2011

Complexity of Minimum Corridor Guarding Problems

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Complexity of Minimum Corridor Guarding Problems

Thesis

submitted in partial fulfillment of
the requirement for the degree

Master of Science (Computer Science)

at

The City College of New York

of the

City University of New York

by

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June 2011

Approved:

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Abstract

In this paper, the complexity of minimum corridor guarding problems is discussed. These problem can be described as: given a connected orthogonal arrangement of vertical and horizontal line segments and a guard with unlimited visibility along a line segment, find a tree or a closed tour with minimum total length along edges of the arrangement, such that if the guard runs on the tree or on the closed tour, all line segments are visited by the guard. These problems are proved to be NP-complete.

Keywords: computational complexity, computational geometry, corridor guarding, NP-complete

Contents

List of Figures

1. Introduction and results

Imagine a guard that patrols in a building and detects emergencies like fire, intruders, etc.. The guard has unlimited visibility, so it need not walk along a whole corridor, but visits some points on the corridor. The guard needs a optimal motion plan to visit all corridors. We called this problem *a minimum corridor guarding problem.* Since a system of corridors in a building can be modeled as a connected arrangement of line segments, a minimum corridor guarding problem can be described as follows:

Minimum corridor guarding problem. Given a connected arrangement A of line segments (corridors) $A = \{A_1, A_2, \ldots, A_n\}$. A *guarding set* P is a set of edges along corridors such that all line segments are visited by P, i.e., $P \cap A_i \neq \emptyset$, $1 \leq i \leq n$. The length of a guarding set is the total length of edges in the set. The minimum corridor guarding problem is to find a minimum length guarding set subject to connectivity condition.

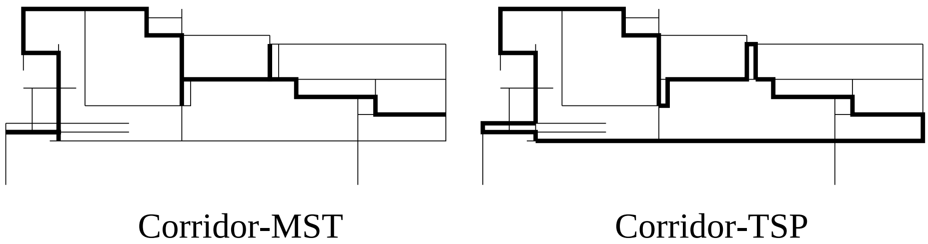

Figure 1: Corridor-MST and corridor-TSP: thick line segments form a solution.

If the guarding must be a tree, the problem is denoted as *corridor-MST*. This tree is denoted as *a guarding tree*. If the guarding set must be a closed tour, the problem is denoted as *corridor-TSP*. This closed tour is denoted as *a guarding tour*. Figure 1 shows an example of corridor-MST and corridor-TSP.

In this paper, two theorems are presented here.

Theorem 1. *Corridor-MST is NP-complete, even all corridors are orthogonal.*

Theorem 2. *Corridor-TSP is NP-complete, even all corridors are orthogonal.*

This paper is organized as follows. Section 2 introduces the related works. Section 3 gives the proof. Section 4 gives the conclusion.

2. Related works

2.1. Geometric connection problems

The minimum corridor guarding problem belongs to a class called *geometric connection problems*: given a set of geometric objects in a plane, find a minimum length network satisfying some constraints that connects all objects. In general, the network is restricted to be a tree or a closed tour.

2.1.1. Minimum corridor connection problem

One geometric connection problem is *minimum corridor connection problems* [7]: given a simple rectilinear polygon, which is rectilinearly decomposed into some rectilinear components (called *rooms*), find a minimum length tree along edges of the decomposition, such that every room is incident to the tree. Figure 2 shows an example of this problem.

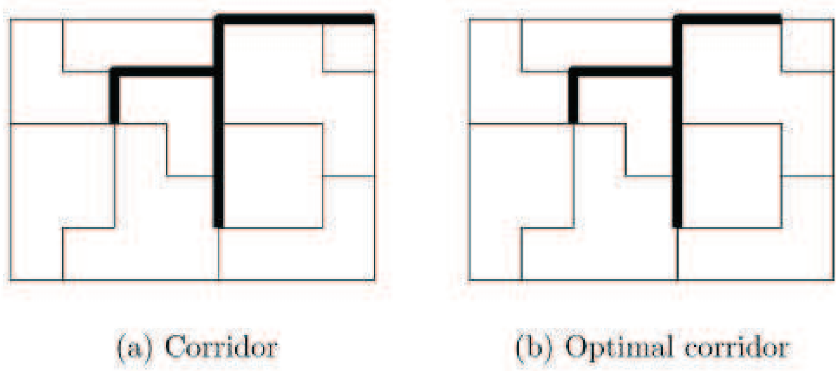

Figure 2: An instance of the minimum corridor connection problem (borrowed from [4])

This problem was firstly presented by Katoh in CCCG 2000 [7]. Bodlaender, et. al. and Gonzalez-Gutierrez, et. al. [4, 13] independently showed this problem to be NP-complete. In addition, Bodlaender, et. al. gave a partial constant factor approximation algorithm for the case that all rooms are fat and have nearly same size [4]. Later, Gonzalez-Gutierrez and Gonzalez gave a partial constant factor approximation algorithm for the case the all rooms are rectilinear c-gons where $c \leq k$ and k is a constant.

2.1.2. Minimum face-spanning problem

Another geometric connection problem is *minimum face-spanning problem on planar graphs*: given a non-negative edge weighted connected planar graph $G = (V, E)$, find a minimum face-spanning tree T of G, that is, a tree $T \subseteq E$ such that the vertices in T contain at least one vertex for every face of G, and the total weight of edges in T is minimum. Figure 3 shows an example of the minimum face-spanning problem.

Patwary and Rahman proved this problem to be NP-complete, even when

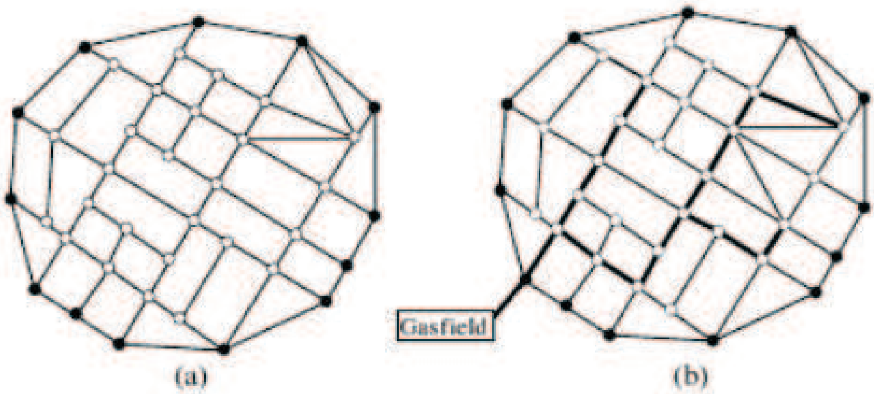

Figure 3: An instance of the minimum face-spanning problem: (a) is a real network of a locality and (b) is a sample setup of gas network drawn by thick lines to supply gas for all regions (borrowed from [19])

the objective is to minimize the number of edges instead of the total weight in the face-spanning tree [19]. Patwary and Rahman also gave $2(\Delta-2)$ ratio approximation algorithm for this problem, where Δ is the maximum degree in the graph G.

2.1.3. Tree and tour covers problem

The third geometric connection problem is *tree and tour covers problem*: given a graph $G = (V, E)$ with non-negative weights on the edges, find a minimum weight tree or a minimum weight closed tour, whose vertices cover all the edges in E.

Arkin, et. al. proved this problem to be NP-complete[1]. In the same paper, Arkin, et. al. also gave a a $1 + r_{st}$ factor approximation algorithm for tree cover problem, and a $2r_{wvc}+r_{tsp}$ factor approximation algorithm for tour cover problem, where r_{st}, r_{wvc} and r_{tsp} are the performance ratio obtained by the algorithm used to approximate a Steiner tree, to approximate a weighted vertex cover, and to approximate a traveling salesman tour.

2.2. Watchman route problem

The *watchman route problem* can be described as: given a polygon with n vertices, find a minimum length closed tour in the interior of the polygon, such that all points in the interior of the polygon are visible from some points on the tour. If the start point of the route is given, the problem is restricted as a *fixed watchman route problem.* One can also consider the minimum corridor guarding problem as a special case of watchman route problem in rectilinear polygon with holes, where the rectilinear polygon collapses into an arrangement of orthogonal line segments. Figure 4 shows an instance of the watchman route problem, where the thick line segments form the solution.

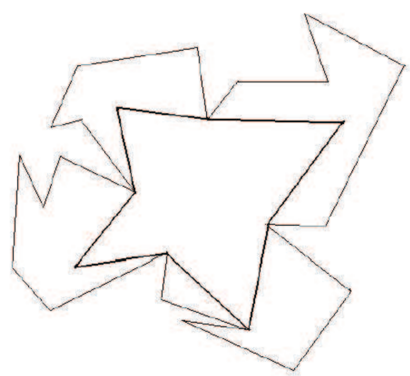

Figure 4: An instance of the watchman route problem(borrowed by [16])

When the polygon has holes, watchman route problem is strongly NP-hard [6], even the polygon is rectilinear. In 1995, Mata and Mitchell published an $O(\log n)$-approximation algorithm to solve this problem [17]. In 2001, Tan gave a linear time 2-factor algorithm for approximating the watchman route problem where the route start from a given point on the boundary [21], which was improved by Tan himself to an approximation ratio $\sqrt{2}$ [23]. In 2007, Tan published a new approximation algorithm to calculate a watchman route of length at most twice of that of the shortest watchman route [24].

A simplified version of the watchman route problem in a simple polygon, i.e., the polygon has no hole, can be solved in polynomial time. For the fixed watchman route problem, in 1991, Chin and Ntafos presented an $O(n^4)$ algorithm to solve fixed watchman route problem [18]. In 1993, Tan, et. al. obtained an $O(n^3)$ solution [26]. In the same year, Tan an Hirata improved the previous work to an $O(n^2)$ algorithm [25]. Unfortunately, in 1997, Hammar and Nilsson pointed out the previous algorithms in [18, 25, 26] were all flawed, and presented a solution to fixed these errors [14]. It is interesting that Tan, et. al. found that the algorithm in [14] was also flawed, and gave an $O(n^4)$ algorithm for the fixed watchman route problem [27], which is, to our best knowledge, the best known algorithm for fixed watchman route problem in a simple polygon.

When the start point is not given, Carlsson and Jonsson gave an $O(n^{12})$ algorithm in 1995 [5]. In the same year, Nilsson proposed an $O(n^6)$ algorithm in his Ph.D. thesis. Tan improved the result to an $O(n^5)$ algorithm in 2001 [22]. Combine the algorithm presented by Tan [22] and the result from Dror, et. al. [8], to our best knowledge, the best known algorithm for watchman route algorithm in a simple polygon is $O(n^4 \log n)$.

2.3. Group Steiner problem

Moreover, the corridor-MST problem is a special case of Group Steiner problem introduced by [20]. This problem can be described as follows. Let $G = (V, E, \omega)$ is a connected undirected edge-weighted graph, where $\omega : E \rightarrow R^+$ is an edge-weighted function. Let $C \subseteq V$ be a non-empty set named *terminals*, which is partitioned into k disjointed groups $C_1, C_2, \ldots, C_k$ be a set of disj of C. The objective of the Group Steiner problem is to find a tree $T = (V', E')$, where $E' \subseteq E$ and $V' \subseteq V$, such that at least one terminal from each set C_i is in the tree T and the total edge-length $\sum_{e \in E'} \omega(e)$ is minimized.

Group Steiner problem is NP-hard because it is a generalization of Steiner problem. Garg, et. al. gave a randomized $O(log^3 n \log k)$-approximation algorithm for this problem, where $n = |V|$ [12]. Bateman, et. al. also presented an algorithm with guaranteed $(1 + \ln k)\sqrt{k}$ approximation ratio [2]. On the negative side, Group Steiner problem is NP-hard to approximate to a performance ratio less than logarithmic times the optimal[9, 15].

3. Proof

3.1. Orthogonal corridor-MST is NP-complete

To show NP-completeness, a reduction from the well known NP-complete problem, *CONNECTED VERTEX COVER problem on planar graph with maximum degree four (CVC-4)*[10, 11], is used in this section. An instance of CVC-4 consists of a planar graph $G = (V, E)$ such that every vertex in V has at most 4 degrees, and a positive integer $R \leq |V|$, the question is whether there exists a connected vertex cover of size at most R for G, that is, a subset $W \subseteq V$ with $|W| \leq R$ such that the subgraph induced by W is connected and for each edge $(u, v) \in E$, $u \in W$ or $v \in W$[4]? Figure 5 shows an example of CVC-4 problem.

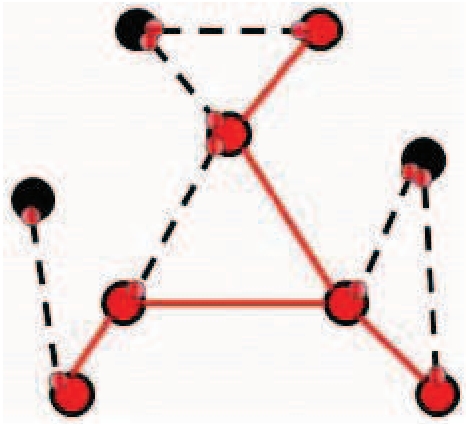

Figure 5: An instance of CVC-4 prolem: thick lines form the optimal solution

Proof. First, a connected orthogonal arrangement A is tranformed from a planar graph $G = (V, E)$ with maximum degree 4 by following steps:

1. Make a rectilinear embedding of G, and enlarge the embedding.
   Use the algorithm presented by Biedl and Kant[3] to construct a rectilinear embedding of G in a grid, and enlarge the embedding vertically and horizontally by a factor $32|V|^2$. In the embedding, an edge of G is represented by a set of connected line segments. We call the set of connected line segments as a *road*. The length of a *road* is total length of line segments in the road. Figure 6 shows an example of transformation from a planar graph to a rectilinear embedding.
2. Make roads equal length.

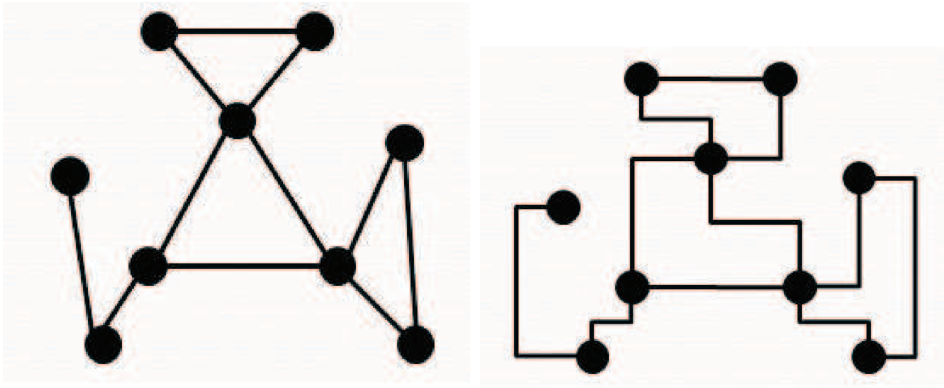

Figure 6: Transform a planar graph with maximum degree four into a rectilinear embedding

> Let K be length of the longest road, and $L = 3K$. Transform all roads equal length. As illustrated in Figure 7, a road (u, v) in Figure 7(a) is transformed to another road with total length $L + 32$ in the Figure 7(b).

Second, It is claimed that a guarding tree P with length at most $(L + 16)|E| + 8(R - 1)$ exists if and only if a connected vertex cover W in G with $|W| \leq R$ exists.

The "if" part is easy. Suppose such W exists. Let T be a tree that exactly spans W. T contains at most $R - 1$ edges. Then build a guarding tree P as follows. For each edge $(u, v) \in E$, add the road (u, v) into P except (c, e) and (d, f). If $(u, v) \notin T$, $u \in W$ or $v \in W$. Suppose $u \in W$, remove (v, b) from P.

The arrangement generated above if formed by connected roads. For each road, observe Figure 7. First, all line segments between a and b are included in P, so these line segments are naturally visited; the line segments (a, c) and

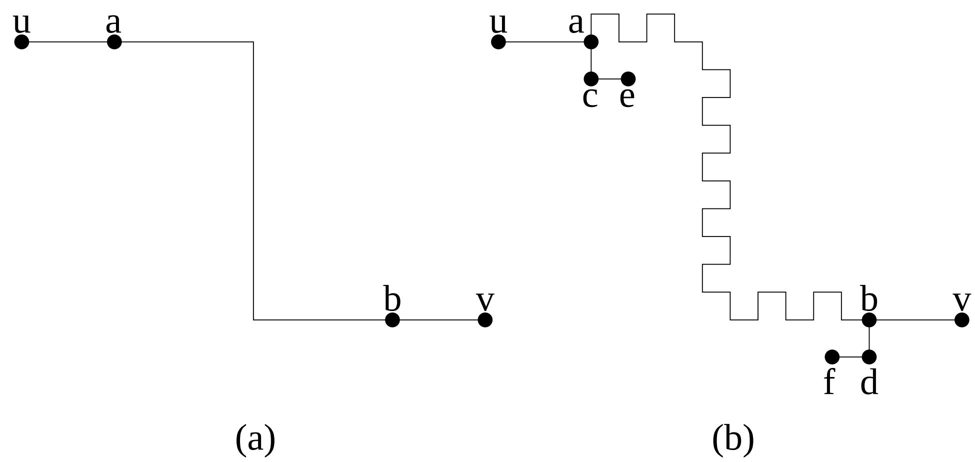

Figure 7: Transform a road to fixed length: $|ua| = |vb| = 8$, $|ac| = |bd| = |ce| = |df| = 4$. In figure (b), each line segment between a and b is no longer than 4, and total length of line segments between a and b is L

(b, d) are also included in P, thus they are visited; the line segments (c, e) and (d, f) are not included in P, but are visited from c and d, respectively; finally, the line segments (u, a) and (v, b) and visited from a and b. It is clear that all line segments in the arrangement are all visited from P, i.e., P is a guarding set.

Next, let us prove P is connected. Let p and q are two different points on P. If p and q belong to the same road, observe Figure 7, it is clear p and q are connected. If p and q belong to different roads. Let p belongs to a road (u_p, v_p), and q belongs to a road (u_q, v_q). If $(u_p, v_p) \in T$, p and u_p is connected. If $(u_p, v_p) \notin T$, $u_p \in W$ or $v_p \in W$. Suppose $u_p \in W$, a point u_p connected to p exists. Similarly, a point u_q connected to q can be founded. The corresponding vertices of u_p and u_q is connected by the tree T in the planar graph G. Follow the path from u_p and u_q in the tree T, because all edges on the path belongs to T, all corresponding roads is connected at both ends, i.e., u_p and u_q is connected. Thus p and q is connected, that is, P is connected.

Therefore, P is a guarding tree. The total length of P is:

$$
\begin{aligned}
&(L+24)(|W|-1)+(L+16)(|E|-|W|+1) \\
=&(L+16)|E|+8(|W|-1) \\
\leq&(L+16)|E|+8(R-1)
\end{aligned}
$$

To prove the "only if" part, suppose such P exists. For any road (u, v) in the arrangement, it is assumed that (1) $(c, e) \notin P, (d, f) \notin P$; (2) $(a, c) \in P, (b, d) \in P$; (3) all line segments between a and b must be in P; (4)$(u, a) \in P$ or $(v, b) \in P$. If (1) is not satisfied, remove (c, e) or (d, f) from P to choose an alternative guarding tree. If (2) is not satisfied, (c, e) or (d, f) can not be visited, P cannot be a guarding tree. If (3) is not satisfied, because P is a guarding tree, at least two consecutive line segments between a and b are not belonging to P, and two line segments (u, a) and (v, b) must be in P. Because every line segment between a and b is no longer than 4, and because $|(u, a)| = |(v, b)| = 8$, remove (u, a) or (v, b) from P and add missing line segments between a and b, an alternative guarding tree can be chosen. If (4) is not satisfied, all line segments between a and b are isolated, that is, P is not a guarding tree.

W is constructed by choosing vertices connected by P. W is connected since P is connected. Moreover, since each road is connected by at least one end, every edge in G must be connected by W. Thus, W is a connected vertex cover.

It is clear that $u \in W$ and $v \in W$ if (u, a) and (v, b) are both in P. Thus the number of roads fully included in P is at least $|W|-1$. The length of P is at least $(L+16)|E|+8(|W|-1)$. Because the length of P is at most $(L+16)|E|+8(R-1)$, $|W| \leq R$.

The reduction can be done in polynomial time. Corridor-MST belongs to NP. Therefore, corridor-MST is NP-complete.

3.2. Orthogonal corridor-TSP is NP-complete

Proof. The CVC-4 problem is also used here for reduction.

First, a connected orthogonal arrangement A is reduced from a planar graph $G=(V,E)$ with maximum degree 4 by following steps:

1. Create an orthogonal embedding of G and enlarge the embedding.

   Use the same method mentioned in 3.1 to create an enlarged rectilinear embedding of G in a grid.

2. Transform vertices.

   Replace each vertex in the embedding by a square with side length 8. Alongside each degree, draw a square with same side length.

   This step is illustrated in Figure 8. Imagine that the figure is a T-intersection. Four thick solid line segments are called *crosswalks*. Thin solid line segments are called *sidewalks*. Thick dot line segments are called *overpasses*.

3. Duplicate all *roads*

   Alongside each road, draw a parallel road with distance 8. If a vertex has only one degree or has exact two opposite degrees, remove two crosswalks. This step can be done as shown in Figure 9.

4. Make roads equal length

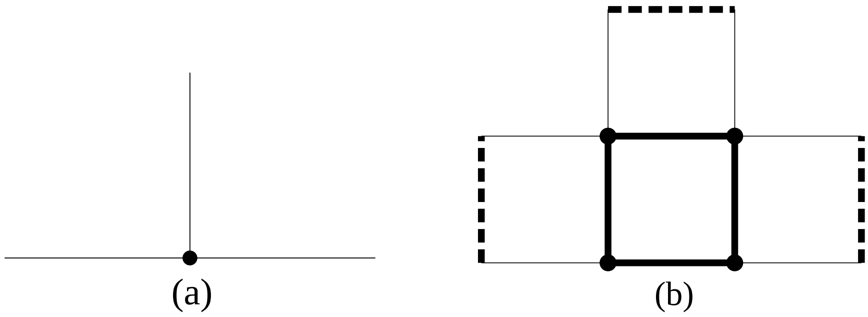

Figure 8: Transform a vertex into squares

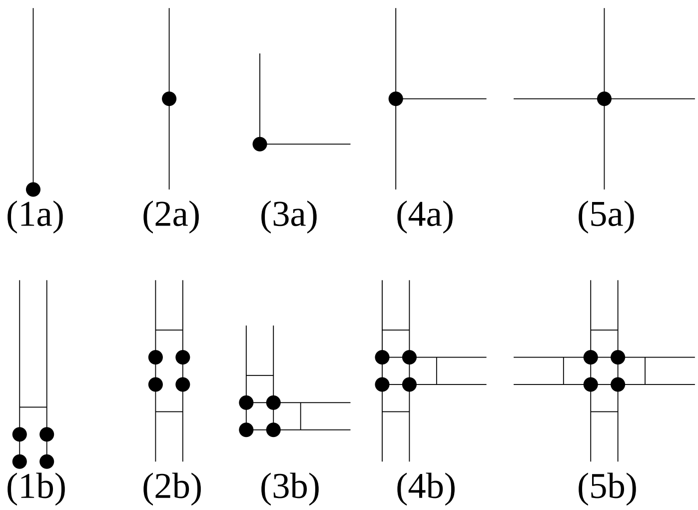

Figure 9: Duplicate roads

Let K be the length of the longest road, and $L = 3K$. Use the same method mentioned in 3.1, make every *road* with equal length L.

Second, it is claimed that a guarding tour P with length at most $2(L + 8)|E| + 32R$ exists if and only if a connected vertex cover W of size at most R in G exists.

To prove the "if" part, suppose such W exists, and T be a tree that exactly spans W. P can be constructed by following steps:

1. Process each edge $(u, v) \in E$. Let $u \in W$.
   Add two roads between u and v into P. Add two sidewalks near u into P. If $(u, v) \in T$, add two sidewalks near v into P; otherwise, add one overpass near v into P.
2. Process each vertex.
   Add some crosswalks illustrated in Figure 10 into P to build a closed tour.

It is clear P includes all roads. Observe Figure 10, P guards all sidewalks, overpasses and crosswalks. Moreover, a guard leaving a vertex through a road eventually returns back from the road on the other side. It is easy to show that P is a guarding tour. In addition, for any vertex $u \in V$ with degree $d(u)$, the number of crosswalks, overpasses and sidewalks included in P near u is $d(u)$ if $u \notin W$, and is $d(u) + 4$ if $u \in W$. The total length of P is:

$$\begin{aligned} & 2L|E| + 8(\sum_{u \in W}(d(u) + 4) + \sum_{u \notin W} d(u)) \\ =& 2(L + 8)|E| + 32|W| \\ \leq& 2(L + 8)|E| + 32R \end{aligned}$$

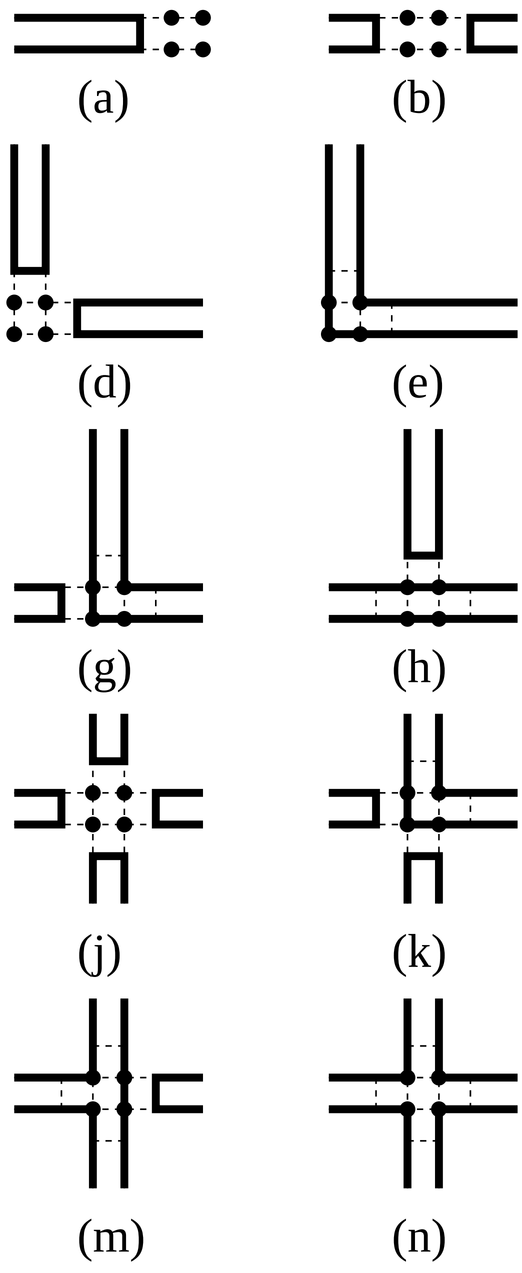

Figure 10: Add crosswalks to build a closed tour: solid line segments form the closed tour

To prove the "only if" part, suppose such P exists. P must include all line segments in each road. Moreover, Figure 10 shows the cases with the least number of crosswalks, overpasses and sidewalks included in P near a vertex. It is assumed that P accords with Figure 10; otherwise, an alternative guarding tour according with this figure can be chosen.

Now W is constructed as follows. An edge (u, v) is *fully connected* if its corresponding roads are connected by sidewalks at both ends in a guarding tour, or *partially connected by* u if its corresponding roads are connected by sidewalks at only one end u. Observe Figure 10. Every edge must be fully connected or partially connected. A connected vertex cover W in G can be constructed by choosing all vertices incident to a fully connected edge. If there is no fully connected edge, all partially connected edges must be connected at the same vertex. W only includes the common vertex.

To prove that W is a connected vertex cover, observe that a guard leaving from a vertex u through a road to a neighbor vertex v will come back before passing any other pair of roads if (u, v) is partially connected. Since P is connected, the fully connected edges must be connected, i.e., W is connected. Moreover, an edge partially connected by one vertex must be incident to a fully connected edge at this vertex unless $|W| = 1$, otherwise P is not a guarding tour. So W is a vertex cover.

Because P accords to Figure 10, the total length of P must be at least $2(L + 8)|E| + 32|W|$. Since the length of P is at most $2(L + 8)|E| + 32R$, $|W| \leq R$ holds.

It is clear that this reduction can be done in polynomial time. Corridor-TSP belongs to NP. Therefore, corridor-TSP is NP-complete.

4. Conclusion

In this paper, both corridor-MST and corridor-TSP are proved to be NP-complete by reduce these two problems from a known NP-complete problem, CVC-4, even all corridors are orthogonal. This result shows that it is strongly difficult to find an optimal guarding set in a system of connected arrangement of orthogonal line segments. However, no constant ratio approximation algorithm for these two problems are known.

Acknowledgment

This work is supported by a NSF grant(No. 1017539). The author thanks Peter Brass for his valuable comments and helpful suggestions related to the paper.

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