International Conference on Hydroinformatics

2014

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ANALYSIS OF COMBINED EFFECTS OF INPUT UNCERTAINTY AND PARAMETER UNCERTAINTY IN HYDROLOGICAL MODELLING

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Uncertainty of a hydrological model mainly stems from a lack of understanding and knowledge about the real hydrological process. Input uncertainty and parameter uncertainty are considered to be the two major uncertainty sources of hydrological model. Until now, enormous studies have aimed at calibrating model parameters and estimating model uncertainty. However, these studies mainly ascribe the model output uncertainty to the unknown non-physical parameters. In fact, rainfall, especially of weather radar rainfall, is widely recognized as a main error source. There are seldom studies that aim to explicitly describe model input and parameter uncertainty simultaneously. For this reason, in this study, we investigate the combined effects of radar rainfall uncertainty and parameter uncertainty on the model output. A radar probabilistic quantitative rainfall scheme (Multivariate Distributed Ensemble Generator, MDEG) is integrated with a rainfall-runoff model (Probability Distributed Model, PDM) to calibrate model parameters and estimate the model uncertainty. Finally, the simulated flows, together with their uncertainty bands are compared with the observed flows to evaluate the proposed scheme.

INTRODUCTION

In a hydrological model, model uncertainty mainly stems from a lack of understanding and knowledge about the real hydrological process, including input uncertainty, output uncertainty, model structure uncertainty, state uncertainty (e.g. moisture conditions and snow cover of catchment), sampling uncertainty and parameter uncertainty. The magnitude of input uncertainty varies from each other and depends on the measuring characteristics. For a hydrological model, the major inputs include precipitation, temperature, soil moisture, pressure and wind. The best way to model the input uncertainty is to analyze all the sources of errors associated with the measurement or to investigate the statistical adjustment method based on the historical data. However, it is hard to illustrate all these input uncertainties and their possible connection. So many studies assume error-free data or the input uncertainties are attributed to the parameter uncertainties [1-3]. Few methods have been developed to explicitly account for input uncertainty [4-6]. Nevertheless, it is a commonly assumed that input uncertainty is independent on model structure and model output. The studies regarding input uncertainty as a parameter of model and estimate it according to the model output surely cannot reflect the realistic characteristic of input uncertainty. For this reason, in this study, we aim to model the uncertainty of radar rainfall individually using the long-term radar and gauge historical data. The model is then integrated with a hydrological model to evaluate the effect of input uncertainty and parameter uncertainty on model output.

MODELS AND DATASET

The Probability Distributed Model (PDM) model

The PDM model is a typical rainfall-runoff model proposed by Moore [7]. The most significant point of PDM is that it regards the soil moisture storage capacity as spatially distributed, which is calculated using:

$$F(c) = 1 - \left(1 - \frac{c}{c_{\max}}\right)^b \quad (1)$$

where c_{max} refers to the maximum storage capacity and b is a factor that controls the degree of spatial variability of the storage capacity. There are 13 parameters in the PDM model, which are described in Table 1. More information about the PDM model can be found at Moore [7].

Table 1. Parameters of the PDM model and their initial values in calibrated procedure

Parameter	Initial values	Description
f_c	0.86	Rainfall factor
τ_d	4.2	Time delay
c_{min}	0	Minimum soil moisture store capacity
c_{max}	105	Maximum soil moisture store capacity
b	1.25	Exponent of the soil moisture distribution
b_e	2.5	Exponent of the actual evaporation function
k_g	90000	Ground water recharge time
b_g	2.65	Exponent of the ground water recharge function
S_t	0	Soil tension storage capacity in the recharge function
k_1	4.04	Time constant of the surface routing
k_2	4.04	Time constant of the surface routing
k_b	8.83	Time constant of the ground water storage routing
q_c	0	Constant flow representing returns/abstractions

Study area and dataset

In this paper, radar and rain gauge datasets are collected from the Hydrology Radar Experiment (HYREX) downloaded from British Atmospheric Data Centre (BADC). The Brue catchment in Somerset, south-west England (51.08°N and 2.58°W), is chosen as the experimental catchment for this study. The radar data are from the Wardon Hill radar, located at a range around 40 km from the center of the catchment. The gauge rainfall is collected from a dense network of 49 tipping bucket gauges (TBRs) with 0.2mm resolution. There is a river gauging station located at Lovington. The radar, gauge, flow and temperature data from October 1993 to March 1994 is regarded as the calibration data, while the dataset covering the period of September and October 1999 are used to evaluate the proposed scheme.

METHODOLOGY

The key point of this study is to estimate the parameters of the hydrological model while taking account of input uncertainty at the same time. We achieve this goal by integrating the radar rainfall error model proposed by Dai *et al*. [8] with the generic error model for calibration and uncertainty estimation presented by Götzinger and Bárdossy [6]. This scheme works as follows. Firstly, according to the Multivariate Distributed Ensemble Generator (MDEG), the systematic error (h) and standard deviation of the random error (ε_R) of radar rainfall can be calculated by [8]:

$$h = a_h R^{b_h} \tag{2}$$

$$Std\left[\varepsilon_R\right] = a_\varepsilon R^{b_\varepsilon} \tag{3}$$

where a_h and b_h are the coefficients of the parameterized model for the deterministic component, and a_ε and b_ε are the coefficients for the variance of the random error. In addition, if we assume the variance of the model output uncertainty caused by parameter is proportional to its corresponding sensitivity, the standard deviation ($\varepsilon_{\theta i}$) is able to be estimated by the first-order approximations, which is given as [6]:

$$Std\left[\varepsilon_{\theta_i}(t)\right] = w_i \frac{\partial Q_i}{\partial \theta_i}(t) \tag{4}$$

where w_i is the coefficient, ∂Q_i is the variation of the output flow due to the change of the certain parameter. Thus the overall variance of the model output is:

$$Var\left[\varepsilon_Q(t)\right] = a_\varepsilon R(t)^{b_\varepsilon} + \sum_i Std\left[\varepsilon_{\theta_i}(t)\right]^2 \tag{5}$$

Then we use the maximum likelihood method to obtain the optimized parameters and w_i. It is expressed as:

$$L\left(\theta, w \middle| \mu, \sigma(t)\right) = \prod_{t=1}^{T} \Phi\left(Q_o(t) - Q_s(t)\right) \tag{6}$$

With the estimated parameters, the total variances of the model output for each time step can be estimated.

RESULTS

Based on the proposed method, we use the calibrated dataset to obtain the optimized parameters and apply these values in the validated dataset to generate the uncertainty bands. We firstly solely consider the effect of parameter variations on the model output, and then investigate the combined effects of input radar rainfall and parameters. Table 2 shows the estimated parameters with and without considering input uncertainty.

Table 2. The calibrated parameters of the PDM model

Parameter	Parameter uncertainty	Combined uncertainty
f_c	0.50	0.50
b	0.20	1.36
k_1	16.55	20.00
k_2	16.55	20.00
k_b	36.97	28.96
b_g	5	5
τ_d	2.50	4.66

The uncertainty bands are generated using the calibrated parameter values. Figure 1 shows the band that only considers the parameter variation, while Figure 2 is the one that simultaneously simulates the input and parameter uncertainties. There is a visual agreement between the simulated flows and observed flows. And we observe that the estimated uncertainty bands can encompass the flow measurements for most time steps.

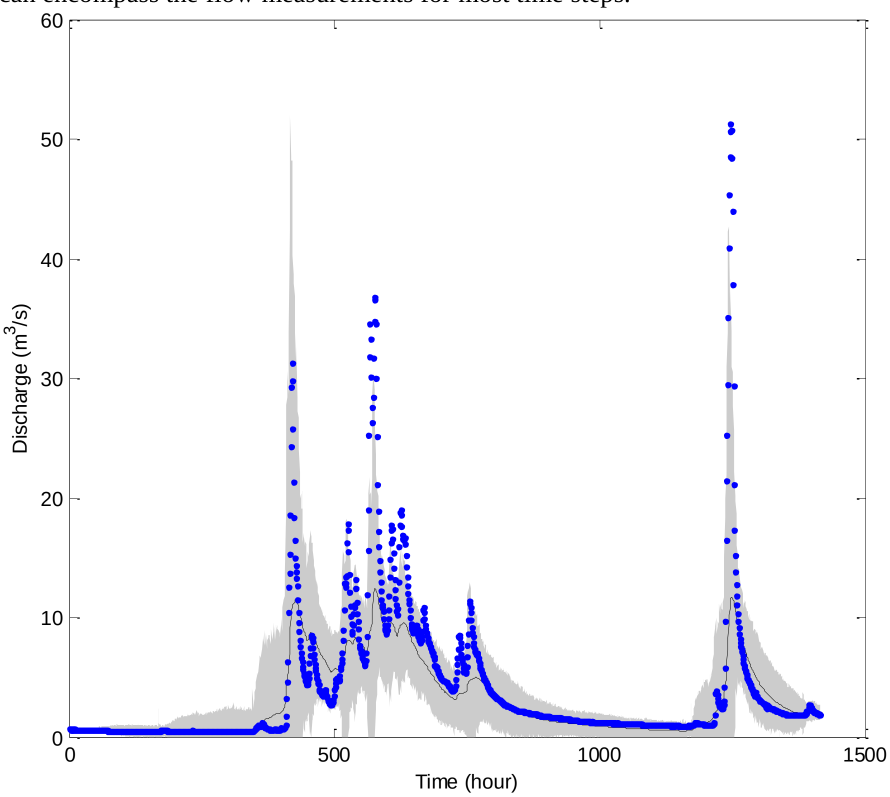

Figure 1. The uncertainty bands of the model output. The blue dots are the observed flow, while the black line refers to the simulated flow.

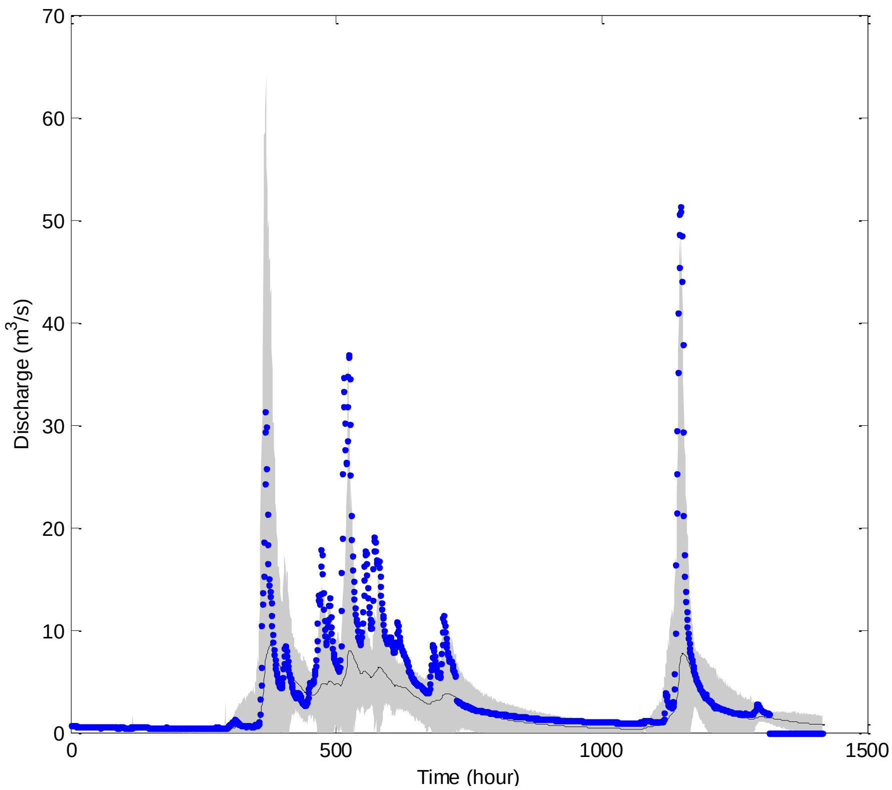

Figure 2. The same as Figure 1, but for the combined input and parameter uncertainties

CONCLUSIONS

This study investigates the combined effects of radar rainfall uncertainty and parameter uncertainty on the model output. The radar rainfall error model MDEG is integrated with a rainfall-runoff model called the PDM to calibrate model parameters and estimate the model uncertainty. The fact that there is a visual agreement between the simulated flows and observed flows and the estimated uncertainty bands can encompass the flow measurements for most time steps proves the accuracy and practicality of the proposed scheme.

ACKNOWLEDGMENTS

The first author would like to thank the University of Bristol and China Scholarship Council for providing the necessary support and funding for this research. The authors would like to thank BADC for providing the data sets used in this study.

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