2018

Mathematics in Contemporary Society - Chapter 5 (Spring 2018)

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Chapter 5

We have an exam next week. Since we will be reviewing for the test, there is only one hour of new material, which reinforces and builds upon what you already learned.

Test Topics for Exam #1

Topics include:

Population, sample, population parameters, sample statistics
Observational study
Sampling techniques
Experiment: treatment group, control group, placebo
Case-control study
Evaluating a statistical study
Selection bias, participation bias
Creating or reading a: frequency table (with relative and cumulative frequency),
histogram, pie chart, line chart, time-series diagram
Quantitative and Qualitative data
Using a multiple bar graph, stack plot, geographical display
Graphic distortion
Positive correlation, negative correlation, no correlation
Creating a scatter plot or scatter diagram, identifying correlation
Causality
Calculating mean, median, mode
Outliers
Describing distributions by peaks, symmetry, skewness and variation
Identifying the shape of a distribution (normal, left-skewed, right -skewed, uniform, bimodal, etc.)

Blackboard Assignment!

(if Blackboard is being used for the class)

To help you study for the test, everyone should participate in Blackboard's Discussion Board. The instructions are very simple:

a)Post two separatereview questions for the exam as a message.

b)Answer two other review questions posted by someone else. (You can add to someone else's answer if you make a meaningful contribution to an incorrect or incomplete answer.)

(Some of the following information was presented in the notes previously—you should remember it. We have new information to consider with it.)

More on Shapes of a Distribution

We often refer to the graph of a data set as a **distribution** of the data set. The overall shape of the distribution is often of interest to the researcher.

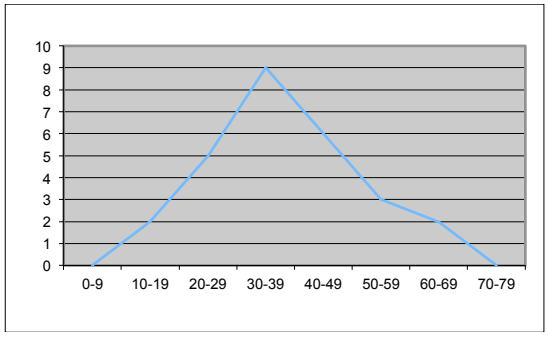
Now that we know about the mean, median and mode, we want to reexamine the different kinds of graphs we can obtain from a data set.

What makes a distribution look the way it does? The values of the data set are distributed in such a way to give the distribution a particular appearance. This appearance tells us much information about the nature of the data set itself.

Peaks in a Distribution

A distribution may have one peak or be a **unimodal distribution.**

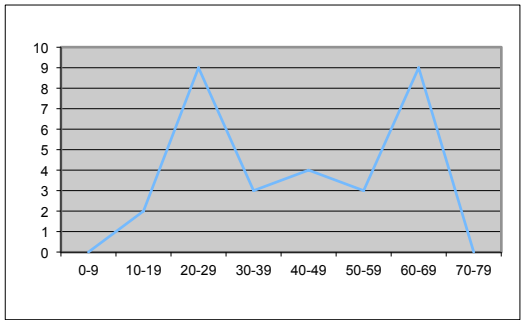
Example 1:



Without knowing what data set we're dealing with here, we can certainly say that there are more values in the category 30-39 than any other category. The mode is a little different here; I don't know which specific value occurs the most often (in fact, the mode of the actual data set may be higher than 39 or lower than 30). Graphically, the peak of the graph represents the mode of the distribution.

We may expect that most data sets will have unimodal distributions. The peak of the graph identifies which category of values best describes the data set.

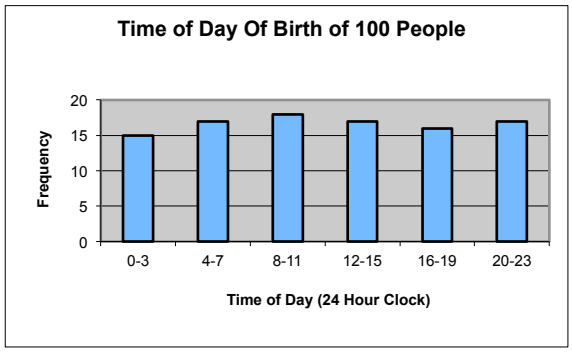
Example 2:



A distribution with two peaks is a **bimodal distribution**. In this case, there are two separate modes that occur with roughly equal frequency.

When a bimodal distribution occurs, it suggests that the data set is divided in such a way that there are two subgroups within the data set, each with its own mode.
If the above example was a set of test scores, it tells us that the data set is divided in such a way as to produce separate results. Perhaps the class in question was made up of different kinds of students (maybe half of them came into the course by passing a placement exam and the other half came from a prerequisite course). Maybe a surprise test was given and half the class was prepared and the other half was not. A bimodal distribution is certainly not an expected result.

Example 3:



A distribution may have no real peak and be a **uniform distribution**. Generally, we say there is no mode here—all values have roughly the same frequency.

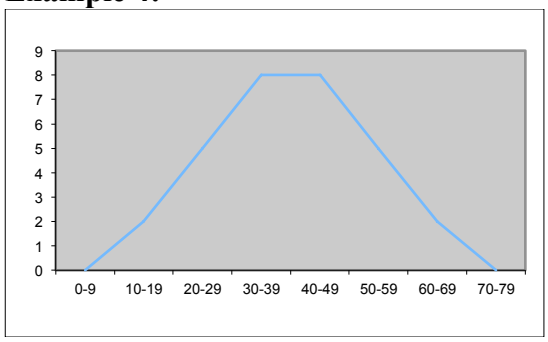
For the above bar graph, 100 people were asked what time of day they were born. 0-3 represents 12:00 AM to 3:59 AM, 4-7 represents 4:00 AM to 7:59 AM, and so on.

In this situation, we really don't expect there to be a time slot that is more frequent than any other time slot. The time of day that we are born is random; there shouldn't be any one time that occurs more often than any other. A person has just as much chance as being born between 8:00 AM and 11:59 AM as 4:00 PM to 7:59 PM.

When a data set has a uniform distribution, it suggests there are a set of values with no overall defining characteristics. While the mean and median may be right in the middle of these values, the lack of a mode tells us that one value is just as good as any other.

Symmetry

Example 4:



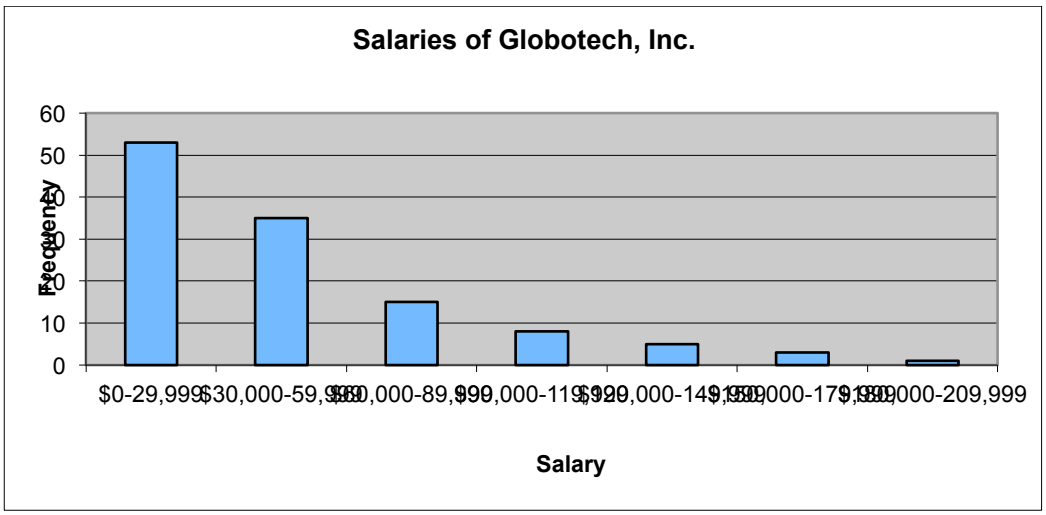
A distribution that can be divided down the middle into two halves that are mirror images of the other is said to be **symmetric**. A symmetric graph shows that the data set is balanced. With the median in the middle, there are just as many values above it as there are below, distributed in like fashion.

In terms of symmetric graphs, we are most interested in the normal distribution. The **normal distribution** has a bell-shaped appearance and is symmetric. In the normal distribution, we say that mean=median=mode. The average of all values is at the center of the graph. The middle term is at the center of the graph. The value that occurs with the most frequency is at the center of the graph.

Many data sets in the real world (height, weight, IQ scores, GPA, shoe size, SAT scores, automobile mileage, cost of shipping a package, etc.) correspond to the normal distribution. When examining a data set, we expect to obtain a normal distribution unless there is a significant reason (some have already been identified) for a different result.

Skewness

Example 5:



A **right-skewed distribution** has a peak on the left side and a "tail" on the right. The outliers (values that are unusually high in comparison to most of the data) of the distribution are on the right side. We say that mode<median<mean. The mode is the smallest value, located at the peak of the graph. The median, the middle term, is higher than the mode. (Remember, the median has to divide the set into 2 equal halves—most of the values are low here.) The mean is the largest value because outliers raise the value of the numerical average.

The above example describes the salaries of 120 employees of Globotech, Inc. As you can see, 75% of the employees earn less that $60,000. But there are a few who make more. We can imagine that the employees earning more than $100,000 represent the executives of the corporation, with the one employee (probably the president) making over $180,000.

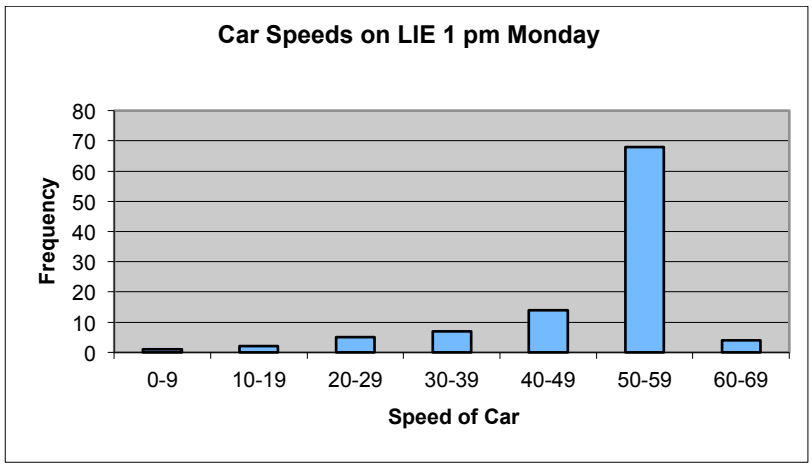
Is salary distributed evenly here? Obviously not.

If you were told that the mean salary of the company was $55,000, you would argue that considerably more than half of the employees make less than $55,000. The high executive salaries cause the mean to be much higher than the 53 employees earning salaries below $30,000. The mean is not the most useful measure of average here.

If the mean is less useful, we could consider the median salary of Globotech, Inc. If the median salary is $33,500 that tells us that 50% of the employees earn less than $33,500 and 50% earn more than $33,500. The median gives the middle salary here. Still, the median does not give much other detail.

Perhaps the mode category of $0 - $29,999 is the most useful piece of information. Almost half of the employees of Globotech make less than $30,000. This may be the most honest representation of "average" employee salary.

Example 6:



A **left-skewed distribution** has a peak on the right side and a "tail" on the left. The outliers (values that are unusually low in comparison to most of the data) of the distribution are on the left side. We say that mean<median<mode. The mode is the highest value, located at the peak of the graph. The median, the middle term, is lower than the mode. (Remember, the median has to divide the set into 2 equal halves—most of the values are high here.) The mean is the smallest value because outliers lower the value of the numerical average.

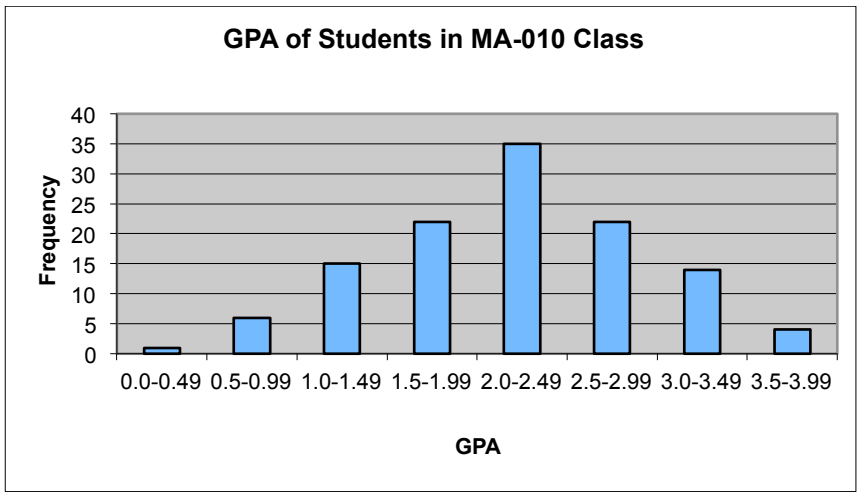
In the above example, we see that most of the cars are traveling at a speed of 50-59 miles per hour (Most people obey the speed limit, right?). However, there are a few slower moving vehicles. Some people prefer lower speeds; there may be construction vehicles; some cars may have their flashers on, etc.

In this case, the mean speed may end up being less than 50-59 mph. But we can clearly see that most drivers are traveling near the speed limit of 55 mph. The mode or median may be more useful measures of "average" speed in this case.

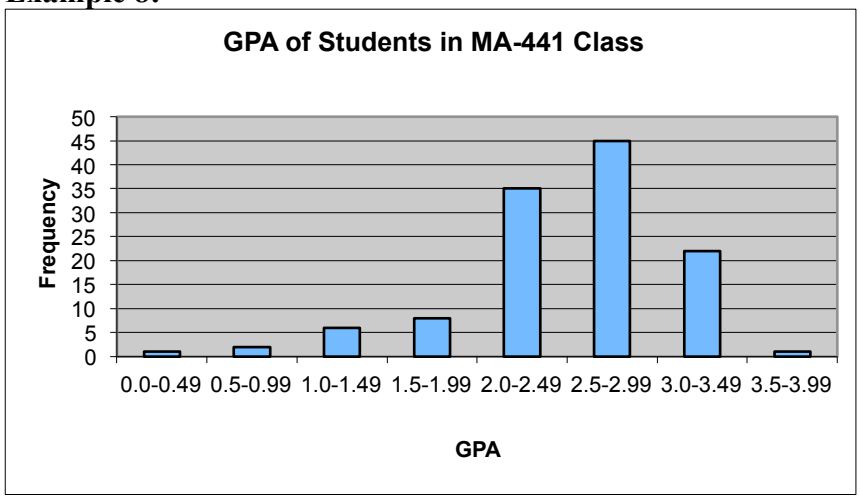
Variation

Variation measures how closely or how far apart the values of a distribution are overall: Suppose we examine the GPAs of students in two different classes:

Example 7:



Example 8:



Both distributions describe data sets of GPAs. The first distribution has greater variation than the second distribution.

Why is this true? MA-010 is an introductory mathematics class that many students have to take, while MA-441 is a calculus class for mathematics and science majors, who have had to pass prerequisites and have been at QCC for one or more semesters. We would expect more diversity among MA-010 students than MA-441 students.

Variation is not always easy to determine from one data set or distribution—it is often easier when making comparisons between graphs.

It is somewhat subjective, but we can say a distribution has low variation if the frequencies of two or three adjacent categories is much higher than all other categories and high variation if the frequencies are closer together overall.

Homework:

For 1)-10), consider the following data sets:
a)Describe the kind of distribution (bimodal, uniform, normal, left-skewed, right-skewed) you would expect to get from a graph of the data set.
b) Give a brief reason why you expect that kind of distribution.

1) The salaries of New York Yankees baseball players
2) The salaries of computer programmers in New York City
3) The SAT scores of May 2004
4) The weights of 100 football players
5) The GPAs of 100 students in the Honors Program at QCC
6) The closing prices of 100 random stocks this Friday
7) The last digit of the social security number of 500 people
8) The CPE exam scores of 200 QCC students this fall
9) The annual income of a sample of 50 day students and 50 evening students
10) The amount paid by the first 100 visitors to the Museum of Natural History, where the suggested donation is $10

For 11)-20), consider the following data sets:
a) Give an example of such a data set with low variation
b) Give an example of such a data set with high variation
(For example, if the data set was "Height". I might say that 50 members of the QCC male basketball team would have low variation (everyone is tall and over 6 feet) as compared to 50 students chosen randomly from the parking lot outside the Humanities Building.)

11) Annual income
12) GPA
13) Temperature
14) Weight
15) Age
16) Closing price of stock
17) IQ score
18) SAT score
19) Home Runs
20) Miles per Hour

Lab Assignment #5–Using Your Own Data Set

1) Collect a data set of 30 **related** scores from the Internet. **(Print out the data set.)** Some sources you might try are:
Weather sites: www.weather.com/ , www.weather.yahoo.com/ , http://www.cnn.com/WEATHER/ for temperatures, precipitation, etc.
Financial sites: www.nasdaq.com/ , www.morningstar.com , finance.yahoo.com/ for stock prices and business information
Sports sites: www.espn.com/ , sports.yahoo.com/, www.si.com for tons of sports related values.

2) Enter the 30 values into Excel in column A in cells A1 to A30.

3) Calculate measures of average using Excel's Paste Function button:
   a) Type the word Mean in a cell, such as D1.
   b) Click on the cell right next to it, such as E1.
   c) Click on the f_x button.
   d) Under **Select a Category**, click on **Statistical**:
   e) Under **Select a Function**, click on **AVERAGE**. Click OK.
   f) For the first field in the box (Number 1), use the mouse to highlight all 30 values in A1 to A30.
   g) Click on OK. The value that appears is the mean!
   h) Repeat the process in cells D2 and E2 using the **MEDIAN** function.
   i) Repeat the process in cells D3 and E3 using the **MODE** or **MODE.MULT** function. Be careful! Excel will only give you one value for the mode even if there are multiple modes. Note any other modes in F3. If there is no mode, #N/A will appear.

4) Create a simple frequency table (as we had in Lab#3) from your data. Include at least 6 equally spaced categories.

5) Create a line chart from the frequency table.

6) Answer the following questions (you may refer to the notes):
   1) Describe your data set. What is the population you are examining? How did you obtain your sample?
   2) Describe your distribution from the line chart. How many peaks do you have? Is it symmetric? Does it appear to be left-skewed, right-skewed, or normal?
   3) How do the mean, median and mode compare to the peak of your line chart? That is, if your chart has a peak, does it match any of the three?