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Mathematical Inquiry with Loops

Jeanne Funk
CUNY La Guardia Community College

First Year Seminar Lesson Plan - LMF101 - Liberal Arts: Math & Science

Abstract

Mathematical Inquiry with Loops is a fun activity designed to introduce students in LMF101 (First Year Seminar for Liberal Arts: Math and Science) to mathematical exploration and hypothesis formation via hands-on exploration with paper loops. It should make students understand that the "facts" that make up mathematics, are not arbitrary, but arise from observation and experience. It should also encourage students to view mathematics as a science rooted in exploration and problem solving, rather than an esoteric exercise in symbol manipulation.

This activity is designed to be very low stakes (ungraded) and to take one hour of class time. There are no mathematical pre-requisites beyond counting, and it addresses the Inquiry and Problem Solving competency, as well as the following learning objectives.

Instructional Objectives

- Introduce students to the foundational knowledge and key concepts of mathematics and science
- Introduce students to types of problems successfully addressed by mathematics and science, along with the methodology.

Performance Objectives

- Demonstrate understanding of foundational knowledge in mathematics and science (eg, observation, interpretation, critical thinking, synthesis, analysis, reflection and evaluation).
- Explain types of research and methods within mathematics and science and the ways these are applied in the disciplines.

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Time: 1 faculty hour

Materials/Resources

- scrap paper (to create 11" strips (minimum – you may want some that are longer for demo purposes)), scissors, and tape for 5-8 groups
- 'Möbius and Beyond' Worksheet (attached)

1. **Instructional Objectives**
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2. **Performance Objectives**
   - Demonstrate understanding of foundational knowledge in mathematics and science (eg, observation, interpretation, critical thinking, synthesis, analysis, reflection and evaluation).
   - Explain types of research and methods within mathematics and science and the ways these are applied in the disciplines.

3. **Rationale**

   This is a fun activity designed to introduce students to mathematical exploration and hypothesis formation. It should make students understand that the "facts" that make up mathematics, are not arbitrary, but arise from observation and experience. It should also encourage students to view mathematics as a science rooted in exploration and problems solving, rather than an esoteric exercise in symbol manipulation. This activity has no mathematical pre-requisites beyond counting.

4. **Presentation of New Information**
   - Hands on creation and manipulation of interesting geometric objects
   - Cooperative examination of data and development of hypotheses

5. **Activity/Practice**
   - Arrange the students in groups of 3-5 and give each group some scrap paper, scissors, and tape. At least two people in each group should have pens/pencils. Suggest that each group have one person to tape, one person to trace, one person to cut, and one person to record data.
   - Model the first example (# of half-twists = 0)
     - Create an untwisted loop, making sure that each group is able to follow along.

  - Ask the class how many sides the loop has and what they think will happen when they cut the loop in half. Their intuition will probably be correct for this example
  - Trace along the inside of the loop and note how you only trace the inside of the loop before getting back to where you started, so the loop has two distinct sides. Bisect the loop and note that you now have two loops that are pretty much the same as the one you started with.
  - Encourage each group to record the data
- Model the second (Möbius) example (# of half-twists = 1)
  - Create a loop with one half-twist, making sure each group can follow along. Be extra careful to assure that the students making the loops understand what a half-twist is.
  - Instruct the class to repeat the tracing/cutting procedure with this loop and record what happens
  - After most of the class is finished, discuss the results as a class
- At this point you can either:
  - (what I do) Repeat the process for the Möbius example half-twists 2-5 (at least) OR
  - (worth trying) Assign each group a # of half-twists and compile all the data when everyone is finished
- Once everyone has a completed chart, ask the class to propose conjectures about loops given the number of half-twists. They should at least be able to hypothesize that:
  - If the # of half-twist is even: the loop will have two sides and produce two loops when bisected
  - If the # of half twists is odd: the loop will have one side and produce one loop when bisected
- Discuss briefly
  - The difference between conjecture/hypothesis and proof (this is not enough to prove a statement to the satisfactions of mathematicians
  - The value of exploration and conjecture in the mathematical process
  - Where does the activity they just completed appear in the Cycle of Scientific Inquiry (I like the image at http://www.emc-square.org/emc2/?p=4910)?

6. Notes/ Other (to self and/or others using the lesson)

Bonus conjecture, time permitting: If unknown, you can count the number of half-twists in a loop by carefully untwisting and counting the number of "moves" required to obtain an untwisted loop. There is a relationship between the number of half-twists in the original loop and the number of half-twists in the loop(s) obtained via bisection.

Möbius and Beyond - Mathematical Exploration with Paper Loops

Loops with varying amounts of twist are interesting geometric objects. The Möbius strip, which has one half-twist, has been studied since the 19th century. In this exercise, you will build and collect data on various paper loops, and use this data to hypothesize about the structure of loops in general.

For each number of half-twists, twist a strip of paper that many times and tape the ends together to form a loop. Trace along the center of the loop without lifting your pen to determine how may sides it has; then cut it in half along this line. Record the results in the chart below.

# of Half-Twists	# of Sides	Result when strip is bisected
0		
1		
2		
3		
4		
5		
6		

Based on this information, what hypotheses do you have about loops, based on number of half-twists?

J. Funk