TEACHING REMEDIAL MATHEMATICS FOR CONCEPTUAL UNDERSTANDING: STUDENT RESPONSE

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INTRODUCTION

This paper describes student responses to a voluntary anonymous questionnaire administered at the end of the fall 2009 semester by the author to her students in Hostos' Elementary Algebra course (Math 020). The questionnaire was conducted pursuant to a COBI grant awarded in June 2009 to the author and Professor Olen Dias, also of the Hostos Mathematics Department, for Teaching Remedial Mathematics Through Sense-Making Strategies. Hostos' Institutional Review Board granted its approval for the publication of this article.

At Hostos, Elementary Algebra is aimed at preparing for the second level of the COMPASS, a City University of New York exit examination necessary to graduation and college-level mathematics courses. The author's approach is to teach for understanding by showing the sense behind the rules, with the goal of promoting long-term concept retention rather than short-term rule memorization. As can be seen from the questionnaire (a copy of which is attached), the approach is largely pictorial. The questionnaire explored the students' response to this approach. Student responses are quoted verbatim, without adjustments for grammar, punctuation, and spelling.

ANALYSIS OF RESULTS

The questionnaire addressed four of the multiple topics covered in the course in the order in which those topics were taught: operations with signed numbers (numbers with positive or negative signs); the rules of exponents; scientific notation; and finding the greatest common factor of a list of exponents raised to powers. Of the 30 active students (those still enrolled after the date to withdraw without penalty), 19 (or close to two-thirds of those students) responded to the questionnaire. A description of the results follows.

OPERATIONS WITH SIGNED NUMBERS

All 19 respondents answered this question. One student replied, without elaboration, that he hadn't found the method helpful. Two others responded that the strategy wasn't new to them, although one of these added that "It was simple this way." The remaining responses were entirely favorable. Many emphasized the value of a diagrammatic approach for determining whether the result of an operation with signed numbers should fall on the positive or negative side of the number line (to the right or left of zero). As one student remarked, "it is helpful because it makes it easier to count and understand it better in an illustrated diagram like the number line." Another stated, "I found it helpful because a picture will show you how much places you move either positive or negative." Similar responses include: "it's easier to understand with graphs"; "the graph basically shows you how many spaces in between"; and "the explanation is a clear image." Other responses found it helpful to have operations with signed numbers broken down piece-meal. One student indicated that "I found it helpful because everything is broken down step by step in the explanation," while another said "it explain step by step how to found the larger size and the smaller one."

OPERATIONS WITH EXPONENTS

Of the 19 respondents, 18 answered the question regarding exponents. Of these, one expressed a preference for a hard and fast rule: for multiplying variables, add the exponents; for dividing variables, subtract the exponents. The remaining 17 responses suggested that trying to work out the result by using variables raised to small powers assisted comprehension. Sample responses include: "It gives us a way to see why we have certain results"; this approach "makes the rule self-explanatory"; "you see everything"; "the method show me the step to follow"; and "it helps you not miss a power."

SCIENTIFIC NOTATION

The answers to this question necessitate a recitation of the question. As can be seen from the attached questionnaire, it reads:

Scientific Notation: For determining how to move the decimal point in converting back and forth between Scientific and Standard Notation and whether to use a positive or negative exponent, instead of memorizing rules, we tried to focus on whether the conversion was making the number look bigger or smaller than it actually is. For example, in converting the number 8403 from Standard to Scientific Notation, we would write the number as , because the number is actually larger by three powers of ten than the way it is shown in Scientific Notation. Similarly, if we were converting the number from Scientific to Standard Notation, we would write .008403, because the number is actually smaller by three powers of ten than the way it appears in Scientific Notation. Do you find this type of explanation helpful? Why or why not?

In teaching this topic, I complemented the verbal explanation described above with a pictorial description illustrating movement of the decimal point by the number of powers of ten indicated by the exponent. Nevertheless, the same individual who expressed a preference for a hard and fast rule in the case of operations with exponents expressed the same preference here. In his words (alluding to a conversion from Scientific to Standard Notation but inverting the rule), "some students will understand it much better if it is said, when it is , then the decimal point has to be moved from right to left, and when it's then the decimal point will be moved from left to right." Two additional students, who were enthusiastic about the pictorial methods used for the other three topics, disliked the strategy used for Scientific Notation. One student felt that the approach involved "too much explaining," while the other said, "I am a visual person. I would have to see it broken down."

The remaining respondents favored the approach. A number of students found that the strategy assisted their understanding of decimals. In their words, "it helped me understand decimals, which I disliked"; "the exponents explain how many times the decimal has to be moved"; and "It helps me find if I'm supposed to move negative or positive." One student stated that the approach "Explains by steps how to get the result." Others described the strategy as "a simple explanation that can not be mistaken"; "useful and simple"; and "very easy to remember." Thus, although the sole non-pictorial method described in the questionnaire drew the most unfavorable responses, on balance, the comments were favorable (15/18, or approximately 83%).

GREATEST COMMON FACTOR OF A LIST OF EXPONENTS

As for the strategy shown for finding the greatest common factor of a list of exponents raised to powers, 18 of the 19 respondents answered this question, all favorably. One student found the strategy helpful "because it was broken down visually, which is the best way to explain any type of math." Other comments include: "You see a picture of what you need in your head and like this no mistake is made"; "when you write it out like that you know exactly which is the GCF"; and "I understand that factors are the most numbers or variables that you can take out from all the groups. The detailed explanation helped me a lot." Additional responses include: "it explains how many variables there are"; "the explanation shows how easy it is"; and "I have always gotten confuse with the GCF, this is excellent." One student commented, "By using this rule the students will learn more easy and less stressful. During the semester this was my favorite topic learned."

SUMMARY AND CONCLUSION

In the case of two of the four questionnaire topics (operations with exponents and Scientific Notation), one student expressed a preference for rules rather than sense-making explanations. Scientific Notation, the sole topic on the questionnaire not explained largely pictorially, drew the most criticism. By and large, however, student responses emphasized the benefit of explaining the derivation of the rules. To quote one student, "The professor didn't just give the problems and solved them, she explained the rules and broke it down for us to understand better." Thus, as to those students who remained in the course and who were willing to take the ques-

tionnaire, their responses suggest that teaching remedial mathematics for conceptual understanding in this demanding course by showing the sense behind the rules facilitated both comprehension and appreciation for the subject. The implications of these results are that teaching for sense-making through a combination of pictorial and verbal explanations facilitates both conceptual understanding and subject matter enjoyment.