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MODEL PREDICTION CONTROL FOR WATER MANAGEMENT USING ADAPTIVE PREDICTION ACCURACY

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In the field of operational water management, Model Predictive Control (MPC) has gained popularity owing to its versatility and flexibility. The MPC controller, which takes predictions, time delay and uncertainties into account, can be designed for multi-objective management problems and for large-scale systems. Nonetheless, a critical obstacle, which needs to be overcome in MPC, is the large computational burden when a large-scale system is considered or a long prediction horizon is involved. In order to solve this problem, we use an adaptive prediction accuracy (APA) approach that can reduce the computational burden almost by half. The proposed MPC scheme with this scheme is tested on the northern Dutch water system, which comprises Lake IJssel, Lake Marker, the River IJssel and the North Sea Canal. The simulation results show that by using the MPC-APA scheme, the computational time can be reduced to a large extent and a flood protection problem over longer prediction horizons can be well solved.

INTRODUCTION

A number of real-time control techniques have been proposed over the past decade in field of the operational water management. For a detailed review see Malaterre *et al.*[1]. The control method best fit for operating water systems is Model Predictive Control (MPC)[2].

Although MPC outperforms other control methods and has become more popular, the computational burden could become a significant disadvantage which hinders its development. Because an optimization needs to be run in real-time in the MPC over each prediction horizon, the more state variables are involved in the optimization, the more computational time is taken. Especially, the computational burden is extremely heavy in a management problem of a large-scale system, e.g. the northwestern Dutch water system considered in this study. However, a longer prediction involvement, for instance eight days, is indeed required for better controlling the system. This longer prediction naturally brings more states into the system and increases the computational time. How to settle this long prediction issue in MPC is the main focus of this paper. In this paper, we investigate how a MPC scheme using so-called Adaptive Prediction Accuracy (APA) could improve the performance, as proposed in [3]. The key idea of this method is that the whole prediction horizon can be divided into several phases. The first phase is the beginning stage of the prediction horizon, which uses the finest prediction time step. It means this phase uses the prediction as precisely as possible. In the second phase, the prediction time step is increased to two times of the one used in the first phase. It is obvious that the number of states is reduced accordingly so that the computational time is reduced as well. The third and following phases use the time step larger than the one used in the previous phase. These phases are the later stage of the prediction horizon so that prediction is allowed to be coarse.

The outline of this paper is as follows. Section 2 describes the dynamics of the water system in the northwestern Netherlands and the water-related structures. Section 3 demonstrates the formation of the proposed MPC controller with the APA scheme. The results, computational time and model performance are discussed in Section 4. Section 5 concludes the paper and gives the direction for future work.

DESCRIPTION OF THE SYSTEM

The water system in the northern part of the Netherlands is chosen as the study area. This system comprises Lake IJssel, Lake Marker, the River IJssel and the North Sea Canal. The present system is protected in a man-made environment with water-related infrastructures. As shown in Figure 1, Lake IJssel is separated from the Wadden Sea with a long dike in the north, while water can flow into the Wadden Sea via the Lorentz Gate and Stevin Gate. A gate and a pump station have been constructed at IJmuiden where water can be diverted from the North Sea Canal to the North Sea. Water exchange between Lake IJssel and Lake Marker, Lake Marker and the North Sea Canal can be operated via the Gates Krabbersgat, Houtrib and Schellingwoude respectively. Note that in the case of multiple gates in parallel, the total width is taken together and the gates are considered to be moving in a synchronized way so that they can be represented by one large gate instead.

Figure 1. Study area – the northern Dutch water system

In order to describe the dynamic of the system, the linearized De Saint Venant equations is applied in this study, which use a robust wind-up method [4,5].

$$\frac{dA^{(f)}}{dt} = \frac{Q_{i-1/2} - Q_{i+1/2}}{\Delta x} + q_i^{(l)} \tag{1}$$

$$\begin{aligned} &\frac{dv_{i+1/2}}{dt} + g\frac{h_{i+1} - h_i}{\Delta x} + g\frac{v_{i+1/2}\,|\,v_{i+1/2}\,|}{C^2 \cdot R} \\ &+ \frac{1}{A_{i+1/2}^{(f)}}\left(\frac{(Q_{i+1/2} + Q_{i+3/2})\cdot v_{i+1} - (Q_{i+1/2} + Q_{i-1/2})\cdot v_i}{2\Delta x} - v_{i+1/2}\frac{Q_{i+3/2} - Q_{i-1/2}}{2\Delta x}\right) = 0 \end{aligned} \tag{2}$$

where Q is the flow (m^3/s) , h is the water level (m), $A^{(f)}$ is the wetted area of the flow (m^2), $q^{(l)}$ is the lateral inflow (m^3/s), C is the Chézy friction coefficient ($m^{1/2}/s$), R is hydraulic radius (m), and P_f is the wetted perimeter (m), $A^{(f)}$ is the wetted area of the flow (m^2) and subscripts i denote the parameters or variables at staggered grid points i respectively. Details of the linearization can be found in [4,5].

ADPATIVE PREDICTION ACCURACY IN MPC

Standard MPC

MPC is a model-based control technique, which uses a state-space model to predict future states of the system and then solves an optimization problem using an objective function under constraints on control actions and system outputs over a certain prediction horizon [6]. The control actions, which are the optimal solutions from the optimization problem, are implemented on the system until the next control step. Then MPC is run again over next prediction horizon, which is referred to as a receding process. The advantage of this procedure is that the control actions are implemented based on the present and future dynamics of the system.

The linear state-space equations used in the water system for each single river-reach can be written as [2] based on Eq. (1) [5]:

$$\begin{aligned} e_{k+1} &= e_k + \frac{(q_k + \Delta q_k)\cdot \Delta T}{S} + \frac{d_k \cdot \Delta T}{S} \\ q_{k+1} &= q_k + \Delta q_k \end{aligned} \tag{3}$$

where the subscript k is the discrete time step index which belongs to the interval [1, T], T is the prediction horizon, ΔT (s) is the time step length, e (m) is the deviation of the S (m^2) is the surface area, q (m^3/s) is the flow through the structure, Δq (m^3/s) is the change of structure flow and d (m^3/s) is the sum of disturbances and other flows not related to structures, such as rainfall runoff and water abstractions.

An objective function is built up to formalize the goal of the water management. The objective problem is built up at any k step as a quadratic form with linear constraints [2]:

$$\min_{\Delta q_s} J_{k,T} = \sum_{j=1}^{n_S}\sum_{i=0}^{T} p_{k+i,2}^{(j)} \cdot (\Delta q_{k+i}^{(j)})^2 + \sum_{j=1}^{n_S}\sum_{i=1}^{T} p_{k+i,1}^{(j)} \cdot q_{k+i}^{(j)}$$
$$+ \sum_{j=1}^{n_N}\sum_{i=0}^{T} r_{k+i,2}^{(j)} \cdot (e_{k+i}^{(j)})^2 + \sum_{j=1}^{n_N}\sum_{i=0}^{T} r_{k+i,1}^{(j)} \cdot e_{k+i}^{(j)}$$

s.t. (4)

$$h_{\min}^{(j)} \le h_{k+i}^{(j)} \le h_{\max}^{(j)}, \forall i \in \{1,2,...,T\}, \forall j \in \{1,2,...,n_N\}$$
$$q_{\min}^{(j)} \le q_{k+i}^{(j)} \le q_{\max}^{(j)}, \forall i \in \{1,2,...,T\}, \forall j \in \{1,2,...,n_S\}$$

where n_N is the number of nodes, n_S is the number of controlled structures, $h_{\min}$ and $h_{\max}$ are the minimum and maximum allowed water level, $q_{\min}$ and $q_{\max}$ are the minimum and maximum allowed flow through the structures, $p_{*,2}$ and $r_{*,2}$ are quadratic penalties on Δq and e respectively, $p_{*,1}$ and $r_{*,1}$ are linear penalties on q and e respectively Those penalties are chosen according to a Maximum Allowed Value Estimate [2].

For the convenience of future discussions, the matrix form is used to represent the state-space Eq. (3):

$$x_{k+1} = A \cdot x_k + B \cdot u_k + C \cdot d_k \tag{5}$$

where:

$$x_k = \begin{bmatrix} h_k \\ q_k \end{bmatrix},\ A = \begin{bmatrix} 1 & \dfrac{\Delta T}{S} \\ 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} \dfrac{\Delta T}{S} \\ 1 \end{bmatrix},\ C = \begin{bmatrix} \dfrac{\Delta T}{S} \\ 0 \end{bmatrix} \tag{6}$$

Based on the linearity of Eq. (5), the state at any time step k+n can be expressed on the basis of the initial condition of the time step k by recursion:

$$x_{k+n} = A^n \cdot x_k + \sum_{i=0}^{n-1} A^i \cdot B \cdot u_{n-i} + \sum_{i=0}^{n-1} A^i \cdot C \cdot d_{n-i} \tag{7}$$

or in the matrix form:

$$X_{k,T} = \hat{A} \cdot x_k + \hat{B} \cdot U_{k,T} + \hat{C} \cdot D_{k,T} \tag{8}$$

Due to the length limitation of this paper, we refer to [2] for the details of coefficients used here.

As shown in Eq.(5), $\{x_{k+1}, ..., x_{k+T}\}$ is a linear representation of the vectors $\{x_k, u_k, u_{k+1}, ..., u_{k+T}\}$, i.e. the linear space span$\{x_{k+1}, ..., x_{k+T}\}$ is a subspace of the linear space span$\{x_k, u_k, u_{k+1}, ..., u_{k+T}\}$, where the notation *span*{} is defined as:

$$span\{\alpha,\beta\} = \{\gamma \mid \gamma = a\cdot\alpha + b\cdot\beta, \forall a,b \in \mathbb{R}\} \tag{9}$$

By substituting Eq.(8) into Eq. (4), the objective function becomes:

$$J_{k,T} = U_{k,T}{}^{\mathrm{T}} \cdot H \cdot U_{k,T} + 2f \cdot U_{k,T} + K \tag{10}$$

Adaptive prediction accuracy setting in MPC

The MPC-APA approach was first proposed for the control of energy resources [3]. To achieve the goal that different prediction time steps can be used in the water system, we also apply the APA setting here on the state-space Eq. (3). In this setting, the prediction horizon can be divided into three phases. Phase I is the period closest to the present, in which the prediction time step is still ΔT. And $2\Delta T$ and $4\Delta T$ is used later in Phase II and III respectively, as shown in Figure 2.

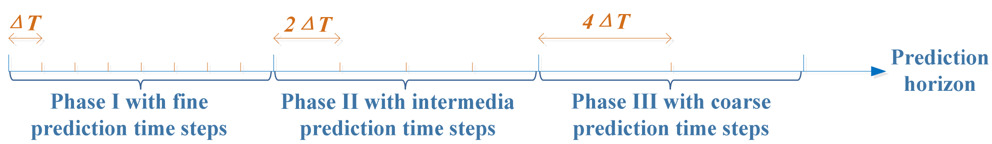

Figure 2. Illustration of the phase division over a prediction horizon

A linear transformation can be built up when $M \cdot \Delta T$ is used as the prediction time step:

$$p : \mathbb{R}^T \to \mathbb{R}^T$$

$$u_{i,j} \to \tilde{u}_{i,j} = \begin{cases} u_{i,j}, \mathrm{mod}(i-1, M) = 0 \\ u_{i-\mathrm{mod}(i-1,M),j}, \mathrm{mod}(i-1,M) \neq 0 \end{cases} \tag{11}$$

The corresponding transformation matrices of these phases form a block matrix which can be expressed by the Kronecker tensor product [7]:

$$P = \begin{bmatrix} P_1 & & \\ & P_2 & \\ & & P_3 \end{bmatrix} \tag{12}$$

where

$$P_1 = \mathrm{kron}(I(T/3), 1) \; P_2 = \mathrm{kron}(I(T/6), \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}) \; P_3 = \mathrm{kron}(I(T/12), \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}) \tag{13}$$

And the objective function using new control variables is given by Eq. (14) as matrices:

$$J_{k,T} = \tilde{U}_{k,T}^{\mathrm{T}} \cdot P^{\text{-T}} \cdot H \cdot P^{-1} \cdot \tilde{U}_{k,T} + 2f \cdot P^{\text{-T}} \cdot \tilde{U}_{k,T} + K \tag{14}$$

After this linear transformation, span$\{x_{k+1}, \ldots, x_{k+T}\}$ becomes a subspace of the linear space span$\{x_k, \tilde{u}_k, \tilde{u}_{k+1}, \ldots, \tilde{u}_{k+T}\}$. The number of the controlled variables is reduced by 5/12 as shown in Eq. (15).

$$\frac{\mathrm{rank}(\mathrm{span}\left\{x_{k,}\tilde{u}_k, \tilde{u}_{k+1}, \ \ldots \ , \tilde{u}_{k+T}\right\})}{\mathrm{rank}(\mathrm{span}\left\{x_{k,}u_k, u_{k+1}, \ \ldots \ , u_{k+T}\right\})} \approx \frac{1+\frac{1}{2}+\frac{1}{4}}{1+1+1} = \frac{7}{12} \tag{15}$$

SIMULATION AND RESULTS

In this section, two simulation experiments are performed. Case 1 uses two days as the prediction horizon while Case 2 uses eight days as prediction horizon. Table 1 presents the important parameters that are used in these two cases. The proposed controller is tested on the flood defense in the northern Dutch water system. A combination of high discharges from the Rivers Rhine and Meuse, 16000 m^3/s and 3500 m^3/s respectively (Figure 3), and regular tidal levels (-1.2 ~ +1.6 mMSL) at sea is made as the lateral conditions. As shown in Figure 3, the period when the Rhine river discharge is above 8000 m^3/s lasts for a month and the discharge peak occurs around time step 750.

Table 1 Simulation parameters

Parameters	**Value**
Control time step	1 (hour)
Prediction time step	1 (hour)
Reference water level	-0.4 or -0.6 (m)
Simulation horizon	2160 (hour)
Prediction horizon (Case 1)	48 (hour)
Prediction horizon (Case 2)	192 (hour)
Quadratic penalty on ΔQ via the Haringvliet Gate	1/5000
Quadratic penalty on ΔQ via the Lorentz Gate	1/8000
Quadratic penalty on ΔQ via the Stevin Gate	1/2000
Quadratic penalty on ΔQ via the Krabbersgat Gate	1/2000
Quadratic penalty on ΔQ via the Houtrib Gate	1/3000
Quadratic penalty on ΔQ via the Schellingwoude Gate	1/200
Quadratic penalty on ΔQ via the IJmuiden Gate	1/1000
Quadratic penalty on ΔQ via the IJmuiden Pump	1/260
Linear penalty on Q via the IJmuiden Pump	1/260

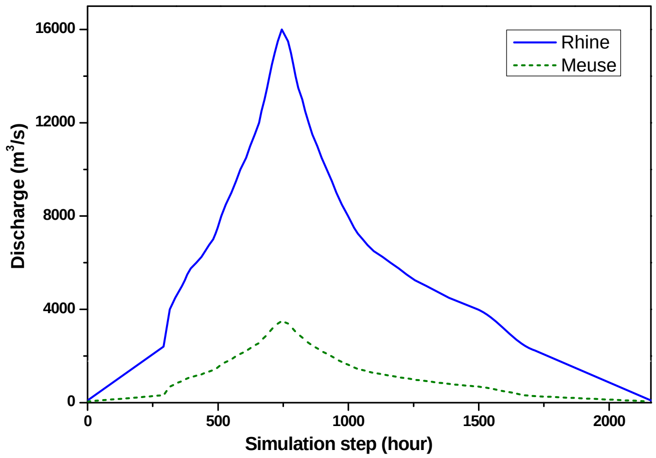

Figure 3. The inflow from the Rivers Rhine and Meuse

Before the discharge peak arrives, the pump at IJmuiden starts pumping water out in order to create more storage room in the North Sea Canal, of which the level goes from -0.4 m to -0.6 m (Figure 4-a). However, due to the longer prediction horizon in Case 2, the IJmuiden Pump starts working 6 days earlier and pumps more water in Case 1 than in Case 2 (Figure 4-b).. Considering Lake IJssel as a large buffer and calculating the balance between the sea level and the lake water level for a longer prediction horizon, the Lorentz Gate and the Stevin Gate, which divert water from Lake IJssel to the Wadden Sea, are operated less frequently in Case 2 than Case 1. Thus, a bit more water is stored in Case 2 than in Case 1. In fact, this extra amount of water only brings up the water level of Lake IJssel by about 0.15 m, which is still far below the safety level. But the less frequent operations of gates can save more unnecessary energies.

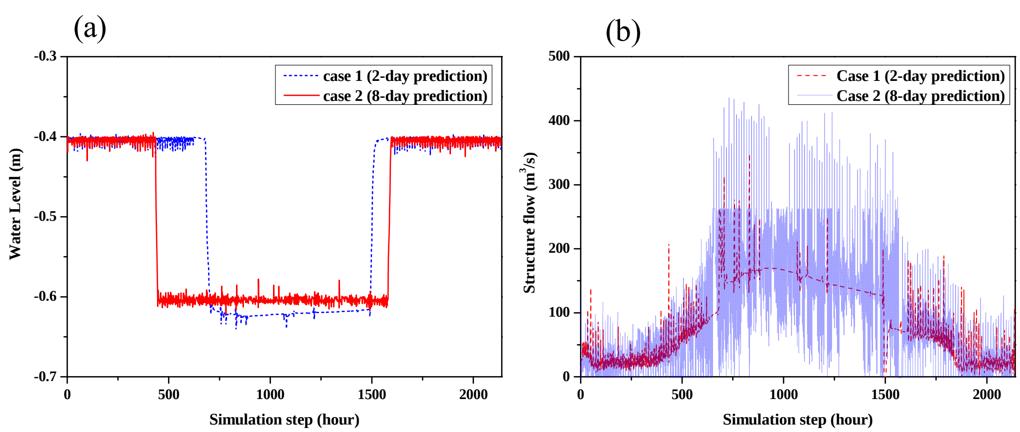

Figure 4. (a) the water levels of the North Sea Canal (b) the sum of structure flows via the pump and gate at IJmuiden

Owing to the reduced number of states in the state-space equations and the optimization, the time of each optimization using the scheme with APA is as low as 22% relative to the scheme without APA, as shown in Table 2. It takes only about 10 seconds to run each optimization and this short response is quite appropriate in real-time control of water systems.

Table 2 Simulation parameters

	Optimization time each step (s)	Simulation time (hour)
8-day prediction with APA	10.34	6.1
8-day prediction without APA	47.33	28.1

CONCLUSION AND FUTURE WORK

The issue of large computational burden has been an obstacle in MPC for a long time. Two cases were compared in this study, one of which used 2 days as the prediction horizon while one of which used 8 days as the prediction horizon. It showed that the performance is much higher using longer prediction, which allows the control to get information further ahead and prepare and respond better. However, the longer prediction also results in heavy computational burden. We propose to use the adaptive prediction accuracy scheme that can be used in MPC to reduce the number of states almost by half. By applying this scheme, 78% of the computational time could be saved. Future research will focus on using the APA scheme on a large-scale system and on a multi-objective problem.

ACKNOWLEDGEMENT

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