's move, who anticipates this and acts accordingly.)

Definition 3. *The set of static equilibria for ROR players with common $\beta_i = \beta$, and common knowledge of type and preferences, is denoted by* $\text{Nash}(\beta)$. *In sequential games for which Player i is the first stage player, the set of subgame perfect equilibria for Player i is denoted* $\text{SPE}_i(\beta)$.

Note that $\overline{x}_i = x_i^*(\overline{x}_{-i}; \infty)$; hence the optimal solution $(\overline{x}_1, \overline{x}_2)$ is a Nash equilibrium when $\beta_1 = \beta_2 = \infty$. Since the utility function has a quadratic specification, as opposed to the linear functional form often used since Fehr and Schmidt 1999, the best-response functions depend on the opponent's strategy over some subinterval of the opponent's range of strategies. This allows for interior solutions, such as a sender in the trust game sending a fractional offer.

The departure from expected utility theory, in that the utility function is applied to expected payoffs rather than evaluating the expectation of utilities for each outcome, perhaps needs justification. It may be that the most fair outcome requires each player to play a mixed strategy. Consider a two-player version of the Platonia Dilemma, which was introduced to illustrate the concept of "superrationality" (Hofstader 1983). Two people have been offered a chance to earn a large lottery prize, and it will be awarded to whomever claims it by the next day provided that exactly one person claims it. No communication is permitted prior to or after the deadline, and no attempt may be made to identify the other potential claimant and share the prize if it is received.

Normalize the payoffs so that earning the prize provides a payoff of 1, and not earning it yields a payoff of 0. This game is represented in normal form by

	Ignore	Claim
Ignore	$(0,0)$	$(0,1)$
Claim	$(1,0)$	$(0,0)$

In the unique Nash equilibrium, both players choose action A, which is also a weakly dominant strategy. However, in the egalitarian solution, each player chooses A with probability 1/2, leaving both with expected payoffs of 1/4. Suppose the game is played sequentially, with Player 1 playing first; also suppose Player 2 observes Player 1 (or some trusted third party) flip a coin to choose between the actions, but does not observe the result. A reciprocating player would be expected to also select an action by a coin toss. Therefore, we should have $x_2^*(1/2, \infty) = 1/2$. While this is the case with the preferences defined here, by instead applying the utility function to each pure outcome, Player 2 would actually be indifferent between actions.

3.2.1 Mini-Ultimatum Game

Consider a mini-ultimatum game in Example 4. If both players are rational and other-regarding with parameter β, this game has the unique Nash equilibrium

$$\text{Nash}(\beta) = \begin{array}{ll} \{(0,1)\}, & \beta \leq \frac{1}{4} \\ \left\{\left(1 - \frac{1}{4\beta}, 1\right)\right\}, & \beta \geq \frac{1}{4} \end{array}$$

where the first strategy in each strategy pair is the probability that a high offer is made, and the second strategy is the probability that a low offer is accepted.

The higher the proposer's parameter β, the more likely the higher offer is made given the opponent has the same β, provided the proposer is rational and believes both players are similarly other-regarding. If in fact $\beta_2 > \beta_1$, then the low offer is accepted with probability β_1/β_2 if $\beta_1 \geq \frac{1}{4}$, and with probability $\frac{1}{4\beta_2}$ if $\beta_1 \leq \frac{1}{4}$. If the mixed strategy chosen by the proposer is known to the responder, than the lower the probability with which the low offer is made, the more likely the responder is to accept the low offer, since

$$x_2^*(x_1;\beta_2) = \begin{cases} \frac{1}{4\beta_2(1-x_1)}, & x_1 \leq 1 - \frac{1}{4\beta_2} \\ 1, & x_1 \geq 1 - \frac{1}{4\beta_2} \end{cases}$$

If mixed strategies are regarded as fractional offers in the continuous version of the game, then this implies that higher offers are more likely to be accepted, as is observed in experiments.

3.3 Quasi-Magical Thinking in Symmetric Games

I present the commonly used and most natural definition of quasi-magical thinking, adapted to this model:

Definition 4. *In a symmetric game, a quasi-magical thinker (QMT) plays*

$$\dot{x}_i(\beta_i) = \underset{x\in[0,1]}{\operatorname{argmax}}\, u_i(x, x; \beta_i)$$

Thus the QMT simply maximizes utility (which in this case is simply the payoff, because the other-regarding term in the utility function is zero) under the assumption that the opponent chooses the same action.

The presence of QMTs in the population can affect the choices made by the rational players, if the proportion of QMTs in the population is known. Suppose that players are either QMTs or rational, with the term "rational" allowing for other-regarding preferences. Also suppose that rational players believe the opponent is a QMT with probability θ. This may reflect a belief that QMTs make up this proportion of the population from which players are drawn, or a posterior probability based on specific information that might be known about the opponent. The game then becomes a Bayesian game in which a QMT plays a fixed strategy and the rational players strive to maximize utility while according for the possibility of QMT behavior in the opponent.

There may even be good reason for a rational behavior to adopt quasi-magical thinking in order to escalate an equilibrium "trap." I will discuss this in the context of the traveler's dilemma, and propose a type of equilibrium in which players mix between QMT behavior and rational behavior.

3.3.1 Traveler's Dilemma

The traveler's dilemma provides an excellent illustration of the impact the presence of irrational players can have on rational players, as demonstrated by the analysis given in (Becker, Carter, Naeve, et al. 2005). It will turn out to be analogous to the prisoner's dilemma with other-regarding preferences, but is studied separately because of its relative simplicity.

Let θ denote the probability that a rational player is paired with a QMT (due to a distribution within the population or to the opponent choosing to play as a QMT with this probability). Suppose the maximum claim is n, but the bonus/penalty is 2. Then the payoff from claiming n, given that a rational opponent claims $n+1$, is $\theta(n-2)+(1-\theta)(n+2)$. The payoff matrix for each player becomes, from the perspective of the row player,

$$
\begin{array}{cccccccc}
 & \text{Claim } n-1 & \text{Claim } n-2 & \cdots & \cdots & \cdots & \text{Claim } 3 & \text{Claim } 2 \\
\text{Claim } n-1\text{:} & n+2\theta & n-3+5\theta & \cdots & \cdots & \cdots & 1+(n+1)\theta & (n+2)\theta \\
\text{Claim } n-2\text{:} & n+2 & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & 1+6\theta & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & 1+5\theta & 6\theta \\
\text{Claim } 3\text{:} & 5 & \cdots & \cdots & 5 & 5 & 3+2\theta & 5\theta \\
\text{Claim } 2\text{:} & 4 & \cdots & \cdots & \cdots & 4 & 4 & 2+2\theta
\end{array}
$$

with the strictly dominated strategy of claiming n omitted. For sufficiently small θ, the Bayesian Nash equilibrium is the usual equilibrium, but if $\theta > 2/n$, the usual equilibrium no longer holds because claiming $n-1$ results in a higher payoff than claiming 2 given that the opponent claims 2 due to potential gain if the opponent happens to be a QMT. Also, when $\theta > 1/2$, it is always profitable to increase one's claim because having a QMT opponent is very likely, so the rational players claim $n-1$ in the Bayesian Nash equilibrium for this case. If $2/n < \theta < 1/2$, then every Bayesian Nash equilibrium requires the rational players to play a mixed strategy. For example, if $n = 20$ and $\theta = 0.4$, then in the payoff-dominant equilibrium, 19 is claimed with probability 0.4, 18 with probability 0.13, and 17 with probability 0.47, yielding an expected payoff of 18.4, as computed with the online program "Game Theory Explorer" (Savani and Stengel 2015). While the equilibria are not unique, one can derive useful facts about their supports (i.e. the set of claims that are made with positive probability) and expected payoffs, as Becker, et al. (2005) do for the case $n = 100$.

It is possible to endogenize θ by allowing players to deliberately choose to adopt QMT behavior, perhaps out of cognizance of the self-defeating nature of rational thinking. This would serve to eliminate a free parameter from the model. A similar model is presented in (Wolpert et al. 2011), in which players are assumed to adopt a persona of either complete rationality, or of complete irrationality, the latter of which is defined as drawing the claim from a uniform distribution over the choice set. I propose a similar approach: rather than selecting a strategy per se, players select a type $\theta_i \in [0, 1]$ representing the probability that a player eschews rational thinking in favor of the QMT strategy. Given θ_i, an equilibrium under this approach exists when given θ_{-i}, Player i has no incentive to change type if the opponent's rational strategy were to update the rational strategy in response to Player i's new type.

To illustrate this, suppose that $\theta \in \{0, 1\}$, so that players choose to be completely rational, or exclusively a QMT. Also, suppose that $n = 100$. If both players choose to be rational, then they will claim \$2, but then if either player decides to be a QMT and claim \$100, that player leaves with \$97 rather than \$2 since the rational opponent will change strategies accordingly. Thus, it is not an equilibrium for both players to be rational, since behaving irrationality can increase either player's payoff, holding constant the opponent's type. In fact, this formulation of the game is a hawk-dove game with payoff matrix

	QMT	Rational
QMT	$(100, 100)$	$(97, 101)$
Rational	$(101, 97)$	$(2, 2)$

This game has two pure equilibrium in which the players choose different types, and a mixed equilibrium in which the players choose to be rational with a small probability. Given the symmetry of the game, the mixed equilibrium is the preferred solution, since neither player has any particular entitlement to the more profitable equilibrium of being rational against a cooperative opponent.

The flaw in this setup is that if a player chooses to be rational, the rational strategy is not known unless the opponent's choice is known, and this cannot be the case in a single-shot game. I conjecture that allowing for a continuum of types resolves this problem, because an equilibrium will allow for a constant $\theta^* \in [0, 1]$ for each player and therefore leaving the rational strategy constant.

Such a solution may help to resolve the puzzle initially posed by this game, that even fully rational players who held their rationality to be common knowledge would be unlikely to play the equilibrium strategy. While this approach may also be useful in analyzing the other games under consideration, the mathematics involved are daunting, and it is of sufficient interest to simply compare ROR strategies with QMT strategies.

3.3.2 Prisoner's Dilemma

In the prisoner's dilemma of Example 1, each player's best response function is

$$x_i^*(x_{-i}; \beta) = \begin{cases} 0, & x_{-i} \leq \frac{1}{9\beta} \\ x_{-i} - \frac{1}{9\beta}, & x_{-i} \geq \frac{1}{9\beta} \end{cases},$$

where x_i is the probability of cooperation (of course, in practice this probability is not known in a one-shot game, but I assume it is known for the sake of analysis. (It is possible to define a continuous game for which players might choose from a continuum between defection and cooperation, although I am not aware of this having been done.). The only Nash equilibrium when $\beta < \infty$ is $(0, 0)$, as with standard preferences, although for sufficiently high β, defecting with certainty is not a dominant strategy.. In fact, this game then becomes analogous to the traveler's dilemma, in which playing an standard equilibrium strategy is suboptimal if the opponent is not rational.

Proposition 1. *The Nash equilibria for Example 1, given two ROR players with identical parameters $\beta_1 = \beta_2 = \beta$, and with all of this information common knowledge, are*

$$\text{Nash}(\beta) = \begin{cases} \{(0, 0)\}, & \beta < \infty \\ \{(x, x) : x \in [0, 1]\}, & \beta = \infty \end{cases}$$

Proof. The best-response function shows that when β is finite, a positive probability of cooperation x_i is not an equilibrium strategy, since each player would cooperate with less probability that the other if possible, but both players cannot do so. That is, given $x_i > 0$, we have $x_i^*(x_{-i}^*(x_i;\beta);\beta) = \max\left(0, x_i - \frac{2}{9\beta}\right) \neq x_i$. On the other hand, $x_i = x_i^*(x_{-i}^*(x_i;\infty);\infty)$ and $x_{-i}^*(x_i;\infty) = x_i$, proving $(x,x) \in \text{Nash}(\infty)$ for all $x \in [0,1]$.

□

3.4 Quasi-Magical Thinking in Asymmetric Games

In this section I propose a definition of quasi-magical thinking that is applicable to all games under consideration. This is intended to generalize the definition for symmetric games, though at the time of writing I have not obtained a complete proof or counterexample of the claim that for symmetric games it is equivalent to Definition 4. The definition recognizes two immediate emotions likely to drive the nonconsequential decision-making of a QMT: hope that one's actions might bring about a more efficient outcome than expected, and fear that one might either be taken advantage of or be punished.

Definition 5. *In the general case of games under consideration, a QMT plays*

$$\dot{x}_i(\beta_i) = \operatorname*{argsup}_{\substack{x\in[0,1] \\ B_i(x)\neq\emptyset}} \inf_{\substack{\beta\in B_i(x) \\ y\in x_{-i}^*(x,\beta)}} u_i(x,y;\beta_i)$$

where

$$B_i(x) = \left\{\beta \in [0,\infty] : x \in \bigcup_{y\in x_{-i}^*(x;\beta)} x_i^*\,(y;\beta)\right\}$$

Given this definition, a QMT believes whatever strategy is selected reveals, perhaps ambiguously, one's other-regarding preferences, so long as it corresponds to an equilibrium under which both players share those preferences. The opponent is believed to respond by behaving according to one of the possible preferences revealed; however, the worst possible such response is believed to be made. This parsimoniously captures both the hope that the opponent might positively reciprocate, and the fear that the opponent might negatively reciprocate, depending on which is relevant to the circumstances.

In the case of the Prisoner's Dilemma in Example 1, it is fairly clear that $\dot{x}_i = 1$; no positive x_i corresponds to an equilibrium for a finite parameter β, so the QMT needs only maximize utility under an equilibrium with $\beta = \infty$.

The following subsections analyze the trust game and ultimatum game of Examples 2 and 3 using the framework of this model.

3.4.1 Trust Game

Because the trust game is sequential, the appropriate solution concept for an ROR first-stage player is that of subgame perfect equilibrium. The static equilibria are relevant to a QMT in the first-stage, so they are presented as well.

Proposition 2. *The subgame perfect equilibria for the trust game given in Example 3 are*

$$\mathrm{SPE}_1(\beta) = \begin{cases} \{(0,0)\}, & \beta < 3/16 \\ \left\{(0,0), \left(1, \frac{4}{9}\right)\right\}, & \beta = 3/16 \\ \left\{\left(1, \frac{2}{3} - \frac{1}{24\beta}\right)\right\}, & 3/16 < \beta < \infty \end{cases}$$

Proof. Player 2's best response to Player 1's strategy is

$$x_2^*(x_1;\beta) = \begin{cases} 0, & x_1 \leq \frac{1}{16\beta} \\ \frac{2}{3} - \frac{1}{24\beta x_1}, & x_1 \geq \frac{1}{16\beta} \end{cases}$$

so Player 1's anticipates a utility of

$$u_1\left(x_1, x_2^*(x_1;\beta);\beta\right) = \begin{cases} 1 - x_1 - 16\beta x_1^2, & x_1 \leq \frac{1}{16\beta} \\ 1 - \frac{3}{16\beta} + x_1, & x_1 \geq \frac{1}{16\beta} \end{cases}$$

Utility is decreasing for $x_1 < \frac{1}{16\beta}$ and increasing for $x_2 > \frac{1}{16\beta}$, so the utility maximizing strategy is either $x_1 = 0$ or $x_1 = 1$. Calculate $u_1(0, x_2^*(0;\beta);\beta) = 1$ and $u_1(1, x_2^*(1;\beta);\beta) = 2 - \frac{3}{16\beta}$, and we have $1 > 2 - \frac{3}{16\beta}$ precisely when $\beta < \frac{3}{16}$, and indifference between the two extremes for $\beta = \frac{3}{16}$.

□

Thus, a rational other-regarding sender will either send all or nothing, depending on how large β_1 is, but this doesn't explain why many people send intermediate amounts. (Also, note that receivers return a greater proportion the more is sent.) To analyze QMT behavior, it is first necessary to compute the static equilibria for the game, including for the case $\beta = \infty$.

Proposition 3. *The static equilibria for the trust game given in Example 2 are*

$$\mathrm{Nash}(\beta) = \begin{cases} \{(0,0)\}, & \beta < 1/4 \\ \left\{\left(1, \frac{2}{3} - \frac{1}{24\beta}\right), \left(\frac{1}{4\beta}, \frac{1}{2}\right), (0,0)\right\}, & 1/4 \leq \beta < \infty \\ \left\{\left(1, \frac{2}{3}\right)\right\} \cup \{(0,0)\}, & \beta = \infty \end{cases}$$

Proof. Player 1's best response to x_2 is

$$x_1^*(x_2;\beta) = \begin{cases} 0; & x_2 \leq \frac{1}{3} \\ \frac{3x_2-1}{8\beta(2-3x_2)^2}, & \frac{1}{3} \leq x_2 \leq \frac{2}{3} - \frac{-1+\sqrt{1+32\beta}}{48\beta} \\ 1, & \frac{2}{3} - \frac{-1+\sqrt{1+32\beta}}{48\beta} \leq x_2 \leq \frac{2}{3} \end{cases}$$

with the altruistic strategy of $x_2 > 2/3$ ignored. Solving the system $x_1 = x_1^*(x_1;\beta);\beta)$ and $x_2 = x_2^*(x_1^*(x_2;\beta);\beta)$ yields the equilibria for $\beta < \infty$ (this is easily done using a computer algebra system, such as *Mathematica*).

For $\beta = \infty$, observe that for $x_2 > 0$, $x_2^*(x_1;\infty) = \lim_{\beta\to\infty} \frac{2}{3} - \frac{1}{24\beta} = \frac{2}{3}$ and $x_1^*(\frac{2}{3};\infty) = \lim_{\to\infty} x_1^*(x_2;\beta) = 1$, proving $(1, 2/3)$ the unique equilibrium for positive x_1. Now, observe that $x_2^*(0,\beta) = 0$ for all $0 < \beta < \infty$, so $x_2^*(0,\infty) = \lim_{\beta\to\infty} 0 = 0$. Similarly, $x_1^*(0;\beta) = 0$ for all $0 < \beta < \infty$, so $x_1^*(0;\infty) = 0$, proving that $(0, 0)$ is an equilibrium for $\beta = \infty$.

□

Proposition 4. *Player 1's QMT strategy in Example 3 is* $\dot{x}_1(\beta_1) = \min\left(1, \frac{1}{4\beta_1}\right)$.

Proof. Sending $x_1 = 1$ is consistent with an equilibrium for any $\beta \geq 1/4$, and we have

$$\inf_{\beta\geq 1/4} u_1(1, x_2^*(1;\beta);\beta_1) = \frac{1}{4},$$

with $x_2^*(1;\frac{1}{4}) = \frac{1}{2}$. Therefore, the QMT maximizes utility by playing $\dot{x}_1(\beta_1) = x_1^*(\frac{1}{2};\beta_1) = \min\left(1, \frac{1}{4\beta_1}\right)$, which is an equilibrium for $\beta = \beta_1$.

□

This provides an explanation for why many players send partial amounts. A sender might hope that sending more would induce reciprocity, but realizes that one cannot reveal any more than $\beta = 1/2$, which corresponds to 1/2 being returned, to which the QMT plays the best-response. While this can motivate an untrusting sender to make an investment, it can also lead a trusting sender to lower the amount sent. This would happen if the opponent is believed to have $\beta_2 > 1/4$ but play as if $\beta = 1/4$.

Proposition 5. *Player 2's QMT strategy in Example 3 is* $\dot{x}_2(\beta_2) = 1/2 + \epsilon$, *where* $\epsilon > 0$ *is an infinitesimally small quantity.*

Proof. Note that because the QMT strategy is the argument of the supremum, including ϵ is unnecessary, but it clarifies that slightly more than 1/2 must be sent to maintain an efficient equilibrium.

□

The theory of equilibrium selection suggests another approach, using the concept of risk-dominance. Using the definition in (Harsanyi, Selten, et al. 1988), given a pair of equilibria, one equilibrium risk-dominates the other if it yields a higher expected payoff than the other, assuming the opponent selects the same equilibrium with some probability and otherwise selects the other equilibrium in the pair. In this game, for $\frac{1}{2} < \beta < \frac{5+\sqrt{29}}{8} \approx 1.3$, the non-payoff-dominant equilibrium $\left(\frac{1}{4\beta}, \frac{1}{2}\right)$ actually risk-dominates the payoff-dominant equilibrium, which is another possible explanation for why a player might send a low but positive amount, especially given that Harsanyi 1995 argues that risk-dominance may be a preferable selection criterion to payoff-dominance. The sequential nature of the game calls this explanation into question; however, it may be applicable to QMTs who viewed the game as simultaneous.

Proposition 6. *For $\beta > 1/2$, $E_1 = (1, \frac{2}{3} - \frac{1}{24\beta})$ payoff-dominates $E_2 = \left(\frac{1}{4\beta}, \frac{1}{2}\right)$, but E_1 risk-dominates E_2 if and only if $\beta > \frac{5+\sqrt{29}}{8} \approx 1.3$.*

Proof. Assume $\beta > 1/4$. Denote $E_1 = (1, \frac{2}{3} - \frac{1}{24\beta})$ and $E_2 = \left(\frac{1}{4\beta}, \frac{1}{2}\right)$. To prove payoff-dominance of E_1 from the sender's point of view, calculate

$$u_1\left(\pi_1(E_1), \pi_2\left(E_1\right); \beta\right) = 2 - \frac{3}{16\beta}$$

and

$$u_1\left(\pi_1(E_2), \pi_2\left(E_2\right); \beta\right) = 1 + \frac{1}{16\beta}$$

Since $2 - \frac{3}{16\beta} > 1 + \frac{1}{16\beta}$, the payoff-dominance of $(1, \frac{2}{3} - \frac{1}{24\beta})$ is established. Now, calculate the utilities resulting from the receiver choosing a different equilibrium strategy:

$$u_1\left(\pi_1\left(1, \frac{1}{2}\right), \pi_2\left(1, \frac{1}{2}\right); \beta\right) = \frac{3-2\beta}{2}$$

$$u_1\left(\pi_1\left(\frac{1}{4\beta}, \frac{2}{3} - \frac{1}{24\beta}\right), \pi_2\left(\frac{1}{4\beta}, \frac{2}{3} - \frac{1}{24\beta}\right); \beta\right) = \frac{(4\beta-1)(4\beta+1)(16\beta+1)}{64\beta^3}$$

Let p_1 denote the risk factor for E_1, defined as the solution to

$$p_1 u_1\left(\pi_1\left(E_1\right), \pi_2\left(E_1\right); \beta\right) + (1-p_1)u_1\left(\pi_1\left(1, \frac{1}{2}\right), \pi_2\left(1, \frac{1}{2}\right); \beta\right) =$$

$$= p_1 u_1\left(\pi_1\left(\frac{1}{4\beta}, \frac{2}{3} - \frac{1}{24\beta}\right), \pi_2\left(\frac{1}{4\beta}, \frac{2}{3} - \frac{1}{24\beta}\right); \beta\right) + (1-p_1)u_1\left(\pi_1\left(1, \frac{1}{2}\right), \pi_2\left(1, \frac{1}{2}\right); \beta\right)$$

and let p_2 denote the risk factor for E_2, defined by

$$p_2 u_1\left(\pi_1\left(E_2\right), \pi_2\left(E_2\right); \beta\right) + (1-p_2)u_1\left(\pi_1\left(\frac{1}{4\beta}, \frac{2}{3} - \frac{1}{24\beta}\right), \pi_2\left(\frac{1}{4\beta}, \frac{2}{3} - \frac{1}{24\beta}\right); \beta\right) =$$

$$= p_2 u_1\left(\pi_1\left(1, \frac{1}{2}\right), \pi_2\left(1, \frac{1}{2}\right); \beta\right) + (1-p_2)u_1\left(\pi_1\left(E_1\right), \pi_2\left(E_1\right); \beta\right)$$

It can be verified numerically that $p_2 > p_1$ if and only if $\frac{1}{4} < \beta < \frac{5+\sqrt{29}}{8} \approx 1.3$, proving risk-dominance of $\left(\frac{1}{4\beta}, \frac{1}{2}\right)$ for these parameter values. □

3.4.2 Ultimatum Game

Proposition 7. *The subgame perfect equilibria for the ultimatum game given in Example 3 are*

$$\text{SPE}_1(\beta) = \begin{cases} \{(0,1)\}, & \beta = 0 \\ \left\{\left(\frac{1}{2} - \frac{-1+\sqrt{1+32\beta}}{32\beta}, 1\right)\right\}, & 0 < \beta \leq 1/4 \\ \left\{\left(\frac{1}{4}, 1\right)\right\}, & \beta = 1/4 \\ \left\{\left(\frac{1}{2} - \frac{1}{16\beta}, 1\right)\right\}, & 1/4 < \beta < \infty \end{cases}$$

Proof. The responder's best response function is

$$x_2^*(x_1; \beta) = \begin{cases} \frac{1}{4\beta(1-2x_1)^2}, & x_1 \notin \left(\frac{1}{2} + \frac{1-\sqrt{1+32\beta}}{32\beta}, \frac{1}{2} + \frac{1+\sqrt{1+32\beta}}{32\beta}\right) \\ 1 & x_1 \in \left[\frac{1}{2} + \frac{1-\sqrt{1+32\beta}}{32\beta}, \frac{1}{2} + \frac{1+\sqrt{1+32\beta}}{32\beta}\right] \end{cases}$$

For $\beta \geq \frac{1}{4}$, the proposer's anticipated utility is maximized for $x_1 = x_1^*(1; \beta) = \frac{1}{2} - \frac{1}{16\beta}$. Otherwise, it is maximized at the smallest offer that is accepted. The limiting case of $\beta = 0$ may be verified using L'Hôpital's Rule.

□

Offers greater than 1/4 are due to inequity-aversion, while offers less than 1/4 are driven by avoidance of rejection. The case $\beta = 0$ may be thought of as $\beta = \epsilon > 0$, an infinitesimally small value, so that $x_2 = 1$ is a strict best response.

Proposition 8. *The static equilibria for the ultimatum game given in Example 3 are*

$$\text{Nash}(\beta) = \begin{cases} \{(0,1)\}, & \beta \le 1/8 \\ \left\{\left(\frac{1}{2} - \frac{1}{16\beta}, 1\right)\right\}, & 1/8 < \beta < 1/4 \\ \left\{\left(\frac{1}{2} - \frac{1}{16\beta}, 1\right), \left(\frac{1}{4}, \frac{1}{4\beta}\right)\right\}, & 1/4 \le \beta < \infty \\ \left\{\left(\frac{1}{2}, 1\right), (0,0)\right\}, & \beta = \infty \end{cases}$$

Proof. The proposer's best response function, given $\beta_i = \beta$, is

$$x_1^*(x_2;\beta) = \begin{cases} 0, & x_2 \le \frac{1}{8\beta} \\ \frac{1}{2} - \frac{1}{16\beta x_2} & x_2 \ge \frac{1}{8\beta} \end{cases}$$

where x_1 is the fraction of the endowment offered, and x_2 is the probability that an offer is accepted. The Nash equilibria are obtained by solving the system $x_1 = x_1^*(x_1;\beta);\beta)$ and $x_2 = x_2^*(x_1^*(x_2;\beta);\beta)$. The case of $\beta = \infty$ is obtaining by noting $x_2^*(x_1;\infty) = 1$ precisely when $x_1 = 1/2$, with $x_1^*(1;\infty) = 1/2$, and $x_2^*(x_1;\infty) = 0$ when $x_1 \neq 1/2$, with $x_1^*(0;\infty) = 0$. □

Proposition 9. *Player 1's QMT strategy in Example 3 is* $\dot{x}_1(\beta_1) = \frac{1}{2} - \frac{1}{16\beta_1}$.

Proof. So long as $x_1 \notin \{0, 1/4\}$, the only strategy by the responder that corresponds to an equilibrium for some β is $x_2 = 1$, to which the proposer plays the best response. Since the QMT strategy is the argument of a supremum, the cases when this best response is 0 or 1/4 can be ignored. □

If the sequence of play is reversed, the responder is faced with an interesting dilemma. The offer must be accepted or refused without knowing what the offer is. Perhaps a QMT responder believes that accepting would allow the proposer to get away with offering nothing, and so such an unfair offer is preemptively rejected. The model does make a prediction for this scenario.

Proposition 10. *Player 2's QMT strategy in Example 3 is* $\dot{x}_2(\beta_2) = \min\{1-\epsilon, \frac{1}{4\beta_2}\}$.

Proof. Playing $x_2 = 1$ is consistent with $\beta = 0$, in which case the QMT fears being sent nothing, while the only offer consistent with an equilibrium in which the responder plays a mixed strategy is $x_1 = 1/4$. Therefore, the QMT plays the best response to this offer (or rejects with an infinitesimal probability if the best response is $x_2 = 1$). Note that including ϵ is unnecessary, but done for clarification. □

4 Conclusion

I proposed a new model integrating other-regarding preferences with quasi-magical thinking. The chief innovations were to introduce concave preferences over relative payoffs, and to define quasi-magical thinking in terms of the effect demonstrating a certain degree of other-regarding preferences is hoped (or pretended) to have on the opponent's choice. The model was applied to the prisoner's dilemma, explaining experimental results that cooperation increases when the opponent is know to have cooperated as opposed to having defected, but tends to be greatest absent any information. It was applied to the ultimatum game, predicting that higher offers are due to inequity-aversion, but lower ones due to profit-maximizing motives with the lowest accepted offer, given beliefs, except in the case of quasi-magical thinkers who believe every offer is accepted and so decide based on inequity-aversion. The trust game was analyzed, suggesting that rational players will send either all or nothing, but magical thinkers will send partial amounts depending on their preferences. The hypothesis that irrational, cooperative behavior can be deliberate was explored in the context of the traveler's dilemma.

This model suggests new experimental approaches to test for nonconsequential decision-making. Reversing the sequence of play in the ultimatum game and trust game, or conducting them as simultaneous games, could yield interesting results and test the predictions of this model for those situations.

The cooperative model, particularly the effect asymmetries have on perceptions of fairness, are an important area of research. Other phenomena not addressed by the model, such as framing effects and imperfect information, deserve focus as they have been observed to significantly affect behavior in the experiments covered by this paper. Extending the model to n-person games, or to larger choices, could be a useful endeavor. The model could be applied to additional games to check the appropriateness of the model's conclusions, or be modified to allow for more generality (such as by assuming a prior distribution over the opponent's preferences). The hypothesis that nonconsequential decision-making might be a strategy of its own within a broader type of equilibrium should be explored further and given a more rigorous treatment.

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