Spring 2018

Mathematics in Contemporary Society - Chapter 9 (Spring 2018)

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Chapter 9

Writing Project #2 – First Draft due _____

Again, we will be using the 50 values that you have collected for your project.

Write a report about your findings from your statistical project. You have worked on this project in multiple stages. First, you gathered 50 values from a topic of interest to you. You presented your data set in the Discussion Board. In the lab assignments, you spent one session creating graphical displays and measures of "average" and variation. For the next assignment, you calculated z-scores and percentiles. There is much you can present.

Your paper should include the following (issues you may address and consider are included):

1) Title/Cover Page
2) Introduction: Discuss the population that your data set was selected from and the parameter that you are examining. Why are you interested in this population? Before you started, what did you believe you were going to find? How and where did you obtain the sample data set? (Be sure to credit the source of the data.) Do you believe the data set you collected has any kind of bias? If so, why is the data biased? Could the bias have been avoided?
3) Body:
    a) Discuss your data set graphically. What kind of distribution is suggested by your frequency table and other graphical displays? If the graph has a peak, what (value range) is it? What is the overall range of values? Discuss your graphs and charts with respect to how they help describe your data set. Which display(s) were the most useful? Which display(s), if any, were not useful?
    b) Discuss your results for mean, median and mode. Which is the most appropriate measure of "average" for your data set? Why? Discuss the range, five number summary and standard deviation. What sort of variation (high, moderate, low) do you see? Why?
    c) Discuss the z-scores and percentiles that you calculated. What range of z-scores and percentiles did you encounter (are the z-scores and percentiles close together or spread apart)? Were there any outliers? Identify them. If there are no outliers, what would an outlier look like for your data set? What do the outliers (or lack of outliers) indicate about you data set?
    d) Would collecting a larger data set (100 values or more) give you any different results from what you already have? How so? Give details. *(Bear in mind that not all samples can be extended in this manner.)*
4) Conclusion: What does the sample indicate about the population and the parameter that you are interested in? Use your sample results to "predict" or describe

the population. Do you believe the population would be a normal distribution or would it be skewed? Do you expect the population to look almost the same as your graphical displays, or very different? What sort of mean will the population parameter have? What sort of variation will it have (as much variation or less)? What do you expect in the future for your population (are your predictions useful only for now or long into the future)?

5) Appendix: Include the following:
    a. **Your original data set with source indicated** (you can scan it, print it or give me the link to it—I need to know which values you used!)
    b. Graded Labs #6 and #7 (with any changes made)

The paper will be two pages in length, double-spaced, size 12 font (this size).

Some thoughts…

In the introduction of the paper, you will write about how you obtained your data set. What population are you examining? How did you get your sample? What do you think the sample will tell you? Is the sample biased?

In the body of the paper, you will write about all the details you discovered in Lab #6 and Lab #7. You can give the different measures of average and variation you calculated and analyze their meaning. You can talk about the shape of the distribution of your data set. You can write about the z-scores and percentiles you calculated.

Finally, in the conclusion of the paper, you summarize what you have learned. A representative sample describes the population for us with its measures of average, variation and distribution shape. The population will have similar characteristics to a well-chosen sample. Of course, if our sample is biased or not representative of the population, we can still speculate about what the population parameters actually are.

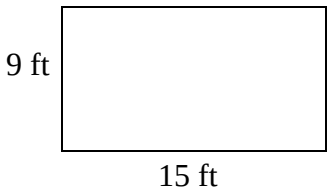
We are moving on to the next subject in the course.

Problem Solving

When presented with a problem, we often make use of our current knowledge and attempt to solve it with what we already know or can remember.

Question 1: Suppose your living room is 15 feet long and 9 feet wide. If carpeting costs $12 per square yard, how much will it cost to carpet the room?
(Write down the answer you obtained without looking at the answer on the next page.)

To solve this problem, we might draw a picture to make it clearer.

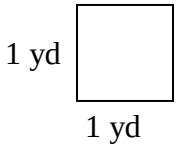


Carpeting a floor is an area problem; we need to know the area of a rectangle:

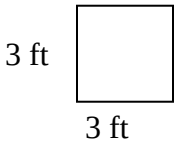
$$\begin{aligned}\text{Area} &= \text{length} \cdot \text{width} \\ &= 15 \text{ ft} \cdot 9 \text{ ft} \\ &= 135 \text{ square ft}\end{aligned}$$

The temptation here may be to multiply by $12 and get an answer, but that is the wrong approach. Notice that the price is given for square yards, not square feet. We need to know how many square feet are in a yard.

So, how many square feet are in a yard? If you look at a square yard:



and remember that 1 yard = 3 feet:



The area of a square yard is 3ft·3ft =9 square feet! This is critical to the problem.

To convert square feet to square yards, we divide by 9:

135 sq ft ÷ (9 sq ft per sq yd) = 15 square yards.

We are now ready to calculate the cost.

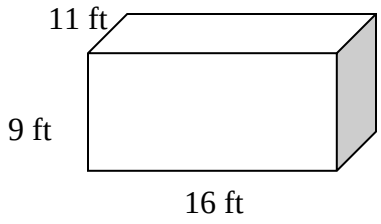
15 sq yd · ($12 per sq yd) = $180

Question 2: Did you get the right answer? If no, why not? If yes, how did you know what to do?

Let's look at the next problem:

Question 3: Suppose an empty pool is 16 feet long, 11 feet wide and 9 feet deep. How many hours will it take to fill the pool with a hose that is flowing at a rate of 4 cubic feet per minute?
(Write down the answer you obtained without looking at the answer on the next page.)

To solve this problem, we might draw a picture to make it clearer.



Filling a pool with water is a volume problem; we need to know the area of a rectangular block:

$$\begin{aligned} \text{Volume} &= \text{length} \cdot \text{width} \cdot \text{height} \\ &= 16 \text{ ft} \cdot 11 \text{ ft} \cdot 9 \text{ ft} \\ &= 1584 \text{ cubic feet} \end{aligned}$$

Since we are filling the pool with a hose flowing 4 cubic feet per minute the next step is to divide:

1584 cu ft ÷ (4 cu ft per minute) = 396 minutes to fill the pool

Some people may stop here and believe that this is the answer. Look again. The answer should be in hours. We need to convert our answer to hours:

396 minutes ÷ (60 minutes per hour) = 6.6 hours

Question 4: Did you get the right answer? If no, why not? If yes, how did you know what to do?

Here are some suggestions and notes on working with units:

1) Drawing a picture can often give you a better understanding of the problem than words alone. If you can visualize it, you can draw it and label it.
2) Identify the units (feet, inches, dollars, pounds, centimeters, yards, ft^2, in^3, etc.) used in a problem. Knowing what you are starting with and having an idea of what you need to finish with will help you answer a problem.
   a) For example, in problems involving area, you should know that two lengths are multiplied together to get square units.
   b) In problems involving cost, clearly identify what the cost represents ($ per sq yard, gallon, quart, mile, etc.)
3) Perform the necessary operations on the units given.
   a) You can only add and subtract numbers with the same units (feet to feet, inches to inches, yd^2 to yd^2, etc.). We add units to accumulate values (as in perimeters or distances) and subtract to find differences between values.
   b) Multiplication of the same kind of unit will lead to square units (area) or cubed unites (volume). Also, we can sometimes multiply to create special

products, such as kilowatt • hour = kilowatt-hour, which is a kilowatt of power usage for an hour of time.

c) Division leads to rates: (For example, miles/hour = miles per hour, cost for 8 apples/8 = cost per 1 apple, 3 feet/1 yard = 3 feet per yard ***-This last rate is a well-known conversion factor.***)

d) We can multiply units by conversion factors to change the kind of unit we have:
(For example, change 40 yards to feet:)

$$\frac{40\ \cancel{\text{yards}}}{1} \bullet \frac{3\ \text{feet}}{1\ \cancel{\text{yard}}} = 40 \bullet 3\ \text{feet} = 120\ \text{feet}$$

4) Determine if the answer that you have has the expected units. (If you were to calculate volume, you should get something like ft^3 at the end. If you were calculating the cost of something, you should end with a dollar amount.)

5) At the end, make sure your answer **makes sense**! If you're calculating the cost of carpeting a room, it probably shouldn't cost $540,000 or 54 cents. You can probably estimate an expected result before you make any calculation.

Conversions

We should be aware of the many conversion factors that we use to convert one kind of unit to another related unit:

Units of Length

1 foot = 12 inches
1 yard = 3 feet
1 yard = 36 inches
1 mile =5280 feet

Units of Volume

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts

Units of Weight

1 pound = 16 ounces
1 ton = 2000 pounds

Units of Time

1 minute = 60 seconds
1 hour = 60 minutes

1 day = 24 hours

These conversion factors can be used to easily convert units:

Example 1 Convert 245 yards to feet. (Using 1 yard = 3 feet)

To convert, we multiply 245 yards by the appropriate conversion factor:

$$\frac{245 \text{ yards}}{1} \bullet \frac{3 \text{ feet}}{1 \text{ yard}} = 245 \bullet 3 \text{ feet} = 735 \text{ feet}$$

Example 2 Convert 216 inches to yards. (Using 1 yard = 36 inches)

To convert, we multiply 216 inches by the appropriate conversion factor:

$$\frac{216 \text{ inches}}{1} \bullet \frac{1 \text{ yard}}{36 \text{ inches}} = \frac{216}{36} \bullet \text{yards} = 6 \text{ yards}$$

Example 3 Convert 5.8 miles to feet. (Using 5,280 feet = 1 mile)

To convert, we multiply 5.8 miles by the appropriate conversion factor:

$$\frac{5.8 \text{ miles}}{1} \bullet \frac{5280 \text{ feet}}{1 \text{ mile}} = 5.8 \bullet 5280 \text{ feet} = 30624 \text{ feet}$$

If we know how to do these length conversions, we should be able to do any others using the conversions factors seen earlier.

Question 5: **Convert 50000 feet to miles.**
Question 6: **Convert 250 gallons to quarts.**
Question 7: **Convert 45.2 pounds to ounces.**
Question 8: **Convert 978 minutes to hours.**

If we need to convert units, but there is no direct conversion factor, we may need to use more than one of them:

Example 4 Convert 15 miles to inches.

To convert, we need to multiply by two conversion factors:

$$\frac{15 \text{ miles}}{1} \bullet \frac{5280 \text{ feet}}{1 \text{ mile}} \bullet \frac{12 \text{ inches}}{1 \text{ foot}} = 15 \bullet 5280 \bullet 12 \text{ inches} = 950{,}400 \text{ inches}$$

You multiply by as many conversions factors as you need to complete the conversion.

Example 5 Convert 1 year to minutes.

To convert, we need to multiply by three conversion factors:

$$\frac{1 \text{ year}}{1} \cdot \frac{365 \text{ days}}{1 \text{ year}} \cdot \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} = 365 \cdot 24 \cdot 60 \text{ minutes} = 525{,}600 \text{ minutes}$$

Broadway fans may recognize the phrase "525,600 minutes" from the show Rent.

Question 9: **Convert the month of April into seconds.**
Question 10: **Convert 200 gallons into fluid ounces.**

We also use conversion factors to convert units that we aren't familiar with, as long as we can look them up and create our own conversion factors.

For example, suppose I want to convert to be able to convert US currency into other foreign currencies and convert foreign currency to US currency.

Currency exchange factors are continuously changing. They can easily be looked up in the newspaper or the internet. In April of 2018, the following could be found at http://www.exchangerate.com/

Currency Exchange Rates April 2018

Currency	Foreign Per Dollar	Dollars Per Foreign
Canadian dollar	1.2590	0.7943
European euro	0.8110	1.2330
British pound	0.7020	1.4245
Japanese yen	107.4346	0.0093
Mexican peso	18.1133	0.0552
Swiss franc	0.9618	1.0397
Australian dollar	1.2865	0.7773

How do we interpret the table?

We see that under **Foreign Per Dollar**, the Canadian dollar value is 1.2590. What does this mean? It means we can buy 1.2590 Canadian dollars (a bit more than one Canadian dollar) with $1 of US currency.

Foreign Per Dollar represents the amount of the foreign currency you can buy for $1 of US currency. So for $1, you can get 0.8110 European euros, or 0.7020 British pounds, or 107.4346 Japanese yen, etc. As you can see, some foreign currencies are worth more than $1 (like the British pound); others are worth less (like the Mexican peso).

We can use the first column to convert our dollars to any foreign currency:

Example 6 Convert $300 to Australian dollars (Using $1 = 1.2865 Australian dollars (AD))
(The left side of the chart tells you how much foreign currency you can get for $1.)

$\frac{\$300}{1} \bullet \frac{1.2865 \text{ AD}}{\$\ 1}$ = 300 • 1.2865 AD = 385.95 Australian dollars

Example 7 Convert $500 to British pounds (Using $1 = 0.7020 British pounds (BP))

$\frac{\$500}{1} \bullet \frac{0.7020 \text{ BP}}{\$\ 1}$ = 500 • 0.7020 BP = 351 British pounds

Question 11: Convert $1,000 to European euros.
Question 12: Convert $5,000 to French francs.

How do we interpret the last column?

We see that under **Dollar Per Foreign**, the Canadian dollar value is 0.7943. What does this mean? It means we can buy $0.7943 (rounded to $0.79 or 79 cents) with 1 Canadian dollar.

Dollars Per Foreign represents the amount of US currency you can buy for 1 unit of foreign currency. So for 1 European euro, you can get $1.2330 US currency. For 1 British pound, you can get $1.4245. As you can see, some foreign currencies are worth a bit more than $1 (you can get $1.42 with one British pound), others are worth quite a bit less (one Japanese yen is worth a little less than a penny).

We can use the last column to convert any foreign currency to US dollars:

Example 8 Convert 50,000 Japanese yen to U.S. dollars (Using 1 yen = $0.0093)
(The right side of the chart tells you how much U.S. currency you can get for 1 unit of foreign currency.)

$\frac{50{,}000 \text{ yen}}{1} \bullet \frac{0.0093}{1 \text{ yen}}$ = 50000 • 0.0093= $465.00

We see that 50,000 yen may sound like a lot, but it's only $465.

Example 9 Convert 6,000 Mexican pesos to U.S. dollars (Using 1 peso = $0.0619)

$\frac{6{,}000 \text{ peso}}{1} \bullet \frac{0.0552}{1 \text{ peso}}$ = 6000 • 0.0552 = $331.20

Question 13: Convert 200 British pounds to U.S. dollars.
Question 14: Convert 1000 European euros to U.S. dollars.

We now want to look at certain **standardized units,** units that are fixed to have a specific measure regardless of where they are used.

The U.S. Customary System

The U.S. Customary System (USCS) of measurement comes from an ancient system of measurement based largely on body parts.

A "foot" was the length of a person's foot. An inch was approximately the width of a thumb. A yard was the distance from the nose to the tip of the thumb. A mile was a walk of 1,000 paces. And so on.

<u>Question 15:</u> Find a ruler and compare the width of your thumb to an inch and the length of your foot to one foot on the ruler? How do they compare?

The system was derived from many sources. Measures from ancient Egyptian, Sumerian, Babylonian, Roman and Hebrew civilizations found their way to Europe. The measures used by England in the 1600s and 1700s were adopted by the English colonies in America, and we still use many of these measures today.

Of course, since every person is different in size and shape, how were standardized units developed?

At one time, these measures were subject to the person doing the measuring. Some standardization may have occurred from time to time if one person (such as the emperor, king or queen) declared measures to be based on his or her own physical characteristics. Of course, this standard could then change from one ruler to the next. In England, attempts to standardize measures can be found in the Magna Carta (1215), and acts of Parliament. Further changes to units occurred in the United States (some seen as late as 1959). Finally, we have the standardized measures we use today!

Here are some of the many measures of the U.S. Customary System:

1 foot = 12 inches	**1 fathom = 6 feet**
1 yard = 3 feet	**1 furlong = 40 rods =0.125 miles**
1 yard = 36 inches	**1 nautical mile = 6076.1 feet**
1 mile =5,280 feet	**1 land league = 3 miles**
1 mile = 1,760 yards	**1 marine league = 3 nautical miles**
1 rod = 5.5 yards	

Some of these measures of length (inch, foot, yard, and mile) are commonly used today. Others, such as rod, fathoms, furlongs, leagues and nautical miles are less often used except in certain fields. Horse racing fans may be aware of certain track lengths being given in furlongs or rods. Boat enthusiasts may be interested in nautical miles or marine leagues.

Even if you're not familiar with all of these measures, conversions are still possible.

Example 1 A racing track has a length of 20 furlongs. How long is that?

We convert furlongs with conversion factors until we get to feet, as seen below:

$$\frac{20 \text{ furlongs}}{1} \cdot \frac{40 \text{ rods}}{1 \text{ furlong}} \cdot \frac{5.5 \text{ yards}}{1 \text{ rod}} \cdot \frac{3 \text{ feet}}{1 \text{ yard}} = 20 \cdot 40 \cdot 5.5 \cdot 3 \text{ feet} = 13{,}200 \text{ feet}$$

OR:

$$\frac{20 \text{ furlongs}}{1} \cdot \frac{0.125 \text{ miles}}{1 \text{ furlong}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} = 20 \cdot 0.125 \cdot 5280 \text{ feet} = 13{,}200 \text{ feet}$$

Question 16: Convert 20 marine leagues to feet.

Table of USCS Weights

AVOIRDUPOIS MEASURES	TROY MEASURES	APOTHECARY MEASURES
1 pound (lb) = 16 ounces	1 grain = 0.0648 gram	1 scruple = 20 grains
1 ounce (oz) = 437.5 grains	1 carat = 3.086 grains = 0.2 grams	1 dram = 3 scruples
1 ton = 2,000 pounds	1 pennyweight = 24 grains	1 apothecary ounce = 8 drams
	1 troy ounce = 480 grains	

As you can see, there are many measures of weight, sorted into categories. Avoirdupois Measures are the measures of commerce; these are the measures (ounce, pound, and ton) we are probably the most familiar with. Troy measures are jewelry measures (note the presence of carat on the list) most useful to jewelers. Apothecary measures are pharmaceutical measures. Pharmacists, doctors and nurses make use of these measures.

Question 17: Convert 400 ounces to pounds.
Question 18: Convert 1000 grains to drams.

Table of USCS Volumes

LIQUID MEASURES	DRY MEASURES
1 tablespoon = 3 teaspoons	1 dry pint = 33.6 $in.^3$
1 fluid ounce = 2 tablespoons	1 peck = 8 dry quarts
1 cup = 8 fluid ounces	1 bushel = 4 pecks
1 pint = 16 fluid ounces	1 cord = 128 ft^3
1 pint = 28.88 $in.^3$	
1 quart = 2 pints	
1 gallon = 4 quarts	

We should be fairly familiar with many of the liquid measures on the left side. The dry measures may be less familiar to us (although a farmer would certainly recognize pecks, bushels and cords).

You may also notice here that a liquid pint is different in size than a dry pint. A dry pint is larger, in terms of cubic inches.

Question 19: A storage container holds 5 liquid pints of milk. How many dry pints of sugar would it hold?

While the United States is mostly engaged in the usage of its own system of measures, most of the rest of the world uses the international metric system (sometimes abbreviated SI for Système International d'Unités).

The Metric System

The metric system was created in France in the late 1700s. At the time, the idea was to create a measuring system that corresponded better with our own number system of units (ones, tens, hundreds, etc.) and decimals (tenths, hundredths, thousandths, etc.). Since then, it has become adopted by many countries as the standard system of measure throughout most of the world.

Features of the Metric System

1) Unlike the USCS, there are fewer base units in the metric system:
   The meter (abbreviated m) is used for length
   The gram (g) is used for mass
   The liter (L) is used for volume
   The second (s) is used for time
2) Prefixes are used to denote size with respect to the base units:

Prefix	Meaning	Power of 10	As a Value
tera	"Trillion"	10^{12}	1,000,000,000,000
giga	"Billion"	10^{9}	1,000,000,000
mega	"Million"	10^{6}	1,000,000
kilo	"Thousand"	10^{3}	1,000
hecto	"Hundred"	10^{2}	100
deca	"Ten"	10	10
deci	"Tenth"	10^{-1}	0.01
centi	"Hundredth"	10^{-2}	0.01
milli	"Thousandth"	10^{-3}	0.001
micro	"Millionth"	10^{-6}	0.000001
nano	"Billionth"	10^{-9}	0.000000001
pico	"Trillionth"	10^{-12}	0.000000000001

A **centimeter** is 1/100th the size of a meter. A **kilogram** is 1,000 times the size of a gram.

These prefixes find their way into other uses. You should all know that a **kilobyte** (KB) of memory is about 1,000 bytes (It's not exactly 1,000, but that's something for a computer course). A **megabyte** (MB) is about 1,000,000 bytes. A **gigabyte** (GB) is about one billion bytes.

3) The metric system is a base 10 system that offers considerable flexibility in unit size.

For example, we can use the meter to understand all other lengths ending in "meter".

1 kilometer = 1000 meters (1 meter = 0.001 kilometers)
1 hectometer (hm) = 100 meters (1 meter = 0.01 hectometers)
1 decameter (dam) = 10 meters (1 meter = 0.1 decameters)
1 meter
1 decimeter (dm) = 0.1 meters (10 decimeters = 1 meter)
1 centimeter (cm) = 0.01 meters (100 centimeters = 1 meter)
1 millimeter (mm) = 0.001 meters (1,000 millimeters = 1 meter)
1 micrometer (μm) = 0.000001 meters (1,000,000 micrometers = 1 meter)

Depending on what is being measured, you may choose on the units above. A meter may be useful for measuring the dimensions of a room, but you may use a centimeter to determine the length of a pencil. You may use a millimeter to measure the width of a pencil or a micrometer to measure the length of a red blood cell. If you want to know how far away your home is from QCC, you would probably use kilometers.

(Certain measures are used more often than others—in the U.S. you are most likely to see meters, centimeters, millimeters and kilometers in use than other measures like the decimeters.)

4) Anything that applies to one base unit can be applied to other base units. Everything previously mentioned about meters could easily be applied to grams and liters as well (just change some letters above).

5) Conversions between measures becomes very easy because the metric system is a base 10 system (all conversions can be done using the powers of 10 (10, 100, 1000, …, 0.1, 0.01, 0.001 etc.).

Consider the following:

	= 0.545 kilometers (km)
	= 5.45 hectometers (hm)
	= 54.5 decameters (dam)
(Start here!) 545 meters	=

= 5,450 decimeters (dm)
= 54,500 centimeters (cm)
=545,500 millimeters (mm)

You should notice that for every step up you divide by a power of 10 (10^1=10 for one step, 10^2=100 for two steps, 10^3=1000 for three steps, etc.) and for every step down you multiply by a power of 10.

Example 2 Convert 3.4 meters to centimeters.

This is two steps down (meters→decimeters→centimeters). Therefore, you should multiply by 10^2=100:

3.4 •100 =3400 centimeters (cm)

You could also use the conversion factor:

$$\frac{3.4 \text{ meters}}{1} \cdot \frac{100 \text{ centimeters}}{1 \text{ meter}} = 3.4 \cdot 100 \text{ cm} = 3400 \text{ cm}$$

Example 3 Convert 458 grams to kilograms.

This is three steps up (grams→decagrams→hectograms→kilograms). Therefore, you should divide by 10^3=1,000:

458 ÷ 1,000 = 0.458 kilograms (kg)

You could also use the conversion factor:

$$\frac{458 \text{ grams}}{1} \cdot \frac{1 \text{ kilogram}}{1{,}000 \text{ grams}} = \frac{458}{1{,}000} \text{ kg} = 0.458 \text{ kg}$$

Example 4 Convert 0.00007 kilometers to millimeters.

This is six steps down (kilometers→hectometers→decameters→meters→decimeters→centimeters→ millimeters). Therefore, you should multiply by 10^6=1,000,000:

0.00007 • 1,000,000 = 70 millimeters (mm)

You could also use the conversion factors:

$$\frac{0.00007 \text{ kilometers}}{1} \cdot \frac{1000 \text{ meters}}{1 \text{ kilometer}} \cdot \frac{1000 \text{ millimeters}}{1 \text{ meter}} = 0.00007 \cdot 1000 \cdot 1000 \text{ mm} = 70 \text{mm}$$

<u>Question 20:</u> Convert 2,578 milliliters to liters.
<u>Question 20:</u> Convert 3.2 grams to centigrams.
<u>Question 21:</u> Convert 0.07 kilometers to millimeters.

We will see more of the metric system in the next chapter!