2017

Study Guide for CUNY Elementary Algebra Final Exam (CEAFE)

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STUDY GUIDE
FOR
CUNY ELEMENTARY ALGEBRA
FINAL EXAM
(CEAFE)

Prepared by Professor Olen Dias, William Baker, & Amrit Singh
Hostos Community College
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**SAMPLE FINAL EXAMINATION for ELEMENTARY ALGEBRA**
LEARNING ENVIRONMENT

If you tell yourself you cannot do math you will be setting out a self fulfilling prophecy.

Many students believe they cannot do math that someone else got the math genes and they were skipped at birth. However, your ability to do mathematics and indeed all your subjects at college need your time and attention to develop and grow within you. When you were taking mathematics previously did you study? Did you study only before the regents, did you study only in class at school? Before concluding that you cannot do math you must first stop using this self defeating language and believing the time you spend studying is a wasted effort. Instead give yourself a chance to grow and learn, only you can do this no one else can open yourself up to you potential but you.

PLANNING YOR STUDY TIME

The second step after you stop listening to the little critical voice inside you telling you math and a college degree are beyond you is to plan out your time and in particular give yourself time to study. On the next page is a daily planner; fill this out with your classes, work schedule, fill out the time you need to spend with your children and most importantly the free time you have to study if you don’t have free time you are probably doing to much you may be setting yourself up for failure. Once you have designated free time for reviewing notes and doing homework stick to it!

GET HELP

If you are having trouble with a math problem struggle with to a point but if it is beyond you don’t let discouragement stop you from trying instead get help. The Hostos Academic Learning Center (HALC - C596) has free walk in tutoring for mathematics, many instructors use software packages like mathxl or put material online in blackboard with help me solve/similar exercise or hint buttons. When seeking help from a tutor or using the help me solve button don’t just copy down what the tutor says or follow blindly the explanation given online instead ask questions or use the similar problem button to redo a similar problem in the textbook in order to test yourself and reinforce the skills and concepts you are trying to learn.

WHY DO I NEED MATH, IN PARTICULAR THIS DUMB A_S ALGEBRA CLASS

This class is a dumb a_s math class because there is a final exam at the end that will ruin your chance at a college degree if you don’t pass it. There is nothing your instructor can do to protect you, good attendance and a minimal effort may have been enough to barely squeak through the algebra regents and the final exam at the end of this class is similar content to the algebra regents exam however the bar is much higher. You see the algebra regents score of 65 to pass is a curved score the actual percent correct you need is much lower. In this course you need 19 out of the 25 questions or 60% on the CUNY Elementary Algebra Final Exam (CEAFE) to pass any less and you automatically repeat this course. Thus CUNY has set a minimal proficiency level with Mathematics and English required by all students to get a degree.

For the most up-to-date information on CEAFE exam CUNY link, please visit http://www.cuny.edu/testing

As to question why do I need mathematics: The top 15 highest-earning college degrees all have one thing in common — math skills. That’s according to a recent survey from the National Association of Colleges and Employers, which tracks college graduates’ job offers.”


"Math is at the crux of who gets paid," said Ed Koc, director of research at NACE. "If you have those skills, you are an extremely valuable asset. We don’t generate enough people like that in this country."

My Schedule:

NAME______________________________ CLASS__________________
When are your classes? When is your family time? When do you work? When do you sleep? When is your free time for studying? How much of this time can you use for math homework?

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Chapter R  Pre-Algebra Review

**Key Focus:** Perform basic mathematical operations with numerals: Addition, Subtraction, Multiplication, and Division.

**Overview of Concepts:**

Here the students learn to perform basic mathematical operations with numerals: Addition, Subtraction, Multiplication, and Division. They would be knowledgably of the basic principle such as the distributive, associative, commutative...

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<td>$a * (b + c) = (a * b) + (a * c)$</td>
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<td>$(a * b) * c = a * (b * c)$</td>
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Students need to get the concepts of the following properties, explaining each principle thoroughly.

*If you follow the rules and abide to the laws, then you will be on safe grounds.*
Addition
To add whole numbers, we can perform the operation directly or we use the method of carrying: **Examples:**

\[
\begin{array}{c}
234 \\
\downarrow \\
435 \\
\hline \\
669 \\
\end{array}
\quad \begin{array}{c}
114 \\
\downarrow \\
769 \\
\downarrow \\
354 \\
\hline \\
1123 \\
\end{array}
\]

Subtraction
To subtract whole numbers, we can perform the operation directly or use the method of borrowing: **Examples:**

\[
\begin{array}{c}
\phantom{-}659 \\
\downarrow \\
427 \\
\hline \\
232 \\
\end{array}
\quad \begin{array}{c}
637 \\
\downarrow \\
542 \\
\hline \\
95 \\
\end{array}
\]

Multiplication
To multiply whole numbers, we can perform the operation directly or we use the method of carrying: **Examples:**

\[
\begin{array}{c}
521 \\
\times \\
3 \\
\hline \\
1563 \\
\end{array}
\quad \begin{array}{c}
342 \\
\times \\
4 \\
\hline \\
1368 \\
\end{array}
\]

Division
To divide whole numbers, we can perform the operation directly or we use long division: **Examples:**

\[
\begin{array}{c}
45 \\
\downarrow \\
15 \\
\hline \\
3 \\
\end{array}
\quad \begin{array}{c}
\phantom{12}5 \\
\overline{\phantom{12}5} \\
\phantom{12}5 \\
\downarrow \\
\phantom{12}6 \\
\downarrow \\
\phantom{12}10 \\
\downarrow \\
\phantom{12}20 \\
\downarrow \\
\phantom{12}25 \\
\phantom{12}5 \\
\end{array}
\]

To reinforce the importance of the basic operations, the following application type of basic operations should be done by the Students individually and then reviewed collectively by the Instructor and Students. Make it fun and interesting.
Exercise: Complete the following Problems:

Addition:

a) \((9 + 7) + 26 = \)

b) \(54 + (-25) = \)

c) \(537 + 589 = \)

d) \(1735 + 295 = \)

e) Find the total of 659, 55, 7894, and 1278

f) Arrange the following numbers in columns and add 235, 3423, 57, 109

g) Jacob spent $1.46 for a notebook, $1.27 for a pen, 99 cents for a ruler, 73 cents for pencils, and $19.28 for a calculator. How much did he spend altogether on his school supplies?

h) How much does a set of household furniture cost if the table cost $1,389, the chairs $650, the writing desk $468, the vanity set $2,312, and a book rack $245?

i) The temperature at four in the morning was \(-30^\circ F\). By noon the temperature had increased by \(27^\circ F\). Determine the temperature at noon.

Subtraction: Complete the following Problems:

a) \(19 - 7 = \)

b) \(34 - 18 = \)

c) \(13 - 65 = \)

d) \(27 + 15 - 21 = \)

e) What is 3831 minus 342?

f) What is 3827 less than 7832?

g) A dog weighed 12 pounds two months ago today he weighed 43 pounds. How many pounds did the dog gain?

h) How much did Clifford pay for a bottle of water if he paid with a $20 bill and received $13.72 change from the cashier?
Multiplication: Complete the following Problems:

a) Multiply 76 x 3

b) Multiply 73 x 24

c) Find the product 642 * 73

d) Find the product 682 * 32

e) A delivery man earns $317 per week. What is his annual earnings?

f) A trader bought 3400 fans, each costing $17. How much did he pay for all of them?

g) A bike dealer sells 56 bikes at $7,830 each. What was his total sale?

h) How many pencils are there in 25 boxes, if each box contains 144 pencils

i) A department store sold the 78 shirts at $16 each. How much money did the store make on shirts?

j) If the area of a rectangle is given by the product of the length times its width. Find the area of a rectangle that has a length of 17 inches and a width of 13 inches.
Division: Complete the following Problems:

a) Divide $936 \div 3$

b) What is the remainder if 518 is Divide by 7

d) What is 14050 divided by 7?

e) If Ricky earns $28,990 per year. What is his average weekly earnings?

f) If 16 ounces equals 1 pound. How many pounds are there in 2960 ounces?

g) On a field-trip, a school orders buses, each of which accommodates 24 students. How many buses should the principal order if there are 531 students?

h) A car is traveling 147 miles in 3 hours. How many miles does it travel per hour?

i) The arithmetic mean is obtained by dividing the sum of the numbers by the number of items. Find the mean of the following test scores: 62, 95, 86, 69, and 93.
Key Focus: Order of the operations

Overview of Concepts:
Perform basic mathematical operations is definitely a prerequisite for the students to follow the idea behind order of the operations

Contents:
Solving problems may involve using more than one operation; to achieve an accurate answer to an algebraic expression/equation it is advisable to follow the following tips:

✓ Perform all operations with parenthesis, brackets, braces, absolute value first.
✓ Then simplify all exponents.
✓ Followed by all multiplication and division in order from left to right.
✓ And finally work on the addition and subtraction from left to right.

The example below will illustrate this concept:
Simplify $3 - 4(6 - 4) + (6 ÷ 2)^2$

Here is the strategy to solve such problem:

$3 - 4(6 - 4) + (6 ÷ 2)^2$

$= 3 - 4(2) + (3)^2$

$= 3 - 8 + 9$

$= 3 - 8 + 9$

$= -5 + 9$

$= 4$

Alert the students that this method is not the same as PEMDAS
Show them that PEMDAS won’t work as effectively and so it should be avoided.
- **Order of Operations**: The operations proceed in the following order:
  1) Parentheses and Absolute Values (if there is more than one parenthesis, you work with the Inner-most first then work your way out)
  2) Exponents and Radicals from left to right
  3) Multiplication and Division from left to right
  4) Addition and Subtraction from left to right

“Learning is to a large extent the elimination of psychological road-blocks the uncovering of what has always been there” (p.236 Koestler)

In class the teacher shows you how to solve problems but you have to make these problems you own. The first step is stop telling yourself you cannot do math, the next step is to do homework to learn how to do problems by uncovering and reviewing what you have seen in class. If you need help visit the tutoring center at your college at Hostos the Hostos Academic Learning Center HALC is on the fifth floor of the C building room C595 and across from this is the student computer lab in which you can do your mathlx, blackboard or any other online homework and review for exams. When doing homework tutors can be a big help but do not ask them to do your homework for you. Instead ask them for hints, to show you the next step, or to explain what you did wrong.

Analogy: An excellent method to learn is to compare new math rules and problems to what you already know. The rules of addition for signed numbers are readily explained using the real life concepts of money i.e. positive–assets and negative –debt.

The real number with zero “0” in the middle describes positive numbers to the right of zero and negative numbers to the left. Thus we say that –5 < 3 (negative 5 is less than 3) because –5 is to the left of 3 or equivalently 3 is to the right of –5. This corresponds the notion that owing five dollars is a condition of less money than having 3 dollars.

\[
-5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\
\longleftrightarrow \overset{\text{-----}}{-} \overset{\text{-----}}{-} \overset{\text{-----}}{-} \overset{\text{-----}}{-} \overset{\text{-----}}{-} \overset{\text{-----}}{-} \overset{\text{-----}}{-} \overset{\text{-----}}{-} \overset{\text{-----}}{-} \overset{\text{-----}}{-} \overset{\text{-----}}{-} \longrightarrow
\]

Thus while 5 is larger than or more than 3 (5 > 3) negative five is less than 3 (–5 < 3). When we want to talk of a the size or magnitude of a number without the sign we use absolute value: \(|-5| = |5| = 5\) because either way both points are five units from the zero
Exercise:

1) Perform the following operation:
   a) $7 \times 8 + 6 - 3$
   b) $53 - 7 \times 6 + 8 - 3$
   c) $6(4 - 7) \div 3 + 8 - 3$
   d) $4(9 \div 3) - 5 \times 6 - 8 + (-3)^2$

2) When necessary, use parenthesis ( ) to make the statement balance:
   a) $16 - 14 = 8 \div 2$
   b) $4 + 2 \times 3 \div 6 = 5$
   c) $15 \div 3 \times 4 \div 2 + 8 = 18$
   d) $2 + 3 \times 8 + 21 \div 7 - 4 = 25$
   e) $24 + 6 \times 16 \div 2 = 72$
   f) $3 \times 4 + 16 = 60$

3) Answer true or false to the following:
   a) $4 + 8(7 + 9) \div 2 = 68$
   b) $25 - (12 \times 1) = 13$
   c) $(9 - 3) \times (3 + 9 - 6) = 36$
   d) $15 - 2 \times 5 = 65$
**Key Focus:** Transforming fractions into equivalent fractions.

**Overview of Concepts:** Students will learn to write fractions and their equivalent fractions. They will accept that different fractions can be used to name the same part of an object.

**Contents:**
Generating a set of fractions that is equivalent is pretty simple. The flow chart below will illustrate this point:

- **To obtain a smaller fraction that is equivalent:**
  
  ✓ Check the numerator and denominator for a common factor greater than 1
  
  ✓ Fraction can’t be reduced any further
  
  ✓ Divide the numerator and the denominator by a common factor

- **Example:**
  \[
  \frac{12}{64} = \frac{12 \div 4}{64 \div 4} = \frac{3}{16}
  \]

- **To obtain a larger fraction that is equivalent:**
  
  ✓ Multiply the numerator and the denominator by a constant greater than 1
  
  ✓ Equivalent fraction

- **Example:**
  \[
  \frac{12}{64} = \frac{12 \times 4}{64 \times 4} = \frac{48}{256}
  \]
  \[
  \frac{12}{64} = \frac{12 \times 5}{64 \times 5} = \frac{60}{320}
  \]
  
  And so on…
Key Focus: Perform basic mathematical operations with fractions: Addition, Subtraction, Multiplication, and Division.

Overview of Concepts:

Here the students learn to perform basic mathematical operations with fractions: Addition, Subtraction, Multiplication, and Division. The knowledgeably of the basic principle such as the distributive, associative, commutative, etc. We will explore the meaning of least common denominator and least common multiple.

Contents:

Least Common Denominator (LCD): as the name suggest, the LCD refers to the least common multiple found by comparing numbers in the denominator of two or more fractions.

Least Common Multiple (LCM): the LCM refers to the smallest whole number that is a multiple of two or more numbers.

The following principles or properties will be reviewed using fractions:

<table>
<thead>
<tr>
<th>Basic Principles</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero Principle</td>
<td>( \frac{2}{3} + 0 = \frac{2}{3} )</td>
</tr>
<tr>
<td>Commutative Principle of Addition</td>
<td>( \frac{3}{5} + \frac{2}{7} = \frac{2}{7} + \frac{3}{5} )</td>
</tr>
<tr>
<td>Associative Principle of Addition</td>
<td>( \left( \frac{5}{6} + \frac{1}{3} \right) + \frac{2}{7} = \frac{5}{6} + \left( \frac{2}{7} + \frac{1}{3} \right) )</td>
</tr>
<tr>
<td>Distributive Principle</td>
<td>( \frac{1}{2} \left( \frac{3}{5} + \frac{4}{7} \right) = \left( \frac{1}{2} \cdot \frac{3}{5} \right) + \left( \frac{1}{2} \cdot \frac{4}{7} \right) )</td>
</tr>
<tr>
<td>Commutative Principle of Multiplication</td>
<td>( \frac{2}{3} \cdot \frac{3}{5} = \frac{3}{5} \cdot \frac{2}{3} )</td>
</tr>
<tr>
<td>Associative Principle of Multiplication</td>
<td>( \left( \frac{1}{3} \cdot \frac{2}{5} \right) \cdot \frac{3}{7} = \frac{1}{3} \left( \frac{2}{5} \cdot \frac{3}{7} \right) )</td>
</tr>
</tbody>
</table>
Addition
To add fractional numbers with like denominator, we can perform the operation directly: follow the flow chart:

✓ Example: like denominator
\[
\frac{3}{5} + \frac{1}{5} = \frac{4}{5}
\]

✓ Example: unlike denominator
\[
\frac{3}{5} + \frac{1}{2} = \frac{3 \times 2}{5 \times 2} + \frac{1 \times 5}{2 \times 5} = \frac{6 + 5}{10} = \frac{11}{10} = 1 \frac{1}{10}
\]

Exercise: Complete the following Problems by performing the indicated operation

1. \[\frac{3}{10} + \frac{9}{11} = \]

2. \[\frac{5}{12} + \frac{3}{18} = \]
**Subtraction**
To subtract fractional numbers with like denominator, we can perform the operation directly: follow the flow chart:

- **Example**: like denominator
  \[
  \frac{3}{5} - \frac{1}{5} = \frac{2}{5}
  \]

- **Example**: unlike denominator
  \[
  \frac{3}{5} - \frac{1}{2} = \frac{3 \times 2 - 1 \times 5}{5 \times 2} = \frac{6}{10} - \frac{5}{10} = \frac{1}{10}
  \]

**Exercise**: Complete the following Problems by performing the indicated operation

1. \[
  \frac{3}{5} - \frac{2}{10} =
  \]
2. \[
  1\frac{5}{12} - \frac{3}{8} =
  \]
3. Multiplication
To multiply fractional numbers, we can perform the operation by horizontal multiplying the numerators and then the denominator, we can perform the operation directly: follow the flow chart:

Example:
\[
\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}
\]

Exercise: Complete the following Problems by performing the indicated operation

1. \(\frac{3}{7} \times \frac{2}{10} = \) 
2. \(\frac{3}{5} \times \frac{1}{8} = \)
**Division**

To divide fractional numbers, we can perform the operation directly: follow the flow chart:

**Reciprocal**: Often called multiplicative inverse of a number. For a fraction, it's obtained by ‘turning’ or ‘flipping’ the fraction over.

✓ **Example:**

\[
\frac{3}{5} \div \frac{7}{2}
\]

\[
= \frac{3}{5} \times \frac{2}{7} = \frac{6}{35}
\]

**Exercise**: Complete the following Problems by performing the indicated operation

1. \(\frac{7}{9} \div \frac{4}{5}\)

2. \(\frac{3}{5} \div \frac{7}{2}\)
Key focus: Transforming fractions into decimals

Overview of Concepts: Here the students learn to Transforming fractions into decimals

Contents:

The flow chart below illustrates the way in which one goes about and convert a fraction to a decimal number

✓ Example:

\[
\frac{25}{8} = \frac{3.125}{8)\ 25\  \begin{array}{c} -24 \\ \hline -10 \\ \hline -8 \\ \hline -20 \\ \hline -16 \\ \hline -40 \\ \hline -40 \\ \hline 0 \\
\end{array}
\]

3.125

Exercise: Write a decimal fraction for each of the following:

1. \(\frac{4}{7} = \)

2. \(\frac{3}{5} = \)
### Exercise:

<table>
<thead>
<tr>
<th>#1) Evaluate: $2 - 15 = ?$</th>
<th>#2) Calculate $18 - (-5) = ?$</th>
</tr>
</thead>
</table>
| a) $-13$  
 b) $-17$  
 c) $13$  
 d) $17$ | a) $13$  
 b) $-13$  
 c) $-23$  
 d) $23$ |

<table>
<thead>
<tr>
<th>#3) Simplify $\frac{18}{-3} - 5 = ?$</th>
<th>#4) Simplify: $12 - (4 - 6) = ?$</th>
</tr>
</thead>
</table>
| a) $-1$  
 b) $1$  
 c) $-11$  
 d) $11$ | a) $10$  
 b) $14$  
 c) $2$  
 d) $-10$ |

<table>
<thead>
<tr>
<th>#5) Simplify $5 - 2(-10)$</th>
<th>#6) Simplify: $(-2)^3 - 2^2$</th>
</tr>
</thead>
</table>
| a) $13$  
 b) $-30$  
 c) $25$  
 d) $-15$ | a) $-12$  
 b) $-4$  
 c) $12$  
 d) $2$ |

<table>
<thead>
<tr>
<th>#7) Simplify: $-48 \div 3 \times 4^2$</th>
<th>#8) Simplify $5 + 7(6 - 8)$</th>
</tr>
</thead>
</table>
| a) $-256$  
 b) $-128$  
 c) $-2$  
 d) $-1$ | a) $19$  
 b) $14$  
 c) $-9$  
 d) $-24$ |

<table>
<thead>
<tr>
<th>#9) Simplify $(6.2 + 0.9) - 4(2.3 - 0.76)$</th>
<th>#10) Simplify $8.1 - \frac{1}{4} - 2 \times (0.11)$</th>
</tr>
</thead>
</table>
| a) $0.69$  
 b) $0.94$  
 c) $5.55$  
 d) $-2.86$ | a) $7.69$  
 b) $7.63$  
 c) $5.65$  
 d) $5.38$ |

<table>
<thead>
<tr>
<th>#11) Simplify $-3 \times 2^2 + 8 \div (-4)$</th>
<th>#12) Find the value of $5 - (-28)$</th>
</tr>
</thead>
</table>
| a) $34$  
 b) $8$  
 c) $-14$  
 d) $-16$ | a) $-23$  
 b) $23$  
 c) $33$  
 d) $-33$ |
Chapter 1  Exponents & Applications

Key Focus:  Understand, Define and give Examples of Exponents

Overview of Concepts:  Here the students learn to appreciate, understand, define and give examples of Exponents

Contents:

Factor:  A factor is basically a number or variable, or term, or expression that is multiplied by another to produce a product, such as:  2 and 5 are factors of 10

Exponents:  An exponent simply tells how many times a base is used as a factor.

\[ B^E \]

Base \hspace{1cm} Exponent/power

\checkmark \textbf{Example:} \quad 5^1 \quad \Rightarrow \quad \text{Read as five to the first power} \\
\hspace{1cm} 5^1 = 5

\checkmark \textbf{Example:} \quad 5^2 \quad \Rightarrow \quad \text{Read as five to the second power or simply five squared} \\
\hspace{1cm} 5^2 = 5 \times 5 = 25

\checkmark \textbf{Example:} \quad 5^3 \quad \Rightarrow \quad \text{Read as five to the third power or simply five cubed} \\
\hspace{1cm} 5^3 = 5 \times 5 \times 5 = 125

\checkmark \textbf{Example:} \quad X^5 \quad \Rightarrow \quad \text{Read as } X \text{ to the fifth power} \\
\hspace{1cm} X^5 = X \times X \times X \times X \times X

And so on…

Whenever the same factor is repeated we use exponential notation to represent the expression, merely for convenience.  Take a look:

\checkmark \textbf{Example:} \quad 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^{12}

\checkmark \textbf{Example:} \quad 5 \times 5 \times 5 \times 5 \times 5 = 5^7

\checkmark \textbf{Example:} \quad X \times X \times X \times X \times X = X^5
Exercise:

1) Write each of the following numbers using exponents
   a) 3 to the third power   b) 9 cubed   c) 8 squared

2) Complete the following table:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Base(s)</th>
<th>Exponent(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>4^5</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>5 X^5</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>(5 X)^5</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>6</td>
<td>X</td>
</tr>
<tr>
<td>e</td>
<td>W</td>
<td>Z</td>
</tr>
</tbody>
</table>

3) Write the factors for each of the following expressions:
   a) 4^5               d) pq^5
   b) (4X)^5            e) p^5q
   c) (XY)^5            f) (2x)^7

4) Find the exact value of X in each of the following equation:
   a) 4^x = 1024         c) X^4 = 81
   b) 6^x = 216          d) 2^x = 64
**Key focus:** Identify and apply the laws of Exponent.

**Overview of Concepts:** Here students can learn to identify and apply the laws of exponent…. summarizing the laws of exponent (see the table below):

**Contents:**

<table>
<thead>
<tr>
<th>Law of Exponents (with like base)</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Multiplication:</strong></td>
<td>(X^2 \times X^3 = X^{2+3} = X^5)</td>
</tr>
<tr>
<td>(X^n \times X^m = X^{n+m})</td>
<td>(2^2 \times 2^3 = 2^{2+3} = 2^5 = 32)</td>
</tr>
<tr>
<td></td>
<td>(2X^2Y^2 \times 6X^3Y^5 = 12X^{2+3}Y^{2+5} = 12X^5Y^7)</td>
</tr>
<tr>
<td><strong>Division:</strong></td>
<td>(X^5 \div X^3 = X^{5-3} = X^2)</td>
</tr>
<tr>
<td>(X^n \div X^m = X^{n-m})</td>
<td>(2^7 \div 2^2 = 2^{7-2} = 2^5 = 32)</td>
</tr>
<tr>
<td></td>
<td>(6X^3Y^5 \div 2X^2Y^3 = \frac{6}{2}X^{3-2}Y^{5-3} = 3X^1Y^2)</td>
</tr>
<tr>
<td><strong>Powers:</strong></td>
<td>(\left(2^3\right)^3 = 2^{3\times3} = 2^9 = 512)</td>
</tr>
<tr>
<td>(\left(X^n\right)^m = X^{n\times m})</td>
<td>(\left(2X^3Y^2\right)^3 = 2^3X^{3\times3}Y^{2\times3} = 8X^9Y^6)</td>
</tr>
<tr>
<td><strong>Zero Exponent:</strong></td>
<td>(2^0 = 1)</td>
</tr>
<tr>
<td>(X^0 = 1)</td>
<td>(\left(25X^3Y^2\right)^0 = 1)</td>
</tr>
<tr>
<td>where: (X \neq 0)</td>
<td>(3X^0 = 3 \times 1 = 3)</td>
</tr>
<tr>
<td><strong>Negative Exponent:</strong></td>
<td>(4^{-2} = \frac{1}{4^2} = \frac{1}{16})</td>
</tr>
<tr>
<td>(X^{-n} = \frac{1}{X^n})</td>
<td>(\frac{25X^{-2}}{5Y^{-4}} = 5Y^4)</td>
</tr>
<tr>
<td>where: (X \neq 0)</td>
<td>(\frac{25X^{-2}}{5Y^{-4}} = \frac{5Y^4}{X^2})</td>
</tr>
<tr>
<td><strong>Radicals:</strong></td>
<td>(\sqrt[3]{X^2} = X^{\frac{2}{3}})</td>
</tr>
<tr>
<td>Index</td>
<td><strong>NOTE:</strong> You would see this rule in more details when we look at radicals.</td>
</tr>
<tr>
<td>Exponent</td>
<td>(\sqrt[3]{25} = \sqrt{25} = 25^{\frac{1}{2}} = 5)</td>
</tr>
<tr>
<td>Equivalent</td>
<td>(\sqrt[3]{8} = 8^{\frac{1}{3}} = 2)</td>
</tr>
</tbody>
</table>
Example #1

Simplify \(\left(\frac{x^8}{x^2}\right)^3\)

A) \(x^{12}\)  B) \(x^{20}\)  C) \(x^{18}\)  D) \(x^{30}\)

Solution:

\[
\left(\frac{x^8}{x^2}\right)^3 = \left(\frac{x^8}{x^2}\right)^3 = (x^6)^3 = x^{18}
\]

Answer: C

Example #2

Simplify , write using only positive exponents: \(\frac{5x^{-10}y^8}{10x^4y^{-3}}\)

A) \(\frac{y^{11}}{2x^{14}}\)  B) \(\frac{y^{11}}{2x^6}\)  C) \(\frac{2y^{11}}{x^{14}}\)  D) \(\frac{y^6}{2x^{14}}\)

Solution:

\[
\frac{5x^{-10}y^8}{10x^4y^{-3}} = \frac{x^{-10-4}y^{8-(-3)}}{2} = \frac{x^{-14}y^{11}}{2} = \frac{y^{11}}{2x^{14}}
\]

Divide–reduce the coefficients and subtract exponents, Rules of addition with the exponents

Note 5/10 = ½ not 2/1, or 2

A negative exponent indicates a fraction \(x^{-n} = 1/x^n\)

Thus: \(\frac{5x^{-10}y^8}{10x^4y^{-3}} = \frac{y^{11}}{2x^{14}}\)

Answer: A

Alternate method:

make the negative exponents positive by applying the rule; by moving term in the numerator to denominator and vice versa:

\[
\frac{5x^{-10}y^8}{10x^4y^{-3}} = \frac{5y^8y^3}{10x^4x^{10}} = \frac{y^{11}}{2x^{14}}
\]
Example #3

Use the rule of exponent to simplify.

\[
\left( \frac{3x^3}{4} \right)^2
\]

Solution

\[
\left( \frac{3x^3}{4} \right)^2 = \frac{(3x^3)^2}{(4)^2}
\]

A quotient raised to a power is equal to the numerator and the denominator raised individually to the power

\[
= \frac{1}{4^2} (3x^{3\cdot2})
\]

A product raised to a power is the product of the powers

\[
= \frac{9x^6}{16}
\]

Answer : D

Exercise:  (Rules of Exponents) Circle the correct answer { True or False }

<table>
<thead>
<tr>
<th>a) ( y^6 y^5 = y^{30} )</th>
<th>b) ( (a^4)^5 = a^{20} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>{ True, False }</td>
<td>{ True, False }</td>
</tr>
<tr>
<td>c) ( x^{-9} x^{11} = x^2 )</td>
<td>d) ( (ab)^4 = a^4b^4 )</td>
</tr>
<tr>
<td>{ True, False }</td>
<td>{ True, False }</td>
</tr>
<tr>
<td>e) ( 2a^0 = 2 )</td>
<td>f) ( (m + n)^2 = m^2 + n^2 )</td>
</tr>
<tr>
<td>{ True, False }</td>
<td>{ True, False }</td>
</tr>
<tr>
<td>g) ( b^3 + b^3 = 2b^6 )</td>
<td>h) ( (2x^2y)^3 = 8x^6y^3 )</td>
</tr>
<tr>
<td>{ True, False }</td>
<td>{ True, False }</td>
</tr>
<tr>
<td>i) ( (3x)^2 = 9x^2 )</td>
<td>j) ( x^{-9} x^9 = 1 )</td>
</tr>
<tr>
<td>{ True, False }</td>
<td>{ True, False }</td>
</tr>
<tr>
<td>k) ( \frac{10^4}{10^{-2}} = 10^2 )</td>
<td>l) ( \frac{-7}{x^{-7}} = \frac{x^7}{7} )</td>
</tr>
<tr>
<td>{ True, False }</td>
<td>{ True, False }</td>
</tr>
</tbody>
</table>
## Exercises:

1) Simplify: \( \frac{5(x^2y)^2}{10x^{-2}y^4} \)

- a) \( \frac{2x^6}{y^2} \)
- b) \( \frac{x^2}{2y^2} \)
- c) \( 2x^6y^2 \)
- d) \( \frac{x^6}{2y^2} \)

2) Simplify: \( \frac{4x^5y^3}{8(xy^2)^2} \)

- a) \( \frac{x^3}{2y} \)
- b) \( \frac{2x^3}{y} \)
- c) \( \frac{x^4}{2y} \)
- d) \( \frac{xy}{2} \)

3) Simplify: \( \frac{x^8x^6}{(x^4)^{-3}} \)

4) Simplify: \( \frac{-12a^5b^{-3}}{8a^5b^{-8}} \)
Key focus: The Concept of Significant Digits

Overview of Concepts:

Here the students learn The Concept of Significant Digits. We’ll develop the logics behind trailing, leading, and squeeze zeros.

Note: Students would not be tested on this concept, however to really understand Scientific Notation, I recommend they learn it thoroughly.

Contents:

Significant digits: All counting numbers are significant; however zero can be significant sometimes. We’ll expand on this by looking at the following definitions:

Leading Zeros: they are never significant.

✓ Example: 0.0000567 ➔ 0.0000567 ➔ 567 are the only numbers that are significant

Squeeze Zeros: they are always significant.

✓ Example: 500067 ➔ 500067 ➔ are the numbers that are significant

Trailing Zeros: they are significant if and only if the number is represented with a decimal point.

✓ Example: 56700000 ➔ 56700000 ➔ 567 are the only numbers that are significant

✓ Example: 567.00000 ➔ 56700000 ➔ 56700000 are the numbers that are significant
**Key focus:** Grasp the Concept of Scientific Notation

**Overview of Concepts:**
Here the students learn to grasp the concept of scientific notation. We'll develop the necessary skill required to accept and modify those extremely large and small number in the form that scientist and engineers would use.

**Contents:**
**Scientific notation:** Is a shorthand way of writing numbers using exponents and powers of ten. Whenever numbers are written in Scientific notation it must be a number between 1 and 10 multiplied by a power of ten.

- The flow chart below illustrates the idea of converting a number into scientific notation:

  ![Flow chart](chart.png)

- **Example 1:** Convert 67,800 to scientific notation
  - 67,800
  - 6.7800
  - 4 places
  - $6.78 \times 10^4$

- **Example 2:** Convert $2.34 \times 10^{-3}$ to scientific notation
  - $2.34 \times 10^{-3}$
  - 3 places
  - 0.00234
Exercise: Write “True” if the statement is True otherwise write the correct answer.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a)</strong></td>
<td>9.87 is between 1 and 10</td>
</tr>
<tr>
<td><strong>b)</strong></td>
<td>−7 is an integer.</td>
</tr>
<tr>
<td><strong>c)</strong></td>
<td>In the scientific notation ( a \times 10^n ), the value of ( a ) can be equal to 76.5</td>
</tr>
<tr>
<td><strong>d)</strong></td>
<td>( 18 \times 10^{-5} ) is written in scientific notation</td>
</tr>
<tr>
<td><strong>e)</strong></td>
<td>( 23.5 = 2.35 \times 10 ) {True or False}</td>
</tr>
<tr>
<td><strong>f)</strong></td>
<td>( 4.032^0 = 1 ) {True or False}</td>
</tr>
<tr>
<td><strong>g)</strong></td>
<td>( 5.3 \times 10^{-4} = 0.000053 ) {True or False}</td>
</tr>
<tr>
<td><strong>h)</strong></td>
<td>( 7.201 \times 10^0 ) is written in scientific notation.</td>
</tr>
<tr>
<td><strong>i)</strong></td>
<td>( 0.0054 = 5.4 \times 10^{-3} ) {True or False}</td>
</tr>
</tbody>
</table>
**Exercise:**

1) Read each question carefully then write your answers in scientific notation.

   a) The ultimate stress of titanium is 135,000 psi.

   b) The New York Mega million lottery was $500,000,000.00

   c) The young’s modulus of normalized steel is 29,000,000 psi.

   d) The Avogadro’s number is 602,000,000,000,000,000,000,000,000 people.

   e) There are 604,800 seconds in a week.

   f) If the cost of a pen is 99 cents. Represent the cost of one million pens in scientific notation.

   g) Mr. Clifford, a welder, have about 45,000 hours of welding experience?

   h) Mr. Fenton, an agricultural worker, have about 1 million minutes of work experience on the field.

   i) In one year, light will travel approximately 5,878,000,000,000 miles.

   j) The reciprocal of a certain number is 0.000040023581

   k) The particle measures 0.00000148 meters.

   l) There were approx. 850,000 bees in the hive.
**Key Focus:** Perform basic mathematical operations involving scientific notation: Addition, Subtraction, Multiplication, and Division.

**Overview of Concepts:** Here the students learn to perform basic mathematical operations involving scientific notation: Addition, Subtraction, Multiplication, and Division.

**Contents:**

We will explore scientific notation by solving problems that require computational skills where the numbers are either in scientific notation or may be in standard numeral.

**Example #1:** Simplify \( \frac{(2 \times 10^6) \times (5 \times 10^7)}{8 \times 10^9} \) leaving your answer in scientific notation.

\[
\frac{(2 \times 10^6) \times (5 \times 10^7)}{8 \times 10^9} = \frac{(2 \times 5) \times 10^{6+7}}{8 \times 10^9} = \frac{10 \times 10^{13}}{8 \times 10^9} = 1.25 \times 10^4
\]
**Example #2**

Multiply and write the answer in scientific notation

\[(4.2 \times 10^4) \times (6 \times 10^{-8})\]

A) 25.2x10^{-4}  
B) 2.52x10^{-4}  
C) 2.52x10^{-5}  
D) 2.52x10^{-3}

Solution:

\[
4.2 \times 10^4 \times 6 \times 10^{-8} = 2.52 \times 10^{-3}
\]

Alternate method:

\[
4.2 \times 10^4 \times 6 \times 10^{-8} = 2.52 \times 10^{-3}
\]

Answer: D

**Example #3**

Divide write your answer in scientific notation \[\frac{4 \times 10^{-5}}{8 \times 10^3}\]

A) 0.5x10^{-8}  
B) 5x10^{-9}  
C) 5x10^{-7}  
D) 5x10^9

Solution:

\[
\frac{4 \times 10^{-5}}{8 \times 10^3} = 0.5 \times 10^{-5-3}
\]

\[
= 0.5 \times 10^{-8} = 0.00000005
\]

\[
= 5 \times 10^{-9}
\]

Alternate method:

\[
\frac{4 \times 10^{-5}}{8 \times 10^3} = 0.5 \times 10^{-5-3}
\]

\[
= 0.5 \times 10^{-8} = 5 \times 10^{-9}
\]

Answer: B
Exercise:

1) Simplify, write your answers in scientific notation.

a) \( \frac{(4 \times 10^6) \times (5 \times 10^7)}{8 \times 10^9} \)

b) \( \frac{(9 \times 10^{-8}) \times (5 \times 10^7)}{3 \times 10^{-3}} \)

c) \( \frac{(1.2 \times 10^6) \times (3 \times 10^{-5})}{6 \times 10^7} \)

d) \( \left[ \frac{(1.2 \times 10^6) \times (2.5 \times 10^7)}{1.2 \times 10^9} \right]^2 \)

e) \( (0.00546711) - (0.3)^3 \)

f) \( (5,000,000) + (3.7541 \times 10^4) - (0.0002)^2 \)

g) \( \frac{(6 \times 10^{-4}) \times (8 \times 10^9)}{2 \times 10^9} \div \frac{(2 \times 10^2) \times (5 \times 10^5)}{8 \times 10^5} \)

h) Approximate wavelengths of light is shown in the table below:

<table>
<thead>
<tr>
<th>Color of light</th>
<th>Approximate Wavelengths (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>( 6.1 \times 10^{-7} )</td>
</tr>
<tr>
<td>Orange</td>
<td>( 6.0 \times 10^{-7} )</td>
</tr>
<tr>
<td>Yellow</td>
<td>( 5.8 \times 10^{-7} )</td>
</tr>
<tr>
<td>Green</td>
<td>( 5.3 \times 10^{-7} )</td>
</tr>
</tbody>
</table>

Determine the approximate value of: \( \left[ \frac{(Yellow) \times (Red)}{(Green)^{(Orange)}} \right]^0 \), showing all necessary steps in your calculations.
Exercise:

1) Simplify the following using the laws of exponents.

   
   \[ a) \ (2X)^5 \] 
   \[ b) \ (-2X)^3 \] 
   \[ c) \ -(2X)^2 \] 
   \[ d) \ -(2X)^2 \]

2) Evaluate the following:

   \[ a) \ \frac{64X^7Y^4}{24X^{99}Y} \] 
   \[ b) \ \left( \frac{6X^7Y^{-2}}{2X^{-3}Y} \right)^2 \]
Exercise:

1) Apply the laws of exponents to intuitively solve the following problems:
   a) \( X^4 \times X^7 \)  
   b) \( 2^4 \times 2^{52} \)  
   c) \((2X)^2 \times X^5 \)  
   d) \( X^{12} \times X^3 \)  
   e) \((5W)^3 \)  
   f) \( 16X^6 \div 4X^4 \)  
   g) \( X^4 \div X^7 \)  
   h) \( 2X^0 \times (X^7)^2 \)

2) Evaluate each problem directly:
   a) \( 1^2 \)  
   b) \( 2^2 \)  
   c) \( 3^2 \)  
   d) \( 4^2 \)  
   e) \( 5^2 \)  
   f) \( 6^2 \)  
   g) \( 7^2 \)  
   h) \( 8^2 \)  
   i) \( 9^2 \)  
   j) \( 10^2 \)  
   k) \( 11^2 \)  
   l) \( 12^2 \)  
   m) \( 13^2 \)  
   n) \( 14^2 \)  
   o) \( 15^2 \)  
   p) \( 16^2 \)  

3) Evaluate each problem directly:
   a) \( 10^0 \)  
   b) \( 10^1 \)  
   c) \( 10^2 \)  
   d) \( 10^3 \)  
   e) \( 10^4 \)  
   f) \( 10^5 \)  
   g) \( 10^{-1} \)  
   h) \( 10^{-2} \)  
   i) \( 10^{-3} \)  
   j) \( 10^{-4} \)  

Key Focus: Compute the square root of a perfect squared number
Overview of Concepts: Learn to compute the square root of a perfect squared number

Contents:

Square root: The square root of a number is one of two equal factors of that number.

Perfect square: A perfect square is the product of a whole number times itself.

✓ Example: $\sqrt{36} = \sqrt{(6)^2} = 6$

✓ Example: $-\sqrt{81} = -\sqrt{(9)^2} = -9$

✓ Example: $\sqrt{0.25} = \sqrt{(0.5)^2} = 0.5$

✓ Example: $\sqrt{\frac{9}{16}} = \frac{\sqrt{(3)^2}}{\sqrt{(4)^2}} = \frac{3}{4}$

✓ Example: $\sqrt{-4} = \text{not a real number}$
**Key Focus:** Compute the square root of a none–perfect squared number.

**Overview of Concepts:** Here the students learn to compute the square root of a none–perfect squared number.

**Contents:**

**None perfect square:** A none perfect square is the product of two completely different whole number. To simplify such problems follow the flow chart below:

- **Example:** \( \sqrt{32} = \sqrt{(5)^4} = \sqrt{(2)^4} \sqrt{2} = \sqrt{(4)^2} \sqrt{2} = 4\sqrt{2} \)
**Key Focus:** Identify and apply the law of Exponent that incorporates radicals.

**Overview of Concepts:**

Here the students learn to Identify and apply the law of Exponent that incorporates radicals. Be aware of the fact that a radical is an exponent in disguise.

**Contents:**

The laws of exponents states that a fractional exponent can be written as a radical and a radical can also be written as a fractional exponent. See the following illustration in the table below:

| Radicals: |  
|---|---|
| Index | Equivalent |
| $\sqrt[n]{X^m}$ | $X^{m/n}$ |

$\sqrt[n]{X^m} = X^{m/n}$

- $\sqrt[3]{25} = \sqrt[3]{25} = 5^{\frac{1}{3}} = \frac{1}{3}$
- $\sqrt[4]{8} = 8^{\frac{1}{4}} = 2$
- $\sqrt[2]{32} = \sqrt[2]{16 * 2} = \sqrt[2]{16} * \sqrt[2]{2} = 4\sqrt{2}$
- $\sqrt[3]{75} + \sqrt[3]{3} = \sqrt[3]{25 * 3} + \sqrt[3]{3} = 5\sqrt[3]{3} + \sqrt[3]{3} = 6\sqrt[3]{3}$
- $5\sqrt[3]{3} + 6\sqrt[3]{3} = (5 + 6)\sqrt[3]{3} = 11\sqrt[3]{3}$
- $5\sqrt{2} + 4\sqrt{2} - 6\sqrt{2} = (5 + 4 - 6)\sqrt{2} = 3\sqrt{2}$
Perform operations on and simplify radicals and roots

The square root of b written using a radical sign as \( \sqrt{b} = a \) means that \( b = a^2 \) (\( a, b \geq 0 \)). The term b under the radical sign is called the radicand. Because equations such as \( a^2 = 9 \) have two solutions (\( a = \sqrt{9} = 3 \)) and (\( a = -\sqrt{9} = -3 \)) we call the positive solution the principal square root.

Note that: \((\sqrt{x})^2 = x \ (x \geq 0)\) likewise \(x^2 = \sqrt{x^2} \ (x \geq 0)\)

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( \sqrt{121} = 11 ) because ( 121 = 11^2 )</td>
</tr>
<tr>
<td>b) ( \sqrt{81} = 9 )</td>
</tr>
<tr>
<td>c) ( \sqrt{49x^2} = 7x )</td>
</tr>
<tr>
<td>d) ( \sqrt{121x^6y^2} = 11x^3y )</td>
</tr>
</tbody>
</table>

**Rule:** \( \sqrt{x^2} = x \) and \((\sqrt{x})^2 = x \)

For all \( x \geq 0 \)

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( (\sqrt{7})^2 = \sqrt{7} \times \sqrt{7} = \sqrt{49} = 7 )</td>
</tr>
<tr>
<td>b) ( \sqrt{3^2} = \sqrt{9} = 3 )</td>
</tr>
</tbody>
</table>

**Addition of Radicals**

Radical terms (any term with a radical) are said to be like terms if they have the same radicand. We can use the distributive property to combine like radical terms.

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( 7\sqrt{2} + 5\sqrt{2} = (7 + 5)\sqrt{2} = 12\sqrt{2} )</td>
</tr>
<tr>
<td>b) ( 3\sqrt{5} - 5\sqrt{5} = (3 - 5)\sqrt{5} = -2\sqrt{5} )</td>
</tr>
</tbody>
</table>

**Products and Quotients of Radicals**

Radical terms can be simplified using the product and quotient rules:

\[
\sqrt{a} \sqrt{b} = \sqrt{ab} \quad (a, b \geq 0) \quad \text{and} \quad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \quad (a \geq 0, b > 0)
\]

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( \sqrt{2} \sqrt{8} = \sqrt{16} = 4 )</td>
</tr>
<tr>
<td>b) ( \sqrt{50} = \sqrt{25\sqrt{2}} = 5\sqrt{2} )</td>
</tr>
<tr>
<td>c) ( \sqrt[4]{\frac{121}{4}} = \frac{\sqrt{121}}{\sqrt[4]{4}} = \frac{11}{2} )</td>
</tr>
<tr>
<td>d) ( \frac{\sqrt{50}}{\sqrt{2}} = \sqrt{\frac{50}{2}} = \sqrt{25} = 5 )</td>
</tr>
</tbody>
</table>

When simplifying or breaking down radicals you must factor the radicand as a perfect square and a

<table>
<thead>
<tr>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIMPLIFY</td>
</tr>
</tbody>
</table>

---

**Created by:** Dr. Olen Dias, Dr. William Baker & Amrit Singh
remainder factor. For this reason you should know the perfect squares!

### Perfect Squares

<table>
<thead>
<tr>
<th>Square Root</th>
<th>Value</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>√4</td>
<td>2</td>
<td>because (2^2 = 4)</td>
</tr>
<tr>
<td>√9</td>
<td>3</td>
<td>because (3^2 = 9) (because = b/c)</td>
</tr>
<tr>
<td>√16</td>
<td>4</td>
<td>b/c (4^2 = 16)</td>
</tr>
<tr>
<td>√25</td>
<td>5</td>
<td>b/c (5^2 = 25)</td>
</tr>
<tr>
<td>√36</td>
<td>6</td>
<td>b/c (6^2 = 36)</td>
</tr>
<tr>
<td>√49</td>
<td>7</td>
<td>b/c (7^2 = 49)</td>
</tr>
<tr>
<td>√64</td>
<td>8</td>
<td>b/c (8^2 = 64)</td>
</tr>
<tr>
<td>√81</td>
<td>9</td>
<td>b/c (9^2 = 81)</td>
</tr>
<tr>
<td>√100</td>
<td>10</td>
<td>b/c (10^2 = 100)</td>
</tr>
<tr>
<td>√121</td>
<td>11</td>
<td>b/c (11^2 = 121)</td>
</tr>
<tr>
<td>√144</td>
<td>12</td>
<td>b/c (12^2 = 144)</td>
</tr>
</tbody>
</table>

### Example

- a) \(\sqrt{810} = \sqrt{81} \times \sqrt{10}\)  
  \[ = 9\sqrt{10}\]
- b) \(\sqrt{150} = \sqrt{25\times6}\)  
  \[ = 5\sqrt{6}\]
- c) \(\sqrt{500} = \sqrt{100\times5}\)  
  \[ = 10\sqrt{5}\]
- d) \(\sqrt{48} = \sqrt{16\times3}\)  
  \[ = 4\sqrt{3}\]
- e) \(\sqrt{98} = \sqrt{49\times2}\)  
  \[ = 7\sqrt{2}\]

---

### Pythagorean Theorem:

In a right triangle with sides \(a\) & \(b\) with the longest side called hypotenuse (across from the 90° angle) \(c\) the Pythagorean theorem states that:

\[
a^2 + b^2 = c^2
\]

**Example:** If \(a = 3\), \(b = 4\) find \(c = ?\)

\[
c^2 = a^2 + b^2
\]
\[
= 3^2 + 4^2
\]
\[
= 9 + 16
\]
\[
= 25
\]

Hence \(c^2 = 25\) and \(c = \sqrt{25} = 5\)
### Reducing Radicals:

A radical expression is in simplest form if it cannot be reduced that means none of its factors are perfect squares.

To reduce a radical expression first, factor the radical with one of the product a perfect square and then, using the product rule simplify by taking the (square) root.

**Example**

To simplify the following write in simplest reduced form: \( \frac{7\sqrt{14} \times 2\sqrt{6}}{\sqrt{3}} \)

\[ \frac{7\sqrt{14} \times 2\sqrt{6}}{\sqrt{3}} = \frac{14\sqrt{84}}{\sqrt{3}} \]

Use the product rule and the quotient rule to simplify the radical.

\[ \frac{14\sqrt{84}}{\sqrt{3}} = 14\sqrt{28} \]

\[ = 14 \times 2 \times \sqrt{7} \]

### Simplify

a) Simplify \( \sqrt{90} \)

\( \sqrt{90} = \sqrt{9 \times 10} \) Factor using the perfect square of 9 and 10 which is not a perfect square.

\( = \sqrt{9} \times \sqrt{10} \) Use the product rule

\( = 3\sqrt{10} \)

Note: The fastest way is to factor out the largest perfect square possible but if you do not then you must repeat the first factor step in order to factor completely.

b) Simplify \( 3\sqrt{48} \)

\[ 3\sqrt{48} = 3\sqrt{4 \times 12} \quad (48=4 \times 12) \]

\[ = 3 \times 2 \times \sqrt{12} \quad \text{He simplifies the radical} \]

\[ = 6\sqrt{12} \quad \text{the product rule} \]

\[ = 6 \times 4 \times \sqrt{3} \quad (12=4 \times 3) \]

\[ = 12\sqrt{3} \]

Alternate method:

\[ 3\sqrt{48} = 3\sqrt{3 \times 16} \quad (48 = 3 \times 16) \]

\[ = 3 \times \sqrt{3} \times 4 \]

\[ = 12\sqrt{3} \]

As noted radicals can be reduced using the product rule, and they can also be reduced using the quotient rule.
Example | Calculate: $\sqrt{300} + 3\sqrt{80} - \sqrt{75} - 2\sqrt{45}$

### Solution

First factor out the perfect radicals

$$\sqrt{300} + 3\sqrt{80} - \sqrt{75} - 2\sqrt{45} = \sqrt{100\sqrt{3}} + 3\sqrt{16\sqrt{5}} - \sqrt{25\sqrt{3}} - 2\sqrt{9\sqrt{5}}$$

Take Sq. Root

$$= 10\sqrt{3} + 3 \times 4\sqrt{5} - 5\sqrt{3} - 2 \times 3\sqrt{5}$$

Multiply whole numbers

$$= 10\sqrt{3} + 12\sqrt{5} - 5\sqrt{3} - 6\sqrt{5}$$

Combine like radicals

$$= 5\sqrt{3} + 6\sqrt{5}$$

---

Example

Simplify: $\sqrt{3} (\sqrt{12} - \sqrt{3})$

D) $-3$

### Solution

$$\sqrt{3} (\sqrt{12} - \sqrt{3})$$

Distributive property $a(b+c) = ab+ac$

$$= \sqrt{3} \sqrt{12} - \sqrt{3} \sqrt{3}$$

Simplify using product rule $\sqrt{a} \sqrt{b} = \sqrt{ab}$ (ab≥0)

$$= \sqrt{36} - \sqrt{9} \sqrt{a} = b \text{ means } a = b^2 \text{ (a,b≥0)}$$

$$= 6 - 3$$

$$= 3$$

Answer: C
Example #3)

Simplify: $5\sqrt{48} - \sqrt{80}$

a) $20\sqrt{3} - 4\sqrt{5}$
b) $16\sqrt{5}$
c) $16\sqrt{3}$
d) $5\sqrt{6} - 4\sqrt{5}$

Solution:

$$5\sqrt{48} - \sqrt{80}$$

$$= 5 \times \sqrt{16} \times \sqrt{3} - \sqrt{16} \times \sqrt{5}$$

Break down the radicands into factors w/ perfect squares

$$= 5 \times 4 \times \sqrt{3} - 4 \times \sqrt{5}$$

Simplify the perfect squares

$$= 20\sqrt{3} - 4\sqrt{5}$$

Multiply and combine only when they are like terms

Example #4)

Simplify Completely $\sqrt{48} - 7\sqrt{3}$

A) $2\sqrt{12} - 7\sqrt{3}$
B) $\sqrt{3}$
C) $-3\sqrt{3}$
D) $3\sqrt{3}$

Solution:

$$\sqrt{48} - 7\sqrt{3}$$

$$= \sqrt{16\sqrt{3}} - 7\sqrt{3}$$

Factor out the radicand 48=16 x 3 with 16 the largest perfect square

$$= 4\sqrt{3} - 7\sqrt{3}$$

Combine like radical terms

$$= -3\sqrt{3}$$

Answer= C

Exercises:
#1) In the right triangle given, if \( b = 3 \), and \( c = 4 \) then find the value of \( a = {?} \)?

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>( \sqrt{7} )</td>
</tr>
<tr>
<td>b)</td>
<td>( \sqrt{7} ) or ( -\sqrt{7} )</td>
</tr>
<tr>
<td>c)</td>
<td>1</td>
</tr>
<tr>
<td>d)</td>
<td>7</td>
</tr>
</tbody>
</table>

#2) In the right triangle above if \( a = 5 \) and \( b = 8 \) find the value of \( c = {?} \)?

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>13</td>
</tr>
<tr>
<td>b)</td>
<td>23</td>
</tr>
<tr>
<td>c)</td>
<td>89</td>
</tr>
<tr>
<td>d)</td>
<td>( \sqrt{89} )</td>
</tr>
</tbody>
</table>

#3) If \( a = 3 \) and \( c = 9 \) find the value of \( b = {?} \)?

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>6( \sqrt{2} )</td>
</tr>
<tr>
<td>b)</td>
<td>2( \sqrt{3} )</td>
</tr>
<tr>
<td>c)</td>
<td>6</td>
</tr>
<tr>
<td>d)</td>
<td>3( \sqrt{10} )</td>
</tr>
</tbody>
</table>

#4) If \( a = 2 \) and \( b = 6 \) then find the value of \( c = {?} \)?

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>4( \sqrt{10} )</td>
</tr>
<tr>
<td>b)</td>
<td>4( \sqrt{2} )</td>
</tr>
<tr>
<td>c)</td>
<td>2( \sqrt{10} )</td>
</tr>
<tr>
<td>d)</td>
<td>10( \sqrt{2} )</td>
</tr>
</tbody>
</table>

#5) Simplify Completely:

\[ \sqrt{45} - 5\sqrt{5} \]

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>2( \sqrt{5} )</td>
</tr>
<tr>
<td>b)</td>
<td>4( \sqrt{5} )</td>
</tr>
<tr>
<td>c)</td>
<td>-5( \sqrt{2} )</td>
</tr>
<tr>
<td>d)</td>
<td>-2( \sqrt{5} )</td>
</tr>
</tbody>
</table>

#6) Simplify \( \sqrt{75x^2} \)

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>25x( \sqrt{3} )</td>
</tr>
<tr>
<td>b)</td>
<td>5x( \sqrt{3} )</td>
</tr>
<tr>
<td>c)</td>
<td>15x</td>
</tr>
<tr>
<td>d)</td>
<td>5x( \sqrt{3} )</td>
</tr>
</tbody>
</table>

#7) Simplify Completely:

\[ \frac{3\sqrt{6\sqrt{10}}}{\sqrt{3}} \]

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>5( \sqrt{20} )</td>
</tr>
<tr>
<td>b)</td>
<td>( \sqrt{60} )</td>
</tr>
<tr>
<td>c)</td>
<td>6( \sqrt{5} )</td>
</tr>
<tr>
<td>d)</td>
<td>5( \sqrt{6} )</td>
</tr>
</tbody>
</table>

#8) Simplify:

\[ \sqrt{50} + \sqrt{8} \]

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>( \sqrt{58} )</td>
</tr>
<tr>
<td>b)</td>
<td>29( \sqrt{2} )</td>
</tr>
<tr>
<td>c)</td>
<td>7( \sqrt{2} )</td>
</tr>
<tr>
<td>d)</td>
<td>7( \sqrt{4} )</td>
</tr>
</tbody>
</table>

#9) Simplify Completely:

\[ \sqrt{48x} + \sqrt{12x} \]

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>( \sqrt{60x} )</td>
</tr>
<tr>
<td>b)</td>
<td>x( \sqrt{60x} )</td>
</tr>
<tr>
<td>c)</td>
<td>6( \sqrt{3x} )</td>
</tr>
<tr>
<td>d)</td>
<td>20( \sqrt{3x} )</td>
</tr>
</tbody>
</table>

#10) Simplify:

\[ \sqrt{490} \]
$$\begin{align*}
(3\sqrt{8})(5\sqrt{20}) & \\
a) 15\sqrt{160} & a) 49\sqrt{10} \\
b) 60\sqrt{10} & b) 7\sqrt{10} \\
c) 30\sqrt{10} & c) 70 \\
d) 30\sqrt{40} & d) 10\sqrt{7} \\
\end{align*}$$

<table>
<thead>
<tr>
<th>#11) Simplify: $2\sqrt{18} + \sqrt{27}$</th>
<th>#12) Simplify Completely: $\frac{2\sqrt{10\sqrt{15}}}{\sqrt{2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $2\sqrt{45}$</td>
<td>a) $\sqrt{150}$</td>
</tr>
<tr>
<td>b) $6\sqrt{2} + 3\sqrt{3}$</td>
<td>b) $2\sqrt{75}$</td>
</tr>
<tr>
<td>c) $6\sqrt{2} + 3$</td>
<td>c) $50\sqrt{3}$</td>
</tr>
<tr>
<td>d) $8\sqrt{5}$</td>
<td>d) $10\sqrt{3}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>#13) Simplify completely: $\frac{\sqrt{75}}{\sqrt{48}}$</th>
<th>#14) Simplify: $\sqrt{3}\sqrt{27}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $\frac{\sqrt{5}}{\sqrt{4}}$</td>
<td>a) 81</td>
</tr>
<tr>
<td>b) $\frac{5\sqrt{3}}{4\sqrt{3}}$</td>
<td>b) $2\sqrt{3}$</td>
</tr>
<tr>
<td>c) $\frac{\sqrt{15}}{\sqrt{12}}$</td>
<td>c) $3\sqrt{3}$</td>
</tr>
<tr>
<td>d) $\frac{5}{4}$</td>
<td>d) 9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>#15) Simplify: $\sqrt{2}(\sqrt{18} - \sqrt{8})$</th>
<th>#16) Calculate: write in simplest form: $5\sqrt{24} - 3\sqrt{6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $2\sqrt{5}$</td>
<td>a) $2\sqrt{18}$</td>
</tr>
<tr>
<td>b) 2</td>
<td>b) $7\sqrt{6}$</td>
</tr>
<tr>
<td>c) $\sqrt{20}$</td>
<td>c) $17\sqrt{6}$</td>
</tr>
<tr>
<td>d) 20</td>
<td>d) $20\sqrt{2} - 3\sqrt{6}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>#17) Simplify: $(3\sqrt{2})^2$</th>
<th>#18) Simplify completely: $(2\sqrt{6})(\sqrt{15})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $9\sqrt{2}$</td>
<td>a) $2\sqrt{90}$</td>
</tr>
<tr>
<td>b) 36</td>
<td>b) $90\sqrt{2}$</td>
</tr>
<tr>
<td>c) 6</td>
<td>c) $18\sqrt{10}$</td>
</tr>
<tr>
<td>d) 18</td>
<td>d) $6\sqrt{10}$</td>
</tr>
</tbody>
</table>
#19) Simplify completely:
\[
\frac{\sqrt{5}\sqrt{30}}{\sqrt{3}}
\]
a) \(5\sqrt{10}\)
b) \(10\sqrt{5}\)
c) \(25\sqrt{2}\)
d) \(5\sqrt{2}\)

#20) Simplify:
\[\sqrt{80} + \sqrt{125}\]
a) \(8\sqrt{10}\)
b) \(41\sqrt{10}\)
c) \(5\sqrt{4} + 5\sqrt{5}\)
d) \(9\sqrt{5}\)

#21) Simplify: \(5\sqrt{8} - 9\sqrt{2}\)
a) \(-4\sqrt{6}\)
b) \(11\sqrt{2}\)
c) \(\sqrt{2}\)
d) \(-180\)

#22) Simplify Completely:
\[\sqrt{20} - 3\sqrt{5}\]
a) \(-5\sqrt{3}\)
b) \(-2\sqrt{5}\)
c) \(2\sqrt{5}\)
d) \(-\sqrt{5}\)

#23) Simplify: \(\sqrt{3}\left(\sqrt{12} - \sqrt{3}\right)\)
a) \(3\sqrt{3}\)
b) \(3\)
c) \(\sqrt{27}\)
d) \(27\)

#24) Simplify Completely:
\[\frac{2\sqrt{10} \times 3\sqrt{18}}{\sqrt{5}}\]
a) \(24\)
b) \(\frac{6\sqrt{180}}{\sqrt{5}}\)
c) \(6\sqrt{36}\)
d) \(36\)

#25) Calculate & write in Simplest form:
\[\frac{3\sqrt{15}\sqrt{10}}{\sqrt{2}}\]
a) \(3\sqrt{75}\)
b) \(75\sqrt{3}\)
c) \(9\sqrt{5}\)
d) \(15\sqrt{3}\)

#26) Simplify:
\[\sqrt{98x^3}\]
a) \(7\sqrt{2}x^3\)
b) \(49x\sqrt{2}x\)
c) \(49\sqrt{2}x^3\)
d) \(7x\sqrt{2}x\)
<table>
<thead>
<tr>
<th>#27) Simplify: $\sqrt{6400}$</th>
<th>#28) Simplify $\sqrt{490000}$</th>
</tr>
</thead>
</table>
| a) 80  
b) 40  
c) 800  
d) 160 | a) $7 \times 10^2$  
b) $7 \times 10^4$  
c) $7 \times 10$  
d) $7 \times 10^3$ |

<table>
<thead>
<tr>
<th>#29) Simplify: $\sqrt{500} - 3\sqrt{80}$</th>
<th>#30) Simplify $3\sqrt{80} - 5\sqrt{45}$</th>
</tr>
</thead>
</table>
| a) $52\sqrt{5}$  
b) $6\sqrt{5}$  
c) $2\sqrt{5}$  
d) $-2\sqrt{5}$ | a) $-2\sqrt{35}$  
b) $6\sqrt{20} - 15\sqrt{5}$  
c) $-3\sqrt{5}$  
d) $3\sqrt{5}$ |

<table>
<thead>
<tr>
<th>#31) Simplify completely: $\sqrt{25x} - 15\sqrt{x}$</th>
<th>#32) Simplify Completely: $10\sqrt{10} - \sqrt{810}$</th>
</tr>
</thead>
</table>
| a) $\sqrt{5x} - 15\sqrt{x}$  
b) $-10\sqrt{x}$  
c) $10\sqrt{x}$  
d) $5\sqrt{x} - 15\sqrt{x}$ | a) $-\sqrt{710}$  
b) $-71\sqrt{10}$  
c) $\sqrt{10}$  
d) $-\sqrt{10}$ |

<table>
<thead>
<tr>
<th>#33) Simplify: $(7\sqrt{2})^2$</th>
<th>#34) Simplify Completely: $\frac{7\sqrt{21}\sqrt{15}}{\sqrt{5}}$</th>
</tr>
</thead>
</table>
| a) 98  
b) 14  
c) $49\sqrt{2}$  
d) 28 | a) $7\sqrt{63}$  
b) $21\sqrt{7}$  
c) $63\sqrt{7}$  
d) $49\sqrt{7}$ |
Exercise:

35) Simplify the following:
   a) $6\sqrt{108} =$
   b) $2\sqrt{8} + \sqrt{72} =$
   c) $9\sqrt{18} - \sqrt{72} =$
   d) $6\sqrt{8} + \sqrt{288} - \sqrt{2} =$

36) Simplify the following:
   a. $\frac{3\sqrt{50}}{5\sqrt{4}}$
   b. $32\sqrt{108} \div 4\sqrt{27}$
   c. $\left(\sqrt{18} \sqrt{18}\right) - 18$
   d. $\left(5\sqrt{18}\right) \left(\sqrt{3}\right)$
   e. $\left(5\sqrt{18}\right) \left(3\sqrt{2}\right)$
   f. $\left(6 - \sqrt{2}\right) \left(4 - \sqrt{2}\right)$
Key Focus:  Rationalizing the denominator.

Overview of Concepts:
Here the students learn to Rationalize the denominator in a complex fraction.

Contents:
Caution!!!  Never leave a radical in the denominator!!!  It’s an impolite way of expressing a mathematical solution.  Whenever we see a radical sign in the denominator of an expression we need to do away with the radical sign, the process of eliminating the radical sign(s) is called rationalizing.  In this course we will focus on the following three flavors:

\[ \frac{m}{\sqrt[n]{X^n}} = \frac{n}{m} \]

Recipe I:  Whenever \( n = m \) we simply multiply the numerator and the denominator by \( \frac{m}{\sqrt[n]{X^n}} \)

✓  Example:  Simplify \( \frac{5}{\sqrt{X}} \)

\[ \frac{5}{\sqrt{X}} \cdot \frac{\sqrt{X}}{\sqrt{X}} = \frac{5 \cdot \sqrt{X}}{\sqrt{X^2}} = \frac{5}{X} \]

Recipe II:  Whenever \( n < m \) we simply multiply the numerator and the denominator by \( \frac{m}{\sqrt[n]{X^{n-m}}} \)

✓  Example:  Simplify \( \frac{5}{\sqrt[3]{X^2}} \)

\[ \frac{5}{\sqrt[3]{X^2}} \cdot \frac{\sqrt[3]{X^{3-2}}}{\sqrt[3]{X^{3-2}}} = \frac{5 \cdot \sqrt[3]{X}}{\sqrt[3]{X^3}} = \frac{5}{X} \]
**Recipe III:** Whenever the denominator is of the form \( \sqrt{X} \pm \sqrt{Y} \) simply multiply the numerator and the denominator by its conjugate \( \sqrt{X} \mp \sqrt{Y} \) and utilize the difference of squares technique.

**Example:** Simplify \( \frac{5}{\sqrt{2} + \sqrt{3}} \)

\[ \frac{5}{\sqrt{2} + \sqrt{3}} \times \left( \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}} \right) = \frac{5\sqrt{2} - 5\sqrt{3}}{\sqrt{4} - \sqrt{9}} = \frac{5\sqrt{2} - 5\sqrt{3}}{2 - 3} = \frac{5\sqrt{2} - 5\sqrt{3}}{-1} \]

\[ = -5\sqrt{2} + 5\sqrt{3} \]

**Exercise:** Simplify the following expressions:

A. \( \frac{3\sqrt{50}}{5\sqrt{4}} \)

B. \( \frac{10\sqrt{75}}{\sqrt{12}} \)

C. \( \frac{X^2 - 25}{\sqrt{X} + 8} \)

D. \( \frac{\sqrt{Y}}{\sqrt{3} - \sqrt{Y}} \)

E. \( \frac{X}{\sqrt{Y}} \)

F. \( \frac{x^2}{\sqrt{5} - 2} \)
Exercise:

1) Evaluate the following problems:

   a) \( \sqrt{8^2} \)  
   b) \( \sqrt{121} \)  
   c) \( \sqrt{X^2} \)  
   d) \( \sqrt{25} \)  
   e) \( \sqrt{(5W)^2} \)  
   f) \( \sqrt{16X^2 \div 4X^4} \)  
   g) \( \sqrt{25 - 9} \)  
   h) \( \sqrt{32 + 4} \)

2) Evaluate each problem:

   a) \( \sqrt{45} \)  
   b) \( \sqrt{108} \)  
   c) \( \sqrt{144} \)  
   d) \( \sqrt{125} \)  
   e) \( \sqrt{(5W)^3} \)  
   f) \( \sqrt{32X^2 \div 4X^4} \)  
   g) \( \sqrt{69 - 5} \)  
   h) \( \sqrt{32 + 8} \)
Key focus: Grasp the Concept of Function Notation

Overview of Concepts:
Here the students learn to grasp the concept of Function Notation. We'll develop the necessary skill required make substitution and evaluate its output based on an input value.

Contents:
Function Notation (one of the first applications of signed numbers on the CEAFE exam is evaluation of functions)

A function can be considered a rule that assigns to each input value a single output value. The concept of a function is often viewed as a box. This function box is a rule f(x) which assigns to each input value x an output value y.

\[ f(x) = y \]

Thus, \( f(x) = x + 2 \) assigns to each value x the output value \( x+2 \). In this case, the output value is always two more than the input value x.

For Example:
\[ f(12) = 12 + 2 = 14 \quad f(-7) = -7 + 2 = -5 \quad f(0) = 2 \quad f(57) = 59 \quad f(-99) = -97 \]

Functions can be assigned any letter but the most common are f, g & h. Let \( g(x) = 2x + 3 \) in this function the output value is always 3 more than twice the input value x.

For example:
\[ g(8) = 2(8) + 3 = 19 \quad g(-5) = 2(-5) + 3 = -10 + 3 = -7 \quad g(0) = 2(0) + 3 = 3 \]

Example
If \( f(x) = x^2 + 5x + 10 \) to find the value of \( f(-3) \) note that \(-3\) is where \( x \) was, this indicates \( x = -3 \)
Step #1: Substitute the value of \( x = -3 \), remember to use parentheses
\[ (-3)^2 + 5(-3) + 10 \]
Step #2: Evaluate or simplify the expression,
\[ = 9 + (-15) + 10 \]
\[ = -6 + 10 \]
\[ = 4 \]

Thus, \( f(-3) = 4 \)
Examples:

**Example #1** Evaluate the expression: \(a^3 + 4(-a + b)\) when \(a = -2, b = -5\)

\[
a^3 + 4(-a + b) = a^3 + 4(-a + b) \quad \text{Substitute } a = -2 \text{ and } b = -5
\]

\[
= (-2)^3 + 4(-(-2)+(-5)) \quad \text{Order of operations: Parentheses have priority over operations.}
\]

Inside the parentheses, the product of two negatives is positive.

\[
= (-2)^3 + 4(2 - 5) \quad \text{Order of Operations: Parentheses have priority over operations.}
\]

Inside the parentheses use rule of addition with different signs.

\[
= (-2)^3 + 4(-3) \quad \text{Order of operations: Exponents precede multiplication and subtraction. A negative number cubed remains negative.}
\]

\[
= -8 + 4(-3) \quad \text{Order of operations: multiplication precedes addition and the product of numbers with different signs is negative.}
\]

\[
= -8 - 12 \quad \text{Rule of addition when the signs are the same.}
\]

\[
= -20
\]

**Example #2** Simplify: \(-0.3(0.4 + 0.1) - (-0.2)^2\)

\[
-0.3(0.4 + 0.1) - (-0.2)^2 \quad \text{Order of operations: Parentheses have priority over operations.}
\]

\[
= -0.3(0.5) - (-0.2)^2 \quad \text{Order of operations: Exponents precede multiplication and subtraction. The square of a negative number becomes positive.}
\]

\[
= -0.3(0.5) - (0.04) \quad \text{Order of operations: multiplication precedes subtraction. The product of two numbers with different signs is negative.}
\]

\[
= -0.15 - 0.04 \quad \text{Rule of addition with the same signs.}
\]

\[
= -0.19
\]
Example #3

Simplify \( 2 - 2\{ 3[3(2 - 3) + 5] \} \)

\[
\begin{align*}
2 - 2\{ 3[3(2 - 3) + 5] \} & = 2 - 2\{ 3[-3 + 5] \} & \text{First, simplify Innermost parentheses.} \\
& = 2 - 2\{ 3[-1 + 5] \} & \text{Next, simplify second set of inner parentheses by order of operations multiplication precedes addition} \\
& = 2 - 2\{ 3[2] \} & \text{Simplifying the innermost parentheses we use rule of addition with different signs.} \\
& = 2 - 12 & \text{Order of operations multiplication precedes subtraction.} \\
& = -10 & \text{Rule of addition with different signs.}
\end{align*}
\]

Example #4 Evaluate the function \( h(x) \) to find \( h(-5) \)

\( h(x) = 2x^2 - 9x - 12 \)

\[
\begin{align*}
h(-5) &= 2(-5)^2 - 9(-5) - 12 & \text{substitute } x \text{ with the input value } x = -5 \text{ be careful to use parentheses} \\
& = 2(25) + 45 - 12 & \text{The square of a negative becomes positive, negative } x \text{ negative } = \text{ positive} \\
& = 50 + 45 - 12 \\
& = 95 - 12 \\
& = 83
\end{align*}
\]

Example #5

If \( g(x) = x^3 + 2x^2 - 3x + 1 \) then find \( g(-3) \)

\[
\begin{align*}
g(-3) &= (-3)^3 + 2(-3)^2 - 3(-3) + 1 & \text{Note always use parentheses when substituting negatives into exponents or multiplication} \\
& = -27 + 2(9) + 9 + 1 & \text{The cube of a negative is negative the square is positive} \\
& = -27 + 18 + 9 + 1 \\
& = 1
\end{align*}
\]
# Chapter 1 Review

### #1) Simplify \((x^3y)^2\)
- a) \(x^9y^2\)
- b) \(x^6y\)
- c) \(x^5y^2\)
- d) \(x^6y^2\)

### #2) Simplify \(x^3x^4x^2\)
- a) \(x^9\)
- b) \(3x^9\)
- c) \(x^{24}\)
- d) \(2x^9\)

### #3) Simplify: \(\frac{x^2y^7}{x^7y^3}\), write your answer with only positive exponents
- a) \(x^{-5}y^4\)
- b) \(x^5y^4\)
- c) \(\frac{y^4}{x^5}\)
- d) \(\frac{x^5}{y^4}\)

### #4) Simplify \(-\frac{4x^5}{28x^9}\)
- a) \(\frac{-1}{7x^4}\)
- b) \(\frac{-7}{x^4}\)
- c) \(\frac{1}{7x^4}\)
- d) \(-7x^{-4}\)

### #5) Simplify \((-3x^3)(-5x^4)\)
- a) \(15x^{12}\)
- b) \(15x^7\)
- c) \(15x^{81}\)
- d) \(-15x\)

### #6) Write 2.345 \(\times 10^3\) in the standard notation.
- a) 2.345
- b) 2345
- c) 234.5
- d) 0.00234

### #7) Write 0.0015 \(\times 10^6\) in scientific notation.
- a) 1500
- b) \(15 \times 10^2\)
- c) \(15 \times 10^{10}\)
- d) \(1.5 \times 10^3\)

### #8) Simplify \(3x^3+5x^3\)
- a) \(15x^6\)
- b) \(8x^6\)
- c) \(8x^3\)
- d) \(15x^9\)
#9). Simplify:

\[
\begin{align*}
\text{a) } & \frac{-2s^6y}{3} \\
\text{b) } & \frac{2y^1}{3x^6} \\
\text{c) } & \frac{-2y}{3x^6} \\
\text{d) } & \frac{-2x^{-6}}{3}
\end{align*}
\]

#10). Simplify: \(\frac{x^3y^5}{(x^2)^3}\)

\[
\begin{align*}
\text{a) } & x \\
\text{b) } & x^2 \\
\text{c) } & x^3 \\
\text{d) } & x^4
\end{align*}
\]

#11) Simplify:

\[
\frac{w^3x^6}{w^3x^{-4}}
\]

\[
\begin{align*}
\text{a) } & x^{10} \\
\text{b) } & wx^{10} \\
\text{c) } & wx^2 \\
\text{d) } & x^2
\end{align*}
\]

#12) Simplify:

\[
\frac{z^5x^{-7}}{z^3x^5}
\]

\[
\begin{align*}
\text{a) } & z^2x^{12} \\
\text{b) } & \frac{x^{12}}{z} \\
\text{c) } & \frac{z^2}{x^{12}} \\
\text{d) } & z^2x^{12}
\end{align*}
\]

#13) Simplify

\[
3x^2 (2x^3y^2)^2
\]

\[
\begin{align*}
\text{a) } & -6x^6y^4 \\
\text{b) } & -12x^8y^4 \\
\text{c) } & 12x^7y^4 \\
\text{d) } & 12x^8y^4
\end{align*}
\]

#14) Simplify

\[
\frac{x^{12}x^6}{(x^3)^2}
\]

\[
\begin{align*}
\text{a) } & x^9 \\
\text{b) } & x^{13} \\
\text{c) } & x^{12} \\
\text{d) } & x^3
\end{align*}
\]

#15) Divide and write your answer in scientific notation.

\[
\frac{2\times10^4}{5\times10^{-2}}
\]

\[
\begin{align*}
\text{a) } & 0.4\times10^6 \\
\text{b) } & 4\times10^5 \\
\text{c) } & 2.5\times10^6 \\
\text{d) } & 4\times10^2
\end{align*}
\]

#16) Divide and give the answer in scientific notation

\[
\frac{6.4 \times 10^8}{2 \times 10^{-6}}
\]

\[
\begin{align*}
\text{a) } & 3.2\times10^{14} \\
\text{b) } & 3.2\times10^2 \\
\text{c) } & 3.2\times10^{-14} \\
\text{d) } & 32\times10^{14}
\end{align*}
\]
#17). Multiply and write in scientific notation:

\[(4.1 \times 10^6)(3.2 \times 10^{-4})\]

a) 13.12 \times 10^2  

b) 1.312 \times 10^3  

c) 1.312 \times 10^2  

d) 1.312 \times 10

#18) Divide and write in scientific notation:

\[
\frac{4 \times 10^4}{8 \times 10^{12}}
\]

a) 2 \times 10^{-8}  

b) 2 \times 10^8  

c) 0.5 \times 10^{-8}  

d) 5 \times 10^{-9}

#19) Simplify: \[\frac{w^{-7}x^6}{w^3x^{-4}}\]

a) \(w^{-4}x^{10}\)  

b) \(\frac{x^{-4}}{w^{10}}\)  

c) \(\frac{x^{10}}{w^{-4}}\)  

d) \(\frac{x^{10}}{w^{10}}\)

#20) Divide

\[
\frac{2 \times 10^2}{8 \times 10^5}
\]

Write your answer in scientific notation only

a) 0.25 \times 10^{-3}  

b) 2.5 \times 10^{-3}  

c) 2.5 \times 10^{-4}  

d) 2.5 \times 10^{-2}

#21) Simplify:

\[
\frac{x^6x^6}{(x^3)^2}
\]

a) \(x^3\)  

b) \(x^{12}\)  

c) \(x^6\)  

d) \(x^9\)

#22) Simplify \((-3x^2)^3 - (-2x)^2\)

a) 108x^8  

b) \(-27x^6 - 4x^2\)  

c) \(-31x^8\)  

d) \(27x^5 + 4x^2\)

#23) Simplify: \[\frac{-3s^5t^2}{-12s^3t^{-4}}\]

a) \(\frac{4t^6}{s^4}\)  

b) \(\frac{-t^6}{4s^4}\)  

c) \(\frac{1}{s^4t^2}\)  

d) \(\frac{t^6}{4s^4}\)

#24) Multiply and write your answer in scientific notation.

\[(6.1 \times 10^{-5})(5 \times 10^3)\]

a) 30.5 \times 10^{-2}  

b) 3.05 \times 10^{-2}  

c) 3.05 \times 10^{-1}  

d) 3.05 \times 10^{-3}
### #25) Simplify: \( \frac{-9x^5y^3}{18(xy^2)^2} \)

a) \( -\frac{2x^3}{y} \)  
b) \( -\frac{x^3}{2y} \)  
c) \( -\frac{x^4}{2y} \)  
d) \( -2x^3y \)

### #26) Solve the following equation for \( x \):

\( P = y + 2x \)

a) \( \frac{P - 2y}{2x} \)  
b) \( \frac{P - y}{2x} \)  
c) \( \frac{P}{2} - y \)  
d) \( \frac{P - y}{2} \)

### #27) Simplify: \( \frac{-10x^4w^7}{5x^{10}w^{-5}} \)

a) \( -\frac{2w^2}{x^6} \)  
b) \( -\frac{2w^{12}}{x^6} \)  
c) \( \frac{2w^{12}}{x^6} \)  
d) \( -\frac{2w^{12}}{x^6} \)

### #28) Given the equation \( 7x - 5y = 35 \)  
Solve for \( y \):

a) \( y = \frac{-7}{5}x + 35 \)  
b) \( y = \frac{7}{5}x - 7 \)  
c) \( y = -\frac{7}{5}x - 7 \)  
d) \( y = \frac{7}{5}x + 7 \)

### #29) Multiply and write your answer in scientific notation: \((5 \times 10^{-5})(3 \times 10^{-7})\)

a) \( 1.5 \times 10^{-11} \)  
b) \( 1.5 \times 10^{-13} \)  
c) \( 1.5 \times 10^{-12} \)  
d) \( 15 \times 10^{-12} \)

### #30) Divide and write your answer in scientific notation: \( \frac{2 \times 10^5}{4 \times 10^{-2}} \)

a) \( 5 \times 10^6 \)  
b) \( 5 \times 10^8 \)  
c) \( 5 \times 10^2 \)  
d) \( 0.5 \times 10^7 \)

### #31) Write in scientific notation.

\( 42 \times 10^5 \)

### #32) Write in scientific notation.

\( 132 \times 10^{-2} \)

### #33) Multiply and write in scientific notation.

\((4.2 \times 10^{-4})(2 \times 10^6)\)

### #34) Multiply and write in scientific notation.

\((0.00042)(2000000)\)
<table>
<thead>
<tr>
<th>#35</th>
<th>Find ( g(-2) ) when ( g(x) = -2x^2 - 5x + 12 )</th>
<th>#36</th>
<th>Given the formula ( F = \frac{9}{5}C + 32 ) converts from degrees Celsius (centigrade) to degrees Fahrenheit. Find the value of ( F ) when ( C = -20^\circ )</th>
</tr>
</thead>
</table>
|     | \begin{align*}
a) & \quad 14 \\
b) & \quad 30 \\
c) & \quad -6 \\
d) & \quad 10 \\
\end{align*} |     | \begin{align*}
a) & \quad -4 \\
b) & \quad 4.26 \\
c) & \quad 93.6 \\
d) & \quad -2.4 \\
\end{align*} |
| #37 | Find \( g(-2) \) when \( g(x) = -2x^2 - 5x + 12 \) | #38 | Given the function \( g(x) = x^2 - 5x - 9 \) Evaluate \( g(-7) \) |
|     | \begin{align*}
a) & \quad -23 \\
b) & \quad 75 \\
c) & \quad 5 \\
d) & \quad 93 \\
\end{align*} |     | \begin{align*}
a) & \quad -23 \\
b) & \quad 75 \\
c) & \quad 5 \\
d) & \quad 93 \\
\end{align*} |
| #39 | Given the function \( f(x) = 2x^2 - 3x + 7 \) Find \( f(-3) \) | #40 | Given the function \( g(x) = x^2 - 9x - 2 \) Evaluate \( g(5) \) |
|     | \begin{align*}
a) & \quad 34 \\
b) & \quad -2 \\
c) & \quad 16 \\
d) & \quad -20 \\
\end{align*} |     | \begin{align*}
a) & \quad 34 \\
b) & \quad -2 \\
c) & \quad 16 \\
d) & \quad -20 \\
\end{align*} |
Chapter 2  Proportions & Applications

Key focus:  Use Ratio to compare two quantities

Overview of Concepts:
Here the students learn to develop the concept of a ratio by using ratio to compare two quantities.
Give many practical examples to the students in this lesson, don’t be complicated, stay simple. For instance, talk about the ratio of male to female in the class, or the number of students that travel by train to the number of students that drive to school, etc…

Contents:

Ratio: is a comparison of two things.
Let's look at an example: what is the ratio of X to Y? There are many ways to do this I would show you all the possible ways I can think of:

\[ \frac{X}{Y} \]

\[ X : Y \]

\[ X \]

\[ Y \]

✓ Example:
What is the ratio of squares to trapeziums?

\[ \frac{9}{5} \]

\[ 9 : 5 \]

\[ 9 \]

\[ 5 \]

And so on…
Exercise:

1) What is the ratio of squares to trapeziums from the representation below?

2) What is the ratio of trapeziums to squares from the representation above?

3) Write the following ratios in simplest form:

   a) \( \frac{2}{8} \)  
   b) \( \frac{12}{86} \)  
   c) \( \frac{24}{148} \)  
   d) \( \frac{25}{625} \)  
   e) \( \frac{55}{100} \)  
   f) \( \frac{8}{64} \)  
   g) \( \frac{23}{123} \)  
   h) \( \frac{14}{58} \)  
   i) \( \frac{32}{56} \)
**Key Focus:** Understand the difference between Ratio vs. Proportion

**Overview of Concepts:**
Here the students learn the difference between Ratio vs. Proportion

**Contents:**

**Proportion:** Basically represent the equality of ratios.

**Cross Product:** Often refers to as cross multiplication (cross multiply), is the procedure used to:
- Check/Compare if ratios are equal.
- Solve proportions.

Follow the flow chart below to solve proportions:

**Example 1:** Richard ate 4 candies in 10 minutes; at the same rate how many candies can he eat in 2 hours (120 minutes)?

Let the number of candies Richard eat in 120 minutes = $N$

$$\frac{4}{10} = \frac{N}{120}$$

$$4(120) = 10(N)$$

$$\frac{4(120)}{10} = \frac{10(N)}{10}$$

$$48 = N$$

Hence Richard could eat 48 candies in 2 hours.
**Example #2** On a map, 2 centimeters represent 300 miles, how many miles do 5 centimeters represent?

Let $x$ represent the unknown number of miles.

\[
\frac{2 \text{ centimeter s}}{300 \text{ miles}} = \frac{5 \text{ centimeter s}}{x} \quad \text{cross-multiplication}
\]

\[
2x = 1500
\]

\[
x = 750 \text{ miles}
\]

---

**Example #3** At the Dunkin Donuts, you paid $5.10 for a dozen donuts. At this rate, how much would you need to pay for 18 donuts?

Solution: since 1 dozen = 12 donuts

Rate of $5.10 for 12 donuts is $5.10 \div 12$ and rate of $x$ for 18 donuts is $\frac{x}{18}$

Hence solve the proportion $\frac{5.10}{12} = \frac{x}{18}$ cross multiplication

\[
x = \$7.65
\]
Exercise:

1) Identify which of the following ratios are proportional:

a) \( \frac{4}{5} = \frac{8}{10} \)

b) \( \frac{3.6}{2} = \frac{32.04}{18} \)

d) \( \frac{16}{17} = \frac{56}{37} \)

c) \( \frac{1.6}{8} = \frac{15.2}{76} \)

2) Fill in the unknown value that makes the ratios a proportion:

a) \( \frac{N}{5} = \frac{15}{60} \)

b) \( \frac{3}{2} = \frac{N}{20} \)

d) \( \frac{5}{6} = \frac{30}{N} \)

c) \( \frac{3}{N} = \frac{45}{70} \)

3) Samantha heart beats 28 times in 20 seconds. How many times does her heart beat in one minute?

4) Ricky serves 7 aces in a tennis match. How many aces can he achieve in at the same rate in 9 matches?

5) Errol made 17 points in a basket ball game. How many points can he score at the same rate after 12 game tournaments?

6) One day Kevin read 2 books. How many books can he read at the same rate after a week?
Key focus: Convert fractions and decimals into percent.

Overview of Concepts: Here the students learn Convert fractions and decimals into percent. Previous knowledge of fractions, decimals and place value will be very important for continuation.

Contents: The flow chart below illustrates the way in which one goes about and Convert fractions and decimals into percent.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{5} )</td>
<td>0.60</td>
<td>( 0.6 \times 100 = 60% )</td>
</tr>
<tr>
<td>( \frac{1}{20} )</td>
<td>0.05</td>
<td>( \frac{0.05 \times 100}{100} = 5% )</td>
</tr>
<tr>
<td>( \frac{3}{2} )</td>
<td>1.50</td>
<td>( \frac{1.5 \times 100}{100} = 150% )</td>
</tr>
</tbody>
</table>
Exercise:

1) Complete the following table, round all answer to the nearest thousandths, when necessary:

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{7}{8}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{5}{2}$</td>
<td>0.67849</td>
<td>65%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{1}{2}$%</td>
</tr>
</tbody>
</table>

2) The 85% of the field were planted with rice. Represent the portion covered with rice as a fraction.

3) 7% of My English class was absent. What decimal part of the class was present?

4) The team scored 89 runs, 22 of them was made by Ronald. Represent Ronald’s score as a percent.

5) The doctor recommend 8 hours of sleep per day, what percent of the day is the doctor telling you to sleep?
Key Focus: Finding the percent of a number.

Overview of Concepts:
Here the students learn to find the percent of a number. A few methods could be used, such as, decimals or fractions, or by setting up a proportion.

Contents:
The flow charts below illustrates the way in which one go about and find the percent of a number.

✓ Example: Evaluate 15% of 45

\[
15\% \times 45 = 0.15 \times 45 = 6.75
\]

∴ 15% of 45 = 6.75

✓ Example: Evaluate 15% of 45

\[
\frac{15}{100} \times 45 = \frac{45}{4} = \frac{675}{100} = \frac{27}{4} = 6 \frac{3}{4} = 6.75
\]

∴ 15% of 45 = 6.75

Write the proportion:
\[
\frac{P}{100} = \frac{M}{N}
\]

P = Percent
M = Part  \(\rightarrow\) ‘is’
N = Total  \(\rightarrow\) ‘of’

Cross multiply and solve
Example: Evaluate 15% of 45

\[
15\% \times 45 = \frac{15}{100} = \frac{M}{45}
\]

\[
\frac{15}{100} = \frac{M}{45}
\]

\[
(15)(45) = M(100)
\]

\[
\frac{675}{100} = \frac{100M}{100}
\]

\[
\frac{675}{100} = \frac{27}{4} = M
\]

\[
M = 6\frac{3}{4} = 6.75
\]

\[
\therefore \quad 15\% \text{ of } 45 = 6.75
\]

Exercise:

1) Read each problem thoroughly before you attempt to solve each of the following:

a) What is 25% of 400?  

b) What is 5% of 60?  

c) 25 is What percent of 400?  

d) 80% of 120 is what number?  

e) 95% of 50  

f) 35% of 175 is what number?  

g) 14% of 62  

h) What is 6% of 56?
i) Clinton got 17 out of 20 questions on his science test, what was his score expressed as a percent?

j) Errol made 14 out of 21 shots in a basketball game. What was his scoring percentage?

k) Chris ate 5 of his 12 chocolates. What percent of chocolate he has remaining?

l) Dolly made 25 cookies and gave 21 to Linda. What percent of cookies did Dolly give away?

m) If Selena took a general knowledge test and missed 10 questions. She achieved a score of 92%. How many questions were given on the test?
Key Focus: Determine the percent of increase.

Overview of Concepts: Here the students learn to determine the percent of increase or decrease.

Contents:

* To find the Percent increase we use the following formula:

\[
\%\text{Increase} = \frac{\text{Increase}}{\text{original}} \times 100
\]

Increase = new amount – original amount

Example: five years ago one gallon of gas was $2.00 today gas sells for $3.00. Calculate the percent increase in the cost of gas.

\[
\%\text{Increase} = \frac{\text{Increase}}{\text{original}} \times 100 = \frac{1}{2} \times 100
\]

\[
\%\text{Increase} = 50
\]

\[
\therefore \text{ The percent of increase from $2.00 to $3.00 = 50%}
\]
**Key Focus:** Determine the percent of decrease.

**Overview of Concepts:** Here the students learn to determine the percent of decrease.

**Contents:**
* On the other hand to find the Percent decrease we use the following formula:

\[
\% \text{Decrease} = \frac{\text{Decrease}}{\text{original}} \times 100
\]

Decrease = original amount – new amount

✓ **Example:** airline ticket for a trip to the virgin islands cost $450.00 last January, in May of the same year tickets cost $375.00. Calculate the percent decrease in the cost of a ticket.

\[
\text{Increase} = ($450.00) – ($375.00)
\]

\[\text{Increase} = $75.00\]

\[
\% \text{Decrease} = \frac{\text{Decrease}}{\text{original}} \times 100
\]

\[\% \text{Decrease} = \frac{75}{450} \times 100\]

\[
\% \text{Decrease} = 16.67
\]

\[\therefore \text{ The percent of decrease, rounded to the nearest whole percent from} \ $450.00 \ \text{to} \ $375.00 = 17\%\]
**Key Focus:** Finding the simple interest

**Overview of Concepts:** Here the student learn to find the simple interest

**Contents:**
* To find the simple interest we use the following formula:

\[ I = PRT \]

Where:
- \( I \) = Interest
- \( P \) = Principal
- \( R \) = Rate
- \( T \) = Time

**Example:** Calculate the simple interest on a principal of $500.00 for 3 years, on a rate of 5%.

**Givens:**
- \( I = ? \)
- \( P = $500.00 \)
- \( R = 5\% = 0.05 \)
- \( T = 3 \) years

\[ I = PRT \]

\[ I = (500)(0.05)(3) \]

\[ I = 75 \]

\[ \therefore \text{ The simple interest} = \$75.00 \]
Exercise:

1) Given $P = 450.00, R = 2\%$, and $T = 3$ years, calculate the simple interest.

2) Given $P = 670.00, R = 2.5\%$, and $T = 2.5$ years, calculate the simple interest.

3) Given $I = 40.00, R = 1.25\%$, and $T = 1$ years, calculate the principal.

4) Determine the percent increase in the level of water in the rectangular tub below

![Rectangular Tub Diagram]

5) Find the percent of increase or decrease to the nearest tenth of a percent.

   a) 11 to 16
   b) 23 to 14
Solving Percent Problems Using Proportions

Percent \( n\% = \frac{n}{100} \) an amount out of a base of 100

To solve a percent problem, we translate as follows as;

\[
\text{Percent Formula:} \quad \frac{\text{Amount}}{\text{Base}} = \frac{\text{Percent Number}}{100} \quad \text{Base is the original quantity and the Amount is the quantity that is a percent of the bases}
\]

**Example #1**  What is 15% of $600? is translated into an equation as \( n = 15\% \times 600 \)

Percent = 15%
Base = 600, this is the quantity we take 15% of.
Amount = \( n \), we are looking for the quantity that is 15% of the base

Solution:

\[
\frac{15}{100} = \frac{n}{600}
\]

Cross multiply or translate ‘% of’ into multiplication to obtain:

\[
n = \frac{15}{100} \times 600 \quad \text{Multiply}
\]

\[
n = 90 \quad \text{Therefore, } 90 \text{ is } 15\% \text{ of } 600
\]

**Example #2**  30 is what percent of 300?
Percent = \( n \)
Base = 300, we take \( n\% \) of 300
Amount = 30, this is the result of \( n\% \) of 300

Solution:

\[
\frac{n}{100} = \frac{30}{300} \quad \text{Cross multiply}
\]

\[
\frac{100 \times 30}{300} = n
\]

\[
10 = n \quad \text{Therefore } 10\% \text{ of } 300 \text{ is } 30.
\]

**Example #3**  50 is 10% of what number?
Percent = 10%
Base = \( n \), we take 10% of an unknown base
Amount = 50, the result of taking 10% of the unknown base is 50

Solution:

\[
\frac{10}{100} = \frac{50}{n} \quad \text{cross multiply to solve}
\]

\[
\frac{50 \times 100}{10} = n \quad \text{reduce}
\]

\[
500 = n \quad \text{Therefore, } 50 \text{ is } 10\% \text{ of } 500.
\]
Percent Examples using proportion lines

**Example #4**  What is 15% of $600?

Solution:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$600</td>
<td>100%    [Whole/Base]</td>
</tr>
<tr>
<td>$x</td>
<td>15%     [Part/Amount]</td>
</tr>
<tr>
<td>$0</td>
<td>0%</td>
</tr>
</tbody>
</table>

The proportion is: \[
\frac{x}{600} = \frac{15}{100} \]

Cross multiply

\[100x = 600(15)\]

\[100x = 9000\] \quad \text{divide both sides by 100}

\[x = 90\]

Therefore, $90 is 15% of $600.

**Example #5**  30 is what percent of 3000?

Solution:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>100%    [Whole/Base]</td>
</tr>
<tr>
<td>30</td>
<td>x%      [Part/Amount]</td>
</tr>
<tr>
<td>0</td>
<td>0%</td>
</tr>
</tbody>
</table>

The proportion is: \[
\frac{30}{3000} = \frac{x}{100} \]

Cross multiply

\[3000x = 30(100)\]

\[3000x = 3000\] \quad \text{divide both sides by 3000}

\[x = 1\]

Therefore 30 is 1% of 3000.
Example #6  
50 is 1% of what number?

Solution: 

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>100%</td>
</tr>
<tr>
<td>50</td>
<td>1%</td>
</tr>
<tr>
<td>0</td>
<td>0%</td>
</tr>
</tbody>
</table>

The proportion is: \( \frac{50}{x} = \frac{1}{100} \)  
Cross multiply  

\[ 1x = 50(100) \]

\[ 1x = 5000 \]

\[ x = 5000 \]

Therefore, 50 is 1% of 5000

Example #7  
150% of $80 is what amount?

Solution: 

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>150%</td>
</tr>
<tr>
<td>$80</td>
<td>100%</td>
</tr>
<tr>
<td>$0</td>
<td>0%</td>
</tr>
</tbody>
</table>

The proportion is: \( \frac{x}{80} = \frac{150}{100} \)  
Cross multiply  

\[ 100x = 80(150) \]

\[ 100x = 12000 \]

\[ x = 120 \]

Therefore, 150% of $80 is $120.
Percent Increase

**Example #8**  The marked price of an item is $200 and is on sale at 20% off. What is the new sale price?

Solution:

\[
\begin{array}{c|c|c}
\text{Base} & \text{100%} \\
\hline
\text{X} & 80\% \ [20\% \ off] \\
\hline
\text{0$} & 0\% \\
\end{array}
\]

The proportion is:

\[
\frac{x}{200} = \frac{80}{100}
\]

Cross multiply

\[
100x = 16000
\]

Divide both sides by 100

\[
x = 160
\]

Answer: Therefore, $160 is the new sale price.

**Alternate Method using Percent Formula**

Percent = 20%

Base = Original $200

Amount = The discount or the quantity off given by the 20% discount

\[
\frac{20}{100} = \frac{n}{200}
\]

Cross Multiply

\[
(20)(200) = 100n
\]

\[
4000 = 100n
\]

\[
4000/100 = n
\]

\[
40 = n
\]

This represents the discount to find the new sales price we subtract $200 – $40 = $160 sale price
Percent Decrease

**Example #9**  The price of a skirt was increased from $20 to $30. The increase was what percent of the original price?

**Solution:**

Method 1 (Use of Proportion)

\[
\begin{array}{c|c|c}
\text{Amount} & \text{Whole/Base} \\
\hline
30 & 100% \\
20 & 0% \\
0 & 0%
\end{array}
\]

The proportion is: \[ \frac{30}{20} = \frac{x}{100} \]

Cross multiply

\[ 20x = 3000 \]

Divide both sides by 20

\[ x = 150 \]

Percent Increase = 150% – 100% = 50%

Answer: Therefore; the increase was 50% of the original price.

**Alternate Method** Percent Formula

The base is the original price = $20

The amount is the increase in sale price: $30 – $20 = $10

\[
\frac{n}{100} = \frac{10}{20}
\]

Cross Multiply \( (100)(10) = 20n \)

\[ 1000 = 20n \]

\[ 1000/20 = n \]

\[ 50 = n \]

Answer: Therefore, the increase was 50% of the original price.
### Review Exercises:

<table>
<thead>
<tr>
<th>#1</th>
<th>An item that costs $720 is discounted 20%. Find the reduced sale price.</th>
</tr>
</thead>
</table>
|     | a) $144  
|     | b) $700  
|     | c) $576  
|     | d) $676  |

<table>
<thead>
<tr>
<th>#2</th>
<th>A refrigerator that usually costs $120 is on sale for $90. Find the percent discount?</th>
</tr>
</thead>
</table>
|     | a) 30%  
|     | b) 33%  
|     | c) 75%  
|     | d) 25%  |

<table>
<thead>
<tr>
<th>#3</th>
<th>An e−Pad that usually costs $800 is on sale for $560. Find the percent decrease.</th>
</tr>
</thead>
</table>
|     | a) 240%  
|     | b) 30%   
|     | c) 24%   
|     | d) 70%   |

<table>
<thead>
<tr>
<th>#4</th>
<th>A bus travels 240 miles in 4 hours how far in miles will it go at this same rate in 9 hours?</th>
</tr>
</thead>
</table>
|     | a) 600  
|     | b) 480  
|     | c) 720  
|     | d) 540  |

<table>
<thead>
<tr>
<th>#5</th>
<th>The price of a small house increased from $400,000 to $480,000. What is the percent increase?</th>
</tr>
</thead>
</table>
|     | a) 40%  
|     | b) 80%  
|     | c) 20%  
|     | d) 16%  |

<table>
<thead>
<tr>
<th>#6</th>
<th>If 1&quot; on a map represents 75 miles then how far apart in miles are two towns 2 ¼ inches apart on the map?</th>
</tr>
</thead>
</table>
|     | a) 150  
|     | b) 150 ¾  
|     | c) 162 ½  
|     | d) 168 ¾  |

<table>
<thead>
<tr>
<th>#7</th>
<th>If 6 lb of cheese costs $24 then how much will 10 lb cost?</th>
</tr>
</thead>
</table>
|     | a) $60  
|     | b) $30  
|     | c) $96  
|     | d) $40  |

<table>
<thead>
<tr>
<th>#8</th>
<th>If 9 mg. of a cancer drug are mixed with 21 mg of an enzyme inhibitor for a dose to be used in an IV solution. Then how many mg of enzyme inhibitor are required to for 12 mg of the cancer drug?</th>
</tr>
</thead>
</table>
|     | a) 24  
|     | b) 33  
|     | c) 25  
|     | d) 28  |
**#9)** A bookstore observes that 2 out of 9 books sold are returned, if 360 books are sold how many will be returned?  

| a) 40 | b) 80 | c) 180 | d) 162 |

**#10)** Females need 44g of protein a day. If every 2 tablespoons of peanut butter provides 8g of protein, how many tablespoons of peanut butter would a female need to eat in order to get all of her protein from peanut butter?  

| a) 5 ½ | b) 18 | c) 10 | d) 11 |

**#11)** A car that originally sold for $20,000 is reduced to $12,000 find the percent discount.  

| a) 8% | b) 80% | c) 4% | d) 40% |

**#12)** If 6 oz of butter and 2 eggs are required for a cake. Then how many eggs are required to make a larger cake with 15 oz?  

| a) 45 | b) 30 | c) 5 | d) 6 |

**#13)** An item that costs $200 is discounted 35% find the reduced sales price.  

| a) $165 | b) $199.65 | c) $70 | d) $130 |

**#14)** The number of students in the Fall semester was 5,000 and in the following Spring was 5500. Find the percent increase  

| a) 5% | b) 50% | c) 11% | d) 10% |

**#15)** How many small containers are needed if 16 2/3 lbs. of sugar is divided into small 1 2/3 lb. container?  

| a) 7 | b) 16 | c) 70 | d) 10 |

**#16)** A camera that sells regularly for $280.00 is discounted by $89 in a sale. What is the sale price?  

| a) $369 | b) $201 | c) $191 | d) $89 |
Chapter 3  Algebraic Expressions & Applications

Key Focus: Explain and identify the difference between an Algebraic expression and an equation.

Overview of Concepts: Here the students learn to explain and identify the difference between an algebraic expression and an equation.

Contents:

Variable: A variable is basically a symbol used to represent an unknown numeral.

Coefficient: A coefficient is basically another word that means number, or number part of a term.

Term: A term is a combination of a variable and a number/coefficient. A variable or a number are also considered as a term whenever they stand independently.

Expression: The putting together of one or more terms forms what we call an algebraic expression. It does not have an equal signs separating any term.

Equation: The direct comparison of term is called algebraic equation, i.e. two expressions separated by an equal sign.
Follow the following flow chart below to guide you through the process of identifying Equations vs. Expressions:

✓ Example: By inspection:

<table>
<thead>
<tr>
<th>Algebraic Expression</th>
<th>Number of Terms</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2X$</td>
<td>1</td>
<td>A number multiplied by a variable</td>
</tr>
<tr>
<td>$7j - 5$</td>
<td>2</td>
<td>Subtraction signs separate two terms</td>
</tr>
<tr>
<td>$7X + 5Y - Z$</td>
<td>3</td>
<td>Addition and subtraction signs separate three terms</td>
</tr>
<tr>
<td>$6X^3$</td>
<td>1</td>
<td>A number multiplied by a variable raised to the third power</td>
</tr>
<tr>
<td>$\frac{3PQR}{S}$</td>
<td>1</td>
<td>A number multiplied by three variable divided by another variable</td>
</tr>
</tbody>
</table>
Exercise:

1) Identify the following as an equation or as an expression.

   a) \(2X + 5Y\)
   
   c) \(6X = 5Y + (-4X)\)

   b) \(7XY \div 9Y^2\)
   
   d) \(4XY = 3XY - 5^2\)

2) Fill in the blank:

   b) \(2(\_\_) + 5 = 21\)
   
   d) \(5X(\_\_Y)^2 = (\_\_)\)

   c) \(27 \div (\_\_\_) + 4 = 7\)
   
   e) \((4XY)^2 = (\_\_)\)

3) Evaluate the following by combining like terms:

   a) \(2X + 4Y - 7X - 5Y + 3\)
   
   c) \(9Z^3 - 2K + 6X + 5Y - 5Z - 2K\)

   b) \(2Y^2 + 5W + 7X - 8Y - 3X\)
   
   d) \(3S - 7Y + 4XY + 7Y - S\)

4) State the variable, the exponent and the coefficient of each of the following:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Coefficient</th>
<th>Variable</th>
<th>Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>5X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-7Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8W^6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9G^2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{P}{2})</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Key Focus: Substituting values into algebraic expressions

Overview of Concepts: Learn to substituting values into algebraic expressions for the purpose of evaluation and simplification.

Contents:

A method used in algebra to simplifying algebraic expressions, where you’re to replace the variable(s) with an assigned value. Let’s look at the example below.

✓ Example 1: Evaluate the expression: $5X^2Y - 2Y^3$: when $X = 3$ and $Y = -2$.

\[
5X^2Y - 2Y^3 = 5(3)^2(-2) - 2(-2)^3 \\
= 5(9)(-2) - 2(-8) \\
= -90 + 16 \\
= -74
\]

✓ Example 2: Evaluate the expression based on the given formula: the classical Pythagorean’s theorem is given by $C = \sqrt{A^2 + B^2}$ use the formula to calculate the exact value of $C$ when $A = 3$ and $B = 4$

\[
C = \sqrt{3^2 + 4^2} \\
= \sqrt{9 + 16} \\
= \sqrt{25} \\
= 5
\]

Always Use Parentheses whenever you’re substituting numbers in an algebraic expression.
Exercise:

1) Given that $X = -2$, $Y = 2$, and $Z = -1$. Evaluate the following algebraic expression:
   a) $8 + X - XY$
   b) $6XZ + 2(X)^2$
   c) $9Z^3 - XYZ$
   d) $XY - 5Z$

2) The formula for computing the area of a triangle is given by $A = \frac{1}{2}bh$. Find the area of a triangle if the base, $b = 6$ inches and the height $h = 13$ inches.

3) The simple interest $I$ on principal $P$ dollars at an interest rate $R$, for time $T$ in years, is given by the formula $I = PRT$. Find the simple interest on a principal of $12,000$ at $6\%$ for $1$ years ($Hint: \ 6\% = 0.06$)

4) Temperature conversion: The formula that relates Celsius and Fahrenheit temperature is $F = \frac{9}{5}C + 32$, if the temperature of the day is $-35\degree C$, what is the Fahrenheit temperature?

5) Find the value of $1 - 5^X$ where $X = 2$

6) Find the value of $(1 - 7)^X$ where $X = 2$
**Key Focus:** Interpreting/translating word problems into algebraic expressions

**Overview of Concepts:** Here the students learn identifying, interpreting/translating word problems into algebraic expressions. I will develop a list of to help you comprehend and order the way in which you translate a phrase/expression into an algebraic expression.

**Contents:**

One of the first steps in solving word problems is translating the conditions of the problem into algebraic symbols. The following tables illustrate some key word/phrase we need to learn:

<table>
<thead>
<tr>
<th>Common Phrases Describing Addition</th>
<th>Algebraic Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eight <em>more than</em> some number</td>
<td>X + 8</td>
</tr>
<tr>
<td>The <em>sum of</em> a number and eight</td>
<td></td>
</tr>
<tr>
<td>A number <em>increased by</em> eight</td>
<td></td>
</tr>
<tr>
<td>Eight is <em>added to</em> a number</td>
<td>X + 8</td>
</tr>
<tr>
<td>Eight <em>greater than</em> a number</td>
<td></td>
</tr>
<tr>
<td>A number <em>plus</em> eight</td>
<td></td>
</tr>
<tr>
<td>The <em>total</em> of a number and eight</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Common Phrases Describing Subtraction</th>
<th>Algebraic Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>A number <em>decreased by</em> 2</td>
<td>X − 2</td>
</tr>
<tr>
<td>Two <em>less than</em> some number</td>
<td></td>
</tr>
<tr>
<td>Two is <em>subtracted</em> from a number</td>
<td></td>
</tr>
<tr>
<td>Two <em>smaller than</em> a number</td>
<td></td>
</tr>
<tr>
<td>A number <em>diminished</em> by 2</td>
<td></td>
</tr>
<tr>
<td>A number <em>minus</em> 2</td>
<td></td>
</tr>
<tr>
<td>The <em>difference between</em> a number and 2</td>
<td></td>
</tr>
</tbody>
</table>
### Common Phrases Describing Multiplication

<table>
<thead>
<tr>
<th>Common Phrases Describing Multiplication</th>
<th>Algebraic Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Double</em> a number</td>
<td>[ 2x ]</td>
</tr>
<tr>
<td><em>Twice</em> a number</td>
<td></td>
</tr>
<tr>
<td>The <em>product of</em> a number and 2</td>
<td></td>
</tr>
<tr>
<td>Two <em>of</em> a number</td>
<td></td>
</tr>
<tr>
<td>Two <em>times</em> a number</td>
<td></td>
</tr>
</tbody>
</table>

### Common Phrases Describing Division

<table>
<thead>
<tr>
<th>Common Phrases Describing Division</th>
<th>Algebraic Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>A number <em>divided by</em> 5</td>
<td>[ \frac{X}{5} = X \div 5 ]</td>
</tr>
<tr>
<td>The <em>ratio</em> of some number and five</td>
<td></td>
</tr>
<tr>
<td>The <em>quotient</em> of a number and five</td>
<td></td>
</tr>
</tbody>
</table>
Key Focus: Interpreting/Translating word problems into algebraic expressions involving more than one operation.

Overview of Concepts: Here the students learn to interpret/translate word problems into algebraic expressions involving more than one operation.

Contents:

The key to this is reading the question carefully and apply critical thinking. Use the previous information collectively and summarize the words into symbolic form.

The following table illustrates some key concepts we need to learn:

<table>
<thead>
<tr>
<th>Question</th>
<th>Algebraic Expression</th>
<th>Summarize</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 more than the quotient of a certain number and 2.</td>
<td>( \frac{N}{2} + 6 )</td>
<td>6 more than something ( \Rightarrow ) something + 6 quotient ( \Rightarrow ) means to ( \div )</td>
</tr>
<tr>
<td>8 less than 2 times a number</td>
<td>( 2Y - 8 )</td>
<td>8 less than something ( \Rightarrow ) something – 8 Times ( \Rightarrow ) means to ( \times )</td>
</tr>
<tr>
<td>Seven Times the sum of a number and 4</td>
<td>( 7(W + 4) )</td>
<td>Times ( \Rightarrow ) means to ( \times ) Sum of ( \Rightarrow ) means to + some number and 4 collectively</td>
</tr>
<tr>
<td>The square of the difference between a number and 4</td>
<td>( (W - 4)^2 )</td>
<td>Difference ( \Rightarrow ) something – Squared ( \Rightarrow ) means to use exponent/power</td>
</tr>
</tbody>
</table>
Exercise:

1) Make appropriate translation to convert each phrase into an algebraic expression:

   a) A certain number plus 6
   b) The quotient of 4 and a number
   c) The product of a number a 9
   d) A number multiplied by 5
   e) 45% of a certain number
   f) The difference between a number and 7
   g) 6 subtracted from the quotient of 3 and a number
   h) Twice the difference between a number and 7
   i) the ratio of 7 and some number
   j) 3 more than the product of a number and 4
   k) 3 more than twice a number
   l) Twice the sum of a number and 4
   m) The quotient of a number and 8
   n) 5 decreased by a number

2) Make appropriate translation to convert each algebraic expression into a phrase

   a) $7Y - 3$
   b) $2(Y - 8)$
   c) $\frac{(Y - 8)}{5}$
   d) $2Y + 8$
**Key Focus:** Identifying and setting up problems involving consecutive integers, consecutive odd integers and consecutive even integers

**Overview of Concepts:** Here the students learn to identify and set up problems involving consecutive integers, consecutive odd integers and consecutive even integers.

**Contents:**

**Consecutive Numbers:** Consecutive numbers are integers that are arranged one after the other from least to greatest: i.e. 1, 2, 3, 4, 5, 6, 7, 8 … Hence, if we consider the first of the consecutive integer to be $X$ then the second would be $X+1$, $X+2$, $X+3$, and so on…

**Consecutive Odd Numbers:** Consecutive odd numbers are integers that are not multiples of two, arranged one after the other from least to greatest: i.e. 1, 3, 5, 7 … Hence, if we consider the first of the consecutive integer to be $X$ then the second would be $X+2$, third would be $X+4$, and so on…

**Consecutive Even Numbers:** Consecutive even numbers are integers that are multiples of two, arranged one after the other from least to greatest: i.e. 2, 4, 6, 8 … Hence, if we consider the first of the consecutive integer to be $X$ then the second would be $X+2$, third would be $X+4$, and so on…

<table>
<thead>
<tr>
<th>Problem</th>
<th>Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consecutive Numbers</td>
<td>1&lt;sup&gt;st&lt;/sup&gt; ➞ $X$</td>
</tr>
<tr>
<td></td>
<td>2&lt;sup&gt;nd&lt;/sup&gt; ➞ $X+1$</td>
</tr>
<tr>
<td></td>
<td>3&lt;sup&gt;rd&lt;/sup&gt; ➞ $X+2$</td>
</tr>
<tr>
<td></td>
<td>4&lt;sup&gt;th&lt;/sup&gt; ➞ $X+3$</td>
</tr>
<tr>
<td></td>
<td>5&lt;sup&gt;th&lt;/sup&gt; ➞ $X+4$</td>
</tr>
<tr>
<td></td>
<td>And So on …</td>
</tr>
<tr>
<td>Consecutive Odd Numbers</td>
<td>1&lt;sup&gt;st&lt;/sup&gt; ➞ $X$</td>
</tr>
<tr>
<td></td>
<td>2&lt;sup&gt;nd&lt;/sup&gt; ➞ $X+2$</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td></td>
<td>4&lt;sup&gt;th&lt;/sup&gt; ➞ $X+6$</td>
</tr>
<tr>
<td></td>
<td>5&lt;sup&gt;th&lt;/sup&gt; ➞ $X+8$</td>
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<td>1&lt;sup&gt;st&lt;/sup&gt; ➞ $X$</td>
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</tr>
<tr>
<td></td>
<td>And So on …</td>
</tr>
</tbody>
</table>
Example 1: Give an algebraic expression for the sum of three consecutive integers.

Consecutive Numbers:

\[ \begin{align*}
1^{\text{st}} & \rightarrow X \\
2^{\text{nd}} & \rightarrow X+1 \\
3^{\text{rd}} & \rightarrow X+2 \\
4^{\text{th}} & \rightarrow X+3 \\
5^{\text{th}} & \rightarrow X+4 \quad \text{And So on …}
\end{align*} \]

Hence the algebraic expression:

\[ (X) + (X + 1) + (X + 2) \]

Example 2: Give an algebraic expression for the sum of four consecutive odd integers.

Consecutive Odd Numbers:

\[ \begin{align*}
1^{\text{st}} & \rightarrow X \\
2^{\text{nd}} & \rightarrow X+2 \\
3^{\text{rd}} & \rightarrow X+4 \\
4^{\text{th}} & \rightarrow X+6 \\
5^{\text{th}} & \rightarrow X+8 \quad \text{And So on …}
\end{align*} \]

Hence the algebraic expression:

\[ (X) + (X + 2) + (X + 4) + (X + 6) \]

Example 3: Give an algebraic expression for the sum of five consecutive even integers.

Consecutive Even Numbers:

\[ \begin{align*}
1^{\text{st}} & \rightarrow X \\
2^{\text{nd}} & \rightarrow X+2 \\
3^{\text{rd}} & \rightarrow X+4 \\
4^{\text{th}} & \rightarrow X+6 \\
5^{\text{th}} & \rightarrow X+8 \quad \text{And So on …}
\end{align*} \]

Hence the algebraic expression:

\[ (X) + (X + 2) + (X + 4) + (X + 6) + (X + 8) \]
Key focus: Define and understand the meaning of algebraic term(s) as related to polynomials.

Overview of Concepts: Here the students learn the definition and understand the meaning of algebraic term(s) as related to polynomials

Contents:

Monomial: One term.
✓ Example: \(X^2\)

Binomial: Two terms
✓ Example: \(X^2 + 2\)

Trinomial: Three terms
✓ Example: \(X^2 + X + 2\)

Polynomials: Polynomial meaning many terms.
✓ Example: \(X^3 + X^2 + X + 2\)

Like Terms: Like terms are terms that have the same variable raised to the same powers. Only the like terms can be combined

✓ Example: Combine like terms in the given expression:
\[
2X^3 + 4 + 3X + 5X^2 + X^3
\]
\[
\frac{2X^3 + 4 + 3X + 5X^2 + X^3}{\text{LIKE TERMS}}
\]
\[
3X^3 + 5X^2 + 3X + 4
\]

➢ We usually write the final answer from largest to smallest (descending order)
Exercise:

1) Simplify the following polynomials by combining like terms

   a) \(2X^2 + 3X + 7X^2 - 6 - 9X\)

   b) \((2X^2 - 2X) + (5X - 3X^2)\)

   c) \((6X^2 + 2X) - (4X - X^2)\)

   d) \((9X^2 - X + 3) - (5 + X - 3X^2)\)

2) Evaluate the following:

   a) Subtract \(5a^2 - 3a\) from the sum of \(3a - 3\) and \(5a^2 + 5\)

   b) A rectangle has sides of \(8x + 9\) and \(6x - 7\). Find the polynomial that represents its perimeter.

3) Simplify: \(\left[(2X^2 - 2X - 4) + (5X - 3X^2)\right] - (3X + 7)\)

4) The length of a rectangle is given by \(7X + 3\) yards and the width is given by \(4X - 5\) yards. Express the area of the rectangle in terms of \(x\).

5) Suppose an orchard is planted with trees in straight rows. If there are \(12X\) rows with \(9X + 2\) trees in each row, how many trees are there in the orchard?

6) Perform the following operations:

   a) \((9W^3 - 2W + 4) + (5W - 3W^2)\)

   b) \((9W^3 - 2W + 4) - (5W - 3W^2)\)

   c) \(4(5W^3 - 9W^2 + 3)\)

   d) \((2W + 4)(5W - 4)\)
Key Focus: Adding polynomials.

Overview of Concepts: Here the students learn to add polynomials.

Contents:

Reinforce to the students that polynomials are like numbers. If we see two polynomials in a question we can always add them to get another polynomial. Like we previously discussed before, we need to focus our attention on the idea of combining like terms. The following example will illustrate what I mean:

✓ Example 1: \((5X - 3) + (8X + 3)\)
1st re-write the expression without parentheses \(\Rightarrow 5X - 3 + 8X + 3\)
2nd combine like terms \(\Rightarrow 13X\)

✓ Example 2: \((5X^2 - 7) + (8X + 3)\)
1st re-write the expression without parentheses \(\Rightarrow 5X^2 - 7 + 8X + 3\)
2nd combine like terms \(\Rightarrow 5X^2 + 8X - 4\)

And so on…
Exercise:

1) Add the following polynomials:

   I \((8X + 9) + (5X + 4)\)

   II \((8X^2 + 9X + 6) + (5X^2 + 4X + 2)\)

   III \((9X^2 - 4X + 3) + (5X^2 - 2X + 7)\)

   IV \((9X + 6) + (5X^2 + 9X + 4)\)

   V \((-8X^2 + 9X - 5) + (-2X^2 + 8X + 2)\)

   VI \((4X^2 - 3X + 6) + (-4X^2 + 3X - 6)\)

   VII \((5W - 7X + 6Y) + (8X + 3W - 6Y)\)

   VIII \((4X^3 - 7X^2 + 9) + (2X^3 + 8X - 3)\)

   IX \((4X^3 - 7X^2 + 9) + (2X^3 + 8X - 3) + 4X - 9\)
**Key Focus:** Subtracting polynomials.

**Overview of Concepts:** Here the students learn to subtract polynomials.

**Contents:**

Reinforce to the students that polynomials are like numbers. If we see two polynomials in a question we can always subtract them to get another polynomial. Like we previously discussed before, we need to focus our attention on the idea of combining like terms. Remembering many times in subtracting polynomials we are required to change signs for obvious reasons. The following example will illustrate what I mean:

✓ **Example 1:** \((5X - 3) - (8X + 3)\)

1\textsuperscript{st} re–write the expression without parentheses \(\Rightarrow 5X - 3 - 8X - 3\)

2\textsuperscript{nd} combine like terms \(\Rightarrow -3X - 6\)

✓ **Example 2:** \((5X^2 - 7) - (8X - 3)\)

1\textsuperscript{st} re–write the expression without parentheses \(\Rightarrow 5X^2 - 7 - 8X + 3\)

2\textsuperscript{nd} combine like terms \(\Rightarrow 5X^2 - 8X - 4\)

And so on…
Exercise:

1) Simplify the following polynomials:

a) \((8X + 9) - (5X + 4)\)

b) \((8X^2 + 9X + 6) - (5X^2 + 4X + 2)\)

c) \((9X^2 - 4X + 3) - (5X^2 - 2X + 7)\)

d) \((9X + 6) - (5X^2 + 9X + 4)\)

e) \((-8X^2 + 9X - 5) - (-2X^2 + 8X + 2)\)

f) \((4X^2 - 3X + 6) - (-4X^2 + 3X - 6)\)

g) \((5W - 7X + 6Y) - (8X + 3W - 6Y)\)

h) \((4X^3 - 7X^2 + 9) - (2X^3 + 8X - 3)\)

i) \((4X^3 - 7X^2 + 9) - (2X^3 + 8X - 3) + 4X - 9\)

ej) \[
\frac{13X^2 + 3X - 9}{4X + 7}
\]
Key Focus: Multiplying Polynomials

Overview of Concepts:
Here the students learn to multiplying Polynomials.

Contents:
Reinforce to the students that polynomials are like numbers. If we see two polynomials in a question we can always multiply them to get another polynomial. Like we previously discussed before, we need to focus our attention on the concepts behind the laws of exponents. The following example will illustrate what I mean:

✓ Example 1: \(7(2X + 5)\)

1\(^{st}\) re-write the expression without parentheses \(\Rightarrow 7(2X + 5)\)

2\(^{nd}\) Multiply to get \(\Rightarrow 14X + 35\)

✓ Example 2: \((X + 7)(X - 3)\)

1\(^{st}\) Apply the distributive property the \(\Rightarrow (X + 7)(X - 3)\)

2\(^{nd}\) Multiply to get \(\Rightarrow X^2 - 3X + 7X - 21\)

3\(^{rd}\) combine like terms \(\Rightarrow X^2 + 4X - 21\)

And so on…
Exercise:

1) Find the product of the following polynomials:

   a) \((8X + 9)(5X + 4)\)  
   b) \(6(5X^2 + 4X + 2)\)
   c) \((4X + 3)(5X^2 + 7)\)
   d) \((X + 6)(X - 4)\)
   e) \((X - 3)(X + 8)\)
   f) \((X + 5)(X - 5)\)
   g) \((X + Y)(X + Y)\)
   h) \((X - 3)^2\)
   i) \((2X + 3)^2\)
   j) \((X + 1)^3\)
Key Focus: Dividing Polynomials

Overview of Concepts: Here the students learn to divide Polynomials.

Contents:
Reinforce to the students that polynomials are like numbers. If we see two polynomials in a question we can always divide them to get another polynomial. Like we previously discussed before, we need to focus our attention on the concepts behind the laws of exponents. The following example will illustrate what I mean:

✓ Example 1: \((2X + 10) ÷ 2\)

1st Re-write the expression without parentheses \(\frac{2X}{2} + \frac{10}{2}\)

2nd Divide to get \(X + 5\)

✓ Example 2: \((2X^2 + 10X) ÷ 2X\)

1st Apply the distributive property the \(\frac{2X^2}{2X} + \frac{10X}{2X}\)

2nd Divide to get \(X + 5\)

And so on…
Exercise:

1) Divide the following polynomials:

a) \((8X + 4) \div 4\)  

b) \((25X^2 + 45X + 15) \div 5\)

c) \((5X^2 + 7) \div 35X\)

d) \((9X + 6) \div (9X + 6)\)

e) \((5 - X) \div (X - 5)\)

f) \((X - Y) \div (Y - X)\)

g) \(9y^5 - 15y^3\)

h) \(\frac{12b^2 - 24}{6}\)

i) \(\frac{2x^2 + 2x}{2x}\)

j) \(\frac{28y^4}{7y}\)
Key Focus: Factoring Prime polynomials

Overview of Concepts:
Here the students learn to identify polynomials that are Prime

Contents:
Prime polynomials are algebraic expressions that are not factorable.

✓ Example 1: Factor: \( X^2 + 9 \)

\[ X^2 + 9 \rightarrow \text{Prime} \]

✓ Example 2: Factor: \( X^2 + 3X + 7 \)

\[ X^2 + 3X + 7 \rightarrow \text{Prime} \]

Note: Prime! meaning the expression cannot be factored.

And so on...
Key Focus: Factoring polynomials using Greatest Common Factor

Overview of Concepts:
Here the students learn to factor polynomials using the Greatest Common Factor.

Contents:
Factor: Is defined as a number or an expression that is multiplied by another to produce a product.

Greatest Common Factor (GCF): Is the greatest: whole number or a variable or both a whole number and a variable that is a factor of each term in an algebraic expression.

✓ Example 1: Factor out the greatest common factor: \(2X^2 + 2X\)
1\(^{st}\) Identify the GCF of \(2X^2 + 2X\) \(\Rightarrow 2X\)
2\(^{nd}\) Factor the expression to get \(\Rightarrow 2X(X + 1)\)

✓ Example 2: Factor out the greatest common factor: \(6Z^3 + 3Z^2 − 12Z\)
1\(^{st}\) Identify the GCF of \(6Z^3 + 3Z^2 − 12Z\) \(\Rightarrow 3Z\)
2\(^{nd}\) Factor the expression to get \(\Rightarrow 3Z(2Z^2 + Z − 4)\)

✓ Example 3: Factor: \(-28m^3n^4− 14m^2n^3\)

Step 1: Find the GCF of numbers and the common variables
GCF(28,14)=14 because 14 is the greatest number that divides evenly into both numbers
GCF \(\{m^3, m^2\}\)= \(m^2\) as \(m^2\) is the smaller of the two terms it divides evenly into both.
GCF \(\{n^4 ,n^3\}\) = \(n^3\) as \(n^3\) being the smaller of the two terms divides into both.
Hence the GCF for \(-28m^3n^4, − 14m^2n^3\) = \(-14 \times n^3\)

Step 2: Divide each term by the GCF.
\[
\frac{-28m^3n^4}{-14m^2n^3} = 2mn \quad \text{and} \quad \frac{-14m^2n^3}{-14m^2n^3} = 1
\]

Step 3: Write the GCF outside the parentheses and write the quotients from step−2 inside the parentheses
Hence \(-28m^3n^4− 14m^2n^3 = -14m^2n^3( 2mn + 1)\)
✓ **Example 4:** Factor: $8x^2y^9+12x^5y^7$ (note there are two terms)

**Step 1:** Find the GCF of numbers and the common variables
- $4$ is the GCF of $8$ and $12$.
- $x^2$ is the GCF of $x^2$ and $x^5$.
- $y^7$ is the GCF of $y^7$ and $y$.

The GCF = $4x^2y^7$.

**Step 2:** Write $8x^2y^9+12x^5y^7$ as product of the GCF with (two terms—in this example) parenthesis carry down the addition operation between the terms:

$$8x^2y^9+12x^5y^7 = 4x^2y^7( _____ + _____ )$$

**Step 3:** to find the terms inside the parentheses divide each term by the GCF

$$\frac{8x^2y^9}{4x^2y^7} = 2y^2 \quad \text{and} \quad \frac{12x^5y^7}{4x^2y^7} = 3x^3.$$  

**Step 4:** Write the answer

$$8x^2y^9+12x^5y^7 = 4x^2y^7(2y^2+3x^3).$$

✓ **Example 5:** Factor: $27a^5 + 18a^4 - 36a^6$

**Step 1:** Find the GCF of numbers and the common variables
- GCF (27, 18, 36) = 9
- GCF(a^5, a^4, a^6) = a^4
- GCF = 9a^4

**Step 2:** Write the gcf product with parentheses leaving 3 terms and carry down the two operations:

$$27a^5 + 18a^4 - 36a^6 = 9a^4 ( _____ + _____ - _____ )$$

**Step 3:** Divide by GCF to find the terms inside the parentheses

$$\frac{27a^5}{9a^4} = 3a, \quad \frac{18a^4}{9a^4} = 2, \quad \frac{-36a^6}{9a^4} = -4a^2.$$  

**Step 4:** Write the answer:

$$27a^5 + 18a^4 - 36a^6 = 9a^4(3a + 2 - 4a^2).$$
**Exercise:** Find the GCF (Greatest common factor) for each pair of terms.

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>a)</td>
<td>GCF of ( { x^6, x^{10} } ) = ?</td>
</tr>
<tr>
<td>b)</td>
<td>GCF of ( { a^5b^3, a^2b^6 } ) = ?</td>
</tr>
<tr>
<td>c)</td>
<td>GCF of ( { 16, 20 } ) = ?</td>
</tr>
<tr>
<td>d)</td>
<td>GCF of ( { 15, 14 } ) = ?</td>
</tr>
<tr>
<td>e)</td>
<td>GCF of ( { (x+2)(x+7), (x+2)(x-5) } ) = ?</td>
</tr>
<tr>
<td>f)</td>
<td>GCF of ( { 40x^3y^5, 16x^8y^4 } ) = ?</td>
</tr>
<tr>
<td>g)</td>
<td>GCF of ( { 8m^7n^3, 10m^2 } ) = ?</td>
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### Factor each of the following expression completely:

#### #1) Factor Completely:
\[
30a^3x^2 - 18a^4x^9
\]

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<tbody>
<tr>
<td>a)</td>
<td>( 6a^2x^2 )</td>
</tr>
<tr>
<td>b)</td>
<td>( 6a^2x(5x - 3a^2x^8) )</td>
</tr>
<tr>
<td>c)</td>
<td>( 6a^2x^2(5 - 3a^2x^7) )</td>
</tr>
<tr>
<td>d)</td>
<td>( 6a^2x^2(5 - 3ax^7) )</td>
</tr>
</tbody>
</table>

#### #2) Factor Completely:
\[
3x^2y - 6xy^2
\]

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<tbody>
<tr>
<td>a)</td>
<td>( 3(x^2y - 2xy^2) )</td>
</tr>
<tr>
<td>b)</td>
<td>( 3xy(x - 2y) )</td>
</tr>
<tr>
<td>c)</td>
<td>( 3x^3y(1 - 2y) )</td>
</tr>
<tr>
<td>d)</td>
<td>( 3xy^2(x - 2) )</td>
</tr>
</tbody>
</table>

#### #5) Factor Completely:
\[
32xy - 18x^2
\]

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<thead>
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</thead>
<tbody>
<tr>
<td>a)</td>
<td>( 2(16xy - 9x) )</td>
</tr>
<tr>
<td>b)</td>
<td>( 2x(16y - 9) )</td>
</tr>
<tr>
<td>c)</td>
<td>( 2x(16y - 9x) )</td>
</tr>
<tr>
<td>d)</td>
<td>( 2x(4y - 3x)(4y + 3x) )</td>
</tr>
</tbody>
</table>

#### #6) Factor Completely:
\[
5n^3m - 15n^2m + 10nm^2
\]

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<thead>
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<tbody>
<tr>
<td>a)</td>
<td>( 5nm(n^3m - 3n^2m + 2nm^2) )</td>
</tr>
<tr>
<td>b)</td>
<td>( 5nm(n^2m - 3nm + 2nm) )</td>
</tr>
<tr>
<td>c)</td>
<td>( 5nm(n^3 - 3m + 2m) )</td>
</tr>
<tr>
<td>d)</td>
<td>( 5nm(n^3 - 3n + 2m) )</td>
</tr>
</tbody>
</table>

#### #7) Factor Completely:
\[
14r^7s^6 + 21r^6s^7 - 28r^5s^8
\]

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<thead>
<tr>
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<tbody>
<tr>
<td>a)</td>
<td>( 7rs(2r^5s^5 + 3r^5s^5 - 4r^4s^7) )</td>
</tr>
<tr>
<td>b)</td>
<td>( 7r^2s^2(2r^5s^4 + 3r^5s^5 - 4r^3s^6) )</td>
</tr>
<tr>
<td>c)</td>
<td>( 7r^7s^6(2 + 3s - 4s^2) )</td>
</tr>
<tr>
<td>d)</td>
<td>( 7r^5s^6(2r^2 + 3rs - 4s^2) )</td>
</tr>
</tbody>
</table>

#### #8) Factor Completely:
\[
6a^2b^3 + 8a^3b^2 + 12a^3b^3
\]

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<tbody>
<tr>
<td>a)</td>
<td>( 2ab(3ab^2 + 4a^2b + 6a^3b^2) )</td>
</tr>
<tr>
<td>b)</td>
<td>( 2a^2b(3b^2 + 4a + 6ab^2) )</td>
</tr>
<tr>
<td>c)</td>
<td>( 2a^3b(3b + 4a + 6ab) )</td>
</tr>
<tr>
<td>d)</td>
<td>( 2a^2b^3(3 + 4a + 6ab) )</td>
</tr>
</tbody>
</table>

#### #9) Factor Completely:
\[
4x^2y + 12x^3y
\]

#### #10) Factor Completely:
\[
28x^3y^3 - 21x^2y^2 + 35x^3y^2
\]
Key Focus: Factoring polynomials of the form $AX^2 + BX + C$, where $A = 1$

Overview of Concepts:

Here the students learn to factor polynomials of the form $AX^2 + BX + C$, where $A$, $B$, and $C$ are constants. First we’ll look at it when $A = 1$, then later on we’ll look at it when $A > 1$.

Contents:

Polynomials of the form $AX^2 + BX + C$ are referred to as trinomials. We factor them by asking what two numbers have a product of $C$ and a sum of $B$. Pay attention, if each term of the trinomial has a GCF, we’ll need to factor it out as our first step and then work with the parentheses if it is factorable.

✓ Example 1: Factor the Trinomial: $X^2 + 11X + 30$

1st Identify if a GCF exists ➔ No GCF

2nd Factors of 30 ➔ $X^2 + 11X + 30$

3rd Factors to get ➔ $(X + 6)(X + 5)$

✓ Example 2: Factor the Trinomial: $2X^2 + 18X + 36$

1st Identify if a GCF exists ➔ $2(X^2 + 9X + 18)$

2nd Factors of 18 ➔ $2(X^2 + 9X + 18)$

3rd Factors to get ➔ $2(X + 6)(X + 3)$

And so on…
Exercise:

1) Factor the following polynomials completely:

A. \( X^2 + 19X + 18 \)

B. \( X^2 + 12X + 36 \)

C. \( X^2 + 37X + 36 \)

D. \( X^2 + 31X + 30 \)

E. \( 2X^2 + 26X + 72 \)

F. \( X^2 + 5X + 4 \)

G. \( X^2 + 3X + 2 \)

H. \( X^2 - 10X + 16 \)

I. \( X^2 - 8X + 16 \)

J. \( X^2 - 8X + 15 \)

K. \( X^2 - 8X + 20 \)

L. \( 2X^2 - 4X + 48 \)

M. \( 3X^2 + 9X - 84 \)

N. \( X^2 - 3X - 28 \)
**Key Focus:** Factoring polynomials of the form $AX^2 + BX + C$, where $A \geq 1$

**Overview of Concepts:**

Here the students learn to factor polynomials of the form $AX^2 + BX + C$, where $A$, $B$, and $C$ are constants. We’ll look at it when $A > 1$.

**Contents:**

Polynomials of the form $AX^2 + BX + C$ are usually referred to as trinomials. We factor them by the trial and error method or by the grouping method. We factor them by asking what two numbers have a product of $(A \times C)$ and a sum of $B$. Pay attention, if each term of the trinomial has a GCF, we’ll need to factor it out as our first step and then work with the parentheses if it is factorable.

✓ **Example 1:** Factor the Trinomial: $6Y^2 + 11Y - 10$

1\textsuperscript{st} Identify if a GCF exists ➞ No GCF, Other than 1

2\textsuperscript{nd} $A \times C = 6Y^2$ ➞ $6Y^2 + 11Y - 10$

3\textsuperscript{rd} Re–write the polynomial as ➞ $6Y^2 + 15Y - 4Y - 10$

4\textsuperscript{th} Factor the polynomial by Grouping ➞ $3Y(2Y + 5) - 2(2Y + 5)$

5\textsuperscript{th} Factor the polynomial by Grouping ➞ $(2Y + 5)(3Y - 2)$

And so on…
Exercise:

1) Factor the following polynomials completely:
   
a) \( 8X^2 - 24X + 18 \)  
   
b) \( 3X^2 + 5X - 2 \)  
   
c) \( 5X^2 + 20X + 15 \)  
   
d) \( 2X^2 - 28X + 96 \)  
   
e) \( 2Z(X - Y) + 3(X - Y) \)  
   
f) \( 15X^2 - 3X + 10X - 2 \)  
   
g) \( 4ad - 25c - 5d + 20ac \)  
   
h) \( 2X^3 - X^2 - 6X \)
**Key Focus:** Factoring polynomials using Special Products

**Overview of Concepts:**
Here the students learn to factor polynomials using Special Products

**Contents:**
Pay attention, if each term of the trinomial has a GCF, we'll need to factor it out as our first step and then work with the parentheses if it is factorable.

Difference of squares \( A^2 - B^2 = (A + B)(A - B) \): if by inspection the expression fits into your special case formula, then use it. It can be a very effective and efficient method of factoring.

✓ **Example:** Factor: \( X^2 - 100 \)

1\(^{st}\) Identify if a GCF exists \( \Rightarrow \) GCF = 1 Don't bother using it!

2\(^{nd}\) \( A^2 - B^2 = (A + B)(A - B) \) \( \Rightarrow \) \( X^2 - 100 \)

3\(^{rd}\) Re-write the polynomial as \( \Rightarrow \) \( X^2 - 10^2 \)

4\(^{th}\) Factor the polynomial \( \Rightarrow (X + 10)(X - 10) \)

And so on…
Perfect squares trinomial: if by inspection the expression fits into your special case formula, then use it. It can be a very effective and efficient method of factoring.

✓ Example 1: Factor: \(25X^2 + 10X + 1\)

1st Identify if a GCF exists  \(\Rightarrow\) GCF = 1  Don't bother using it!

\[
\begin{align*}
A^2 + 2AB + B^2 &= (A + B)(A + B) = (A + B)^2 \\
A^2 - 2AB + B^2 &= (A - B)(A - B) = (A - B)^2
\end{align*}
\]

\(25X^2 + 10X + 1\)

3rd Re-write the polynomial as  \(\Rightarrow (5X)^2 + 2(5)(1)X + (1)^2\)

4th Factor the polynomial  \(\Rightarrow (5X + 1)^2\)

✓ Example 2: Factor: \(49X^2 - 42XY + 9Y^2\)

1st Identify if a GCF exists  \(\Rightarrow\) GCF = 1  Don't bother using it!

\[
\begin{align*}
A^2 + 2AB + B^2 &= (A + B)(A + B) = (A + B)^2 \\
A^2 - 2AB + B^2 &= (A - B)(A - B) = (A - B)^2
\end{align*}
\]

\(49X^2 - 42XY + 9Y^2\)

3rd Re-write the polynomial as  \(\Rightarrow (7X)^2 - 2(7)(3)XY + (3)^2\)

4th Factor the polynomial  \(\Rightarrow (7X - 3Y)^2\)

And so on…
Exercise:

1) Factor the following polynomials completely:

   a) \( 8X^2 - 24X + 18 \)
   e) \( 2Z(X - Y) + 3(X - Y) \)

   b) \( 3X^2 + 5X - 2 \)
   f) \( 15X^2 - 3X + 10X - 2 \)

   c) \( 5X^2 + 20X + 15 \)
   g) \( 4ad - 25c - 5d + 20ac \)

   d) \( 2X^2 - 28X + 96 \)
   h) \( 2X^3 - X^2 - 6X \)
**Exercise:** Apply appropriate technique to factor completely.

<table>
<thead>
<tr>
<th>#1) Factor completely</th>
<th>#2) Factor Completely:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5a^3b^3 - 20ab^3$</td>
<td>$5x^3 + 10x^2 - 175x$</td>
</tr>
<tr>
<td>a) $5ab^3(a -2)^2$</td>
<td>a) $5(x + 2x -35)$</td>
</tr>
<tr>
<td>b) $5ab^3(a^2-6a)$</td>
<td>b) $5(x + 5)(x - 7)$</td>
</tr>
<tr>
<td>c) $5ab^3(a+2)(a+b)$</td>
<td>c) $5(x^2 + 4x -35)$</td>
</tr>
<tr>
<td>d) $5ab^3(a+2)(a-2)$</td>
<td>d) $5(x - 5)(x + 7)$</td>
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<table>
<thead>
<tr>
<th>#3) Factor Completely:</th>
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</thead>
<tbody>
<tr>
<td>$32x^3 - 50xy^2$</td>
</tr>
<tr>
<td>a) $2x(16x^2-25y^2)$</td>
</tr>
<tr>
<td>b) $2x(4x -5y)^2$</td>
</tr>
<tr>
<td>c) $2(16x^2-25y^2)$</td>
</tr>
<tr>
<td>d) $2(4x + 5y)(4x -5y)$</td>
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</table>

<table>
<thead>
<tr>
<th>#4) Which of the following is a factor of:</th>
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<tbody>
<tr>
<td>$12ax + 15bx -20ay -25by$</td>
</tr>
<tr>
<td>a) $(3x + 5y)$</td>
</tr>
<tr>
<td>b) $(4a - 5b)$</td>
</tr>
<tr>
<td>c) $(4x -5y)$</td>
</tr>
<tr>
<td>d) $(4a + 5b)$</td>
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<table>
<thead>
<tr>
<th>#5) Factor completely:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$9xy^3 - 36xy$</td>
</tr>
<tr>
<td>a) $9xy(y^2+4)$</td>
</tr>
<tr>
<td>b) $9xy(x^2-4)$</td>
</tr>
<tr>
<td>c) $9xy(y-2)(y+2)$</td>
</tr>
<tr>
<td>d) $9x(x-2y)(x+2y)$</td>
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</table>

<table>
<thead>
<tr>
<th>#6) Which of the following is the complete factorization of</th>
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</thead>
<tbody>
<tr>
<td>$2x^2 - 13x - 24$ ?</td>
</tr>
<tr>
<td>a) $(2x - 6)(x + 4)$</td>
</tr>
<tr>
<td>b) $(2x - 3)(x - 8)$</td>
</tr>
<tr>
<td>c) $(2x + 3)(x - 8)$</td>
</tr>
<tr>
<td>d) $2(x + 3)(x - 4)$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>#7) Factor completely:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$32x^2y^2 - 16x^3y$</td>
</tr>
<tr>
<td>a) $16xy(2xy -x^2)$</td>
</tr>
<tr>
<td>b) $16x^2y^2(2 -x)$</td>
</tr>
<tr>
<td>c) $8x^2y(4y - 2x)$</td>
</tr>
<tr>
<td>d) $16x^2y(2y - x)$</td>
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</table>

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<tr>
<th>#8) Factor completely:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8x^6 -24x^5 + 16x^4 -8x^3$</td>
</tr>
<tr>
<td>a) $8( x^6 - 6x^5 + 4x^4 - 2x^3)$</td>
</tr>
<tr>
<td>b) $8x^3( x^3-3x^2 + 2x )$</td>
</tr>
<tr>
<td>c) $8x^3( x^3-3x^2 + 2x - 1)$</td>
</tr>
<tr>
<td>d) $4x^3(2x^3- 6x^2 + 4x - 2)$</td>
</tr>
</tbody>
</table>
#9) Factor Completely
54x^3 − 6x^2

a) 3x(18x^2 − 2x)
b) 6x(9x^2 − x)
c) 6x^2(9x − 1)
d) 6x^2(9x)

#10) Factor Completely
2x^4 − 4x^3 + x^2

a) 2(x^4 − 2x^3 + x^2)
b) 2x(x^3 − 2x^2 + x)
c) 2x^2(x^2 − 2x + 1)
d) x^2(2x^2 − 4x + 1)

#11) Factor Completely
128x^2 − 162y^2

a) 2(64x^2 − 81y^2)
b) 2(8x − 9)(8x + 9)
c) 2(8x − 81y)(8x + y)
d) 2(8x − 9y)(8x + 9y)

#12) Factor Completely:
50x^2 − 98

a) 2(25x^2 − 49)
b) 2(5x − 7)(5x + 7)
c) 2(5x + 7)(5x − 7)
d) 2(5x − 7)(5x − 7)

#13) Factor Completely:
6x^5y^3 − 18x^4y^2 + 36x^3y^3

a) 6(x^5y^3 − 3x^4y^2 + 12x^3y^3)
b) 6xy(x^4y^2 − 3x^3y + 12xy)
c) 6x^2y^2(x^3y − 3x^2 + 12xy)
d) 6x^3y^2(x^2y − 3x + 12y)

#14) Factor Completely:
12x^3 − 27x

a) 3(4x^3 − 9x)
b) 3x(4x^2 − 9)
c) 3x(2x − 3)(2x + 3)
d) 3x(2x + 3)(2x + 3)

#15) Factor Completely:
32x^2 + 50

a) 2(16x^2 + 25)
b) 2(4x + 5)(4x + 5)
c) 2(2x + 5)(4x + 5)
d) 2(2x − 5)(4x − 5)

#16) Factor completely:
4x^3 + 4x^2 − 8x

a) (4x^2 − 4x)(x + 2)
b) 4(x + 2)(x − 1)
c) 4x(x − 1)(x + 2)
d) x(4x + 4)(x − 1)

#17) Factor completely:
4x^2 − 4

a) (2x − 2)(2x + 2)
b) 4(x^2 − 1)
c) 4(x + 1)(x − 1)
d) 2(2x + 2)(x − 1)

#18) Factor completely:
y^2 + 49

a) y( y + 49)
b) (y + 7)(y + 7)
c) (y + 1)(y + 49)
d) Prime
Key Focus: Complex Fraction.

Overview of Concepts:
Here the students learn to simplify complex fractions.

Contents:
As in regular fractions, we appreciate the effort of having our answers in its lowest term, i.e... Reduced form. Polynomials can also be written in fractional form, to simplify such expressions we'll need to factor the numerator and the denominator individually, and then make the appropriate cancellation(s). All the tactics we have learned in factoring thus far would be beneficial in simplifying these kinds of problems. The following example will illustrate this concept:

✓ Example: simplify: \( \frac{X^2 + 13X + 40}{X + 8} \)

1st Factor \( \Rightarrow \frac{(X + 5)(X + 8)}{(X + 8)} \)

2nd Reduce \( \Rightarrow \frac{(X + 5)(X + 8)}{(X + 8)} \)

3rd Final result \( \Rightarrow (X + 5) \)

And so on…
Exercise:

1) Simplify the following completely:

a) \[ \frac{X^2 - 2X - 24}{X + 4} \]

b) \[ \frac{X^2 + 5X}{X^2 - 25} \]

c) \[ \frac{X^2 + 12X + 20}{X^2 - 5X - 14} \]

d) \[ \frac{2X^2 - 24X + 72}{4X - 24} \]

e) \[ \frac{2Z(X - Y) + 3(X - Y)}{6Z + 9} \]

f) \[ \frac{X^2 - 4}{4 - X^2} \]

g) \[ \frac{W + 7}{X^2 + 8X + 7} \]

h) \[ \frac{X^2 - 12X + 36}{6 - X} \]
**Mixed Exercise:**

**Fill in the blanks:**

a) List all the factors of 12 in increasing order. _____, _____, _____, _____, _____, _____.

b) 2 and 6 are called the ______________ of 12.

c) 12 = 4 × 3 is ______ of 12 but not complete factor of 12.

d) Prime factors of 12 = _____ × _____ × _____

e) 35 = _____ × 7

f) \( x^2 - 9 = (____)(x - 3) \)

g) one factor of \( y^2 + 6y + 8 \) if \( y + 2 \), what is the other factor ____________

h) \( (m + 5)(____) = m^2 + 10m + 25 \)

i) GCF of 3, 6, 9 is ____________

j) GCF of \((x+2)(x +7)\) and \((x - 4)(x + 2)\) is ______________

k) \( (2x + 1)(x +__) = 2x^2 + 5x + 3 \)

l) \( (5x - 3)(x + ____) = 5x^2 + 7x - 6 \)

m) \( (x + 5)(____ +____) = 2x^2 + 13x + 15 \)
<table>
<thead>
<tr>
<th></th>
<th>#1) Factor Completely: $2x^2 + 11x + 15$</th>
<th>#2) Factor Completely: $2x^2 + 13x + 15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>$(2x + 15)(x + 1)$</td>
<td>a) $(2x + 15)(x + 1)$</td>
</tr>
<tr>
<td>b)</td>
<td>$(2x + 5)(x + 3)$</td>
<td>b) $(2x + 5)(x + 3)$</td>
</tr>
<tr>
<td>c)</td>
<td>$(2x + 1)(x + 15)$</td>
<td>c) $(2x + 1)(x + 15)$</td>
</tr>
<tr>
<td>d)</td>
<td>$(2x + 3)(x + 5)$</td>
<td>d) $(2x + 3)(x + 5)$</td>
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<tr>
<th></th>
<th>#3) Factor Completely: $2x^2 + 31x + 15$</th>
<th>#4) Factor Completely: $2x^2 + 17x + 15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>$(2x + 15)(x + 1)$</td>
<td>a) $(2x + 15)(x + 1)$</td>
</tr>
<tr>
<td>b)</td>
<td>$(2x + 5)(x + 3)$</td>
<td>b) $(2x + 5)(x + 3)$</td>
</tr>
<tr>
<td>c)</td>
<td>$(2x + 1)(x + 15)$</td>
<td>c) $(2x + 1)(x + 15)$</td>
</tr>
<tr>
<td>d)</td>
<td>$(2x + 3)(x + 5)$</td>
<td>d) $(2x + 3)(x + 5)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>#5) Factor Completely: $2x^2 − 15x + 21$</th>
<th>#6) Factor Completely: $2x^2 − 9x + 21$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>$(2x − 3)(x − 7)$</td>
<td>a) $(2x − 3)(x − 7)$</td>
</tr>
<tr>
<td>b)</td>
<td>$(2x − 21)(x − 1)$</td>
<td>b) $(2x − 21)(x − 1)$</td>
</tr>
<tr>
<td>c)</td>
<td>$(2x − 7)(x − 3)$</td>
<td>c) $(2x − 7)(x − 3)$</td>
</tr>
<tr>
<td>d)</td>
<td>$(2x − 1)(x − 21)$</td>
<td>d) $(2x − 1)(x − 21)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>#7) Factor Completely: $2x^2 − 43x + 21$</th>
<th>#8) Factor Completely: $2x^2 − 23x + 21$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>$(2x − 3)(x − 7)$</td>
<td>a) $(2x − 3)(x − 7)$</td>
</tr>
<tr>
<td>b)</td>
<td>$(2x − 21)(x − 1)$</td>
<td>b) $(2x − 21)(x − 1)$</td>
</tr>
<tr>
<td>c)</td>
<td>$(2x − 7)(x − 3)$</td>
<td>c) $(2x − 7)(x − 3)$</td>
</tr>
<tr>
<td>d)</td>
<td>$(2x − 1)(x − 21)$</td>
<td>d) $(2x − 1)(x − 21)$</td>
</tr>
</tbody>
</table>
#9) Factor Completely: $2x^2 - 7x - 15$

a) $(2x - 3)(x + 5)$  
b) $(2x + 3)(x - 5)$  
c) $(2x - 5)(x + 3)$  
d) $(2x + 5)(x - 3)$

#10) Factor Completely: $2x^2 - 29x - 15$

a) $(2x - 1)(x + 15)$  
b) $(2x + 1)(x - 15)$  
c) $(2x + 15)(x - 1)$  
d) $(2x - 15)(x + 1)$

#11) Which of the following is a factor of: $2x^2 + 13x - 15$

a) $(2x - 1)$  
b) $(2x + 1)$  
c) $(2x + 15)$  
d) $(2x - 15)$

#12) Which of the following is a factor of: $2x^2 - x - 15$

a) $(x - 3)$  
b) $(2x - 5)$  
c) $(x + 5)$  
d) $(2x - 3)$

#13) Which of the following is a factor of: $2x^2 - 13x - 7$

a) $2x - 1$  
b) $x + 7$  
c) $2x + 7$  
d) $x - 7$

#14) Factor Completely: $5x^3 + 10x^2 - 175x$

a) $5x(x + 2x - 35)$  
b) $5x(x + 5)(x - 7)$  
c) $5x(x^2 + 4x - 35)$  
d) $5x(x - 5)(x + 7)$

#15) Which is a factor of: $3x^2 + 11x + 10$

a) $(3x + 2)$  
b) $(x + 5)$  
c) $(3x + 5)$  
d) $(x + 10)$

#16) Which of the following is a factor of: $10ax + 15bx - 6ay - 9by$

a) $2a - 3b$  
b) $5x + 3y$  
c) $5x - 3y$  
d) $2ax - 3by$

#17) Factor completely: $9xy^3 - 36xy$

a) $9xy(y^2 + 4)$  
b) $9xy(x^2 - 4)$  
c) $9xy(y - 2)(y + 2)$  
d) $9x(x - 2y)(x + 2y)$

#18) Which of the following is the complete factorization of $2x^2 - 13x - 24$?

a) $(2x - 6)(x + 4)$  
b) $(x - 6)(2x + 4)$  
c) $(2x - 3)(x - 8)$  
d) $(2x + 3)(x - 8)$
<table>
<thead>
<tr>
<th>Question</th>
<th>Polynomial</th>
<th>Options</th>
</tr>
</thead>
</table>
| #19)     | $3x^2 + 13x - 10$ | a) $x - 2$  
            b) $3x + 2$  
            c) $x - 5$  
            d) $3x - 2$  |
| #20)     | $3x^2 - 13x - 10$ | a) $x - 2$  
            b) $3x + 2$  
            c) $x + 5$  
            d) $3x - 2$  |
| #21)     | $3x^2 + x - 10$ | a) $x - 2$  
            b) $3x - 5$  
            c) $x + 5$  
            d) $3x + 5$  |
| #22)     | $3x^2 - x - 10$ | a) $3x + 2$  
            b) $3x - 5$  
            c) $x + 5$  
            d) $3x + 5$  |
| #23)     | $3x^2 + 17x + 8$ | a) $(3x + 2)(x + 4)$  
            b) $(3x + 4)(x + 1)$  
            c) $(3x + 1)(x + 8)$  
            d) $(3x + 8)(x + 1)$  |
| #24)     | $15nx + 20mx - 6ny - 8my$ | a) $5x + 2y$  
            b) $5x - 2y$  
            c) $3n - 2m$  
            d) $3x + 4m$  |
| #25)     | $3x^2 - 16x - 35$ | Factor:  
            a) $3x^2 - 16x - 35$ |
| #26)     | Factor Completely using grouping:  
            $28rx + 21sx - 20ry - 15sy$ | a) $3x^2 - 16x - 35$ |
| #27)     | $9xy^3 - 64x^3y$ | Factor completely:  
            $9xy^3 - 64x^3y$ |
| #28)     | Factor completely:  
            $16a^2b - 100b^3$ | $9xy^3 - 64x^3y$ |
**Key Focus:** Identifying and solving quadratic equations.

**Overview of Concepts:**

Here the students learn to identifying and solving quadratic equations of the form $AX^2 + BX + C = 0$, where $A$, $B$, and $C$ are constants. Factoring polynomials are a prerequisite for this topic.

**Contents:**

A quadratic equation of the form $AX^2 + BX + C = 0$, sometimes considered as a second degree polynomial equation. We will utilize the knowledge of factoring to solving quadratic equations in most cases

✓ **Example:** Solve the equation: $X^2 + 11X + 30 = 0$

1st Identify if a GCF exists  ➞ GCF = 1  Don't bother using it!

2nd Factors of 30  ➞ $X^2 + 11X + 30 = 0$

3rd Factors to get  ➞ $(X + 6)(X + 5) = 0$

4th Solve the Equation  ➞ $(X + 6) = 0$ ; $(X + 5) = 0$

$X + 6 = 0$ ; $X + 5 = 0$

$X = -6$ ; $X = -5$

5th Hence the solution set:  ➞ $\{-6, -5\}$

And so on…
Exercise:

1) Solve the following quadratic equations:

a) \( X^2 + 19X + 18 = 0 \)
b) \( X^2 + 12X + 36 = 0 \)
c) \( X^2 + 37X + 36 = 0 \)
d) \( X^2 + 31X + 30 = 0 \)
e) \( 2X^2 + 26X = -72 \)
f) \( X^2 + 5X = -4 \)
g) \( X^2 = -3X - 2 \)
h) \( X^2 - 10X = -16 \)
i) \( X^2 - 8X + 16 = 0 \)
j) \( X^2 - 8X + 15 = 0 \)
k) \( (X + 2)(X^2 - 4) = 0 \)
l) \( 2X^2 - 4X + 48 = 0 \)
m) \( 3X^2 + 9X = 84 \)
n) \( X^2 - 3X = 28 \)
**Key Focus:** Application quadratic formula.

**Overview of Concepts:**

Here the students learn to apply quadratic formula to solve problems

**Contents:**

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

What do we do with this formula? Simple! We do just the same thing we would do with any other formula, that is to say we use substitution to solve for an unknown.

We have probably seen equations of the form \( ax^2 + bx + c = 0 \) if we have then that’s great! If we don’t we are surely seeing it now, it represents the standard form of the quadratic equation so we will use it to match the equation against it to evaluate the value(s) of \( x \).

**Example:** Solve the equation: \( x^2 + 2x - 3 = 0 \)

By observation we see that the equation is in standard form, so we identify some of the known things, that is:

\[
ax^2 + bx + c = 0
\]

Notice that the variables \( a, b \) and \( c \) is related to: \( x^2 + 2x - 3 = 0 \)

\( a = 1; b = 2; c = -3 \)

Substitute these values in the equation (it works like a recipe!):

\[
x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)}
\]

\[
x = \frac{-2 \pm \sqrt{4 + 12}}{2}
\]

\[
x = \frac{-2 \pm \sqrt{16}}{2}
\]

\[
x = \frac{-2 \pm 4}{2}
\]

\[
x = \frac{-2 + 4}{2} \Rightarrow x = \frac{2}{2} \Rightarrow x = 1
\]

\[
x = \frac{-2 - 4}{2} \Rightarrow x = \frac{-6}{2} \Rightarrow x = -3
\]
Exercise:

1) Use the quadratic formula to solve the following equations, ALL ANSWERS MUST BE EXACT (DO NOT ROUND/APPROXIMATE):

   a) \( Y^2 + Y - 2 = 0 \)  
   b) \( Y^2 - 3Y + 2 = 0 \)  
   c) \( Y^2 + 4Y - 21 = 0 \)  
   d) \( X^2 - 8 = 0 \)

2) Find the roots of the following equations, express your answer(s) to the nearest tenth, (when possible):

   a) \( 2Y^2 + 3Y - 3 = 0 \)  
   b) \( (X + 3)(X - 2) = 1 \)  
   c) \( (X + 4)(X - 6) = 2 \)  
   d) \( X^2 + 10 = 8X \)

3) Solve the equation: \( W^2 = 6W - 1 \)
Chapter 4  Coordinate Geometry & Applications

Key Focus:  Apply the Pythagorean’s theorem

Overview of Concepts:
Here the students learn to apply the Pythagorean’s theorem to find the missing side of a right angle triangle.

Contents:

✓ Example 1: Consider the triangle below where $B = 3$ ft and $A = 4$ ft. What is the dimension in feet for $C$?

✓ Example 2: Consider the triangle below where $C = 13$ ft and $A = 5$ ft. What is the dimension in feet for $B$?
Exercise:

1. Find the length of the indicated side of the right triangle.

![Diagram of a right triangle with sides 6 and 8 and options A) 10, B) 12, C) 14, D) 2, E) 196]

2. In the rectangle below the length of the diagonal $D$ is

![Diagram of a rectangle with sides 12 and 9 and options A) 6, B) 10, C) 13, D) 15, E) 225]

3. If a 10 ft ladder leans against a wall, and it is given that the distance from the foot of the ladder to the wall is 8 ft. What is the total vertical distance the ladder meet up the wall?

   (A) 2
   (B) 4
   (C) 6
   (D) 36
   (E) $\sqrt{6}$

4. Prove that the hypotenuse $AC$ of the triangle shown on the plane below is $\sqrt{261}$
**Key Focus:** Solving linear equation in two variables by the process of elimination and by the method of substitution.

**Overview of Concepts:**
Here the students learn to solve linear equation in two variables by the process of elimination and by the method of substitution.

**Contents:**
In this section we will only focus on system of linear equations in two variables. We will discuss two methods for solving such equations, later you will be introduced with a third method (Graphically). Practice all the methods they are equally important.

We’ll now explore system of equations algebraically: This Chapter is about solving the system of linear equations containing two equations in two variables using **Addition (Elimination) method** and **Substitution method**.

Fact: The solution of a system of equations is the point of intersection, or the points which are common to both graphs. In other words, the solution a point (x, y) will satisfy both equations. Look at the three figures below:

<table>
<thead>
<tr>
<th>Figure #1</th>
<th>Figure #2</th>
<th>Figure #3</th>
</tr>
</thead>
</table>
| \[
\begin{align*}
  y &= -x + 4 \\
  y &= x - 2 \\
\end{align*}
\]
| Or \[
\begin{align*}
  x + y &= 4 \\
  x - y &= 2 \\
\end{align*}
\]
| In Figure 1, The point at which these lines meet is called a solution to the system of equations. It appears that the lines meet at the point (3,1) in order to verify that (3,1) is a solution to this system of equations we substitute these values into both equations we obtain: \[
\begin{align*}
  1 &= -(3) + 4 \\
  1 &= 3 - 2 \\
\end{align*}
\]
| Since both these statements are true the point (3,1) lies on both lines and is a **unique** solution to this system of equations. |
| \[
\begin{align*}
  y &= 2x + 4 \\
  y &= 2x + 2 \\
\end{align*}
\]
| Or \[
\begin{align*}
  4x - 2y &= -4 \\
  2x - y &= -4 \\
\end{align*}
\]
| Notice there is not solution or point of intersection of these parallel lines, which have the same slope. This system of equation has no solution and the system is called **Inconsistent**. |
| \[
\begin{align*}
  y &= x - 2 \\
  2y &= 2x - 4 \\
\end{align*}
\]
| Or \[
\begin{align*}
  -x + y &= -2 \\
  2x - 2y &= 4 \\
\end{align*}
\]
| In Figure 3, both equations have same graph and hence each and every point is the point of intersection. This system is called **Consistent** and has infinite solutions. |
Method I: Elimination of one Variable by Substitution

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Example</th>
</tr>
</thead>
</table>
| **This method is very useful especially when one variable is expressed in terms of the other in one of the two given equations.** | **Question:** Solve the system of equations:  
\[
\begin{align*}
y &= 3x \\
5x - y &= 20
\end{align*}
\]  
**Solution:** We would substitute (replace) \( y \) by \( 3x \) in the 2nd equation:  
\[
\begin{align*}
5x - y &= 20 \\
5x - (3x) &= 20 \\
2x &= 20 \\
x &= 10
\end{align*}
\] now we substitute (replace) \( x = 10 \) into one of the ‘original equations’ and solve for \( y \):  
\[
\begin{align*}
y &= 3x \\
y &= 3(10) \\
y &= 30
\end{align*}
\] check both values of \( x \) and \( y \) into each ‘original equations’: hence;  
\( x = 10, y = 30 \) or \( (10, 30) \)  

1. Replace one variable in terms of the other; this will reduce the problem to one equation in one variable.  
2. Solve for the remaining variable.  
3. Substitute the now known value in an equation involving both variables, and solve for the second variable.  
4. Check both values in each ‘original equation.’
# Method II: Elimination of one Variable by Addition or Substitution

### Procedure

This method is also very useful; it requires that the coefficient of one of the variables (x or y) be equal in **absolute value**. Your job is to identify or use multiplier(s) to attain what is desired.

1. **By observation:** decide on the variable that is to have equal or opposite coefficients. If necessary we will use multipliers.

2. **We will now add or subtract to combine the equations:** this reduces the pair of equations to one equation in one variable.

3. **Solve for this variable.**

4. **Take this new value and use it to substitute it in one of the 'original' equation:** the second variable can now be solved for.

5. **As always we will check both values in each 'original equation.'**

### Example

<table>
<thead>
<tr>
<th>Question:</th>
<th>Solve the system of equations:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(2x + 3y = -4)</td>
</tr>
<tr>
<td></td>
<td>(5x + 2y = 1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution:</th>
<th>We will multiply the 1(^{st}) equation by 2 and the 2(^{nd}) by 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(2 \updownarrow 2x + 3y = -4)</td>
</tr>
<tr>
<td></td>
<td>(3 \downarrow 5x + 2y = 1)</td>
</tr>
</tbody>
</table>
|           | \[\begin{align*}
|           | 4x + 6y &= -8 \\
|           | 15x + 6y &= 3 \\
|           | -11x + 0y &= -11 \\
|           | -11x &= -11 \\
|           | x &= 1 \\
|           | \end{align*}\]                                                 |
|           | \[\begin{align*}
|           | 2x + 3y &= -4 \\
|           | 2(1) + 3y &= -4 \\
|           | 3y &= -4 - 2 \\
|           | 3y &= -6 \\
|           | y &= -2 \\
|           | \end{align*}\]                                                 |
|           | now we substitute (replace) \(x = 1\) into one of the 'original equations' and solve for \(y\): |
|           | \(\begin{align*}
|           | x &= 1 \\
|           | y &= -2 \\
|           | \end{align*}\)                                                 |
|           | check both values of \(x\) and \(y\) into each 'original equations': |
|           | hence; \(x = 1, y = -2 \) or \((1, -2)\)                      |
**Exercise:** Use an appropriate method to solve the following system of equations

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 1) Solve the system: \[
\begin{align*}
  x + y &= 8 \\
  x - y &= 4 \\
\end{align*}
\] | 2) Solve the system: \[
\begin{align*}
  x + 3y &= 5 \\
  x - 3y &= -7 \\
\end{align*}
\] |
| a) (2, 6) | a) \(x = 2, y = 1\) |  |
| b) (10, −2) | b) \(x = 1, y = 2\) |  |
| c) (6, 2) | c) \(x = -1, y = 3\) |  |
| d) (4, 4) | d) \(x = -1, y = 2\) |  |

| 3) Solve the system: \[
\begin{align*}
  4x + 2y &= 8 \\
  4x - y &= 2 \\
\end{align*}
\] For the \(y\)-value | 4) The value of \(y=\)? \[
\begin{align*}
  3x - 2y &= 1 \\
  2x + 5y &= 7 \\
\end{align*}
\] |
| a) 1 | a) 0 |  |
| b) 2 | b) 1 |  |
| c) −2 | c) 2 |  |
| d) 2.5 | d) −1 |  |

| 5) Find the \(x\)-value of the solution to the system of equations: \[
\begin{align*}
  2x + y &= 3 \\
  5x + 2y &= -3 \\
\end{align*}
\] | 6) Find the \(y\)-value for the system of linear equations: \[
\begin{align*}
  x + 3y &= 2 \\
  3x + 8y &= -4 \\
\end{align*}
\] |
| a) 3 | a) −2 |  |
| b) −1 | b) 10 |  |
| c) 1 | c) 6 |  |
| d) −3 | d) −10 |  |
7) Find the value of $y$
\[
\begin{align*}
3x - 2y &= 1 \\
2x + 5y &= 7
\end{align*}
\]
\[
\begin{array}{llll}
a) 0 \\
b) 1 \\
c) 2 \\
d) -1
\end{array}
\]

8) Find the $y$ value of the solution to the system of equations:
\[
\begin{align*}
x - 3y &= -3 \\
x + 6y &= 15
\end{align*}
\]
\[
\begin{array}{llll}
a) -3 \\
b) 2 \\
c) 3 \\
d) -2
\end{array}
\]

9) Find the $x$ value to the solution of the system of equations:
\[
\begin{align*}
5x + 3y &= 8 \\
3x + y &= 4
\end{align*}
\]
\[
\begin{array}{llll}
a) 1 \\
b) 2 \\
c) -1 \\
d) -2
\end{array}
\]

10) Find the value of $x$?
\[
\begin{align*}
3x - 2y &= 1 \\
2x + 5y &= 7
\end{align*}
\]
\[
\begin{array}{llll}
a) 0 \\
b) 1 \\
c) 2 \\
d) -1
\end{array}
\]
**Key Focus:** Understand the difference between equations and inequalities.

**Overview of Concepts:**

Here the students learn to represent the solution of equations and inequalities. Also comprehend the inclusion of the absolute value symbol.

**Contents:**

**Absolute Value Equations:** An absolute value is defined as the nonnegative equivalent number that is equal numerically to a given real number. Mathematically this is represented as:

\[
|x| = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0 
\end{cases}
\]

- **Example 1:** If \( x = 5 \), then \(|5| = 3\), since \( 3 > 0 \)

- **Example 2:** If \( x = -5 \), then \(|-5| = -(-3) = 3\) since \( 3 < 0 \)
✓ Example 3: solve: \(|x - 1| = 2\)

By definition absolute value leads us to two possible equations:

\[
x - 1 \geq 0 \iff \text{replaced by} \quad x - 1 = 2 \quad x = 3
\]

\[
x - 1 < 0 \iff \text{by subtracting } x - 1 = 2 \quad x = -1
\]

Then we check both values in the original equation:

\[
|x - 1| = 2
\]

\[
|3 - 1| = 2 \quad \text{and} \quad |(-1) - 1| = 2
\]

\[
|2| = 2 \quad \text{and} \quad |-2| = 2
\]

\[
2 = 2
\]

Hence the solution set is \(\{3, -1\}\)

Solving and Graphing Inequalities

As in the case of solving equations, there are certain manipulations of the inequality which do not change the inequality symbol. Here is a list of "permissible" manipulations:

| Rule 1. Adding/subtracting the same number on both sides. | Example: Solve the inequality \(x - 3 < 5\)  
\[
x - 3 < 5 \quad \text{(add 3 to both sides)}
\]
\[
x - 3 + 3 < 5 + 3
\]
\[
x < 8
\] |
| Rule 2. Dividing or multiplying by a positive number | Example: Solve \(4x > 24\)  
\[
4x > 24 \quad \text{(divide both sides by 4)}
\]
\[
\frac{4x}{4} > \frac{24}{4}
\]
\[
\text{Hence} \quad x > 6
\] |

Note: You can change the orientation of the sides of inequality as long it points and opens up to the same values.

Example: \(x < 5\) is all numbers smaller or less than 5 while \(5 > x\) means 5 is bigger than or more than the unknown number \(x\) both these statements are equivalent.
Rule 3
However, when you divide or multiply by a negative number you change the sense of the inequality

Example: \(-x < 7\)
becomes \(\frac{-x}{-1} > \frac{7}{-1}\)

or \(x > -7\)

Note: To understand why to switch the inequality, consider this, while $50 < $70 however \(-$50 > -$70\)
Thus a debt of $50 is a higher value of money than a debt of $70.

Example: Solve the inequality \(2x + 5 < 15\) and graph it on the number line.

The basic strategy for inequalities and equations is the same: isolate \(x\) on one side, and put the number terms on the other side.

\[
2x + 5 < 15 \quad \text{Subtract 5 on both sides}
\]

\[
2x + 5 - 5 < 15 - 5
\]

\[
2x < 10 \quad \text{Divide by 2 both sides}
\]

\[
\frac{2x}{2} < \frac{10}{2}
\]

Hence \(x < 5\)

To graph it we draw the number line and locate 5 we place an unfilled circle at 5. Then we shade in the numbers less or to the left of 5:

\[
-5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5
\]

EXERCISES:

Part-1, Solve

<table>
<thead>
<tr>
<th>#1) Solve (x - 5 &gt; 3)</th>
<th>#2) Solve (-2x \leq 8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (8 &lt; x)</td>
<td>a) (x \leq 10)</td>
</tr>
<tr>
<td>b) (x &gt; 2)</td>
<td>b) (x \leq -6)</td>
</tr>
<tr>
<td>c) (2 &lt; x)</td>
<td>c) (x \leq -4)</td>
</tr>
<tr>
<td>d) (x &gt; 8)</td>
<td>c) (x \geq -4)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>#3) Solve (9y + 8 \geq 7y)</th>
<th>#4) Solve (18 - x &lt; 9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (-4 \leq y)</td>
<td>a) (x &gt; -9)</td>
</tr>
<tr>
<td>b) (4 &gt; y)</td>
<td>b) (x &lt; -9)</td>
</tr>
<tr>
<td>c) (-4 \geq y)</td>
<td>c) (x &lt; -2)</td>
</tr>
<tr>
<td>d) (2 \geq y)</td>
<td>d) (x &gt; 9)</td>
</tr>
</tbody>
</table>
### Part 2, Solve and Graph

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>#1)</strong></td>
<td>Graph the solution.</td>
<td>Graph the solution.</td>
</tr>
<tr>
<td></td>
<td>$8 &lt; x$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>←→</td>
</tr>
<tr>
<td><strong>#2)</strong></td>
<td>Solve and graph the solution.</td>
<td>Graph the solution.</td>
</tr>
<tr>
<td></td>
<td>$7x &gt; -21$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>←→</td>
</tr>
<tr>
<td><strong>#3)</strong></td>
<td>Solve and graph the solution.</td>
<td>Graph the solution.</td>
</tr>
<tr>
<td></td>
<td>$-4x \geq 12$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>←→</td>
</tr>
<tr>
<td><strong>#4)</strong></td>
<td>Graph: “A number is less than or equation eight”</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>←→</td>
</tr>
<tr>
<td><strong>#5)</strong></td>
<td>Solve and graph the solution.</td>
<td>Graph the solution.</td>
</tr>
<tr>
<td></td>
<td>$2x + 5 &lt; 11$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>←→</td>
</tr>
<tr>
<td><strong>#6)</strong></td>
<td>Graph: “Five is greater than a number”</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>←→</td>
</tr>
</tbody>
</table>
#1) Graph the inequality
\[ 9x - 2 > 6x - 5 \]

- a)
- b)
- c)
- d)

#2) Graph the inequality
\[ -x + 5 < 3x - 3 \]

- a)
- b)
- c)
- d)

#3) Graph the inequality:
\[ 5x - 1 > 6x - 3 \]

- a)
- b)
- c)
- d)

#4) Graph the inequality
\[ 5x + 1 > 4x - 2 \]

- a)
- b)
- c)
- d)

#5) Graph the inequality
\[ -x + 2 \geq -2 \]

#6) Graph the inequality
\[ -x + 2 \leq -3 \]
Draw a line to match the sentence on the left with the algebraic expressions, inequalities etc. on the right.

<table>
<thead>
<tr>
<th>Sentence</th>
<th>Algebraic Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 less than a number</td>
<td>( n + 5 )</td>
</tr>
<tr>
<td>Number is less than five</td>
<td>( 5n - 5 )</td>
</tr>
<tr>
<td>A number is greater than five</td>
<td>( 5n )</td>
</tr>
<tr>
<td>Sum of a number a five</td>
<td>( n - 5 )</td>
</tr>
<tr>
<td>The difference between five and a number</td>
<td>( n &lt; 5 )</td>
</tr>
<tr>
<td>The product of five and a number</td>
<td>( 5 - n )</td>
</tr>
<tr>
<td>The quotient of 5 and a number</td>
<td>( \frac{5}{n} )</td>
</tr>
<tr>
<td>Five times a number is decreased by five</td>
<td>( n &gt; 5 )</td>
</tr>
</tbody>
</table>

Give one value of the variable which makes the inequality true

a) \( 4 > x \)

b) \( y \leq -7 \)

c) \( -8 < m \)

d) \( -2 \geq n \)
**Linear Inequalities:** A linear inequality is similar to equality algebraically. However we must make sure whenever we are multiplying or dividing by a negative number the order of the inequality sign must be reversed.

✓ **Example 1:** Find the solution set and graph: 
\[-2x + 4 > 6\]

\[
\begin{align*}
-2x + 4 &> 6 \\
-2x &> 6 - 4 \\
-2x &> 2 \\
x &< \frac{2}{-2} \\
x &< -1
\end{align*}
\]

solution \_set : \{x | x < -1\}

*Note the ‘open hole’ this is very critical, in that it distinguishes between inequality signs, in this particular example (<) \(\rightarrow\) open hole means \(-1\) is not included in the solution set.

✓ **Example 2:** Find the solution set and graph: 
\[|2x + 4| \leq 6\]

(*Use the skills learn when we did absolute values*)

\[
\begin{align*}
2x + 4 &\geq 0 \leftrightarrow replace \\
|2x + 4| &\text{ by} \\
2x + 4 &\rightarrow \\
2x &\leq 6 - 4 \\
2x &\leq 2 \\
x &\leq \frac{2}{2} \\
x &\leq 1
\end{align*}
\]

\[
\begin{align*}
2x + 4 &< 0 \leftrightarrow replace \\
|2x + 4| &\text{ by} \\
- (2x + 4) &\rightarrow \\
- 2x - 4 &\leq 6 \\
- 2x &\leq 6 + 4 \\
x &\geq \frac{10}{-2} \\
x &\geq -5 \leftrightarrow -5 \geq x
\end{align*}
\]

Substitute a value between 1 and \(-5\) in the original equation (let’s pick 0) to check:

\[
\begin{align*}
|2(0) + 4| &\leq 6 \\
4 &\leq 6
\end{align*}
\]

solution \_set : \{x | -5 \leq x \leq 1\}

*Note the ‘filled hole’ this is very critical, in that it distinguishes between inequality signs, in this particular example (\(\leq\)) \(\rightarrow\) filled hole means \(-5\) and 1 is included in the solution set.
Exercise:

1) Graph the inequality:
   a) \(-x + 2 \leq -3\)
   
   b) \(9x - 2 > 6x - 5\)
   
   c) \(2x + 5 < 11\)
   
   d) \(2x + 5 < 11 + 4x\)
   
2) The solution of \(3X - 2 < 4\) is:
   
3) Find the solution set and graph: \(|2X - 1| > 7\)
Key Focus: To describe the coordinate plane and the axes.

Overview of Concepts:

Here the students learn to describe the coordinate plane and the axes.

Contents:

What the Plane and its Axes Look Like
Below is a blank Cartesian plane. It’s pretty useless until something is plotted on it.

The Cartesian plane is also divided in four quadrants. The quadrants are identified as shown below:
**Key Focus:** Identify the x− coordinate and the y− coordinate.

**Overview of Concepts:**

Here the students learn to Identify the x− coordinate and the y− coordinate.

**Contents:**

**Points on a Graph**

A point is the basic relationship displayed on a graph. Each point is defined by a pair of numbers containing two coordinates. A coordinate is one of a set of numbers used to identify the location of a point on a graph. Each point is identified by both an x and a y coordinate. In this unit you will learn how to find both coordinates for any point. You will also learn the correct notation for labeling the coordinates of a point. You will first begin by identifying the x−coordinate of a point.

**Identifying the x−coordinate:** The x−coordinate of a point is the value that tells you how far from the origin the point is on the horizontal, or x−axis. To find the x−coordinate of a point on a graph:

1. Draw a vertical line from the point directly to the x−axis.
2. The number where the line hits the x−axis is the value of the x−coordinate.

The figure above shows a graph with two points, B and D. In this figure:

- The x−coordinate of point B is 100.
- The x−coordinate of point D is 400.

**Identifying the y−coordinate:** As we already mentioned, each point is defined by two coordinates, the x and the y coordinate. Now that you know how to find the x−coordinate...
of a point, you have to be able to find the \( y \)-coordinate. The \( y \)-coordinate of a point is the value that tells you how far from the origin the point is on the vertical, or \( y \)-axis. To find the \( y \)-coordinate of a point on a graph:

1. Draw a horizontal line from the point directly to the \( y \)-axis.

2. The number where the line hits the axis is the value of the \( y \)-coordinate.

Looking back at the graph with our points B and D, we now identify the \( y \)-coordinate for each.

- The \( y \)-coordinate of point B is 400.
- The \( y \)-coordinate of point D is 100.
**Key Focus:** Identify point(s) on the coordinate plane.

**Overview of Concepts:**

Here the students learn to identify point(s) on the coordinate plane.

**Contents:**

**Notation for Identifying Points:** Once you have the coordinates of a point you can use the ordered pair notation for labeling points. The notation is simple. Points are identified by stating their coordinates in the form of (x, y). Note that the x-coordinate always comes first. For example, in the figure we’ve been using, we have identified both the x and y coordinate for each of the points B and D.

- The x-coordinate of point B is 100.
- The y-coordinate of point B is 400.
- (100, 400) are the coordinates of point B
- The x-coordinate of point D is 400.
- The y-coordinate of point D is 100.
- (400, 100) are the coordinates of point D
Points on the Axes: If a point is lying on an axis, you do not need to draw lines to determine the coordinates of the point. In the figure below, point A lies on the y-axis and point C lies on the x-axis. When a point lies on an axis, one of its coordinates must be zero.

♥ Point A—If you look at how far the point is from the origin along the x-axis, the answer is zero. Therefore, the x-coordinate is zero. Any point that lies on the y-axis has an x-coordinate of zero.

♥ If you move along the y-axis to find the y-coordinate, the point is 400 from the origin. The coordinates of point A are (0, 400)

♥ Point C—If you look at how far the point is from the origin along the y-axis, the answer is zero. Therefore, the y-coordinate is zero. Any point that lies on the x-axis has a y-coordinate of zero.

♥ If you move along the x-axis to find the x-coordinate, the point is 200 from the origin. The coordinates of point C are (200, 0)
**Key Focus:** Plot points on the coordinate plane and graphing it.

**Overview of Concepts:**
Here the students learn to Plot points on the coordinate plane and graphing it.

**Contents:**

**Plotting Points on a Graph:** There are times when you are given a point and will need to find its location on a graph. This process is often referred to as plotting a point and uses the same skills as identifying the coordinates of a point on a graph. The process for plotting a point is shown using an example.

✓ **Example 1:** Plot the point (200, 300).

<table>
<thead>
<tr>
<th>Step One</th>
<th>Step Two</th>
<th>Step Three</th>
</tr>
</thead>
<tbody>
<tr>
<td>First, draw a line extending out from the x-axis at the x-coordinate of the point. In our example, this is at 200.</td>
<td>Then, draw a line extending out from the y-axis at the y-coordinate of the point. In our example, this is at 300.</td>
<td>The point where these two lines intersect is at the point we are plotting, (200, 300).</td>
</tr>
</tbody>
</table>

![Step One Diagram](image1)

![Step Two Diagram](image2)

![Step Three Diagram](image3)
Exercise:

1) Use the graph below to answer the four questions for this problem.

a) Which point(s) lie on the x-axis?  

b) What is the y-coordinate of point S?  

c) Point Q = ( , )  

d) Point V = ( , )  

2) Fill in the coordinates of each of the points in the triangle illustrated on the grids below.
Key Focus: Determine algebraically the mid-point of a given line segment.

Overview of Concepts:
Here the students learn to determine algebraically the mid-point of a given line segment. It is important to have the students give the mid-point of a line by inspection, then involve the mid-point formula.

Contents:
A midpoint is a point that denotes the middle of any given line segment. The Midpoint Theorem says the x coordinate of the midpoint is the average of the x coordinates of the endpoints and the y coordinate is the average of the y coordinates of the endpoints.

If a line segment has the end points \((X_1, Y_1)\) and \((X_2, Y_2)\), the midpoint is given by the following formula:

\[
\text{mid - point} = \left( \frac{X_1 + X_2}{2}, \frac{Y_1 + Y_2}{2} \right)
\]

✓ Example 1: Find the coordinates \((x, y)\) of the midpoint of the segment that connects the points \((6, 1)\) and \((2, 5)\):

\[
\text{mid - point} = \left( \frac{6 + 2}{2}, \frac{1 + 5}{2} \right) = \left( \frac{8}{2}, \frac{6}{2} \right) = (4, 3)
\]
Key Focus: Determine algebraically the Distance of a give line segment.

Overview of Concepts:
Here the students learn to determine algebraically the Distance of a give line segment.

Contents:
The distance formula says that the distance between any two points with coordinates $(X_1, Y_1)$ and $(X_2, Y_2)$ is given by the following equation:

$$
\text{Distance} = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}
$$

* As learned earlier in the Pythagorean’s Theorems, you should be able to identify the relationship it has with the distance formula.

✓ Example 1: Find the coordinates $(x, y)$ of the distance of the segment that connects the points $(2, 5)$ and $(6, 2)$:

$$
\text{Dis tan ce} = \sqrt{(6 - 2)^2 + (2 - 5)^2}
$$

$$
\text{Dis tan ce} = \sqrt{(4)^2 + (-3)^2}
$$

$$
\text{Dis tan ce} = \sqrt{16 + 9}
$$

$$
\text{Dis tan ce} = \sqrt{25}
$$

$$
\text{Dis tan ce} = 5
$$
**Key Focus:** Determine algebraically the slope of a line.

**Overview of Concepts:**
Here the students learn to determine algebraically the slope of a line.

**Contents:**
The Slope or Gradient of a line is an indication of how steep a line is. To calculate the slope of a line you need only two points from that line, (X1, Y1) and (X2, Y2). The formula used to calculate the Slope is given below as:

![Slope formula graph](image)

There are three steps in calculating the slope of a straight line when you are not given its equation (We will look at equations soon).

- **Step One:** Identify two points on the line.
- **Step Two:** Select one to be (X₁, Y₁) and the other to be (X₂, Y₂).
- **Step Three:** Use the slope formula to calculate slope.

**Example 1:** Find the slope of the line connecting the points (3, 1) and (6, 7):

\[
Slope = \frac{Y_2 - Y_1}{X_2 - X_1}
\]

\[
Slope = \frac{(7) - (1)}{(6) - (3)} = Slope = \frac{6}{3} = 2
\]
**Key Focus:** Grasp the concept of slopes, relation to the quadrants, and the cases of vertical and horizontal slope

**Overview of Concepts:** Here the students learn to compare patterns of slopes

**Contents:**

<table>
<thead>
<tr>
<th>If the line is sloping upward from left to right, so the slope is positive (+).</th>
<th>If the line is sloping downward from left to right, so the slope is negative (−).</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>If this line is extended longer we would see that it always goes from <strong>quadrant III</strong> to <strong>quadrant I</strong></th>
<th>If this line is extended longer we would see that it always goes from <strong>quadrant IV</strong> to <strong>quadrant II</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
</tbody>
</table>

**When the line is horizontal**

![Diagram](image5)

We can see that no matter what two points we choose, the value of the y-coordinate stays the same; it is always 3. Therefore, the change in y along the line is zero. No matter what the change in x along the line, the slope must always equal zero.

\[
Slope = \left( \frac{0}{"Change in X"} \right) = 0
\]

Zero divided by any number is zero. Horizontal lines have a slope of 0, i.e. The line has **no slope**!

**When the line is vertical**

![Diagram](image6)

In this case, no matter what two points we choose, the value of the x-coordinate stays the same; it is always 2. Therefore, the change in x along the line is zero.

\[
Slope = \left( \frac{"Change in Y"}{0} \right) = \infty
\]

Since we cannot divide by zero, we say the slope of a vertical line is infinite. Vertical lines have an infinite slope, i.e. The slope is basically **undefined**!
**Key Focus:** Define and evaluate the x–intercepts and the y–intercepts.

**Overview of Concepts:** Here the students learn to define and evaluate the x–intercepts and the y–intercepts.

**Contents:**
Intercept as the name suggests is basically where an intersection of two lines occurs. In a graph we generally consider two intercepts the X–Intercept and the Y–Intercept. The X intercept of a straight line is simply where the line crosses the X axis, while the Y intercept of a straight line is simply where the line crosses the Y axis. The graph below illustrates this definition.

Two interesting facts to note about the intercept are as follows:
* The X–Intercept always has its point represented as some value of x but the y value is always “Zero” \( (X, 0) \)
* The Y–Intercept always has its point represented as some value of Y but the X value is always “Zero” \( (0, Y) \)
**Example 1:** Determine the x and y-intercepts of the given graph shown below.

- **X-intercept** ➔ (3, 0)
- **Y-intercept** ➔ (0, 5)
Exercise:

1) What is the Mid-point of a line with endpoints \((-3, 4)\) and \((10, -5)\)?

2) Find the Mid-point of \((1, 1)\) and \((4, 4)\).

3) How far apart are the points \((1, -5)\) and \((-10, 9)\)?

4) What is the distance between \((1, 1)\) and \((4, 7)\)?

5) Determine the slope of the line that passes through the points \((6, -2)\) and \((3, 4)\).

6) A spider crawls on a wall, on its way up the wall the spider had to get over two picture frames one of which was located at a point \((5, 0)\) and the other at \((5, -5)\), what is the slope of the line connecting the two picture frame?
7) Which of the following graph(s) below represent a line with a negative slope:

(A) ![Graph A]

(B) ![Graph B]

(C) ![Graph C]

8) If the slope of a bridge is “zero” then that bridge is:

A. At an incline
B. Horizontal
C. Vertical
D. None of the above

9) Determine the x–intercept and the y–intercept for the given graphs below

a) ![Graph a)

b) ![Graph b)
Key Focus: Identify the classical linear equations in two variables.

Overview of Concepts: Here the students learn to identify the classical linear equations in two variables.

Contents:
Graphing linear equations is pretty simple, but only if you work neatly. If you're messy, you'll only make extra work for yourself, and you'll often get the wrong answer. We'll walk you through a few examples. Follow my pattern, and you should do fine.

The general equation of a straight line is $y = mx + c$ (slope–intercept form), below is a clearly illustrates what this equation actually represents:

![Graph of a straight line](image)

Simply expressed, if you solve for $y$, the coefficient of $x$ is the slope and the constant (number by itself) is the $y$–intercept.

The following example will illustrate the way you go about and develop an equation into a graph; we will discuss two ways of obtaining the graph. Here is an equation we will work on:

**Example:** Graph: $y - 2x = 3$

First we re–write the equation in slope–intercept form: $y = 2x + 3$

Then we observe is the slope and the $y$–intercept: **Slope** = 2; **$y$–intercept** = $(0, 3)$

Now we are ready to develop this equation into a graph:

**Method I**

First we would make a table as shown below:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 2x + 3$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td></td>
<td>(−2,  )</td>
</tr>
<tr>
<td>−1</td>
<td></td>
<td>(−1,  )</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>(0,  )</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>(1,  )</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>(2,  )</td>
</tr>
</tbody>
</table>

**Note:** Since you are given nothing but the equation to start with, you will be responsible to choose the $x$–values (you should pick at least 5 numbers) as
shown in the table above. Choosing small numbers as we have done would reduce the chance of making algebraic error(s)!

Then we would complete the last two columns in the table by substitution as illustrated in the table below:

<table>
<thead>
<tr>
<th>x</th>
<th>( y = 2x + 3 )</th>
<th>(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>( y = 2(-2) + 3 ) ➔</td>
<td>-4 + 3 = -1</td>
</tr>
<tr>
<td>-1</td>
<td>( y = 2(-1) + 3 ) ➔</td>
<td>-2 + 3 = 1</td>
</tr>
<tr>
<td>0</td>
<td>( y = 2(0) + 3 ) ➔</td>
<td>0 + 3 = 3</td>
</tr>
<tr>
<td>1</td>
<td>( y = 2(1) + 3 ) ➔</td>
<td>2 + 3 = 5</td>
</tr>
<tr>
<td>2</td>
<td>( y = 2(2) + 3 ) ➔</td>
<td>4 + 3 = 7</td>
</tr>
</tbody>
</table>

➢ Now that the table is completed our job is based on the skills learned in the beginning of this chapter, i.e. plotting points on the axis. The grids on the below illustrates the plotted points:

➢ Now we’re ready to connect the dots! Connecting the dots will produce a line, which simplifies our definition for linear relationship. Since it’s a straight lines (being "linear" equations), you might as well use your ruler for this part, too! The drawing is your answer. Once you’ve connected your dots, you’re done with the exercise. The grids below illustrates it:
Method II

You probably would say this method is easier (in fact it is!), but method I should still be kept in mind; since method II fails when the line is horizontal or vertical or a line through the origin.

➢ From previous experience we know that all it takes to get a line is two points. So all we have to basically do is find two points and draw a line that connects or passes through those points. It happens that those two unique points are such that they always crosses the axis; “the intercepts”

➢ Using our definition of intercepts we compute its coordinate:

<table>
<thead>
<tr>
<th>x-intercept $\Rightarrow$ $y = 0$</th>
<th>y-intercept $\Rightarrow$ $x = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>So; $0 = 2x + 3 \Rightarrow x = -1.5 \Rightarrow (-1.5, 0)$</td>
<td>So; $y = 2(0) + 3 \Rightarrow y = 3 \Rightarrow (0, 3)$</td>
</tr>
</tbody>
</table>

➢ Now that we have the two points, we plot them and then connect the dots to get a line, as shown below:
Key Focus: Graphing parallel lines.

Overview of Concepts: Here the students learn to identify and graphing parallel lines.

Contents: If the slope of two lines are the same and have different intercepts, then the lines are said to be parallel.

So our primary concern is to identify the slope of the line. Constructing the graph is no different from the previous methods learned. The only difference here is you'll need to construct two lines on the same grids. How do you do this? Simple! Just do each graph independently.

Example: Determine if the linear relationships of $y = 2x + 2$ and $y = 2x - 2$ will produce parallel lines.

From observation we can see that the slopes of both equations are the same, therefore the lines must be parallel by definition. The graph below illustrates the definition:
**Key Focus:** Graphing perpendicular lines.

**Overview of Concepts:** Here the students learn to identify and graphing perpendicular lines.

**Contents:**
If the product of the slope of two lines is \(-1\), then the lines are said to be perpendicular. So our primary concern is to identify the slope of the line. In order to constructing the graph we use the knowledge that was previously learned. The only difference here, as in parallel line, is you’ll need to construct two lines on the same grids. How do you do this? Simple! Just do each graph independently, below is an example of what I am talking about:

**Example:** Determine if the linear relationships of \( y = -2x + 2 \) and \( y = \frac{x}{2} - 2 \) will produce perpendicular lines.

Take a look at the first equation; by observation we see that the slope is \(-2\). Similarly the slope of the second equation is \(\frac{1}{2}\).

Now if we take the product of the slopes we get: \((-2) \times \left(\frac{1}{2}\right) = -1\)

Therefore the lines must be perpendicular by definition.

The graph below illustrates the definition:
**Key Focus:** Graphing System of Linear Equations.

**Overview of Concepts:** Here the students learn to identify and graph system of equations in order to find the solution.

**Contents:**
In this section we will develop another way to find the solution(s) for *System of equations or sometimes called simultaneous equations*. A system of equations refers to a number of equations with an equal number of variables. We will only look at the case of two linear equations in two unknowns. The situation gets much more complex as the number of unknowns increases.

Because we will be focusing primarily on linear equations, the graphs will be lines. This can help us visualize the situation graphically. There are three possibilities:

<table>
<thead>
<tr>
<th>Independent Equations</th>
<th>Dependent Equations</th>
<th>Inconsistent Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lines intersect</td>
<td>Equations describe the same line</td>
<td>Lines do not intersect (Parallel Lines; have the same slope)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>One solution</th>
<th>Infinite number of solutions</th>
<th>No solutions</th>
</tr>
</thead>
</table>

There are three possibilities:

1. **Independent Equations**
   - Lines intersect

2. **Dependent Equations**
   - Equations describe the same line

3. **Inconsistent Equations**
   - Lines do not intersect (Parallel Lines; have the same slope)
✓ **Example:** What is the solution set for the representation illustrated below?

![Graph showing two intersecting lines]

In order to determine the solution, we need to identify where on the graph the two lines intersect (cross each other). At this point we determine the coordinate of that intersected point; this is our solution. See the illustration of obtaining the answer (right).

**Hence the solution is:** \((-2, 1)\)

✓ **Example:** Solve the following pair of equations:

\[
\begin{align*}
y + x &= 2 \\
y &= x
\end{align*}
\]

By following the procedures below you would see that graphing the equation pair is a very simple process, as a matter of fact, let’s just say we did it before!

We would first look at the equations separately, making sure they are in the **“slope–intercept form”**

\[
\begin{align*}
1^{st} \text{ equation:} & \quad y + x = 2 \quad \Rightarrow \quad y = -x + 2 \\
2^{nd} \text{ equation:} & \quad y = x \quad \Rightarrow \quad y = x
\end{align*}
\]

Now that we have both equations in the “slope–intercept form” we proceed with the same knowledge we used when we were graphing lines.

![Graph showing the equations \(y = x\) and \(y = -x + 2\)]

**Hence the solution is:** \((1, 1)\)
Exercise: You would need some decent graph paper to do this assignment

1) Use Graphical method to solve the following system of equations:

   a) \[2x + 3y = 5\]
    \[x + 3y = 7\]

   b) \[x + y = 13\]
    \[x = y + 1\]

2) Graph the equations and describe the relationship: \[Y = 3X - 3\] and \[Y = \frac{-X}{3}\]

3) What is the equation of the line that contains the points with (x, y) coordinates \((-3, 7)\) and \((5, -1)\)?

4) Prove that the equation \(2y = 6x - 4\) is a line, illustrate your answer by constructing an appropriate graph

5) An equation for the line \(L\) is:

   ![Graph showing line L with axes labeled X and Y]
Key Focus: Graphing parabolas

Overview of Concepts: Here the students learn to identify and graph parabolas

Contents:

Example: Graph the equation of \( X^2 - Y = 1 \)

As we have seen in graphing a line, it was a wise decision to compute a few points (approximately 5 points) in the table. In graphing a parabola we would apply the same ideas, below is a table that illustrates this concept.

➢ First we would need to put the equation in Y = “Form”, by doing that we obtain:

\[
Y = X^2 - 1:
\]

<table>
<thead>
<tr>
<th>X</th>
<th>( Y = X^2 - 1 )</th>
<th>(X, Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>( (-3)^2 - 1 )</td>
<td>9 - 1</td>
</tr>
<tr>
<td>-2</td>
<td>( (-2)^2 - 1 )</td>
<td>4 - 1</td>
</tr>
<tr>
<td>-1</td>
<td>( (-1)^2 - 1 )</td>
<td>1 - 1</td>
</tr>
<tr>
<td>0</td>
<td>( 0^2 - 1 )</td>
<td>0 - 1</td>
</tr>
<tr>
<td>1</td>
<td>( 1^2 - 1 )</td>
<td>1 - 1</td>
</tr>
<tr>
<td>2</td>
<td>( 2^2 - 1 )</td>
<td>4 - 1</td>
</tr>
<tr>
<td>3</td>
<td>( 3^2 - 1 )</td>
<td>9 - 1</td>
</tr>
</tbody>
</table>

➢ Now that we have the point we plot them on the grids and connect the dots with a smooth curve that passes through the points, as shown below:
Key Focus: Graphing circles

Overview of Concepts: Here the students learn to identify and graph circles

Contents:

<table>
<thead>
<tr>
<th>Circle with center at origin (0, 0)</th>
<th>Circle with center at point (h, k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation: ( x^2 + y^2 = r^2 )</td>
<td>Equation: ( (x - h)^2 + (y - k)^2 = r^2 )</td>
</tr>
<tr>
<td>Center: (0, 0)</td>
<td>Center: (h, k)</td>
</tr>
<tr>
<td>Radius: r</td>
<td>Radius: r</td>
</tr>
</tbody>
</table>

Example:

1. Graph the equation: \( x^2 + y^2 = 9 \)  
2. Graph the equation: \( (x - 2)^2 + (y - 3)^2 = 4 \)

<table>
<thead>
<tr>
<th>1.</th>
<th>2.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center: (0, 0)</td>
<td>Center: (2, 3)</td>
</tr>
<tr>
<td>Radius: ( \sqrt{9} = 3 )</td>
<td>Radius: ( \sqrt{4} = 2 )</td>
</tr>
</tbody>
</table>
**Key Focus:** Graphing ellipses

**Overview of Concepts:** Here the students learn to identify and graph ellipses

**Contents:**

<table>
<thead>
<tr>
<th>Ellipse with center at origin (0, 0)</th>
<th>Ellipse with center at point (h, k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$</td>
<td>Equation: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$</td>
</tr>
<tr>
<td>Center: (0, 0)</td>
<td>Center: (h, k)</td>
</tr>
<tr>
<td>Where $a \neq b$ and $a, b &gt; 0$</td>
<td>Where $a \neq b$ and $a, b &gt; 0$</td>
</tr>
</tbody>
</table>

**Example:**

1. Graph the equation: $\frac{X^2}{16} + \frac{Y^2}{4} = 1$
   
   ![Graph of ellipses](image1)

2. Graph the equation: $\frac{(X+1)^2}{9} + \frac{(Y+2)^2}{16} = 1$
   
   ![Graph of ellipses](image2)
### SAMPLE FINAL EXAMINATION #1 for ELEMENTARY ALGEBRA

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Simplify $8\sqrt{7} - 9\sqrt{28}$</td>
<td>2) Simplify $\sqrt{2} (\sqrt{8} + \sqrt{2})$</td>
<td></td>
</tr>
<tr>
<td>3) Simplify $\frac{x^7 x^5}{(x^4)^3}$</td>
<td>4) Simplify $\frac{-2a^5 b^{-6}}{8a^{-2} b^2}$</td>
<td></td>
</tr>
<tr>
<td>5) Multiply and write your final answer in the scientific notation. $(1.4 \times 10^5)(9 \times 10^{-8})$</td>
<td>6) Divide and write your final answer in the scientific notation. $\frac{4.2 \times 10^{-3}}{4 \times 10^{-5}}$</td>
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<tr>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td><strong>7)</strong> Simplify</td>
<td><strong>8)</strong> Simplify</td>
<td></td>
</tr>
<tr>
<td>$(4x^2 - 3x + 7) - (5x^2 + x - 8)$</td>
<td>$(a + b)^2 - (a^2 + b^2)$</td>
<td></td>
</tr>
<tr>
<td><strong>9)</strong> Multiply</td>
<td><strong>10)</strong> Divide</td>
<td></td>
</tr>
<tr>
<td>$(3x - 1)(x^2 + 5x - 2)$</td>
<td>$\frac{12x^3y^2 - 18x^2y^3 + 6xy}{-6xy}$</td>
<td></td>
</tr>
<tr>
<td><strong>11)</strong> Factor each of the following</td>
<td><strong>12)</strong> Solve.</td>
<td></td>
</tr>
<tr>
<td>a) $3x^2 - x - 15$</td>
<td>a) $a - 2b = 11$</td>
<td></td>
</tr>
<tr>
<td>b) $16a^2b - 49b^3$</td>
<td>b) $3a + b = 40$</td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td>Description</td>
<td></td>
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<tr>
<td>----------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td><strong>13)</strong></td>
<td>Factor $15ax - 10xb + 3ay - 2by$</td>
<td></td>
</tr>
<tr>
<td><strong>14)</strong></td>
<td>Let 'n' represent a number, then translate the following sentence as an algebraic equation &quot;8 less than 3 times a number is 4&quot;.</td>
<td></td>
</tr>
<tr>
<td><strong>15)</strong></td>
<td>Solve for $m$. $3(5 - m) = 2(m - 7)$</td>
<td></td>
</tr>
<tr>
<td><strong>16)</strong></td>
<td>Solve for $W$. $2L + 2W = P$</td>
<td></td>
</tr>
<tr>
<td><strong>17)</strong></td>
<td>Find all the solution(s). $2y^2 = 98$</td>
<td></td>
</tr>
<tr>
<td><strong>18)</strong></td>
<td>Find all solution(s) to the quadratic equation. $x^2 + 4x - 21 = 0$</td>
<td></td>
</tr>
<tr>
<td><strong>19)</strong></td>
<td>Solve and graph the inequality. $2x - 7 &lt; 5x - 1$</td>
<td></td>
</tr>
<tr>
<td><strong>20)</strong></td>
<td>Find the value of $h(-3)$ $\text{If } h(x) = -x^2 + x + 1$</td>
<td></td>
</tr>
<tr>
<td><strong>21)</strong></td>
<td>Find the length of the missing side of the given right angle triangle.</td>
<td></td>
</tr>
<tr>
<td><strong>22)</strong></td>
<td>Find the equation of the line passing through the points $(-1, 5)$ and $(2, -1)$</td>
<td></td>
</tr>
</tbody>
</table>
### 23) Find the equation of a vertical line passing through a point \((7, 3)\).

### 24) Given the linear equation \(2x - y = -10\)

**a)** Write the equation: \(2x - y = -10\) in slope-intercept form

**b)** Find the \(x\)-intercept

**c)** Find the \(y\)-intercept

**d)** Find the slope of the line represented by the linear equation \(2x - y = -10\)

**e)** Graph the linear equation \(2x - y = -10\)

### 25) On a school trip, 16 students must be accompanied by 2 adults. If 7 adults went on the trip, how many student were there on this trip?

### 26) Julie bought a $600 bracelet on sale for $480. What was the percent decrease?

### 27) Find the equation of a horizontal line passing through a point \((7, 3)\).
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1)</strong> Simplify</td>
<td>$4\sqrt{5} - 11\sqrt{20}$</td>
<td><strong>2)</strong> Simplify</td>
</tr>
<tr>
<td><strong>3)</strong> Simplify</td>
<td>$\frac{x^8x^6}{(x^4)^{-3}}$</td>
<td><strong>4)</strong> Simplify</td>
</tr>
<tr>
<td><strong>5)</strong> Multiply and write your final answer in the scientific notation.</td>
<td>$(5 \times 10^{-5}) (7 \times 10^{-6})$</td>
<td><strong>6)</strong> Divide and write your final answer in the scientific notation.</td>
</tr>
<tr>
<td><strong>7)</strong> Simplify</td>
<td>$(3x^2 - x + 5) - (-6x^2 + 2x - 7)$</td>
<td><strong>8)</strong> Simplify</td>
</tr>
<tr>
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<tr>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td><strong>9)</strong> Multiply</td>
<td><strong>10) Divide</strong></td>
<td></td>
</tr>
<tr>
<td>((a - 4b)(a^2 + ab - 2b^2))</td>
<td>(\frac{8x^4 - 16x^3 - 4x^2}{-4x^2})</td>
<td></td>
</tr>
<tr>
<td><strong>11)</strong> Factor each of the following</td>
<td><strong>12)</strong> Solve</td>
<td></td>
</tr>
<tr>
<td>a) (3x^2 - 16x + 21)</td>
<td>(-2x + 5y = 20)</td>
<td></td>
</tr>
<tr>
<td>b) (9xy^3 - 64x^3y)</td>
<td>(x + 3y = -32)</td>
<td></td>
</tr>
<tr>
<td><strong>13)</strong> Factor</td>
<td><strong>14)</strong> Let ‘n’ represent a number, then translate the following sentence as an algebraic equation. &quot;2 more than a number is 5 less than 3 times the number &quot;.</td>
<td></td>
</tr>
<tr>
<td>(7a^2 + 3ab - 6b^2 - 14ab)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>15)</strong> Solve for (x).</td>
<td><strong>16)</strong> Solve for (n).</td>
<td></td>
</tr>
<tr>
<td>(3(n - 1) + 2 = 5n - 9)</td>
<td>(m = 5n + 2p)</td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td>Equation/Problem</td>
<td></td>
</tr>
<tr>
<td>----------</td>
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</tr>
<tr>
<td>17) Find all the solutions.</td>
<td>$3x^3 = 27x$</td>
<td></td>
</tr>
<tr>
<td>18) Find all solutions to the equation.</td>
<td>$9x^2 - 18x = 0$</td>
<td></td>
</tr>
<tr>
<td>19) Solve and graph the inequality.</td>
<td>$5x + 3 \geq 8x - 6$</td>
<td></td>
</tr>
<tr>
<td>20) Find the value of $s(-1)$, if $s(x) = 3x^2 - 4x +1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21) Find the length of the missing side of the given right triangle.</td>
<td></td>
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</tr>
<tr>
<td>22) Find the equation of the line passing through the points $(0, -4)$ and $(5, -6)$</td>
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</tr>
<tr>
<td><strong>23a</strong>) Find the equation of a vertical line passing through a point ((6, -3)).</td>
<td><strong>24</strong>) Write: (-2x + 3y = -12)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) In slope–intercept form ((y = mx + b))</td>
<td></td>
</tr>
<tr>
<td><strong>23b</strong>) Find the equation of a horizontal line passing through a point ((-5, 1)).</td>
<td>b) Find the (x)–intercept</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Find the (y)–intercept</td>
<td></td>
</tr>
</tbody>
</table>
25) During the special sale, if you buy 2 chocolate bars at the regular price, you will receive three bars free. How many free chocolate bars for free if you bought 8 bars?

24a) Find the slope of the line represented by the linear equation
\[-2x + 3y = -12\]

26) An electronic note book is on sale for $210. Find the percent decrease if the original price of the note book was $280?

24b) Graph the linear equation.
\[-2x + 3y = -12\]