2018

Mathematics in Contemporary Society - Chapter 13 (Spring 2018)

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Chapter 13

Functions

Example 1
Suppose you are going to the gas station to buy gas for your car. Depending on how much gas you purchase, you will pay a certain amount:

Gallons of Gas	Cost
0	$0.00
1	$2.50
2	$5.00
3	$7.50
4	$10.00
5	$12.50
6	$15.00
7	$17.50
8	$20.00
9	$22.50
10	$25.00

For any whole number of gallons from 0 to 10, we know the corresponding cost. We can also approximate decimal values in between. A purchase of 7.8 gallons would cost somewhere between $17.50 and $20.00.

If we wanted to know much 14 gallons would cost, we might be able to figure out that it would cost $10 more (each gallon of gas seems to be $2.50), or $35. Perhaps a formula would be more useful:

Cost = $2.50·Gallons

This allows me to calculate the cost of any number of gallons.

If I purchased 14 gallons of gas:

Cost = $2.50·(14 Gallons)

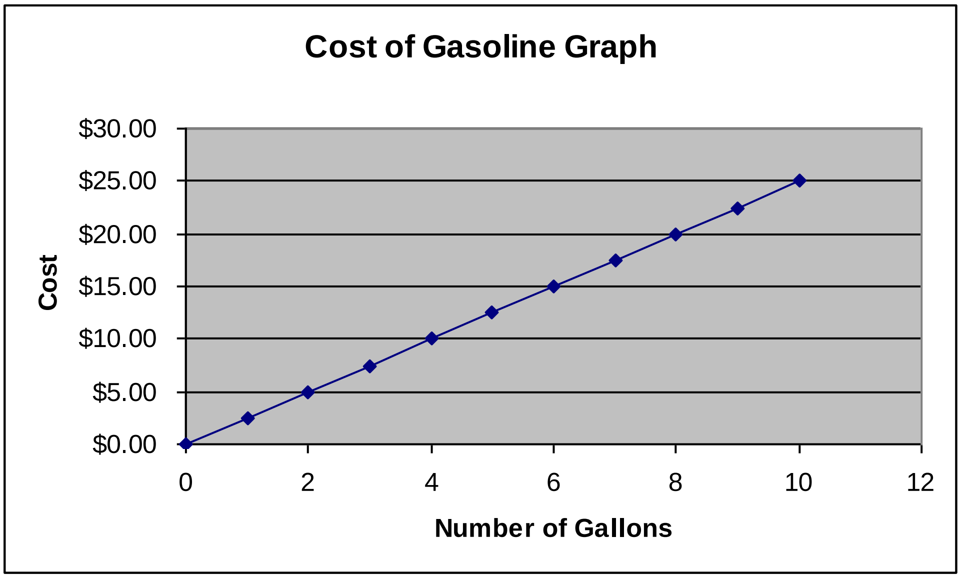
Cost = $35.00

If I purchased 7.8 gallons of gas:

Cost = $2.50·(7.8 Gallons)

Cost = \$19.50

If we don't like a formula, we can plot points on a graph (just like we did with a scatter plot) and connect the dots:



The graph gives us a visual sense of the cost of gasoline rising steadily as the number of gallons rises.

The relationship between the number of gallons purchased and the cost is a **function**.

A **function** is a relationship between at least two variables such that, for every one (or sometimes more) independent variable(s) there is one and only one corresponding dependent variable.

In this case, the number of gallons of gas we buy is the **independent variable**, because you can buy as many gallons of gas as you would like. The cost of gas is the **dependent variable,** because the cost depends on the number of gallons. For any number of gallons of gas you want to purchase, there is one and only one corresponding cost.

Very often we use x to represent the independent variable and y to represent the dependent variable. Instead of :

Cost = \$2.50·Gallons

We may write:

y = 2.50·x

where x = the numbers of gallons and y = the cost

We may also simply write:

y = f(x) (We say "y equals f of x")

to imply that "y is a function of x."

Let's look at another example:

Example 2

Suppose you are taking a cab ride into the city. Usually, there is a charge for being picked up by the cab and a charge per mile. Suppose the pickup charge is $2 and the cost per mile is $0.75. We can make a chart of the cost of different cab rides:

Number of Miles	Cost
0	$2.00
1	$2.75
2	$3.50
3	$4.25
4	$5.00
5	$5.75
6	$6.50
7	$7.25
8	$8.00
9	$8.75
10	$9.50

In this example, the number of miles you travel is the independent variable, because you travel as far as you would like. The cost of the trip is the dependent variable**,** because the cost depends on far you go. For any number of miles you would like to travel, there is one and only one corresponding cost.

For any whole number of gallons from 0 to 10, we know the corresponding cost. We can also approximate decimal values in between. A cab ride of 6.4 miles would cost somewhere between $6.50 and $7.25.

If we wanted to be more exact, or take a longer trip like 23 miles, a formula would be more useful:

Cost of Cab Ride= $0.75·Miles + $2

(We may also use function notation of y = 0.75x + 2, where x = miles and y = cost.)

This allows me to calculate the cost of any number of miles.

If I rode for 6.3 miles:

Cost of Cab Ride= $0.75·(6.4 Miles) + $2

Cost = $4.80 + $2.00

Cost = $6.80

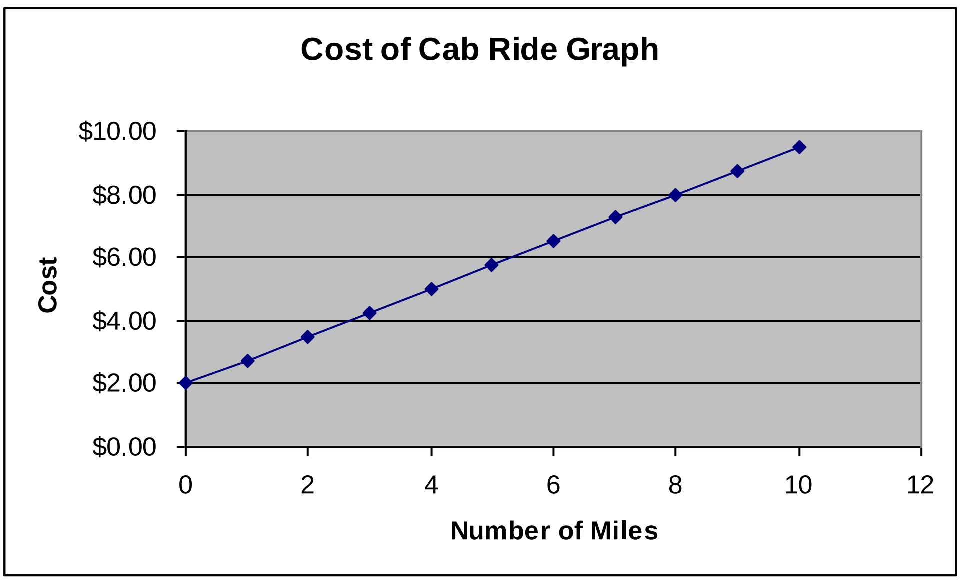
If I rode for 23 miles:

Cost of Cab Ride= $0.75·(23 Miles) + $2

Cost = $17.25 + $2.00

Cost = $19.25

If we don’t like a formula, we can plot points on a graph, as we did earlier:



We see a similar graph as before, a steadily increasing straight line.

Not all functions are as simple as this, however.

Example 3

Suppose every morning at 8 am, a soda machine in the Science Building lobby is filled with 200 cans of soda. At the beginning of every hour, the number of sodas in the machine is counted. We can make a chart of the hourly observations:

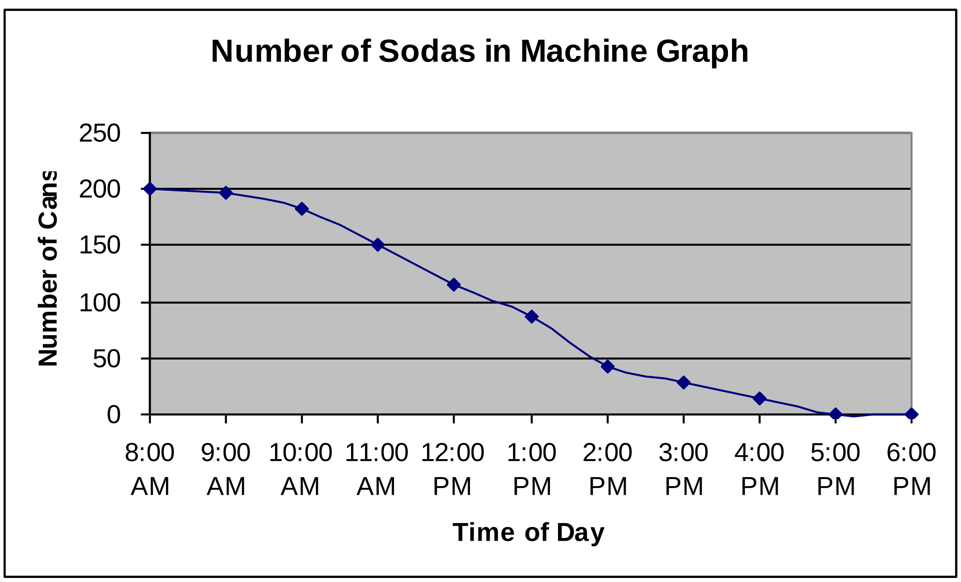
Time of Day	Number of cans
8:00 AM	200
9:00 AM	197

10:00 AM	182
11:00 AM	151
12:00 PM	115
1:00 PM	86
2:00 PM	43
3:00 PM	28
4:00 PM	14
5:00 PM	0
6:00 PM	0

In this example, the time of day is the independent variable, because the time of day is not controlled by anything else. The number of cans in the machine is the dependent variable**,** because it depends on what time it is.

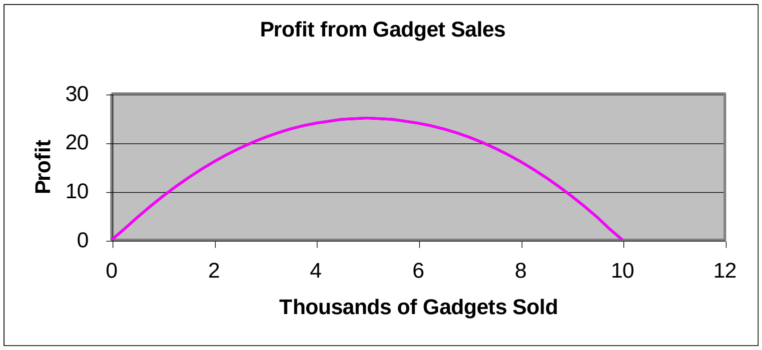
For any hour from 8 am to 6 pm, we know the number of cans in the machine. We can also approximate for any time in between. At 2:43 pm, there were between 43 and 28 cans in the machine.

In this case, it does not seem that we can create a simple formula, as we did in the last two examples. We can create a graph, however:

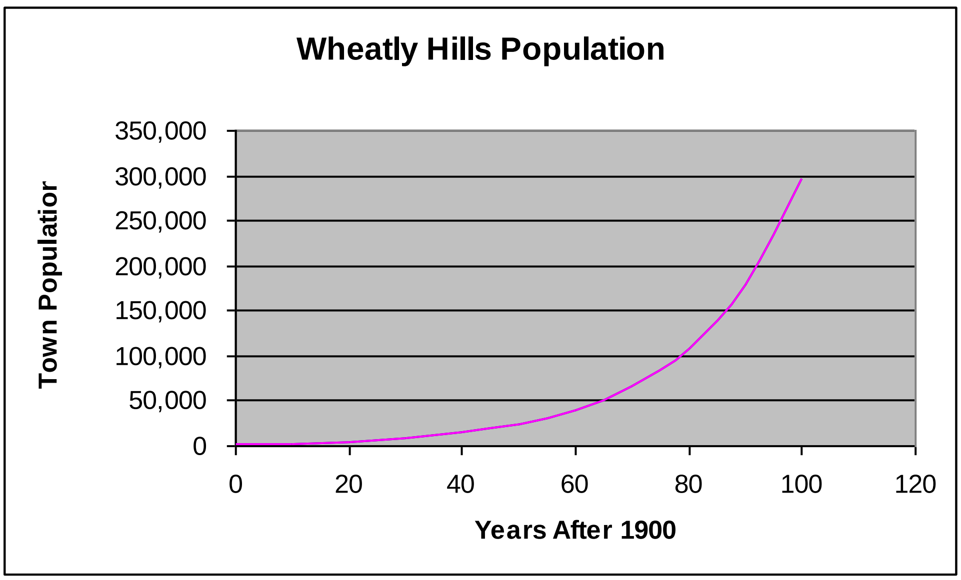


The graph and chart help us see that the number of soda does not drop at a constant rate. Certain hours (between 11 am and 2 pm) are obviously busier than others and more cans are sold. The early hours are slow for business and apparently nothing is sold after 5 pm because the machine is empty!

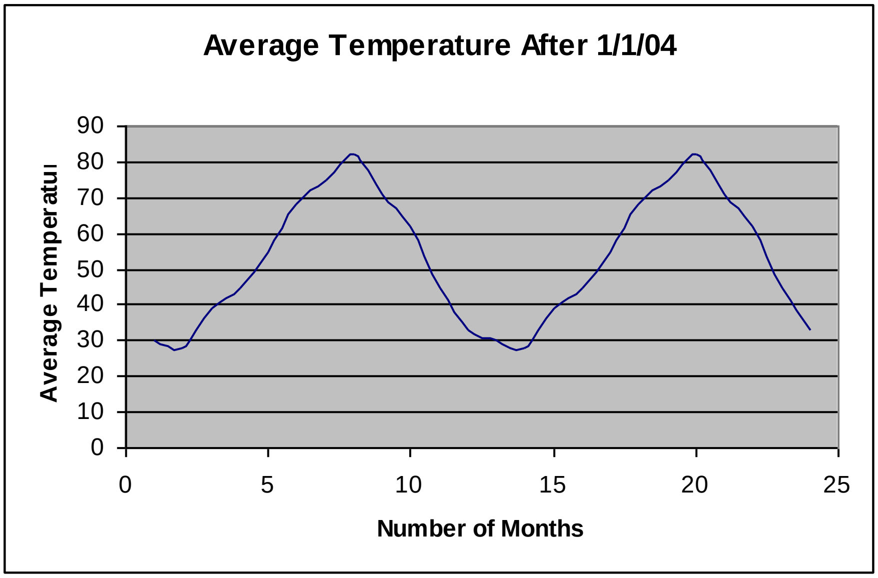
I have given you three examples of functions. The gasoline and cab examples are referred to as **linear functions**, because they appear graphically as a straight line. Here are some others, with details omitted:



Quadratic functions have a parabolic (curved U) shape**,** reaching a maximum or minimum, depending on which way the parabola is curved.



Exponential functions show rapid growth that accelerates quickly to larger and larger amounts, as often seen in population growth.



Periodic functions have a repeating pattern, as can be seen in the above graph. Every year, the average temperature will follow a similar pattern.

The only assignment left is the Week 13/14 Quiz. It's the last one!