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UNCERTAINTY CHARACTERIZATION OF A CONCEPTUAL RAINFALL-RUNOFF MODEL BY USING GREY NUMBERS

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A procedure for characterizing global uncertainty of a rainfall-runoff simulation model based on the use of grey numbers is presented. Through this methodology, the uncertainty is characterized by intervals; once the parameters of the rainfall-runoff model have been properly defined as grey numbers, use of the grey mathematics and functions allows for obtaining simulated discharges in the form of grey numbers whose envelope defines a band which represents the vagueness/uncertainty associated with the simulated variable. The grey numbers representing the model parameters are estimated in such a way that the band obtained from the envelope of simulated grey discharges includes an assigned percentage of observed discharge values and is, at the same time, as narrow as possible. The approach is applied to a real case study and the uncertainty bands obtained both downstream of the calibration process and downstream of the validation process are compared with those obtained by using a well-established approach, like the GLUE approach, for characterizing uncertainty. The results of the comparison show that the proposed approach may represent a valid tool for characterizing the global uncertainty associable with the output of a rainfall-runoff simulation model.

INTRODUCTION

Characterization of uncertainty in the context of hydrological modelling is very important since natural phenomena are typically characterized by an intrinsic variability which cannot be described with a deterministic approach and, on the other hand, the very structure and parameters of hydrological models are a source of error and hence of uncertainty, the models themselves always being a more or less simplified approximation of reality. Also the data measured and used as inputs to the model for its calibration or validation can be affected by measurement errors. Given the presence of these various causes of error, it is possible to define different types of uncertainty, namely, a) inherent randomness (e.g. the weather), b) uncertainty related to the structure of the model, c) uncertainty related to the model parameters and d) uncertainty related to the data. The various causes of error, i.e. these different components of uncertainty, lead to total error, i.e. the formation of total uncertainty associable with the model. It is important to observe, however, that the contribution of the different sources of error to total error is generally not known and breaking down total error into its individual components is often difficult, particularly in a hydrological context, where the models are not linear (Shrestha and Solomatine [1]).

Many methods have been developed and proposed in the scientific literature to characterize the individual components or the total uncertainty. Based on the indications of Shrestha and Solomatine [1], these methods can be classified in general into six categories, namely, (a) analytical methods (e.g Tung [2]), (b) approximation methods (e.g. First Order Analysis, Dettinger and Wilson [3]), (c) resampling-based methods (e.g. Kuczera and Parent [4]), (d) Bayesian methods (e.g. Krzysztofowicz [5]), (e) methods based on the analysis of model errors (e.g. Montanari and Brath [6]) and (f) grey number or fuzzy set theory-based methods (e.g. Maskey et al. [7]; Alvisi and Franchini [8]).

In this study we present a method for characterizing uncertainty in a context of simulation of the rainfall-runoff process, where the attention is focused on total model uncertainty. The technique used is based on grey number theory (Deng [9]) and, with reference to the classification proposed above, it thus falls within category (f). This choice is based on the fact that grey number theory represents an appropriate tool for modelling uncertainties that do not originate from randomness but are caused by imprecise (or incomplete) knowledge about a real system (Jacquin and Shamseldin [10]). When developing a rainfall-runoff simulation model, the representation of the process is in general incomplete/simplified, particularly where a conceptual model is used. It is this imprecise representation of the process which is the major source of uncertainty in the simulation phase. Thus, grey theory can be used as a valid tool for modelling the related uncertainties.

In the sections that follow, we briefly present the structure of the conceptual rainfall-runoff model adopted and how it is greyified in order to represent the total uncertainty of the model. Finally, we present and discuss the results obtained when the proposed method was applied to a specific case study and compared them with those obtained applying the GLUE method (Beven and Binley [11]).

THE GREY ADM MODEL

The rainfall-runoff model considered in this study is of conceptual type and is called ADM - A Distributed Model - proposed by Franchini [12]. The model consists of two main blocks, the first representing the water balance and the second the component of transfer to the basin outlet. Overall, the block representing the water balance is characterized by 7 parameters, whereas the one representing transfer is characterized by 4 parameters, so that the model has a total of n_{par}=11 parameters. Using the ADM model it is possible to represent the rainfall-runoff transformation process in a basin in a simplified manner. In practice, the simulated crisp discharge $Q_{sim,i}$ at a generic time t_i (with i=1:n_{sim}) is a function of (a) the inputs, rainfall and evapotranspiration, observed in the time intervals preceding time t_i until the system gains "memory", (b) the initial system conditions (whose effect is reduced until disappearing as i increases, i.e. as we "move away" from the initial point in time) and (c) the n_{par}=11 parameters of the model.

Let us now assume that each of the inputs, parameters and initial conditions of the model is represented by a grey number. A grey number, hereinafter indicated as $x^{\pm}$, is a number whose exact value is unknown but which falls within an interval, $[x^-, x^+]$ that is known (Liu and Lin [13]). With a grey number, therefore, the uncertainty associated with a given numerical quantity is represented by means of an interval whose upper x^+ and lower x^- limits are known, whereas its distribution within the interval is not (see Alvisi and Franchini, 2010 for further details on grey numbers).

Since, at each computational time step the model output, i.e. runoff at the basin outlet at the generic time t_i, is a function of the inputs, parameters and initial conditions, if each of these is represented by a grey number, the output will likewise be a grey number. In other words, at each time step the grey model does not furnish a crisp value of simulated discharge, but rather a grey number which reflects all the uncertainties of the parameters, inputs and initial conditions, given the selected model, and thus the envelope of the lower and upper extremes of the grey discharges defines a band, which can be interpreted as the total model vagueness/uncertainty.

Operatively, the grey discharges can be computed relying on the definition of the function of grey numbers (see Alvisi and Franchini [14]): with reference to the generic time t_i (where i=1:n_{sim}), we look for a set of real/crisp values of the inputs, parameters and initial conditions, each included in the corresponding grey number, such that the crisp simulation conducted starting from the initial time t_0 provides the minimum simulated discharge $Q^-_{sim,i}$ at the time t_i; similarly, we look for a set of real/crisp values of the inputs, parameters and initial conditions such that the crisp simulation provides the maximum simulated discharge $Q^+_{sim,i}$ at the time t_i. For each simulation time point it is thus necessary to solve two optimisation problems, one of minimisation and one of maximisation. The discharge values obtained through such optimisation processes, $Q^-_{sim,i}$ and $Q^+_{sim,i}$, respectively represent the lower and upper extremes of the grey number $Q^{\pm}_{sim,i}$ representing the simulated discharge at time t_i.

Calibration of the grey ADM model

In the grey ADM model previously described, inputs, parameters and initial conditions are assumed to be grey numbers, thus highlighting a situation where uncertainty is present in all of them. The model output, at each time instant, is a grey discharge which reflects the uncertainties in the parameters, inputs and initial conditions, given the model. This uncertainty is thus the total model uncertainty. It is clear that if the uncertainty in the inputs, parameters and initial conditions is increased, the grey discharge at each time instant will have a larger amplitude. Now, in order to calibrate the ADM model we would like to fix the level of model uncertainty, i.e. we would like to fix a level of model uncertainty we consider acceptable. This acceptable level could be defined in such a way that at least a given percentage (e.g. PI = 95%) of observed discharges are included in the corresponding grey discharges produced by the model and, at the same time, these grey discharges are as narrow as possible. Once we have fixed this acceptable level of uncertainty in the model output (model uncertainty), we are interested in defining the grey parameters, inputs and initial conditions producing such an output. This would imply breaking down the global model uncertainty into the different components (input uncertainty, parameter uncertainty, initial condition uncertainty) but, as pointed out in the introduction, this would be very complex, if not impossible. However, this is not our purpose since we are not interested in characterising the single uncertainty components but only the total model uncertainty. Thus, we assume crisp inputs and initial conditions (as usually assumed in practical applications) and we delegate the grey parameters to produce grey discharges which overall respect the total uncertainty level requested. In this situation the grey parameters do not represent the parameter uncertainty alone since other sources of uncertainty are included within them. In other words, these grey parameters are those which, within the framework of grey mathematics, allow the predefined level of total model uncertainty to be obtained when other sources of uncertainty (inputs and initial conditions) are considered as crisp values.

Formally speaking, the calibration process described above consists in looking for the lower and upper extremes of each of the parameters of the model such that:

$$\sum_{i=1}^{n_{cal}} \left| Q^{+}_{sim,i} - Q^{-}_{sim,i} \right| \text{ will be minimum} \tag{1}$$

subject to

$$\left(\frac{1}{n_{cal}} \sum_{i=1}^{n_{cal}} \delta_i \right) \cdot 100 \geq PI$$

where (2)

$$\delta_i = \begin{cases} 1 & \text{if } Q^{-}_{sim,i} \leq Q_{obs,i} \leq Q^{+}_{sim,i} \\ 0 & \text{otherwise} \end{cases}$$

where $Q_{obs,i}$ is the observed crisp discharge at the basin outlet at each of the time points t_i of the calibration time window (where i=1:n_{cal}) and PI is the pre-assigned percentage of observed to be included within the simulated grey discharges.

CASE STUDY

The proposed approach was applied to a real case, the Sieve river, whose main course is 56 km long and the time of concentration is about 10 hours. The available data consist in the hourly time series of discharges at the basin outlet, areal rainfall and evapotranspiration pertaining to two distinct periods: the first extending from 1 December 1959 to 31 March 1960; the second from 1 January 1992 to 31 December 1996. The calibration of the grey ADM model was repeated twice, the first time by using a calibration time window T_{cal}^{59-60} extracted from the period between December 1959 and March 1960, and the second time by using a calibration time window T_{cal}^{92-96} extracted from the period between January 1992 and December 1996; the reaming data of each period (T_{val}^{59-60} and T_{val}^{92-96}) were used in the procedure validation phase. The choice of using the data regarding these periods to perform two different calibrations and validations was based on the consideration that the quality of the data is different; in particular the series relating to the period between January 1992 and December 1996 can be considered of "lower quality", as significantly discordant patterns can be identified for a number of events as regards the entity of rainfalls and the corresponding runoffs.

Operatively, the calibrations were performed by using the expeditious procedure proposed by Alvisi et al. [15] in order to reduce the computational times. This procedure is based on the observation that (a) some model parameters may be more significant than others for the purpose of characterising the variability of simulated discharges and (b) the crisp parameter values which produce the minimum and maximum values of simulated discharges over time tend to coincide with the extreme values of the corresponding grey numbers. Thus, via the Hornberger-Spear-Young method (Hornberger and Spear [16]; Young [17]) $n^{*}_{par}=8$ most significant parameters of the ADM model were identified and the corresponding grey values calibrated. For more details concerning the expeditious procedure the reader can refer to Alvisi et al. [15].

ANALISYS AND DISCUSSION OF THE RESULTS

Table 1 shows the average widths (AW) of the bands obtained from the envelope of the grey numbers representing the simulated grey discharges and the percentage of observed values included within the bands (POC) (see Alvisi and Franchini [8] for a formal definition of these metrics). In particular, Table 1 shows the results of the two calibrations performed on the time windows T_{cal}^{59-60} and T_{cal}^{92-96}, and of the corresponding validations performed respectively on the time windows T_{val}^{59-60} and T_{val}^{92-96}.

Table 1. Average width (*AW*) of the bands and corresponding percentages of observed values included (percentage of coverage *POC*) obtained using the procedure based on grey numbers (assuming *PI*=95% in the calibration phase) in relation to the different calibration and validation time windows.

	Calibration		Validation	
	***AW* (m^3/s)**	***POC* (%)**	***AW* (m^3/s)**	***POC* (%)**
Cal. on $T_{cal}^{59-60} \rightarrow$ Val. on T_{val}^{59-60}	68.4	95.1	51.9	81.0
Cal. on $T_{cal}^{92-96} \rightarrow$ Val. on T_{val}^{92-96}	74.3	95.0	35.5	75.2
Cal. on $T_{cal}^{59-60} \rightarrow$ Val. on T_{val}^{92-96}	68.4	95.1	17.8	49.5
Cal. on $T_{cal}^{92-96} \rightarrow$ Val. on T_{val}^{59-60}	74.3	95.0	96.1	93.3

Analysing the results obtained in the calibration phase, we may observe that the percentage of observed values actually included corresponds to the one imposed, thus demonstrating the correctness of the calibration procedure adopted. If we consider, on the other hand, the validation results obtained for the time windows T_{val}^{59-60} and T_{val}^{92-96} using the parameters calibrated on the time windows T_{cal}^{59-60} and T_{cal}^{92-96}, respectively, we note that the percentage of coverage falls to around 81% for the first data set (relative to 1959-1960) and around 75% for the second data set (relative to 1992-1996, that is the one of "inferior quality"). Figure 1 shows the corresponding trend in the band obtained for the validation time window T_{val}^{59-60} using the grey procedure with parameters calibrated on the time window T_{cal}^{59-60}. It may be observed that the fact that the percentage of coverage reported in Table 1, equal to 81%, is lower than the expected 95% is mainly due to the presence of some well-defined time intervals (see intervals highlighted in Figure 1) for which the outputs of the model in terms of grey runoffs deviate significantly from the observed values. These time intervals are concentrated in the recession phases (see for example intervals 1 and 3) or are associated with simulated events, of modest entity, which do not have any corresponding observed/recorded event (see for example intervals 2 and 4). Similar considerations apply to the validation time window T_{val}^{92-96}.

Table 1 also shows the results of the model validation for the time window T_{val}^{92-96}, which was performed on the basis of the grey parameters calibrated on the time window T_{cal}^{59-60}, and results of the model validation for the time window T_{val}^{59-60}, which was performed on the basis of the grey parameters calibrated on the time window T_{cal}^{92-96}. These results further highlight the effect of the different "data quality" on the percentage of coverage in the validation phase: in fact if we consider the results of the validation on the time window T_{val}^{92-96}, performed using the calibrated parameters for the time window T_{cal}^{59-60} the percentage of coverage falls to around 50%. This is understandable considering that the grey parameters calibrated with a "good" dataset (i.e. a data set with consistent rainfall-runoff data and without data relative to phenomena which cannot be described by the rainfall-runoff model used) are "narrow" and therefore, once used in a simulation with a poorer quality dataset, are able to explain the trend in observed values to a lesser degree (as also shown by the average width of the corresponding simulated band, which, the time window (T_{val}^{92-96}) being equal, is reduced by about one half, from around 35 m^3/s in the case of parameters calibrated on the time window T_{cal}^{92-96} to around 18 m^3/s in the case of parameters calibrated on the time window T_{cal}^{59-60}). The opposite applies for the validation results with respect to the time window T_{val}^{59-60}, where the validation was performed using the parameters calibrated on the time window T_{cal}^{92-96}. In this case the average bandwidth obtained in the validation phase (T_{val}^{59-60}), based on parameters calibrated on a dataset (rainfall-discharge) affected by a high degree of uncertainty, increases significantly (from around 52 m^3/s for the parameters calibrated on the time window T_{cal}^{59-60} to around 96 m^3/s for the parameters calibrated on the time window T_{cal}^{92-96}), so that a high percentage of observed values, close to 93% can be included within the band.

In short, percentages of observed values included depend on the quality of the data and if the procedure is parameterised with a good dataset, using a dataset of inferior quality in the validation phase will make it difficult to correctly represent the vagueness/uncertainty associated with these latter data.

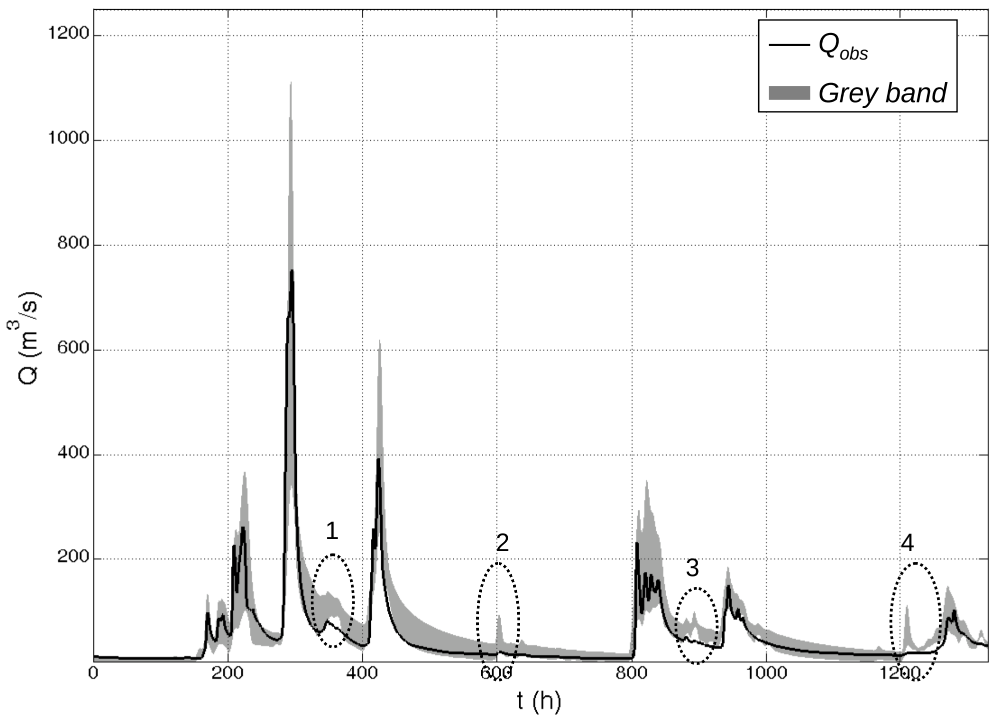

Figure 1. Validation time window T_{val}^{59-60}. Band produced by the grey ADM model with grey parameters calibrated on the time window T_{cal}^{59-60}

To conclude the analysis of the results obtained with the proposed procedure, we present, by way of comparison, the results that would be obtained using a method widely employed in the field of hydrology to characterise global uncertainty in the simulation phase, namely, GLUE (Generalized Likelihood Uncertainty Estimation, Beven and Binley [11]).

This procedure was calibrated on the time windows T_{cal}^{59-60} and validated on the corresponding time windows T_{val}^{59-60} (see Figure 2).

Keeping in mind the different conceptual background characterising the GLUE and grey bands, i.e. in the GLUE case the band is an ensemble of distinct behavioural realizations of the crisp model whereas in the grey approach the band is a single grey realization produced by the grey ADM model, if we compare the band of Figure 2 obtained using the GLUE procedure with the corresponding band of Figure 1 obtained using the grey model we can observe that the two procedures provide similar results in terms of band trends and widths. In particular, the problems encountered with the grey method in relation to the four numbered time intervals, already commented with reference to Figure 1, are still present when the GLUE method is applied: in fact the GLUE method also gives bands that do not include the observed data for these time intervals and thus this situation can properly be ascribed to some inconsistencies in the data or to the difficulties of the rainfall-runoff model (due to some internal limitation or inaccuracy) in effectively reproducing the discharges in these time windows.

Also the average bandwidths and percentages of coverage show that the GLUE method, and the grey method give similar performances. However, with respect to the calibration time window, the GLUE method gives a greater *AW* (81.9 m^3/s vs. 68.4 m^3/s) (*POC* being the same), whereas, for the validation time window, the GLUE method gives a slightly higher *POC* (85.9% vs. 81.0%) but also a greater *AW* (65.3 m^3/s vs. 51.9 m^3/s; see also Figure 1 and figure 2, peak at $t \cong 300$ h and peak at $t \cong 450$ h).

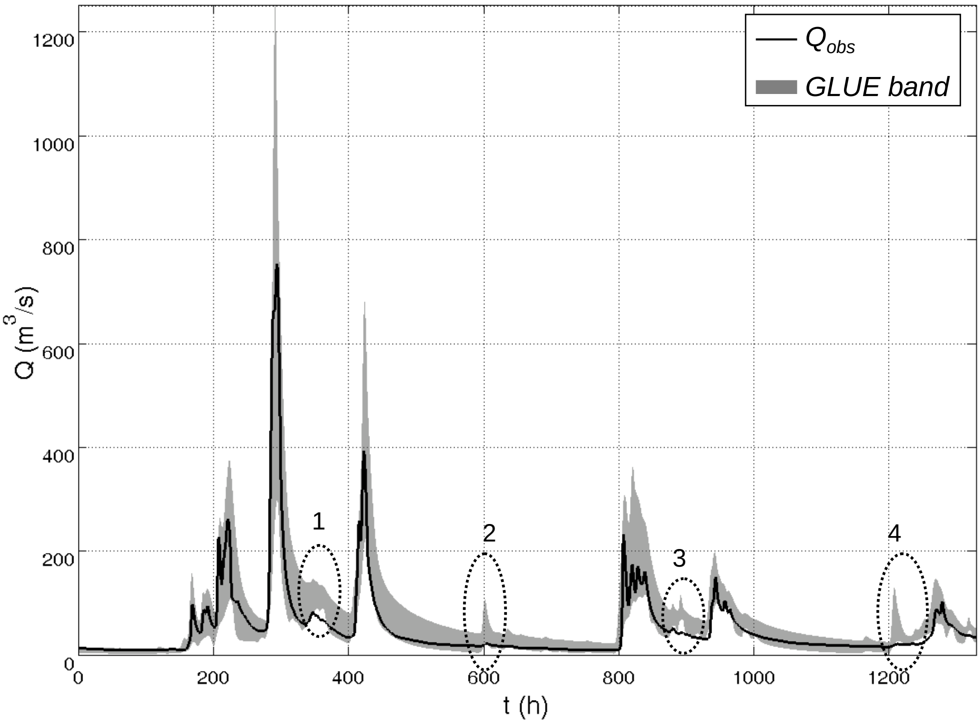

Figure 2. Validation time window T_{val}^{59-60}. Band produced by the GLUE procedure calibrated on the time window T_{cal}^{59-60}

CONCLUSIONS

This paper proposes a new procedure for characterising total uncertainty in rainfall-runoff simulation model based on using grey numbers to parameterise the model. The model so parameterised provides not a crisp value of the simulated discharge at each time step, but rather an interval representing the total model vagueness/uncertainty.

The grey parameterisation of the rainfall-runoff model and its subsequent validation on different sets of real data revealed that the procedure leads to the definition of uncertainty bands which, in the calibration phase, perfectly cover a percentage of observed values equal to the one imposed, but which in validation tend to underestimate these percentages. This tendency becomes more marked as the "quality of the data" used declines, i.e. when the data show inconsistencies and/or are relative to phenomena which cannot be described by the rainfall-runoff model used given its structure, and is particularly accentuated when the procedure is calibrated on datasets of good quality and validated on datasets of poor quality.

Finally, a comparison between the bands obtained respectively with the grey procedure and GLUE method shows a strong similarity between the two approaches, although the grey approach produces slightly narrower bands, the percentage of coverage being the same.

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