Quantum tunneling of the interfaces between normal-metal and superconducting regions of a type-I Pb superconductor

Eugene M. Chudnovsky
CUNY Lehman College

S Vélez
Universitat de Barcelona

A García-Santiago
Universitat de Barcelona

J A. Hernandez
Universitat de Barcelona

How does access to this work benefit you? Let us know!

Follow this and additional works at: http://academicworks.cuny.edu/le_pubs

Part of the Physics Commons

Recommended Citation
Quantum tunneling of the interfaces between normal-metal and superconducting regions of a type-I Pb superconductor

E. M. Chudnovsky,1,2 S. Vélez,2,3 A. García-Santiago,2,3 J. M. Hernandez,2,3 and J. Tejada2,3

1Physics Department, Lehman College, The City University of New York, 250 Bedford Park Boulevard West, Bronx, NY 10468-1589, USA
2Departament de Física Fonamental, Facultat de Física, Universitat de Barcelona, Avinguda Diagonal 645, ES-08028 Barcelona, Spain
3Institut de Nanociència i Nanotecnologia IN2UB, Universitat de Barcelona, c. Martí i Franquès 1, ES-08028 Barcelona, Spain

(Received 7 December 2010; published 15 February 2011)

Evidence of a nonthermal magnetic relaxation in the intermediate state of a type-I superconductor is presented. It is attributed to quantum tunneling of interfaces separating normal and superconducting regions. Tunneling barriers are estimated, and the temperature of the crossover from the thermal to the quantum regime is obtained from the Caldeira-Leggett theory. Comparison between theory and experiment points to tunneling of interface segments of a size comparable to the coherence length, by steps of the order of 1 nm.

DOI: 10.1103/PhysRevB.83.064507

PACS number(s): 74.25.Ha, 74.50.+r, 75.45.+j

I. INTRODUCTION

Quantum tunneling of relatively macroscopic solid-state objects such as flux lines in type-II superconductors1,2 and domain walls in magnets3 has been the subject of intensive research in the past. The corresponding energy barriers and spatial scales are nontrivially determined by statistical mechanics of the pinning potential.1,2,4,5 Interaction with environment makes this a problem of macroscopic quantum tunneling with dissipation.6 The latter is especially important for the tunneling of flux lines because of their predominantly dissipative dynamics.7–11 Type-I superconductors, when placed in the magnetic field, do not develop flux lines. Instead, they exhibit an intermediate state in which the sample splits into normal and superconducting regions separated by planar interfaces of positive energy.12–14 Recently, there has been a renewed interest in the equilibrium structure, pinning, and dynamics of interfaces in type-I superconductors.15–20 In the presence of pinning centers, the interfaces adjust to the pinning potential by developing curvature, as is schematically shown in Fig. 1. Pinning by point or small-volume defects should result in a broad distribution of energy barriers. It is therefore plausible that, at low temperature, type-I superconductors continue to relax toward equilibrium via quantum diffusion of interfaces. This situation is similar to the diffusion of domain walls in disordered ferromagnets,22 however, to our knowledge, no literature evidence of quantum diffusion of domain walls in disordered ferromagnets.22 However, to our knowledge, no literature exists on nonthermal magnetic relaxation in type-I superconductors. Experimental evidence of such a relaxation and its theoretical treatment are subjects of this paper.

II. EXPERIMENT

Two samples (A and B) in the shape of an octagonal disk of thickness 0.2 mm and surface area 40 mm$^2$ were prepared by cold-rolling of short cylinders cut from a commercial Pb rod of purity 99.999%. They were annealed during one hour at 290 °C and 280 °C (melting temperature of lead is 327.5 °C), respectively, in glycerol and nitrogen atmosphere to reduce the mechanical stress from defects that might have been introduced during preparation of the sample. Magnetic measurements were performed with the use of a commercial superconducting quantum interference device (SQUID) magnetometer in the field up to 1 kOe in the temperature range 1.8–8 K. The magnetometer was equipped with a continuous low temperature control (CLTC) and enhanced thermometry control (ETC) and showed thermal stability better than 0.01 K. Isothermal magnetization curves had the same shape for samples A and B (see Fig. 2). The fit of the data by $B_s(T) = B_s(0)[1 - (T/T_c)^2]$ produced identical values of $B_s(0) = 802 ± 2$ Oe and $T_c = 7.23 ± 0.02$ K for both samples, in accordance with the values of the critical field and transition temperature reported for lead. These values of the parameters, together with the high purity of our samples, confirm that we are dealing with a conventional type-I superconductivity in lead. The observed magnetization curves are typical of a pure type-I Pb superconductor in a weakly pinned intermediate state (see, e.g., Refs. 15 and 18). In such a state, the type-I superconductor has the magnetization curve that is qualitatively similar to $M(H)$ of the type-II superconductor. This is because many of the physical processes involved in the formation of the intermediate state of a type-I superconductor are conceptually similar to the physical processes responsible for the formation of the mixed state of a type-II superconductor. The essential difference is that the magnetic field penetrates into a type-I superconductor in the
form of normal domains versus quantized vortices in a type-II superconductor. The maxima in the virgin magnetization curves in Fig. 2 (not to be confused with the $H_{c1}$ effect in a type-II superconductor) are due to the surface barriers for the nucleation of normal domains. These barriers and the pinning of interfaces separating normal and superconducting regions are responsible for the magnetic hysteresis. At some higher field (not to be confused with $H_{c2}$ in a type-II superconductor), the magnetization goes to zero due to the complete expulsion of superconducting domains by normal domains.

Magnetic relaxation was measured by first applying the field $B > B_{c1}(T)$, then subsequently switching the field off and recording (for more than one hour) isothermal temporal evolution of the remnant magnetization $M_{\text{rem}}(t)$ in a zero field. Figure 3 shows the time evolution of $M_{\text{rem}}(t)/M_{\text{rem}}(0)$ in sample A between 2 and 6 K. Similar data with slightly different slopes were obtained for sample B. At all temperatures, the observed slow relaxation followed very well the logarithmic time dependence $M_{\text{rem}}(t) = M_{\text{rem}}(0)[1 - S(T) \ln t]$, where $S(T)$ is the so-called magnetic viscosity. Temperature dependence of the viscosity for samples A and B is shown in Fig. 4. Remarkably, it does not extrapolate to zero in the limit of $T \to 0$ but, instead, tends to a finite-temperature-independent limit as the sample is cooled down.

Conceptually, the slow relaxation of interfaces separating normal and superconducting regions is similar to the magnetic after-effect due to relaxation of domain walls in bulk ferromagnets. In the latter case, the logarithmic time dependence of the relaxation is usually considered an indication of the broad distribution of energy barriers. The same result can be obtained in a model with a single barrier if the height of the barrier is affected by the global relaxation. Regardless of the model, the finite value of $S(0)$ points toward the quantum mechanism of the escape from metastable states. By analogy with type-II superconductors, where nonthermal magnetic relaxation is due to quantum tunneling of flux lines, and with ferromagnets, where nonthermal relaxation implies tunneling of domain walls, it is reasonable to assume that, in type-I superconductors, the effect is due to quantum tunneling of interfaces separating normal and superconducting regions. The structure of the interface (see Fig. 5) is determined by two parameters: the coherence length $\xi$ and the London length $\lambda_L$. 

FIG. 4. (Color) Temperature dependence of the magnetic viscosity for samples A (black) and B (red). $S(T)$ tends to a nonzero value in the limit of $T \to 0$. Experimental error is less than the size of the points.
condensate wave function changes on a scale $\xi$, while the modulus of the Cooper-pair condensate wave function changes on a scale $\xi$.

Type-I superconductivity corresponds to $\kappa < 1/\sqrt{2}$. Concentration of Cooper pairs $|\Psi|^2$ gradually goes to zero on a distance $\xi$ as one moves through the interface from the superconducting to the normal region. When crossing the interface in the opposite direction, one would see the magnetic field going down from its thermodynamic critical value $B_c$ to zero on a distance $\delta = \sqrt{\lambda L \xi} < \xi$.

The energy of the unit area of the interface is\(^2\)

$$E = \sigma \int dxdy \left[ \sqrt{1 + \left( \frac{dZ}{dx} \right)^2 + \left( \frac{dZ}{dy} \right)^2} + U(x,y,Z) \right]$$

(2)

where $\sigma = \xi B_c^2/(3\sqrt{2}\pi)$.\(^1\)

Pinning provides curvature of the interface (see Fig. 1). We shall describe such an interface by a singled-valued function $Z(x,y)$. The energy of the interface

$$E = \sigma \int dxdy \left[ \sqrt{1 + \left( \frac{dZ}{dx} \right)^2 + \left( \frac{dZ}{dy} \right)^2} + U(x,y,Z) \right]$$

(2)

consists of two parts: elastic energy and energy due to the pinning potential $U(x,y,z)$. Metastable equilibrium is achieved through the balance of these two energies that corresponds to the minimum of Eq. (2). Magnetic relaxation occurs due to the decay (or formation) of the bumps in the interface shown in Fig. 1. We shall describe such a bump by the lateral size $L$ and height $a$. For a particular bump, these parameters are determined by the local pinning potential. Since the latter is unknown, we shall test self-consistency of the approach based upon theory of tunneling with dissipation\(^6\) by extracting the average values of $L$ and $a$ from experiment.

Let us first estimate the energy barrier associated with the bump. It is easy to see that the change in the elastic energy of the interface due to formation of the bump (see Fig. 6) is independent of $L$ and is generally of the order $\sigma \pi a^2$.\(^7\) This follows from the fact that the area of a spherical segment above any cross section of a sphere differs from the area of that cross section by $\pi a^2$. This energy must be balanced by the negative energy of the pinning to make the bump an equilibrium state of the interface. Consequently, $U_B \approx \pi \sigma a^2$,

FIG. 5. Structure of the interface between normal and superconducting regions of type-I superconductor. The magnetic field decays on a scale $\xi$, while the modulus of the Cooper-pair condensate wave function changes on a scale $\xi$.

FIG. 6. Flattening (or formation) of a bump via quantum tunneling of a pinned interface ($I$) separating normal ($N$) and superconducting (SC) regions.

where the average value of $a$ should represent the typical amplitude of the random pinning potential and, thus, the height of the energy barrier. Note that the transport current would tilt the pinning potential and lower the barriers. In this paper, however, we consider quantum relaxation toward equilibrium in the absence of the transport current (similar to the magnetic relaxation of a ferromagnet in a zero magnetic field), rather than quantum creep of the interfaces caused by the transport current.

We want to find the WKB exponent $I_{\text{eff}}/\hbar$ for the tunneling of $Z(x,y)$ between two configurations of the interface corresponding to the local energy minima (see Fig. 6). As with the flux lines,\(^7\) we shall assume that the tunneling probability is dominated by the dissipation part of the Caldeira-Leggett effective action\(^5\)

$$I_{\text{eff}} = \frac{\eta}{4\pi} \int_0^{\hbar/\tau} d\tau \int_{-\infty}^{\infty} dxdy \int dxdy \left[ \frac{Z(t) - Z(t')}{(t - t')^2} \right],$$

(3)

where $\eta$ is a viscous drag coefficient describing dissipative motion of the interface and $\tau = it$ is imaginary time. For a segment of the interface of size $L$, which tunnels by a distance $a$, the $T = 0$ value of the effective action in Eq. (3) can be estimated as

$$I_{\text{eff}} \approx \frac{\eta L^2 a^2}{4\pi}.$$

(4)

The drag coefficient $\eta$ can be obtained from the argument similar to that of Bardeen and Stephen for the flux lines.\(^8\) Let the magnetic field be in the $y$ direction. In the presence of the current of density $j$ in the $x$ direction, the magnetic force experienced by the $dxdy$ element of the interface in the $z$ direction is

$$dF = \frac{1}{c} \int dxdy dz jB.$$

(5)

Writing $j$ via the electric field and normal-state resistivity $\rho_n$ as $j = E/\rho_n$, and substituting here $E = (V/c)B$ for the electric field produced inside the interface moving at a speed $V$ in the $z$ direction, one has $j = (V/c)(B/\rho_n)$. This gives

$$dF = \frac{V}{\rho_n c^2} \int dxdy dz B^2(z)$$

(6)

for the force per unit area of the interface. Substitution into this formula of $B \approx B_c \exp(-z/\delta)$ finally yields

$$dF = \eta V, \quad \eta = \frac{\sqrt{\lambda L \xi} B_c^2}{2\rho_n c^2}.$$

(7)

As has been explained in the introduction, the crossover from thermal to quantum diffusion of the interface should
occur around \( T_Q = \hbar U_B / I_{\text{eff}} \). With the help of Eqs. (1), (4), and (7), one obtains

\[
T_Q \approx \frac{4\pi^2 \hbar \sigma}{\eta L^2} = \frac{4\pi \sqrt{2\hbar \rho_n c^2}}{3\sqrt{k} L^2}.
\]

Notice that, due to the dimensionality of the problem, \( T_Q \) does not depend on the size of the tunneling step \( a \). Recalling that \( \lambda_L = (m c^2 / (4\pi e^2 n))^{1/2} \) in terms of the effective mass \( m \) and concentration \( n \) of the electrons and writing \( \rho_n = (m v / e^2 n) = 4\pi \kappa L / c^2 \) in terms of the normal electron collision frequency \( v \), the crossover temperature can be presented in the form

\[
T_Q \approx \frac{16\pi^2 \sqrt{2}}{3} \kappa^{3/2} \left( \frac{\xi}{\lambda_L} \right)^2 \hbar \nu,
\]

which shows its explicit dependence on the microscopic parameters of the material.

**IV. DISCUSSION**

\( T_Q \) can be estimated from experiment, based upon the following argument. At finite temperature, the magnetic viscosity shown in Fig. 4 has contributions from both thermal activation and quantum tunneling \( S = S_T + S_Q \), where \( S_T = S(0) \). The parameter \( T_Q \) is defined as the temperature at which the two contributions are equal, that is, \( S_T = S_Q \) and \( S(T_Q) = 2S_Q \). This gives \( T_Q \) in the ballpark of 4–5 K. The values of \( \lambda_L \) and \( \xi \) in lead are 37 and 83 nm, respectively, giving \( \kappa = \lambda_L / \xi = 0.45 \). For the energy of the unit area of the interface, Eq. (1) with \( B_r \approx 800 \, \text{G} \) gives \( \sigma \sim 0.4 \, \text{erg/cm}^2 \). Normal resistivity of lead at 4 K is of the order\(^3\) \( 5 \times 10^{-11} \, \Omega \cdot \text{m} \approx 5.6 \times 10^{-21} \, \text{s} \). Equation (7) then gives for the drag coefficient \( \eta \approx 0.35 \, \text{erg/s/cm}^4 \). We shall now check the self-consistency of our model by computing the average size of the tunneling segment \( L \) and the tunneling step \( a \). From Eq. (8), one obtains \( L \approx 90 \, \text{nm} \sim \xi \), which is rather plausible. Indeed, \( L \approx \xi \) describes the segment of the interface inside which Cooper pairs are strongly correlated and, therefore, they can collectively participate in a coherent tunneling event. For the tunneling transition to occur in our experimental time window of one hour, \( I_{\text{eff}} \) can not significantly exceed 25\( h \). According to Eq. (4), this condition is satisfied by tunneling steps \( a \) below 1 nm, which is also quite plausible. The typical energy barrier \( U_B \approx \pi \sigma a^2 \) must be then of the order 100 K in accordance with the fact that thermal activation dies out below 4 K.

In conclusion, we have observed nonthermal magnetic relaxation in lead that we attribute to quantum tunneling of small segments of interfaces separating normal and superconducting regions. A theory of such a tunneling has been developed. Comparison between theory and experiment suggests macroscopic quantum tunneling of interface segments comparable in size to the coherence length, by steps of the order of 1 nm.

**ACKNOWLEDGMENTS**

The work of E. M. C. was supported by the US Department of Energy Grant No. DE-FG02-93ER45487 and by Catalan ICREA Academia. The work at the University of Barcelona was supported by the Spanish Government Project No. MAT2008-04535. S. V. acknowledges financial support from Ministerio de Ciencia e Innovación de España. J. M. H. and A. G.-S. thank Universitat de Barcelona for supporting their research. J. T. acknowledges financial support from ICREA Academia.

---