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Assessment Of The Sensitivity Of GompitZ

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ON THE UNCERTAINTY OF PROBABILISTIC SEWER DETERIORATION FORECAST MODELLING

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Decision support for sewer rehabilitation programs is dependent on reliable information about the asset conditions. Inference about the condition development can be made by combining condition observations with deterioration forecasting models. GompitZ is a Non-Homogenous Markov Chain model, which enables waste water utilities to predict the sewer deterioration at a network segment level, based on closed-circuit TV (CCTV) inspection and normalized condition grading [1]. The predictions can be used to forecast sewer renewal needs. Successful calibration and prediction is however dependent on the validity of the model assumptions, sufficient amounts of CCTV observations, and that the condition grading system used is capable of describing the actual sewer conditions.

Two research questions are addressed in this paper:

1. How is the uncertainty of GompitZ predictions dependent on the amount of inspections available for calibration?
2. Is it possible to assess this uncertainty if one has an incomplete dataset of inspections?

To answer the first question, Monte Carlo simulations were applied, and the uncertainty of predictions was assessed for inspection samples of varying size. It was found that the uncertainty may be considerable even for datasets where 40-60 % of the network has recently been inspected; a bias in the portions of the predicted condition classes was also detected. The second question has been addressed by considering the combination of the variance in the model parameters, and the variance of the sample (distribution of pipe segments in each condition), and it has been concluded that it is only possible to estimate the uncertainty of an incomplete inspection dataset if the sample variance is small.

INTRODUCTION

Condition assessments are paramount input information for any infrastructure asset management process in which asset condition is a decision-influencing factor. Condition measurements on a subset of an asset stock can, in combination with understanding of deterioration processes and application of appropriate models, be used to assess the condition of the whole asset stock both for present time and the future. The value of performing condition assessments are manifested in better understanding of the overall state of the network, and input to decision support for prioritising renewal resources on the asset stock. However, the value of condition data is not absolute, and is dependent on many factors, both related to the properties of the assets (condition variability), and the nature of the asset owners' renewal strategy (renewal intensity, desired level of service, etc.).

CCTV condition assessments on sewers are associated with considerable costs, and it is therefore important to provide guidlines to help decide how much and where to perform inspctions. Given that the quality of the collected inspectiosn is good enough, one can use such guidelines to assess the amount of CCTV inspections necessary to support renewal investment decisions, and thereby balancing the costs of inspection with improved decision support [2]. This paper presents a case study where the quality of information generated from CCTV sewer condition inspections and Markov chain modelling has been evaluated, in terms of uncertainty, as a function of how large a portion of the sewer network has been inspected. The following research questions have been addressed:

- How is the uncertainty of GompitZ predictions affected by how large the portion of the network has been inspected?
- Is it possible to estimate this uncertainty for datasets where only a portion of the network has been inspected?

GompitZ

GompitZ is a sewer deterioration model that was developed under the research project CARE-S [3], and is a Non-Homogenous Markov Chain model [1], where the probability of future condition states of pipes are calculated based on past condition observations; the condition class (CC) of a pipe is modelled as a Markov Chain state probability in GompitZ. The Markov transition probabilities are calculated using Gompertz' distribution, as derived from Gompertz-Makeham's law, which states that the decay rate is the sum of a time-dependent and a time-independent deterioration rate component [4]. GompitZ has been defined with these properties, and both the time-dependent and the time-independent elements may be modified with explanatory covariate vectors ($\mathbf{Z_0}$ and $\mathbf{Z_1}$ in Eq. (1), respectively); selected covariates may thus affect both the initial condition and the deterioration rate [1]. The survival function S_{ik} for a pipe i, which expresses the probability that a pipe is in CC k or better (of c possible CCs), is written in Eq. (1). The regression parameters in GompitZ ($\boldsymbol{\alpha}_k$'s, $\boldsymbol{\beta_0}$ and $\boldsymbol{\beta}_1$), are determined by maximising the likelihood function, employing a mixed general linear regression method.

$$\begin{aligned}
&S_{ik}\left(t\,|\,u_i\right) = P\left\{Y\left(t\right)\le k\right\} = \exp\left[-\exp\left[\alpha_k + \mathbf{Z}_{0i}^T\boldsymbol{\beta}_0 + t\cdot\exp\left(\mathbf{Z}_{1i}^T\boldsymbol{\beta}_1 + u_i\right)\right]\right]\\
&k = \left\{1,2,...,c-1\right\}\\
&S_{ic}\left(t\,|\,u_i\right) = 1\\
&u_i \sim N\left(0,\sigma^2\right), \text{individual frailty factor}
\end{aligned} \qquad (1)$$

One may note that the transition probabilities in GompitZ are restricted so that a pipe segment may only make a transition to the adjacent "worse" condition. In this way, it is not possible for a pipe to improve from a worse to a better condition, which has been known to occur when modelling sewer deterioration with regression or machine learning models [5]. For a more thorough review of the theoretical background of GompitZ, it is referred to Le Gat [1].

The output from an analysis with GompitZ may consist of (1) the calibrated parameters ($\boldsymbol{\alpha}_k$, $\boldsymbol{\beta_0}$, $\boldsymbol{\beta}_1$ and σ; here forth termed as $\boldsymbol{\theta}$), (2) the predicted condition probabilities for each pipe and time in the prediction period ($\mathbf{P}_i(t)$), and a list of pipes of recommended pipes for renewal over the prediction period (given a user-defined condition-based renewal strategy).

RESEARCH METHOD

Monte Carlo uncertainty assessment method

The first research question to be answered in this paper is: How does the uncertainty of GompitZ predictions depend on the availability of condition observations? A Monte Carlo approach has been employed to perform this uncertainty assessment.

First, a dataset F where all N pipes i have been observed once is presented in Eq. (2), where $\mathbf{Z}_{0i}$, $\mathbf{Z}_{1i}$ represent the vectors of explanatory covariates for pipe i, t_i is the time of inspection, y_i is the condition observation, and w_i is the weight of that observation. Based on the full set of observations F in Eq. (2), a random R_m subset of size n is selected, see Eq. (3).

$$F = \{O_i,\ i = 1,2...,N\}, \quad O_i = \{\mathbf{Z}_{0i}, \mathbf{Z}_{1i}, t_i, y_i, w_i\} \tag{2}$$

$$R_m = \{O_j,\ j = 1,2...,n\}, \quad R_m \subset F, \quad O_j = \{\mathbf{Z}_{0j}, \mathbf{Z}_{1j}, t_j, y_j, w_j\} \tag{3}$$

For each selected subset R_m the conditional state probabilities for a pipe j can be calculated according to Eq. (4), while the conditional and marginal likelihood functions may be expressed according to Eq. (5):

$$P_{\boldsymbol{\theta}}\left\{Y(t_j) = k \middle| u_j\right\} = S_k\left(t_j \middle| u_j\right) - S_{k-1}\left(t_j \middle| u_j\right) \tag{4}$$

$$\begin{gathered} L\left(\boldsymbol{\theta} \middle| O_j, u_j\right) = P_{\boldsymbol{\theta}}\left\{Y(t_j) = y_j \middle| u_j\right\}^{w_j} \Rightarrow L\left(\boldsymbol{\theta} \middle| O_j\right) = \int_{-\infty}^{+\infty} L\left(\boldsymbol{\theta} \middle| O_j, u_j\right) \phi\left(u, \sigma^2\right) du \\ \phi \sim N\left(u, \sigma^2\right) \end{gathered} \tag{5}$$

$$\text{maximise } L(\boldsymbol{\theta}, R_m) = \prod_{j=1}^{n} L\left(\boldsymbol{\theta}, O_j\right) \Leftrightarrow \text{maximise } \ln\left[L(\boldsymbol{\theta}, R_m)\right] \tag{6}$$

Where $\boldsymbol{\theta}_m$ represents the vector of model parameters in GompitZ (α_k's, $\boldsymbol{\beta}_0$ and $\boldsymbol{\beta}_1$). GompitZ can then be calibrated by maximizing the logarithm of the marginal likelihood function for all pipes j in subset R_m, as expressed in Eq. (6). For each time one generates a random subset R_m, one will be able to obtain one parameter vector $\boldsymbol{\theta}_m$. Based on each $\boldsymbol{\theta}_m$ one can calculate the survival function for all N pipes according to Eq. (1), together with the individual frailty factor for the pipes that have been observed, and further calculate the condition probabilities of each pipe $\mathbf{P}_m(t_p)$ at a given time of prediction t_p. If selecting random subsets R_m of the same size n, performing calibrations and predictions are carried out sufficiently many times, one will be able to estimate confidence intervals and describe the variability of the model parameters $\boldsymbol{\theta}$ and the survival functions for all N pipes. The variability of the calibrations and predictions have been described by recording 2.5 %, 50.0 %, 97.5 % percentiles, expected value and variance of the parameter vector $\boldsymbol{\theta}$, the condition probabilities matrix $\mathbf{P}$, the deterioration index (*DI*), the individual frailty factor. Only results for the condition probabilities are presented in this paper.

The procedure described above has been performed for subset sizes $n/N = 10, 20, ..., 90\%$ for each stratum, with 1 000 repetitions for each subset size.

Uncertainty assessment for incomplete datasets

The second research question presented in the introduction was whether one could assess the prediction uncertainty in GompitZ under limited inspection rates; e.g. if one has inspected only say 30 % of a sewer network – is it then possible to assess the uncertainty of predictions? There are two sources of uncertainty related to GomptiZ predictions: (1) the sample uncertainty (multivariate hyper-geometric distribution), (2) the uncertainty of the model parameters.

The sample uncertainty arises from the fact that not all pipes have been observed, and that the distribution of observed condition states does not necessarily faithfully picture the whole population distribution. The best estimator for the number of pipes *m* in condition *k*, when *y* pipes in CC *k* have been observed, and *n* out of a total *N* pipes have been observed can be expressed as Eq. (7) [6], which further means that the probability of observing that combination of CC's, can be expressed as in Eq. (8), which is the multivariate hyper-geometric probability mass function, where the observations $\mathbf{y}=[y_1,y_2,\ldots,y_{c-1},y_c]$ have been made on the state space [1,c]. It is in principle possible to assess the variance of the expression in Eq. (8), and may be solved by Markov Chain Monte Carlo techniques. Due to the large binomial coefficients (N>1000) one will have to calculate the value of the binomial coefficients by Stirling's approximation [7], and thereby get an estimate for the sample variance (Eq. 9).

$$\tilde{m}_k = \left\lfloor \frac{(N+1)\,y_k}{n} \right\rfloor \tag{7}$$

$$P\left(\mathbf{K}=\tilde{\mathbf{m}}\middle|\mathbf{y}\right)=\frac{\prod_{k=1}^{c-1}\binom{\tilde{m}_k}{y_k}\cdot\binom{N-\sum_{i=1}^{c-1}\tilde{m}_k}{n-\sum_{i=1}^{c-1}y_k}}{\binom{N}{n}} \tag{8}$$

$$\boldsymbol{\sigma}^2_{\mathbf{K},\text{sample}} \simeq \operatorname{var}(\mathbf{K})\Big/ N^2 \tag{9}$$

The covariance matrix ($\boldsymbol{\Sigma}(\boldsymbol{\theta})$) for the parameters (2) is calculated in the calibration process. It is possible to evaluate the propagation of the uncertainty from the model parameters to the survival function by applying the delta method, as shown in Eq. (10).

$$\begin{gathered}\tau_{ik}^2=\sum_o\sum_q\boldsymbol{\Sigma}(\boldsymbol{\theta})_{o,q}\frac{dS_{ik}}{d\theta_o}\cdot\frac{dS_{ik}}{d\theta_p}\\ \sqrt{n}\left(S_{ik}\left(\hat{\boldsymbol{\theta}}\right)-S_{ik}\left(\boldsymbol{\theta}\right)\right)\to\lim_{n\to\infty}N\left(0,\tau_{ik}^2\right)\quad\Rightarrow\sigma_{ik,\theta}^2=\tau_{ik}^2\end{gathered} \tag{10}$$

This method is applicable to assess the variance in CC's distribution for each pipe. However, when considering the whole stock of pipes, one has to consider the dependency between the pipes, since their CC probability distributions are governed by the same covariates. It is therefore simpler to assess the uncertainty of the predictions for the whole asset stock by considering the calibrated parameters as asymptotically normal distributed, and repeat predictions many times with multivariate normal random covariates generated based on the covariance matrix: $\hat{\boldsymbol{\theta}}^*\sim\mathrm{N}_d\left(\hat{\boldsymbol{\theta}},\boldsymbol{\Sigma}_{\boldsymbol{\theta}}\right)$. The uncertainty σ_k^2 of the predictions can then be estimated based on the repeated predictions. If one assumes independence between the parameter uncertainty and the sample uncertainty, one can express the total uncertainty in terms of the sum of sample and parameter variance: $\sigma_k^2=\sigma_{k,\theta}^2+\sigma_{k,sample}^2$

Case study dataset

The data that has been considered in this case study is from Oslo VAV (Oslo municipality, Norway). In total 12 003 CCTV condition assessments were considered in GompitZ, amounting to a total network length of 499 km or 27 % of the complete Oslo VAV sewer network. All CCTV inspections in this dataset were conducted between 2002 and 2012, and approximately 85 % of the inspections were carried out in the period 2008-2012. The CCTV inspections have been evaluated and coded into condition grades from 1 to 5 (best to worst), according to the standardised Norwegian sewer condition classification system [8]. Four strata were calibrated in the research project *Secured and Monitored Service from Oslo VAV (SMS)*, where the goal was to use condition monitoring efforts as an aid for rehabilitation planning [9]. The strata were divided as concrete pipes (< Ø 600 mm; 3 783 pipes), concrete culverts (>= Ø600 mm; 564 pipes), non-concrete pipes (< Ø 600 mm; 6 989 pipes), and other culvert materials (>= Ø 600 mm; 667 pipes). The two culvert strata were not considered in this paper due to their small sample sizes (hence was the model calibrations based on 10 772 pipe segments).

Pipe diameter, type of effluent, construction period, presence of road traffic, type of bedding soil, and presence of trees were used as model covariates [9]; all of which are known to show a significant impact on sewer deterioration in other studies in the literature [10-12].

Virtual datasets with 2 500 and 5 000 pipes were also selected, with diameter and effluent type as explanatory covariates. The individual frailty factors were generated as standard normal random numbers.

RESULTS

Overall CC variability results

By considering Figure 1 and Table 1 one may observe that the uncertainty in sewer CC predictions may be significant when a lesser extent of a sewer network has been observed. Even with 40 % of the network inspected, the uncertainty in the CC properties may vary greatly. One may also notice that the smaller datasets have greater uncertainty than the larger, but that this effect is smaller for the virtual datasets. The overall uncertainties of the virtual datasets are also smaller than for the real datasets.

When only a portion of the network has been inspected and predicted with GompitZ, there is also a bias in the expected distribution of the CC's; i.e. the condition proportions will be under- or overrepresented; this can be explained by the fact that an increasing amount of the variability will be accounted for in the individual frailty factor, and that this will have an increasing impact on the shift in the probability distributions as more of the network is observed. This effect is present both for virtual and real datasets.

Variability at pipe level

Figure 2 shows the condition predictions for each pipe segment for three different situations, one where predictions have been made on a calibration from 40 % of the data (left), one where several 40 % subset predictions have been aggregated (middle), and one where all pipes have been used for calibration. Each vertical line represents a pipe segment. One may firstly note that there is less certainty in the left plot compared to the right plot, one may hence be much less certain about predictions based on 40 % of the observations. One may also note that some CC's are underrepresented, and that some are overrepresented.

Further, one may notice the differences between the left and the middle plot; if one aggregates predictions based on several subsets, the prediction for each pipe segment becomes much more uncertain. We may interpret the difference between the left and the middle plot as a manifestation of the fact that the individual variability of each condition observation is high compared to the explanatory power of the model covariates, and that the predictions therefore are much more dependent on which pipes were observed, than the model covariates. If one had observed only 40 % of the pipe segments in this stratum, one would not be able to predict the condition of unobserved pipes well.

Table 1. 95 % confidence intervals for four different inspection portions (2018)

Material	CC	20 %	40 %	60 %	80 %
Non-concrete pipes	CC1	34.8 - 45.0 %	38.3 - 42.7 %	38.8 - 41.5 %	39.1 - 40.5 %
	CC2	8.6 - 12.6 %	8.9 - 11.1 %	8.7 - 10.2 %	8.5 -9.3 %
	CC3	16.8 - 27.4 %	17.0 - 22.6 %	16.4 - 19.6 %	15.8 -16.9 %
	CC4	11.3 - 18.6 %	12.0 - 15.2 %	12.1 - 13.9 %	12.2 - 13.0 %
	CC5	7.4 - 19.9 %	12.2 - 20.2 %	16.9 - 21.5 %	21.5 - 23.0 %

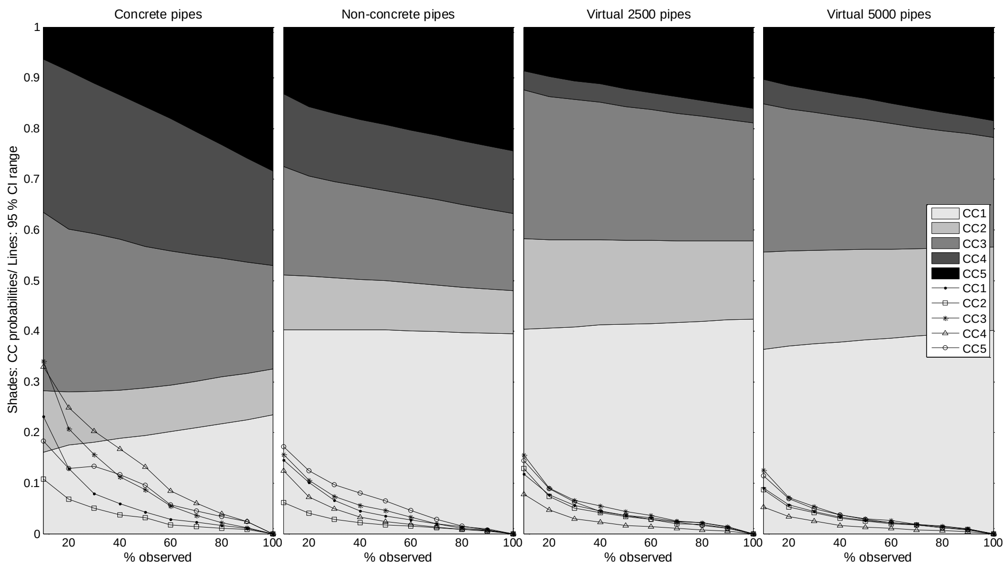

Figure 1. CC probabilities as a function of inspection rate (shades) and 95 % confidence interval range (lines)

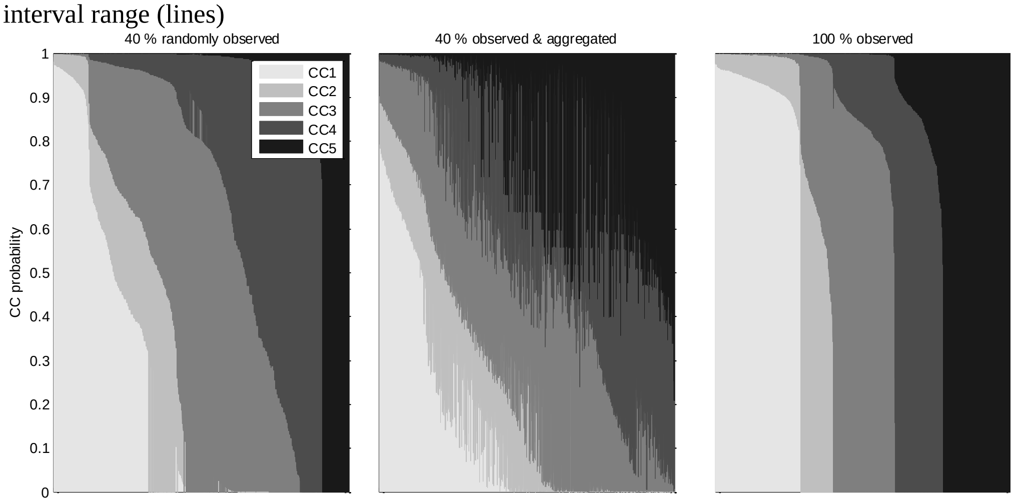

Figure 2. Predicted (2018) CC probabilities distributions for the concrete pipes

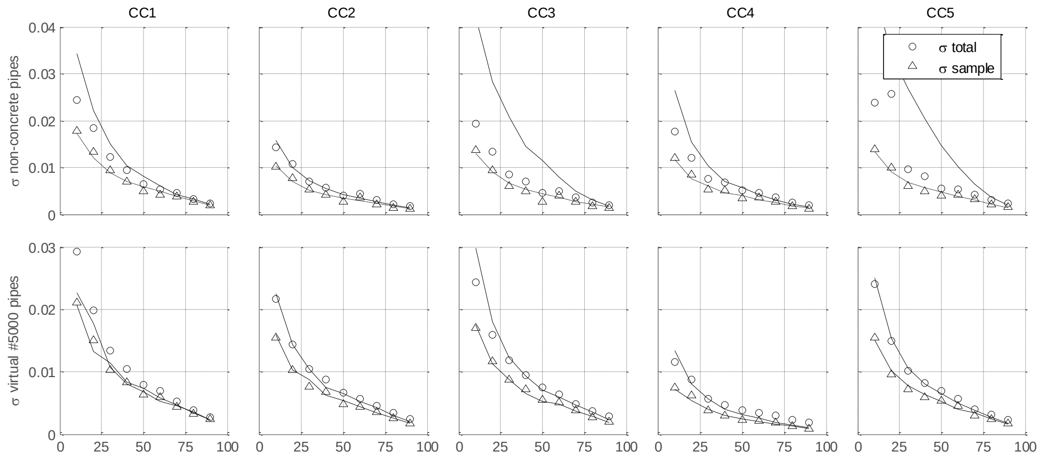

Figure 3. Comparison between standard deviation from Monte Carlo simulations (lines) and the uncertainty assessment method for incomplete datasets (points), as a function of the portion of inspected network. Sample and total standard deviations are shown.

Uncertainty estimates for incomplete datasets

An uncertainty point estimate approach has also been implemented, and compared with the results of the results found in the Monte Carlo simulation approach. The results are shown in Figure 3, only the largest of the real and the virtual dataset results are shown, due to space limitations. The uncertainty estimates for the real dataset are fairly good CC 1, 2, and 4, but not for 3 and 5, and may give an indication of the uncertainty of predictions, but not accurately. The uncertainty estimation for the virtual dataset shows a far better fit.

The potential deviations between the two uncertainty assessment methods are (1) that the sample uncertainty can only be estimated (we do not know how many pipes are in each condition when we have only inspected a portion of the network), and (2) that he assumption of independence between parameter and sample variance is not reasonable. From the result, one may see that the estimate of the sample uncertainty is fairly good for most situations, but that the estimate of the remaining uncertainty generally is not well estimated for the real datasets. This is because the assumption of independence between sample and parameter uncertainty is not reasonable, because there is an interaction between the sample and parameter variance when the parameter estimate values are sensitive to which observations are used to calibrate the model. This effect will be stronger when the covariates are insufficiently explanatory; therefore one may observe that the uncertainty estimates for the real datasets are more relevant, because the virtual dataset covariates are set to fit the model assumptions.

It is, due to the aforementioned reasons, not possible to accurately assess the uncertainty of an incomplete dataset if the sample variance is large, and the individual variability in condition observations is large. The mean and variance of the individual frailty factors one has observed may be an indicator of whether or not the independence assumption is reasonable.

CONCLUSION

The uncertainty of GompitZ predictions under varying availability of inspection data has been assessed by Monte Carlo simulation, and it has been shown that this uncertainty may be considerable, both with respect to predicting the overall condition distribution of the network and individual pipes. In order to assess the uncertainty of predictions, one must consider the variance in parameters, the sample variance, and the interaction between the two. Assessment of individual pipe uncertainty can be done by considering the calibration parameters as asymptotically normal distributed, and applying the delta method.

Assessing the uncertainty in predictions datasets where only a portion of the pipes have been inspected can be done by considering the parameters as asymptotically normal distributed, generate normal distributed random numbers with variances according to the parameter covariance matrix, and repeat the predictions many times. The variability in the condition probabilities observed in the repeated predictions can be used to express the prediction uncertainty of the whole asset stock, if one adds the sample uncertainty. This procedure is only valid if the uncertainty in the calibration parameters is not dominated by sample uncertainty. One should therefore consider the variance of the individual frailty factor one has observed.

The case study data shows that the uncertainty of sewer prediction may be considerable for datasets where a large portion has not been inspected. The authors would therefore not advise to base renewal decisions on GompitZ predictions for pipes that have not been inspected; unobserved pipes which score badly in GompitZ should rather be programmed for inspection. The balance between the necessary number of “random” and “targeted” inspections should however be assessed; this will be the topic for a following paper.

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