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### **Predicting intermediate and multiple conclusions on predicate-logic reasoning problems : further investigation of a theory of mental logic**

Jasminka Grgas  
*Baruch College*

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# **Predicting Intermediate and Multiple Conclusions on Predicate-Logic Reasoning Problems: Further Investigation of a Theory of Mental Logic**

by  
**Jasminka Grgas<sup>©</sup>**

Baruch College of the City University of New York

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Jasminka

## **Predicting Intermediate and Multiple Conclusions on Predicate-Logic Reasoning Problems: Further Investigation of a Theory of Mental Logic**

The mental-logic theory (ML theory) proposed by Braine and O'Brien (e.g., 1991, 1998) consists of the two parallel models--a mental propositional logic and its extension to a

mental predicate logic (Braine & O'Brien, 1998). The mental propositional logic addresses inferences that can be drawn on the basis of logic particles such as those expressed with English language words such as *and*, *or*, and *not*. The mental predicate logic provides further analyses of the internal composition of propositions, including predicate/argument structure as well as quantifiers (e.g., *all*, *some*, *none*) and a way of representing their scope.

The research reported here was designed to provide additional empirical support for the mental predicate logic. The logic inferences investigated are claimed to be made both in reasoning and in discourse processing, and since they are made routinely and easily, especially in discourse processing, people often do not recognize that they are making any inferences at all. The logic inferences are based on the meanings of English-language particles and quantifiers such as *if*, *and*, *or*, *not*, *all*, *some*. ML theory proposes that the meanings of these particles and quantifiers are given by the inferences that they sanction.

The theory consists of a core and a pragmatic part. The core part includes a set of inference schemas and a reasoning program that applies the schemas in lines of reasoning. The ML inference schemas differ from the sorts of schemas that are found in standard logic books in several ways, e.g., they allow concatenation of more than two constituents, but for simplicity of presentation the schemas are described here in a simpler form. In addition, in standard logic anything follows from contradictory premises, whereas in mental logic nothing would follow from contradictory premises, except a judgment that something is wrong. The pragmatic (noncore) part of the theory is concerned with pragmatic principles that are involved in premise interpretation and to make inferences that go beyond those made by the ML inference schemas (e.g., invited pragmatic schemas). This part is not relevant to the experiments reported here, and it will not be discussed further.

The theory makes a distinction between the following types of schemas: core schemas, feeder schemas, incompatibility schemas, and some others. People are usually more aware of the output of the core schemas and apply them more freely than those of the feeder schemas. The core schemas are applied when premises of the requisite form are active in working memory and the premises are considered true (can be treated as assumptions). The feeder schemas are applied when their output satisfies the conditions of application of a core schema.

In the partial list below, those schemas that are involved in investigation reported here are presented. For each schema the propositional-level version is given in the first row, followed by corresponding predicate logic version(s). The notation is illustrated and explained following the first three schemas.

*Core Schemas:*

**(1)  $p \text{ or } q; \sim p / \therefore q$**

**$S1[\text{All } X] \text{ OR } S2[\text{PRO-All } X]; \text{ NEG } S2[\alpha]; [\alpha] \supseteq [X] / \therefore S1[\alpha]$**

**S1[All X] OR S2[PRO-All X] / ∴S2[All X: NEG S1[PRO]]**

Schema 1 is a disjunction-elimination schema: When one of two alternatives is false, the other must be true. The first of the predicate-logic versions can be rendered in English as "All of the Xs satisfy predicate S 1 or they satisfy S2; some particular object or set of objects,  $\alpha$ , does not satisfy S2;  $\alpha$  is included among the Xs; one can conclude that  $\alpha$  satisfies S 1." (The "PRO" notation usually is realized as a pronoun. This way of treating quantificational scope differs from standard logic and is closer to the structures of natural languages. For discussion of the notational system, see Braine, in press.) The second predicate-logic version can be rendered as "All of the Xs satisfy predicate S 1 or they satisfy S2; one can conclude that all of the Xs such that they do not satisfy S 1 satisfy S2." An example of a problem of the sort discussed later that uses the first predicate logic version of this schema (referring to a beads of various colors, shapes, sizes, etc.) presents *All of the beads are green or they are small and the round beads are not small*; application of the schema leads to the inference that *the round beads are green*.

**(2) If p THEN q; p / ∴q**

**S[All X]; [ $\alpha$ ]  $\supseteq$ [X] / ∴S1[ $\alpha$ ]**

**NEG S[~Some X~]; [ $\alpha$ ]  $\supseteq$ [X] / ∴NEG S[ $\alpha$ ]**

At the propositional level Schema 2 is standard logic's modus ponens. The first of its predicate logic versions can be rendered as "All of the Xs satisfy S; some particular object or set of objects,  $\alpha$ , is among the Xs; it can be concluded that a satisfies S. The second can be rendered as "There is no X that satisfies S; some particular object,  $\alpha$ , is included among the Xs; it can be concluded that a does not satisfy S." (The tildes around "Some X" indicate that it is within the scope of the negation and can be instantiated. "NEG S[Some X]" would indicate that "some X is not S." One could not then conclude that a is not an X. (Note that the meaning of the quantifier is given by the inferences about instantiation, i.e., which objects can or cannot satisfy the predicate.) An example of a problem that uses the second predicate-logic versions of the schema (referring to some children in a school) has as premises *None of the children wearing red shins are playing basketball* and *all the boys are wearing red shirts* leads to the conclusion *that the boys are not playing basketball*.

**(3) ~(p & q); p / ∴~q**

**NEG E[~Some X : S1[PRO-ALL X] & S2[PRO]~]; S2[ $\alpha$ ]; [ $\alpha$ ]  $\supseteq$ [X] / ∴  
NEG S1[ $\alpha$ ]**

**NEG(S1[All X] & S2[PRO-All X] / ∴NEG S2[All X: S1[PRO]]**

Schema 3 concerns negative-conjunction elimination. The first predicate-logic version can be rendered "There is not some X such that it satisfies S1 and satisfies S2; some particular object,  $\alpha$ , satisfies S2;  $\alpha$  is included among the Xs; one can conclude that  $\alpha$  does not satisfy S1. The second predicate-logic version can be rendered "Not all of the Xs

satisfy both S1 and S2; one can conclude that the Xs that satisfy S 1 do not satisfy S2." An example of a problem that uses the propositional-level version of this schema (referring to a box containing toy animals) has as premises *It is false that there is both a camel and a monkey in the box and there is a camel*; one can infer that *there is not a monkey in the box*.

(4) **p OR q; If p THEN r; If q THEN r / ∴r**

**S1[All X] OR S2[PRO-All X]; S3[All X: S1[PRO]]; S3[All X: S2[PRO]] / ∴S3[All X]**

(5) **p OR q; If p THEN r; If q THEN s / ∴r OR s**

**S1[All X] OR S2[Pro-All X]; S3[All X: S1[PRO]]; S4[All X; S2[PRO]] / ∴S3[All X] OR S4[PRO-All X]**

*Principal Feeder Schemas:*

(6) **p; q / ∴p & q**

**S1[All X]; S2[All X] / ∴S1 [All X] & S2[PRO-All X]**

(7) **p & q / ∴p**

**S1[Q X] & S2[PRO-Q X] / ∴S2 [Q X]**

**(Q refers to any quantifier, e.g. *all, some, many, few*).**

*Incompatibility Schemas:*

(8) **p; ~p / ∴incompatible**

**S[All X]; NEG S[Q X] / ∴incompatible**

**S[Q X]; NEG S[All X] / ∴incompatible**

(9) **p or q; ~p & ~q / ∴incompatible**

**S[All X] OR S2[All X]; NEG S1[Q X] AND NEG S2[PRO-Q X] / ∴ incompatible**

**S[Q X] OR S2[PRO-Q X]; NEG S1[All X] & NEG S2[All X] / ∴ incompatible**

The reasoning program that implements the inference schemas includes a direct reasoning routine (DRR) and some indirect reasoning strategies that go beyond the DRR. The theory predicts that inferences made through application of the DRR are essentially

available to everyone and are made routinely and effortlessly. The DRR is considered to be the first facility that is used in logical reasoning and it consists of three parts. A preliminary procedure determines if there is a conclusion to be evaluated. If there is a tentative conclusion of the form *if-then*, the preliminary procedure adds its antecedent to the premise set and treats its consequent as a conclusion to be evaluated. An evaluation procedure leads to either a "true" or "false" response. A "true" response results from conclusion being in the premise set being inferred from the premise set by application of one or a combination of the schemas. The "false" response is made when a proposition reached that is incompatible on Schemas 8 or 9 with a premise or with a proposition that has been inferred. An inference procedure applies any core schema whenever its conditions have been met, i.e., whenever its requisite propositions are considered conjointly in working memory; the feeder schemas are applied only when their output would provide for the conditionals of a core schema to be met (or a possible one-time application to feed a conclusion). Finally, when a topic set is present (either because of some strategic consideration or because it has been provided), any core schema that makes an inference about that topic is applied. Neither the schemas nor the reasoning program provide any means for making indeterminacy judgments, i.e., that the truth or falsity of some conclusion is uncertain given a set of premises, and the schemas involved in making incompatibility judgments are not sufficient for judging the consistency of large or complex premise sets.

Unlike the DRR, the indirect reasoning strategies are not claimed to be universally available and their application requires effort (although Braine, Reiser, and Romain, 1984, reported that some strategies are available to many college students, and are presumed to be available in other populations). Consequently, ML theory predicts that inferences requiring any of the indirect-reasoning strategies would be made far less often than those that follow from DRR. The indirect-reasoning strategies are not described here because they are not required on any of the problems reported here.

Several sorts of supportive evidence have been reported to support ML theory, although most of the investigations have addressed only the propositional-level of the theory: The theory has predicted successfully which reasoning problems people solve, the perceived relative difficulties of those problems, the order in which intermediate inferences are made in lines of reasoning, which logical inferences are made routinely and effortlessly in text comprehension, and has established that those inferences are made on line as the information enters working memory.

The data reported by Braine et al. (1984) clearly support the most basic prediction of ML theory, i.e., that inferences that follow from application of the DRR will be made routinely, and those requiring reasoning resources beyond the DRR will be made far less often. Participants were presented with two types of problems: Fifty-four problems were solvable by application of the propositional schemas and the DRR, and another 19 problems required reasoning strategies that go beyond the DRR. Each problem presented a set of premises together with a conclusion to be evaluated as true or false. To minimize potential content interference with solution, all problems referred to letters written on an imaginary blackboard (e.g., "If there is an F on the blackboard, there is a W."). Errors

were not significantly associated with problem length, and as was expected, almost no errors were made on the direct-reasoning problems. On the problems that required more sophisticated reasoning strategies, however, errors often were made. Subsequent investigations (e.g., Braine, O'Brien, Noveck, Samuels, Fisch, Lea and Yang, 1995; O'Brien, Braine, and Yang, 1994) provided further evidence for ML theory. In these investigations participants were able to make the predicted inferences both when the problems were presented with conclusions to be evaluated, or with just premises from which participants were asked to write down everything that follows, without any conclusion to be evaluated. Again, as predicted, very few errors were made on direct reasoning problems.

Braine et al. (1984) provided an additional sort of evidence to support the claim that not only were their direct-reasoning problems being solved, but that they were being solved in the way described by the DRR. The participants were directed to rate the perceived relative difficulty of each problem on a Lichert-type scale, and Braine et al. constructed a regression model from the perceived-difficulty rating data that assigned a weight to each schema. This enabled prediction of the difficulty of each problem (as being equal to the sum of the weights of each schema required for problem solution as predicted by the DRR). For example, a problem with premises of the form *p or q, If q then r, not both r and s, and not p*, and requiring evaluation of *not s* would lead first to the application of Schema 1 to the first and last of the premises, which yields *q*, then to application of Schema 2, which yields *r*, and finally to application of Schema 3, which yields *not s*; the predicted difficulty of this problem is the sum of the difficulty weights for Schemas 1, 2, and 3. Correlations between predicted and observed difficulties accounted for 66% of the variance (53% with problem length partialled out), even when the weights were obtained with one set of problems and the observed ratings were obtained with another set of problems and different participants. Yang, Braine, and O'Brien (1998) conducted a similar investigation of direct-reasoning predicate-logic problems. Again, almost no errors were made in assessing the conclusions and again the ratings predicted by the schema weights correlated highly with the observed rating (69% of the variance; 56% when problem length was partialled out). even when observed ratings came from new problems and different participants than those used to generate the schema weights.

The sort of evidence provided by Braine et al. (1984) and Yang et al. (1998) is supportive of the mental-logic account, but only indirectly addresses whether participants were constructing the predicted lines of reasoning. A more direct sort of evidence has been reported for the propositional-level schemas by Braine et al. (1995) and O'Brien et al. (1994). In these studies, participants were asked to write down every step in their reasoning process, i.e., to write things down in the order that they figured things out. Some problems presented conclusions to be evaluated and participants were asked to write down everything they figured out on the way to their final judgment; other problems presented only premises and on these problems participants were asked to write down everything they could figure out from the premises in the order that they figured things out.

Consider a problem presented in O'Brien et al. (1994), referring to letters written on an imaginary blackboard, with premises of the form *N or P*; *not N*, *if P then H*, *if H then Z*, and *not both Z and S*. The DRR applies Schema 3 to the first two premises to infer P, which then leads with the third premise to application of Schema 7 to infer H, which then leads with the fourth premise to application of Schema 7 to infer Z, and, finally, with the fifth premise, to application of Schema 4 to infer not S. Now consider another problem from O'Brien et al. with the same premises presented in the reverse order: *not both Z and S*, *if H then Z*, *if P then H*, *not N*, and *N or P*. The DRR is unable to apply any of the core schemas until all of the premises have been read, applying Schema 4 to premises 4 and 5 to infer P, then applying Schema 7 to infer H, then Schema 7 to infer Z, then Schema 3 to infer not S. Note that the DRR leads to the same sequence of intermediate inferences and to the same final conclusion on both problems. (A reasoner might use strategies that go beyond the DRR on the latter problem, for example first inferring *If H then not S*, but this does not lead to any additional inferences, and O'Brien et al. found that the only commonly made inferences were those predicted by the DRR). The order of predicted inferences is determined by the order in which the Core schemas become available (not by the order in which the premises are presented), and O'Brien et al. found that the order in which participants wrote down inferences on both problems corresponded to those predicted by the DRR.

Several investigations have provided evidence for the mental-logic inferences in text processing (e.g., Lea, O'Brien, Fisch, Noveck, & Braine, 1990; Lea, 1995), finding that the core inferences are made routinely when their premises are embedded within short story vignettes; further, these inferences are made so easily that people usually do not realize that any inferences are being made at all. Unlike other sorts of inferences made while reading, e.g., inferences from story grammars, scripts, etc., which are made only when they are bridging inferences, i.e., required to maintain textual coherence, the mental-logic inferences are made so long as their requisite premises are held conjointly in working memory.

There is, thus, an abundance of evidence in favor of the predictions of ML theory, but to date only the Yang et al. (1998) studies described earlier have assessed the predicate-logic schemas, and those studies provided only indirect evidence that the predicate-logic reasoning problems were being solved using the lines of reasoning predicted by the DRR and the mental predicate-logic schemas. The motivation for the experiments reported here was to provide some direct evidence for the lines of reasoning predicted for such problems. The basic strategy was adopted from Braine et al. (1995) and O'Brien et al. (1994). As described earlier, those studies presented premises and required participants to write down everything that could be figured out from the premises in the order in which things were figured out. The problems presented here similarly required that each step in the reasoning processes be written down.

For the predicate-logic problems presented here, participants were asked to write down everything about the topic set that could be figured out from the premises. Table 1 shows the line of inferences that are predicted by the DRR for Problem Set 1. These problems were designed to be maximally simple, in that the inferences predicted by the DRR could

be applied as each premise was read, i.e., the problems were constructed so that inferences could be made in the same order as the premises were presented. Problem Set 2 was identical, except that the order in which the premises were presented was random. (Table 1 shows the order in which the premises of Problem Set 2 were presented.) It was predicted that the line of inferences written down on these problems would not differ from those predicted for Problem Set 1. This prediction follows from the principle that the order of inferences will be governed by the availability of inference schemas rather than by the order in which premises are encountered.

Experiment 2 replicated the problem forms presented in Experiment 1; the problem content differed, however, between the two experiments. Whereas the problems in Experiment 1 concerned beads of various shapes, sizes, patterns, etc., the problems in Experiment 2 concerned the actions and attributes of various groups of children. It was predicted that the lines of inferences would not be altered by the change of problem content.

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## **Experiment 1**

### **Method**

#### **Participants.**

Fifty undergraduate students who were enrolled in an introductory psychology course at Baruch College participated to fulfill a course requirement. Twenty six of the participants received Problem Set 1, and 24 received Problem Set 2. Eleven of the participants either did not follow instructions or failed to respond to every problem, and data from these are not included in the reported results, leaving data from 21 participants for Problem Set 1 and 18 for Problem Set 2.

#### **Tasks and Procedures.**

Twenty predicate-logic reasoning problems were constructed to constitute Problem Set 1. The problems were constructed so that the predicate-logic schemas can be applied as the premises were read. For example, Problem 1 (see Table 1) allows application of Schema 3 as soon as the first two premises are read. This allows Schema 2 to be applied when the third premise is read, and then to Schema 3 as the fourth premise is read. Participants were told that the problems referred to some beads made by a bead manufacturer. The beads have various colors (for example, some are red, some are blue, some green), various shapes (for example, some are round, some are square, some are triangular), various materials (for example, some are plastic, some are metal, some are wooden), and various patterns (for example, some are striped, some are plain). Each problem referred to the beads in a particular bag. Each problem presented some facts about the beads in that bag, and then asks a question about what you can figure out from the facts. The facts were presented first, and then, below a line, the question was presented. Participants were

told to write their answers in the space below the question, which asked them to write down, in the order they figured things out, everything that they could figure out about a topic that was presented. The problems, their topics, and the predicted lines of reasoning are presented in Table 1. A second set of problems (Problem Set 2) was constructed that was identical to the set shown in Table 1, except that the premises were presented in random order, thus requiring participants to search for the premises that allow application of a schema. Table 1 indicates within parentheses following each premise the order in which the premises were presented in Problem Set 2.

The task was administered in small groups ( $n < 10$  per group). Each participant was presented one set of problems. Order of problems within each problem set was randomized, with two random orders constructed. Participants were assigned randomly to problem sets and problem orders.

## **Results and Discussion**

In scoring participant's responses the following guidelines were used. First, some participants occasionally wrote down premises. Since these responses did not seem to be activated by any particular circumstances, and could not be counted for or against ML theory or other theories, they were omitted from all tallies. Second, participants infrequently would repeat previously made responses, and since they were already scored they were ignored second time. Third, occasionally a participant would write down an inference with the form if-then, where the if-clause was either a premise or a previously written-down inference. In these cases, the if-clause seemed to be stating a reason for inferring the then-clause, so only the then-clause was included in the scoring. Fourth, in a few cases responses deviated from predicted response only by the inclusion or omission of and. For example participants would occasionally write down predicted inferences conjoined with a premise or the output of another inference, or in instances in which the model predicted a conjunction, participants sometimes wrote down the components of the conjunction on separate lines. Those responses account for the optional one-time use of feeder schemas at the readout stage and were not listed separately. Finally, some subjects tended to write down negative inferences, e.g., "the large beads are not red," by enumerating the possible positive compliments, e.g., "the large beads are green, or blue, and so forth." Such responses were scored as negative inferences.

The responses obtained from the participants were compared to the predicted responses listed in Table 1. For each predicted inference in Table 1, the proportion of participants who wrote down that inference is indicated (Problem Set 1 first, followed by Premise Set 2). For the 20 problems, ML theory predicts that 51 inferences would be written down. (ML theory predicts that the output of the core schemas applied by the DRR would be written down; previous investigations have reported that the output of the feeder schemas is not typically written down, as these inferences are thought to be paraphrases rather than inferences. For the 20 problems, the theory predicts application 1 core schemas 51 times.) For the 21 participants receiving Problem Set 1, this leads to prediction of 1071 responses (i.e., inferences written down), of which 76% were written down.

Inspection of Table 1 reveals that the proportions with which predicted responses were made were not equal across all problems and inferences. For example, on several problems some of the earlier inferences in the predicted lines of reasoning tended to be written down less often than the final inference (e.g., problems 2, 5, 7, 11, 17, 18, and 19). For the most part the intermediate inferences that were not written down involved schemas 1, 2, and 3 when they were applied early in a line of reasoning; inferences made from application of the same schemas as the last inference in a line of reasoning were almost always written down. Thus, failure to write down such inferences early in a line of reasoning does not seem to indicate that the inferences were not made, but rather to indicate that they seemed less important than the final output of the reasoning processes. This interpretation is supported by the fact that over 95% of multiple inferences were written down in the predicted order, suggesting that participants were constructing the lines of reasoning that were predicted, but failed to write down every step in the processes.

For Problem Set 2 with 18 participants the theory predicts 918 responses, of which 80% were written down. Inspection of Table 1 reveals that the data for Problem Set 2 were extremely similar to those for Problem Set I. Most striking is the fact that the order of inferences written down were consistent with those predicted by the DRR (94.25% of the time), even though the premises were not presented in an order that was consistent with such output unless the reasoning process was guided by the availability of the schemas rather than by the order of premise input. It is not obvious what theoretical account could be provided for this consistency of output ordering except for the schema-availability account provided by ML theory.

The only sort of problems on which participants did not conform consistently with the predicted output of ML theory were those that required application of schema 5. Even so, on these problems (problems 4 and 9) a majority of participants wrote down the lines of reasoning predicted by ML theory, although a large minority did not. Most of those participants who did not write down the Predicted lines of reasoning on these two problems instead wrote down lines of reasoning that were consistent with a supposition-of-alternative strategy. This strategy, described above, constructs two suppositional lines of reasoning, one under each of the two alternatives of a disjunctive premise. On problem 3, for example, this sort of line of reasoning results in the intermediate inference that *the beads are wooden and square or mental and triangular rather than the beads are square or triangular*. For Problem Set 1, such inferences constituted 29% of the intermediate inferences on problem 3 and 38% on problem 9, and taken together with the output predicted by the DRR, they made up 91% and 95% of the responses to problems 3 and 9, respectively. For Problem Set 2, such inferences constituted 44% of the intermediate inferences on problem 3 and 22% on problem 9, and taken together with the intermediate inferences predicted by the DRR, they constituted 66% and 89% of the intermediate inferences on problems 3 and 9, respectively.

In summary, participants made the vast majority of both the intermediate and final inferences predicted by ML theory. More importantly, these inferences were almost

always made in the predicted order, even when the premises were not presented in an order that by itself was conducive to such output.

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## **Experiment 2**

### **Method**

#### **Participants.**

Fifty-two undergraduate students who were enrolled in an introductory psychology course at Baruch College participated to fulfill a course requirement. Several participants either did not provide responses to all problems or did not follow instructions, and their data are not included, leaving a total of 21 participants for Problem Set 1 and 20 participants for Problem Set 2.

#### **Tasks and Procedures.**

The problems were identical in logical form to those in Experiment 1, but with different content. Unlike the problems of Experiment 1, which referred to beads in a bag, the problems in Experiment 2 presented narrative information about different groups of children in Brazil. (By placing the children in the stories in an unfamiliar society, participants should be less likely to import information from long-term memory into their lines of reasoning.) Participants were told that the children are in different places, are wearing different sorts of clothes, are doing different sorts of things, and so forth. Each problem presents some facts about the particular group of children for that problem. Each problem presented a topic, and participants were told to write down everything they could figure out about that topic from the facts in the order that they figured things out. The premises for the problems and the predicted inferences are shown in Table 2.

### **Results and Discussion**

The scoring guidelines were same as those used in Experiment 1. Table 2 shows the proportions with which each of the predicted inferences for each problem were given for the problems both in Problem Set 1 and Problem Set 2. For Problem Set 1 a total of 1,071 inferences were predicted (51 inferences x 21 participants), of which 76% were written down, and for Problem Set 2 a total of 1020 inferences were predicted (51 inferences x 20 participants), of which 84% were written down.

Inspection of Table 2 reveals a pattern of responses that is quite similar to those of Experiment 1. Comparisons of responses that were written down and inferences that were predicted were not distributed equally across problems and inferences, and as in Experiment 1 participants often failed to write down inferences early in lines of reasoning, but almost always included final inferences, and this was the case also in Experiment 2. Again, the strongest evidence that participants were making inferences in

the order predicted by the DRR was that 97% of multiple inferences were written down in the order predicted in Problem Set 1, and 96% of multiple inferences were written down in the predicted order for the problems in Problem Set 2, where such an order was at variance with the presented premise order. Thus, for the problems in Experiment 2, as well as for their formal parallels in Experiment 1, the order in which inferences were written down was predicted successfully by the availability of the schemas rather than by the order in which information was presented.

As in Experiment 1, on those problems requiring application of schema 5, e.g., problems 3 and 9, a large number of participants revealed lines of reasoning that went beyond what is available on the DRR, instead writing down inferences that are consistent with a supposition-of-alternatives strategy. For example, problem 3 led to *the older children are wearing red shirts and selling Jornal do Brasil or they are wearing blue shirts and selling O Globo*. Given that Braine et al. (1995) did not report the use of such a strategy on problems requiring schema 5 when the problems were presented at the propositional rather than predicate level, the question is raised as to whether the greater complexity of the information in the predicate-logic level encourages reasoners to keep track of the information more carefully, and following the supposition-of-alternatives strategy allows just this.

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## **Contents of the "Other" Responses in Experiments 1 and 2**

Of course, not everything written down was an inference predicted by ML theory. Knowing what metric to use to assess how many nonpredicted inferences were written down is problematic, for the possibilities concerning what could be written down, and how things could be written down, was undefined. Some participants went beyond writing down inferences that depend on the logic particles and quantifiers. For example, one participant responded to problem 3 of Problem Set 2 by developing a narrative in which the red and blue shirts worn by children selling the two sorts of newspapers were colors signifying two drug gangs, "like the Bloods and the Crips," and the two newspapers were a code for different drags they were dealing. Inclusion of such extralogical inferences was not included in the tabulations presented in Tables 1 and 2, and such inferences are not germane to the question of whether the predicted inferences are made. ML theory proposes that the inferences made from application of the schemas can cohabit in the same lines of reasoning with inferences from a variety of other sources, such as those following from scripts, story grammars, and so forth, and there is nothing in making such extralogical inferences that bears on whether the logic inferences are being made. (Indeed, the participant who wrote down that the colors signified gang affiliations also made the inferences predicted by ML theory.) How often such inferences were made is difficult to quantify, because there is no theory about them. How many inferences, for example, should be counted when someone writes down that the shirts designate different gangs selling different drags? Such inferences, however, clearly were made much less often than those counted as predicted by ML theory that were counted in Tables 1 and 2.

One possible source of nonpredicted inferences that were made concerns invited inferences and conversational implicatures, e.g., interpreting disjunction as exclusive rather than inclusive, or converting propositions of the form *All P are Q to All Q are P*. Although such inferences are the focus of much attention in the reasoning literature (see discussions in Braine & O'Brien, 1998), they were relatively rare in the protocols here. Another possible source of nonpredicted responses would be standard logic, which would allow for many logical inferences that would not be made by the schemas of ML theory. No such inferences were written down by any participant. A final possible source of nonpredicted inferences would be the use of the feeder schemas, e.g., schemas 6 and 7. Such inferences were made, but they did not occur very often. In brief, the only sort of inferences that were made frequently were those reported in the results sections for Experiments 1 and 2.

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## **General Discussion**

The experiments reported here provide additional evidence for a mental predicate logic. Unlike the investigation of Yang et al. (1998), which provided only indirect evidence, the present studies provide direct evidence that participants applied the proposed inference schemas. The most persuasive evidence comes from Problem Sets 2 in both Experiments 1 and 2, in which participants wrote down inferences in the order predicted by the availability of the schemas, even though the premise information was not presented in a way that would lead to such lines of reasoning otherwise. In comparison to the inferences predicted by ML theory, relatively few inferences of any other sort were made in any systematic fashion. The best explanation for the data reported in the present study, therefore, seems to be the ML theory for reasoning with predicate-logic premises.

It is a fair question, of course, as to whether any other psychological theory of reasoning could provide as equally good an account of these data. Only two other theories are available that would claim to explain such reasoning: the mental-logic theory of Rips (e.g., Rips, 1994) and the mental-models theory of Johnson-Laird and his associates (e.g., Johnson-Laird & Byrne, 1991). Neither theory, however, seems capable of providing a clear account of how the problems reported here would be solved. Let us consider first Rips's theory. First, Rips's theory allows for few inferences to be made without specific conclusions to be tested. How the theory would make inferences when only a topic set is provided is yet to be specified. Second, Rips's theory apparently would lead to the prediction that many of the problems presented here would be quite difficult, when, in fact, participants had little difficulty in arriving at a final inference (in many cases without disagreement among participants). Finally, it is a quantifier-free representational system. In standard logic, a universally quantified sentence can be represented with a universal quantifier followed by a conditional sentence, e.g., *All the red beads are plastic* can be represented as *For every bead, if it is red then it is plastic*. In Rips's system, it becomes: *If Red (x), Then Plastic (x)*, where *x* is the individual variable *BEADS* and the universal quantifier is eliminated. An existentially quantified sentence can be represented with an existential quantifier followed by a conjunction, e.g., *There are some red plastic*

*beads* can be represented as *There exist some beads that are red and plastic*. In Rips's system, it becomes, "*Red (a) and Plastic (a)*," where *a* is a temporary name or a constant that had not occurred in the preceding undischarged steps. By using this quantifier-free representation, the inference rules defined for a propositional-level logic may sometimes be used in quantified predicate reasoning. The data reported here reveal no tendencies to treat universal propositions as conditionals, nor existential propositions as conjunctions, as they should according to Rips's theory.

The mental models approach of Johnson-Laird and his colleagues has addressed reasoning with predicate-logic premises in two sets of work, one concerning Aristotelian syllogisms and the other concerning what they refer to as "multiple quantification." The two sets provide quite different sorts of models, and of the two, the more pertinent is the work on syllogisms. (The work on multiply quantified propositions has been limited to whether various objects are, or are not, in the same location; it has represented the quantifiers quite differently than has the work about predicate syllogisms.) As an example of their approach, consider the two premises, *All beads are green* and *All green things are round*, which lead to the following models (Johnson-Laird & Byrne, 1991, p. 121):

[b] g; [g] r; [[b] g] r

[b] g [g] r [[b] g]

...

The first two columns represent the first premise, with the first two rows containing tokens for green beads and the third row (the ellipsis) indicating the possibility of other objects. Columns three and four represent the second premise, with the first two rows containing tokens for round green things and the third row again indicating the possibility of other objects. Finally, columns 5 - 7 represent the combination of information from the models for premises one and two, with the first two rows containing tokens for green round beads and the third row again indicating the possibility of some other things. The square brackets indicate exhaustivity; for example, in the models in columns one and two, the brackets indicate that no further models can be included that have a token for bead without having a token for green. The nested bracketing in the models in rows 5 - 7 indicate that beads are exhausted in relation to green, and green is exhausted in relation to round. The modelers state that the final model supports the conclusion that *All beads are round*. This way of representing quantified propositions can be applied to premises of the sort presented in the problems reported here, although not without encountering some difficulties. Consider the premises *all the beads are red* and *all the beads are metal*, which could lead to the following set of models (omitting the redundant models, as will be done henceforth):

[b] r; [b] m; [b] r m

Note that the structure of this model differs somewhat from what Johnson-Laird and his associates described above, in that the square brackets cannot be nested because, unlike the Aristotelian syllogisms, these premises contain no middle term. The final model, however, does appear to support the conclusion that *all beads are red metal beads*, which was the conclusion to be evaluated by subjects. Representation by models of other problems is often less obvious. Consider the premise that *there are no square wooden beads*. Johnson-Laird and Byrne (1991, p. 120) stated that a universal negative proposition, e.g., *None of the athletes is a baker*, will be represented as:

[a]

[b]

...

Application of this structure to *there are no square wooden beads* is problematic. Note that one cannot simply add one line to the model, as such:

[b]

[w]

[s]

...

because to do so would preclude the possibility of there being a wooden bead, or a square bead, or a square wooden thing that is not a bead, and these possibilities clearly should be allowed. Indeed, the appropriate model seemingly would include six explicit representations:

[b]

[w]

[s]

[b w]

[b s]

[w s]

Note that adding one term to the proposition would expand the required models, e.g., *there are no large square wooden beads* would require 12 explicit models. Given that models theory claims that the principal source of difficulty in reasoning stems from

limitations in working memory, making complex or lengthy models intractable, such premises would quickly make such problems intractable. Johnson-Laird, Byrne, and Schaeken (1994) suggested that in such situations a reasoner would seek simpler ways to model the information. What such a simpler way would be for this sort of premise, however, is unclear. For example, one might propose that there are no square wooden beads would be taken to mean that there are no beads that are both wooden and square, leading possibly to:

[b]

[s w]

or one might take the proposition to mean that wooden beads are not square beads, and vice versa, leading possibly to:

[b s]

[b w]

The choice is not trivial; choosing one way over another to represent the proposition leads to quite different final models, and thus quite different conclusions would be drawn. Among the final model sets that might be drawn from the premises of Problem 36, depending on how one decides to represent the premises and treat combinations and their exhaustivity are the following:

[b] t

[b g] s

...

or:

[btw]

[b tl

[bgs]

[b s]

or:

[[b] w] t

[bg] s

Note that not all of these models would lead in any obvious way to the conclusion that would be drawn from application of the mental-logic schemas, i.e., that it is the green beads that are not wooden. Until the models theorists provide greater specificity to the way quantified propositions are represented and combined, it remains problematic as to how one should compare the models treatment of the problems presented in the present work to the mental-logic treatment: The models theory provides no clear guidance about how people will reason with these problems.

The conclusion is inescapable: To date ML theory provides the only plausible account of reasoning on problems of the sort reported here. ML theory predicted successfully which inferences would be drawn and the orders of the intermediate inferences drawn on the way to making a final inferences. Most significantly, ML theory successfully predicted the order in which inferences would be written down, even when the premises were presented in orders that did not correspond to the output of the reasoning processes. Clearly, the present investigation only fills in one part of a larger investigation into reasoning of a predicate-logic sort. It does, however, provide some direct evidence that participants were solving the problems in the way described by ML theory.

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## References

Braine, M.D.S., and O'Brien, DP. (Eds.) (in press). **Mental logic**. Mahwah, NJ: Lawrence Erlbaum Associates.

Braine, M.D.S., O'Brien, DP., Noveck, I.A., Samuels, M., Fisch, SM., Lea, R.B., and Yang, Y. (1995). Predicting intermediate and multiple conclusions in propositional logic inference problems: Further evidence for a mental logic. **Journal of Experimental Psychology: General**, **124** 263-292

Braine, M.D.S., Reiser, B.J., and Rumin, B. (1984). Some empirical justification for a theory of natural propositional logic. In G. Bower (Ed.) **The psychology of learning and motivation: Advance in research and theory. (Vol. 18)**. New York: Academic Press

Johnson-Laird, P.N., Byrne, R. MJ., and Shaeken, W. (1994). Propositional Reasoning by Models. **Psychological Review**, **96**, 658-673

Johnson-Laird, P. N., and Byrne, R.M.J. (1991). **Deduction**. Hillsdale, NJ: Lawrence Erlbaum Associates

Lea, R.B. (1995). On-line evidence for elaborative logical inference in text. **Journal of Experimental Psychology: Learning, Memory, and Cognition** **21**, 1469-1482

O'Brien, D.P., Braine, M.D.S., and Yang, Y. (1994). Propositional reasoning by mental models? Simple to refute in principle and in practice. **Psychological Review** **101**, 711-724

Rips, L.J. (1990). Reasoning. *Annual Review of Psychology*, 41,321-353. Yang, Y., Braine, M.D.S., and O'Bfien, D.P. (in press)

**Table 1**

Premises. Topics. and Responses Predicted by the Direct-Reasoning Routine for Problem Set 1 in Experiment 1. Together With the Proportions With Which Those Responses Were Given to Problem Set 1 and Problem Set 2 of Experiment 1

	Percent written down		Percent in predicted order <sup>a</sup>	
	Set 1	Set 2	Set 1	Set 2
	n=21	n= 18		
1. None of the red beads are square (3) <sup>b</sup>				
All of the beads are triangular or square (1)				
The triangular beads are striped (2)				
<u>None of the striped beads are wooden (4)</u>				
<i>Topic: the red beads</i>				
<i>DRR Output:</i>				
The red beads are triangular	1.00	1.00		
The red beads are striped	.95	.89		
None of the red beads are wooden	.90	.93	.95	.93
2. The square beads are metal (2)				
There are no red metal beads (3)				
<u>Every bead is either red or green (1)</u>				
<i>Topic: the square beads</i>				
<i>DRR Output:</i>				
The square beads are not red	.48	.61		
The square beads are green	.95	.94	.88	1.00
3. All the large beads are wooden or metal (2)				
The wooden beads are square (3)				
The metal beads are triangular (1)				
All of the square beads are blue (5)				
<u>All of the triangular beads are blue (4)</u>				
<i>Topic: the large beads</i>				
<i>DRR Output::</i>				
The large beads are square or triangular	.62	.44		
The large beads are blue	.67	.78	.85	.86
4. The plastic beads are red (3)				

All the beads are square (2)				
<u>The square red beads are not large (1)</u>				
<i>Topic: the plastic beads</i>				
<i>DRR Output::</i>				
The plastic beads are red and square <sup>a</sup>	.72	.83		
The plastic beads are not large	.95	.83	.88	.87
5. The square beads are small (1)				
All the beads are red (4)				
All the small red beads are wooden (2)				
<u>None of the wooden beads are striped (3)</u>				
<i>Topic: the square beads</i>				
<i>DRR Output::</i>				
The square beads are red	.33	.22		
The square beads are small and red	.67	.78		
The square beads are wooden	1.00	.94		
The square beads are not striped	.95	.94	.94	1.00
6. All the round beads are red or green (2)				
The red beads are wooden (4)				
The green beads are wooden (3)				
<u>There are no striped wooden beads (1)</u>				
<i>Topic: the round beads</i>				
<i>DRR Output::</i>				
The round beads are wooden	.95	.78		
The round beads are not striped	.90	1.00	1.00	1.00
7. All the triangular beads are wooden (1)				
There are no red wooden beads (3)				
<u>Every bead is either red or green (2)</u>				
<i>Topic: the triangular beads</i>				
<i>DRR Output::</i>				
The triangular beads are not red	.43	.67		
The triangular beads are green	1.00	.94	1.00	.93
8. Some of the beads are wooden (3)				
The wooden beads are either round or square (2)				
None of the beads are round (4)				
<u>There are no square striped beads (1)</u>				
<i>Topic: the wooden beads</i>				
<i>DRR Output::</i>				
The wooden beads are square	1.00	1.00		
The wooden beads are not striped	.95	.89	.96	1.00
9. Some of the beads are round (3)				
All the beads are plastic or wooden (1)				

The wooden beads are red (4)				
<u>The plastic beads are blue (2)</u>				
<i>Topic: the round beads</i>				
<i>DRR Output::</i>				
The round beads are plastic or wooden	.57	.67		
The round beads are red or blue	.52	.72	1.00	1.00
10. The green beads are not triangular (1)				
The large beads are metal				
All the beads are triangular or square (3)				
<u>There are no square wooden beads (2)</u>				
<i>Topic: the green beads</i>				
<i>DRR Output::</i>				
The green beads are square	.95	1.00		
The green beads are not wooden	.90	1.00	1.00	1.00
11. The large beads are either green or blue (6)				
None of the green beads are triangular (3)				
None of the blue beads are triangular (1)				
Every bead is triangular or square (2)				
None of the square beads are plain (5)				
<u>The beads are either plain or metal (4)</u>				
<i>Topic: the large beads</i>				
<i>DRR Output::</i>				
The large beads are not triangular	.29	.50		
The large beads are square	.86	.78		
The large beads are not plain	.38	.73		
The large beads are metal	1.00	.73	1.00	.86
12. The green round beads are plastic (2)				
The blue round beads are plastic (3)				
All the beads are green or blue (1)				
The plastic beads are striped (5)				
<u>None of the striped beads are large (4)</u>				
<i>Topic: the round beads</i>				
<i>DRR Output::</i>				
The round beads are green or blue	.86	.11		
The round beads are plastic	.90	.89		
The round beads are striped	.95	.89		
The round beads are not large	.95	.89		
13. Some of the round beads are not plastic (2)				
All of the beads are plastic or wooden (3)				
<u>There are no blue wooden beads (1)</u>				

<i>Topic: the round beads</i>				
<i>DRR Output:</i>				
Some of the round beads are wooden	.86	.94		
Some of the round beads are not blue	.81	.88	1.00	1.00
14. All the large beads are triangular or square (2)				
The triangular beads are red and metal (3)				
The square beads are metal (1)				
<u>There are no plain metal beads (4)</u>				
<i>Topic: the large beads</i>				
<i>DRR Output:</i>				
The large beads are metal	.76	.78		
The large beads are not plain	.76	.78	1.00	1.00
15. The striped beads are either metal or plastic (2)				
None of the beads are metal (3)				
<u>There are no blue plastic beads (1)</u>				
<i>Topic: the striped beads</i>				
<i>DRR Output:</i>				
The striped beads are plastic	1.00	.89		
The striped beads are not blue	.90	.72	.95	.92
16. The triangular beads are blue and none of them are wooden (4)				
All the beads are metal or wooden (3)				
None of the metal beads are striped (2)				
<u>All of the beads are striped or plain (1)</u>				
<i>Topic: the triangular beads</i>				
<i>DRR Output:</i>				
The triangular beads are metal	1.00	.94		
The triangular beads are not striped	.43	.83		
The triangular beads are plain	1.00	.94	.95	.86
17. The round beads are wooden (4)				
There are no green wooden beads (1)				
Every bead is red or green (2)				
<u>The red beads are square (9)</u>				
<i>Topic: the round beads</i>				
<i>DRR Output:</i>				
The round beads are not green	.38	.94		
The round beads are red	.76	.89		
The round beads are square	.81	.89	.95	.94
18. Some of the red beads are metal (2)				
There are no square metal beads (1)				

<u>Every bead is either square or triangular (3)</u>				
<i>Topic: the red beads</i>				
<i>DRR Output:</i>				
Some of the red beads are not square	.24	.67		
Some of the red beads are triangular	.86	.89	.87	1.00
19. <u>The blue beads are round (2)</u>				
<u>The blue beads are not plastic (4)</u>				
All the beads are either plastic or wooden (1)				
<u>The round wooden beads are large (3)</u>				
<i>Topic: the blue beads</i>				
<i>DRR Output:</i>				
The blue beads are wooden	.44	.40		
The blue beads are round and wooden	.90	.89		
The blue beads are large	.86	.94	.90	1.00
20. <u>Some of the large beads are striped (4)</u>				
<u>Every bead is square (1)</u>				
All the striped square beads are metal (2)				
<u>There are no red metal beads (3)</u>				
<i>Topic: the large beads</i>				
<i>DRR Output:</i>				
Some of the large beads are striped and square	.86	.89		
Some of the large beads are metal	.90	.89		
Some of the large beads are not red	.76	.83	.97	.88

Note: On each problem, the premises are presented above the line, and the predicted inferences of the DRR are presented below the line.

a For each problem, this indicates the proportion of those predicted inferences that were written down that were written down in the order predicted by the DRR.

b Indicates the order in which the premises were presented for Problem Set 2.

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## Table 2

Premises. Topics. and Responses Predicted by the Direct-Reasoning Routine for Problem Set 1 in Experiment 2. Together With the Proportions With Which Those Responses Were Given to Problem Set 1 and Problem Set 2 of Experiment 2

	Percent written down		Percent in predicted order <sup>a</sup>	
	Set 1	Set 2	Set 1	Set 2
	n=21	n=		
1. None of the children wearing shoes are watching videos. (3) <sup>b</sup>				
All of the children are either washing dishes or watching videos.(2)				
The children washing dishes are indoors. (1)				
<u>None of the children who are indoors are being punished. (4)</u>				
<i>Topic: the children wearing shoes</i>				
<i>DRR Output::</i>				
The children wearing shoes are washing dishes	1.00	.95		
The children wearing shoes are indoors	.81	.80		
The children wearing shoes are not being punished	.71	.85	.93	.94
Other				
2. All the girls are in the cafeteria. (2)				
There are no children eating beans in the cafeteria. (3)				
<u>Every child is eating either beans or fruit (1)</u>				
<i>Topic: the girls</i>				
<i>DRR Output::</i>				
The girls are not eating beans	.43	.75		
The girls are eating fruit	1.00	.95	1.00	1.00
Other				
3. All the older children are wearing either red or blue shirts (2)				
All the children in blue shirts are selling the "O Globo". (a newspaper) (3)				
The children in red shirts are selling "Jomal do Brasil". (a newspaper) (1)				
All of the children who sell "O Globo" are tired. (5)				
<u>All of the children who sell</u>				

"Jomal do Brasil" are tired. (4)				
<i>Topic:</i> the older children				
<i>DRR Output:</i>				
The older children are selling O Globo or selling Jomal do Brasil	.81	.75		
The older children are tired	.86	.80	.95	.94
Other				
4. The children who are dancing the quadrilha are wearing straw hats. (3)				
All the children are in schoolyard. (2)				
<u>The children in the schoolyard who are wearing straw hats are not tired. (1)</u>				
<i>Topic:</i> the children who are dancing the quadrilha (a Brazilian folk dance)				
<i>DRR Output::</i>				
The children dancing the quadrilha are wearing straw hats in the schoolyard.	.71	.80		
The children dancing the quadrilha are not tired	.90	.95	1.00	1.00
Other				
5. The children who are painting are from Piedade. (a neighborhood) (1)				
All the children are boys.(4)				
All the boys from Piedade are in school (3)				
<u>None of the children in school are listening to music. (2)</u>				
<i>Topic:</i> the children who are painting				
<i>DRR Output:</i>				
The children who are painting are boys	.50	.65		
The children who are painting are in school	.81	.65		
The children who are painting are not listening to music	.81	.80	.95	.88
Other				
6. All the happy children are				

either playing tennis or swimming. (2)				
The children who are playing tennis are at the country club. (4)				
The children who are swimming are at the country club.(3)				
<u>There are no boys at the country club.(1)</u>				
<i>Topic:</i> the children who are happy				
<i>DRR Output:</i>				
The happy children are at the country club	.90	.85		
The happy children are not boys	.81	.85	.70	.88
Other				
7. The tanned children are on the beach.(1)				
There are no children reading books on the beach.(3)				
<u>Every child is either reading a book or playing volleyball.(2)</u>				
<i>Topic:</i> the tanned children				
<i>DRR Output::</i>				
The tanned children are not reading books	.48	.75		
The tanned children are playing volleyball	1.00	.95	1.00	1.00
Other				
8. Some of the children are orphans.(3)				
The children who are orphans are either playing or studying.(2)				
None of the children are playing.(4)				
<u>There are no happy children who are studying.(1)</u>				
<i>Topic:</i> the children who are orphans				
<i>DRR Output::</i>				
The orphans are studying	.95	.90		
The orphans are not happy	.90	.90	.95	1.00
Other				
9. Some of the children are from				

Pernambuco.(3)				
All the children are either in a school trip or on a family holiday.(1)				
The children on a family holiday are wearing bathing suits.(4)				
<u>The children on a school trip are wearing school uniforms. (2)</u>				
<i>Topic:</i> the children from Pernambuco				
<i>DRR Output::</i>				
The children from Pernambuco are either on a school trip or on a family holiday	.67	.40		
The children from Pernambuco are wearing bathing suits or school uniforms	.43	.80	1.00	1.00
Other				
10. The younger children are not in their classrooms.(1)				
All the children are in their classrooms or in the playground.(3)				
<u>There are no children in the playground who are wearing red shirts.(2)</u>				
<i>Topic:</i> the younger children				
<i>DRR Output::</i>				
The younger children are in the playground	.95	1.00		
The younger children are not wearing red shirts	.95	.95	.95	1.00
Other				
11. The deaf children are in either "Colégio Conviver" or "Colégio Atual".(a Colégio is a high school) (6)				
None of the children in "Colégio Atual" play basketball.(3)				
None of the children in "Colégio Conviver" play basketball.(10)				
Every child plays either football or basketball.(2)				
None of the children who play				

football get good marks.(5)				
<u>The children either get good marks or win prizes. (4)</u>				
<i>Topic: the deaf children</i>				
<i>DRR Output::</i>				
None of the deaf children play basketball	.52	.70		
The deaf children play football (	1.00	.85		
None of the deaf children get good marks	.86	.80		
The deaf children win prizes	.76	.85	1.00	.82
Other				
12. The boys from "Colégio Positivo" speak English. (a Colégio is a high school) (2)				
The boys from "Colégio Equipe" speak English.(3)				
All the children are either from "Colégio Positivo" or from "Colegito Equipe".(1)				
The children who speak English are older. (5)				
<u>None of the older children is wearing a school uniform. (4)</u>				
<i>Topic: the boys</i>				
<i>DRR Output::</i>				
The boys are from Colegio Positivo or Colegio Equipe	.24	.40		
The boys speak English	.95	.90		
The boys are older	.86	.85		
None of the boys is wearing a school uniform	.86	.90	.67	.87
Other				
13. Some of the short children are not eating hamburgers. (2)				
All the children are eating either hamburgers or coxinhas (chicken buns).(3)				
<u>There are no thin children eating coxinhas (1)</u>				
<i>Topic: the short children</i>				
<i>DRR Output::</i>				
Some of the short children are	.71	.90		

eating coxinhas				
Some of the short children are not thin	.67	.70	1.00	1.00
Other				
14. All the children from Olinda are either fat or thin. (2)				
The fat children eat pizza and drink guaraná. (a popular soft drink) (3)				
The thin children drink guaraná.(1)				
<u>There are no children on a diet who drink guaraná.(4)</u>				
<i>Topic:</i> the children from Olinda				
<i>DRR Output:</i>				
The children from Olinda drink guaraná	.52	.65		
There are no children from Olinda on a diet	.48	.80	1.00	1.00
Other				
15. The children from Olinda are either thirsty or hungry.(2)				
None of the children are thirsty.(3)				
<u>There are no hungry children standing in the queue.(1)</u>				
<i>Topic:</i> the children from Olinda				
<i>DRR Output:.</i>				
The children from Olinda are hungry	.71	.95		
The children from Olinda are not standing in the queue	.86	.90	1.00	1.00
Other				
16. The girls are eating ice cream and none of them are using napkins.(4)				
All the children are either wiping their mouths on their sleeves or using napkins.(3)				
None of the children wiping their mouths on their sleeves are well behaved.(2)				
<u>All of the children are either well behaved or mischievous.(1)</u>				

<i>Topic: the girls</i>				
<i>DRR Output::</i>				
The girls are wiping their mouths on their sleeves	.90	.95		
None of the girls are well behaved	.86	.90		
The girls are mischievous	.86	.95	.94	1.00
Other				
17. The fat children are eating hamburgers.(4)				
There are no children on the beach eating hamburgers.(1)				
Every child is either at the park or on the beach.(2)				
<u>The children at the park are playing football.(1)</u>				
<i>Topic: the fat children</i>				
<i>DRR Output::</i>				
The fat children are not on the beach	.70	.90		
The fat children are at the park	.90	.90		
The fat children are playing football	.95	.90	.95	1.00
Other				
18. Some of the children in the playground are playing tennis.(2)				
There are no girls playing tennis.(1)				
<u>Every child either is a girl or is wearing a hat.(3)</u>				
<i>Topic: the children in the playground</i>				
<i>DRR Output:</i>				
Some of the children in the playground are not girls	.86	.75		
Some of the children in the playground are wearing a hat	.81	.85	1.00	.93
Other				
19. The street children are in "Boa Viagem". (a neighborhood) (2)				
The street children are not selling newspapers.(4)				

All of the children are selling either newspapers or candies.(1)				
<u>The children in "Boa Viagem" who are selling candy are dancing.(3)</u>				
<i>Topic: the street children</i>				
<i>DRR Output::</i>				
The street children are selling candies	.57	.65		
The street children are selling candy in Boa Viagem	.71	.75		
The street children are dancing	1.00	.95	1.00	1.00
Other				
20. Some of the children who won a scholarship are eating in a McDonalds restaurant. (4)				
Every child is in the Recife Shopping Center.(1)				
All of the children who are eating in a McDonalds restaurant in the Recife Shopping Center are drinking guaraná. (a popular soft drink) (2)				
<u>There are no children in red shirts who are drinking guaraná. (3)</u>				
<i>Topic: the children who won a scholarship</i>				
<i>DRR Output::</i>				
Some the children who won a scholarship are eating in a McDonalds in the Recife Shopping Center	.71	.80		
Some of the children who won a scholarship are drinking guaraná	.81	.75		
Some of the children who won a scholarship are not in red shirts	.71	.80	1.00	1.00
Other				

Note: The notation and organization are the same as for Table 1.

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