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Performance Indices for Multivariate Ice Hockey Statistics

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7.1 Hockey on ice

It cannot be surprising that the origins of today’s ice hockey game can be traced back to Canadian winters in the 19th century. Not only is much of Canada very cold during winter but, over a century ago, the country was much more agrarian; so the Canadian people were ready for a game to play on their abundant, frozen ponds.

In the mid-1800s, chaos was the best description of the game. The ‘rules’ were local and ad hoc. Games were sometimes played with dozens of players on each side! There is still disagreement about many aspects of the beginnings of early ice hockey. Nevertheless, in the 1880s, students at McGill University in Montreal and Queens University in Kingston began to play an organized game of hockey with consistent rules.

Shortly afterwards, leagues were formed in both Ontario and Quebec. In 1883, Baron Stanley, the governor-general of Canada, purchased a ‘cup’ for less than $50 to be awarded to the amateur champions of Canada. A team from the Montreal Amateur Athletic Association was the first winner. From that day to this, winning the Stanley Cup, currently awarded to the champions of the National Hockey League (NHL), has represented the pinnacle of hockey achievement. Nearly all players, at some time, envision their names engraved on the Cup as champions of all of hockey. Over the years, it has been to Russia, endured a trip to the bottom of the St Lawrence River, and had a vast mix of liquids consumed from it.

The early players were hardy men. Tough physical play was then, as it is now, part of the game of ice hockey. This has drawn criticism, on occasion, but the reluctance to remove this aspect of the game has a very long history. An Ottawa newspaper, describing the first Stanley Cup championship game in 1884, commented that, ‘general rabble predominated’. A later remark, attributed to Conn Smythe, an early NHL mogul with the Toronto Maple Leafs, ‘If you can’t beat ’em in the alley, you can’t beat ’em on the ice’, has not been
forgotten. The game is still not for the faint of heart. In all fairness, the sport does not have a more physical history than some other sports, especially if fan contributions are considered!

The National Hockey League, the world’s premier league, formed on 22 November 1917 with four teams. Toronto was the only team that played on artificial ice! Until the 1960s, for the most part, the NHL consisted of six teams: Boston, Chicago, Detroit, Montreal, New York (Rangers) and Toronto. Most of the early players were Canadian. In 1966, the league doubled in size and since that time has been the fastest growing professional sport. This season, 1997/98, the NHL has 26 teams and has announced future expansion to 30 teams.

While ice hockey has grown quickly in North America, it has grown even more rapidly elsewhere, particularly in Europe. There are now excellent European leagues; especially in Finland, Sweden and Russia. Many countries now enter ice hockey teams in the winter Olympics and produce outstanding hockey players able to play in the NHL. The availability of this large pool of players (in combination with the infusion of television money to lure them from their homelands) has been a major factor in the expansion of the NHL.


7.2 A review of the statistical research

Applications of the methods of mathematical statistics to hockey data have been scarce. This appears to be changing, perhaps paralleling the increasing popularity of hockey. Twenty years ago, Morris (1973) and Mullet (1977) provided two of the earliest applications. Mullet showed that the goals scored by and against teams in the NHL are surprisingly well described by Poisson distributions and, even more surprisingly, that the goals for and against a team seem to be independent. He used these results to predict game outcomes quite accurately. Recently, Lock (1997) applied a variation of this Poisson model technique to college hockey and was also able to predict game outcomes accurately. Danehy and Lock (1993) used regression models to develop ratings of the National Collegiate Athletic Association (NCAA) Division I Men’s Ice Hockey teams. A rating system for individual players is developed in the later sections of this chapter.

As in other sports (soccer and American football, in particular), the method of resolving games tied at the end of regulation time has been a source of controversy. Even today, various methods co-exist. During the regular season, the NHL plays a single ‘sudden-death’ overtime period that ends as soon as one goal is scored; if no goal is scored within five minutes, the game ends in a tie. However, during the NHL playoffs, there are no ties; a game continues until one team scores the winning goal. The International Hockey League (a North American league) and most international competitions, including the Olympics, use a different method called a ‘shoot-out’. In a shoot-out, each team, in turn, sends a player to attack the other team’s goal which is guarded only by the goalkeeper. Goals scored in a shoot-out determine the winning team. Morris (1973)
began the statistical analysis of the different methods for settling ties that was only recently resumed by Liu and Schutz (1994) and Hurley (1995). These studies find that the stronger team does tend to win the game, but Hurley also found that the shoot-out gives the weaker team a better chance to win. Liu and Schutz found that doubling the current five minute overtime would settle 60% to 65% of the games tied at the end of regulation time. Twenty minutes of overtime would settle 80% to 85%.

Fans do not like ties and seem to find the shoot-out exciting; players do not like ties either, but they also feel that shoot-outs often lead to random outcomes. In an attempt to satisfy both groups, Hurley analyzed some interesting combinations of sudden death and the shoot-out to find the means and variances of expected game durations. Arguments for and against the shoot-out are still dominated by purist arguments such as 'real hockey games are settled by team play!' In this issue and others, hockey officials so far seem uninfluenced by analytic arguments about the game; the last NHL rule change in settling ties was in 1983, when the five-minute overtime was introduced. Hopefully, this will change soon in light of promising new research. For example, an innovative new method for measuring power play and penalty-killing efficiency has considerable intuitive appeal (Anderson-Cook and Robles, 1997).

One of the authors (David Williams) played in the NHL for four years. Applying his experience, we addressed two statistical issues uppermost in player interest and concern. A salary study (Williams and Williams, 1997) showed that the salaries of NHL players are strongly related to the countries of their origin. We argued that the league options that players have within their own countries are better in some countries than others. The result is that more money is required to lure the players with the better options into the NHL. A curious, tangential finding of this salary study indicated that NHL talent scouts, as they roam the world searching for ice hockey talent, are very consistent in their draft selections.

The plus/minus statistic concerns players; in fact, it frequently annoys them. A player's plus/minus is the number of goals scored by his team minus the number scored against his team while he is on the ice. Consequently, the reason for player concern is that this statistic depends on much more than a player's own performance; it also depends heavily on the way that each player is used by the coach and the general performance of the player's team. In another study, we demonstrated how a player's plus/minus should be compared from team to team (Williams and Williams, 1996). Unfortunately, current data do not allow adjustment of this statistic for the different ways in which team coaches use their players.

7.3 Statistical performance evaluation in the National Hockey League

Hockey is very much a team sport and, as such, does not lend itself easily to numerical evaluation of individual players. In varying degrees, every player statistic is correlated with the quality of the player's team. On a weak team, even the most talented player will have difficulty scoring goals, and every regular player on the team is likely to have a negative plus/minus. This is discussed more
completely later. Nevertheless, many statistics on individual player performance are gathered, published and analyzed.

The statistics gathered on the teams and players in the North American hockey leagues are quite similar. However, the interpretation of these statistics is not always the same. The Quebec League, for aspiring junior players 18 to 21 years old, is considered to be a wide-open high-scoring league. In this league, a young player would be well advised to score a large number of goals in order to draw attention. In contrast, the Western League, another junior league, is considered to be physically tough and so a hopeful player needs to make sure that his statistics indicate an ‘appropriate’ response to this particular aspect of the game (e.g. a high number of penalty minutes). This chapter focuses on the NHL in which the world’s best players currently perform.

While hockey writers focus on their favorite statistics and publishers print books filled with them, an interesting question remains outstanding: ‘How do NHL players and teams use performance statistics internally?’ Perhaps more than the public realizes, individual players often have statistical targets written into their contracts. These targets may be specific to the player or may be based on team performance. A defenseman may receive a bonus for games in which the team’s opposition is held to one goal or less, or if he is elected to the league All-Star team. A forward may receive a bonus if he scores a stated number of goals during the season. There are many different bonus clauses in player contracts.

Team ‘segment’ bonuses, which are not part of the players’ contracts, are common. All NHL teams use them and some minor league teams have them too. These bonuses are given if a team achieves specified statistical goals during the segment, which is usually five games but is sometimes ten. To illustrate, all players may receive a cash bonus if the team wins a stated number of games, or perhaps if the team holds the opposition below a stated number of goals during the segment. These segment bonuses are used to influence team play, particularly to keep the team playing consistently during the long season.

Salary arbitration is another important internal use of statistics. Arbitration is a defined procedure for settling salary disputes in the case of disagreement between a player and his team. During these negotiations, the use of statistics is adversarial and is not particularly ‘academically’ oriented. While a player’s statistics will be part of the agenda, the actual meeting is a legal one dominated by lawyers and legal protocols. How statistical analyses affect an arbitrator in the privacy of his own deliberations is speculative, but the analyses may well miss the mark if they are overly complex.

The players have an additional problem regarding the use of statistics, sometimes they cannot find out what they are! Generally, the teams gather the game performance data and are not necessarily quick to share them with the players. Some players in the NHL have performance bonuses for total on-ice time during the season. This is not an easy statistic to keep and sometimes players with such bonus clauses have found that their teams are not eager to keep them informed of their progress. The NHL teams keep many statistics that are not systematically shared with the public or players. In one arbitration meeting, a player was surprised to find that he was not scoring enough goals during the last two minutes of games!
7.4 Did Wayne Gretzky have a poor season in 1992/93?

How do the coaches use statistics? This varies considerably, even on the same team during the same year. This is particularly true of the plus/minus. Sometimes, players are told not to be concerned about lower plus/minus and at other times they are criticized for them (Williams and Williams, 1996).

The remainder of this chapter introduces a methodology to evaluate individual players based on multivariate statistical methods. Using this evaluation, it is interesting to observe and speculate about the fact that player salaries are highly correlated with their offensive points, but are not correlated at all with any measure of defensive play (Williams and Williams, 1997).

7.4 Did Wayne Gretzky have a poor season in 1992/93?

A question of multiple player statistics

In sports, writers, fans and even team managements all evaluate players with their favorite statistics. Disagreements about the importance of the various statistics are common. After more than 100 years, baseball fans still argue about the significance of slugging percentages and batting averages. This kind of dispute is common to most sports and hockey is certainly not an exception. In hockey, goals scored by a player may be the most important statistic because, in the last analysis, scoring goals wins games. On the other hand, many excellent goal scorers rely heavily on their play-making linemates to get the puck to them in advantageous positions – so the argument begins that assists are equally, or even more, important.

On any team, not every player has the same role and this may affect the interpretation and importance of the individual statistics. For NHL players who act as enforcers, the number of major fighting penalties is more important than goals. Any goals scored by them are simply bonuses to their teams. In NHL salary arbitration, players and management typically focus on very different statistics since their financial perspectives are quite different. For most players, the relative importance of an individual statistic is subjective.

Statistical analysis is eased by consistent interpretation of data (e.g. agreement that 'bigger' numbers are better, or worse, than 'smaller' ones). Unfortunately, in hockey, this is not always the case; some statistics are alternately viewed from different ends of the telescope. Further ambiguity is created by the fact that hockey is a team game and a player's statistics are affected by both the individual player's performance and that of his team-mates. As a result of this ambiguity, evaluation of hockey players has been dominated by subjective professional judgment, perhaps influenced by a small number of elected statistics.

The ambiguity that exists in the selection and interpretation of sports statistics, and hockey in particular, raises the question of whether the methods of mathematical statistics might be used to compress and summarize the multiple observations made on each NHL player into meaningful performance indices. This section describes an example of the development of indices that not only have useful interpretations, but also have the desirable property that they are totally data driven: they are not subjective. While there has been little use of these advanced methods of statistics in the analysis of hockey, these methods have been used in actual NHL salary arbitration cases. We shall not use such
balanced with his ninth place in goals? In an overall evaluation, should he be ranked third on the team, or closer to ninth, as his goals would suggest? Since it is not clear how to give relative weights to goals and assists, it is not clear where Gretzky stands. Equal weighting would suggest that he be ranked sixth, but why equal weighting? And, even though we may agree that goals and passes that set-up goals are the most important plays, hockey fans certainly know that there is more to the game than just goals and assists.

7.5.3 Penalties

The number of penalties a player incurs is also a relevant statistic. Fans and coaches get upset with ‘dumb’ penalties, but not every penalty is considered ‘dumb’. In a game of intimidation like hockey, some players are expected to get penalties, especially penalties associated with certain types of physical infractions. Popular writers (e.g. Dryden, 1994) have even developed performance indices which evaluate a player’s penalties positively, the more penalties the better. General managers of NHL teams also often interpret penalty minutes positively, but it is difficult to make this argument for all players. Most penalties result in power plays for the opposition during which the penalized team must play short-handed. Opponent’s goals are scored at a higher rate during power plays than when the teams play at even strength. On average, two-minute power plays result in goals scored by the opposition about 20% of the time. So 300 minutes in penalties against a player could result in as many as 30 goals for opponents. A team that receives many penalties is likely to lose games by doing so.

This negative interpretation of penalty minutes may be unfair to the enforcers in the NHL who are paid to intimidate other players and break certain rules. In a study covering more teams, enforcers perhaps should be identified and studied separately, but this was not done here because only two players, Marty McSorley and Warren Rychel – who are not the focus of this analysis – would possibly be affected by it.

A ranking of Kings players in 1992/93 shows that Gretzky had the smallest number of penalty minutes on the entire team. This is a strong positive for Gretzky because he obtained his goals and assists without putting his team in many short-handed situations. Still, we can hardly rank him as the number one Kings player on this basis alone. Not only that, but there are many more statistics available for exploration.

7.5.4 Plus/minus

Goals and assists are very important in hockey and high scoring players are paid well for it. But sometimes, teams score easy goals against another team’s best offensive players because these players do not play well defensively. In fact, some offensive players appear to have very little interest in defending their own end of the rink. Since it takes good defense as well as good offense to achieve victory, hockey uses a statistic called ‘plus/minus’ which purports to measure a player’s offensive versus defensive abilities.

When a team scores an even strength goal, every player on the ice for the scoring team gets a ‘plus’ and every player on the ice for the other team gets a
The accumulated net difference between a player’s pluses and minuses is referred to as his ‘plus/minus’. A negative plus/minus suggests that a player may not be paying attention to his defensive responsibilities and may not be valuable even if he is a high scorer. Unfortunately, interpretations of this statistic can also be ambiguous, because some players who are very good at defensive play are consistently used by coaches against the other teams’ most offensively-skilled players. The result is that these players get few goals, but do collect minuses. Also, some defensemen are known to be very slow leaving the ice for player changes when the forwards on their team are in the middle of promising rushes into the other team’s territory; this tactic tends to inflate their plus/minus ratings. Nevertheless, all things considered, a higher plus/minus is better than a lower one.

So how did the Kings do while Robitaille, Kurri and Gretzky were on the ice? A ranking of Kings players in 1992/93 with respect to plus/minus shows that Gretzky is eighth; Robitaille and Kurri are near the top, but behind whom? So the evaluation of Gretzky continues to be ambiguous – he was ninth in goals, third in assists, first in penalties and now we see that he was eighth in ‘plus/minus’. So where we began with one statistic, goals, we now have four, all of which are potentially important candidates for integration into an overall evaluation of Gretzky’s 1992/93 season.

### 7.5.5 Per season or per game?

Gretzky was injured in 1992/93 and played in only 45 games. In contrast, Robitaille played in all 84 games and, as a result, had much more playing time to assemble his point totals. How should their statistics be compared? How should we adjust the data for this playing time discrepancy? Or should we adjust at all? While injuries are common in hockey, a smaller number of games played during the season often reflects a player who is not really part of the regular line-up. Certainly, what any player’s season would have been like had he played more games is completely a matter of speculation, and not necessarily a matter for simple extrapolation. In professional hockey circles, it is virtually standard to look at a player’s performance statistics over a season, without regard for the reasons for playing in a limited fraction of the schedule. And, it must be admitted, there is a certain cold logic and fairness in this view. So initially, our analysis is based on season totals.

### 7.6 Player evaluation with respect to multiple variables

To overcome the ambiguity that results from examining a number of statistics one at a time, we need a methodology to assess the variables simultaneously in a single, comprehensive overview. To this end, it is useful to study first the variables two at a time rather than starting with all available statistics.

Figure 7.1 displays a log-log scatterplot of the number of goals scored by each of the Kings players against their assists. The top three Kings players, Gretzky, Robitaille and Kurri, are labeled in this and each of the subsequent plots. Robitaille is at the top right corner of Fig. 7.1 because he led the team in both goals and assists. In fact, Robitaille also led the team in shots and would also be in the top right of plots of shots against both goals and assists.
So clearly on the basis of these offensive statistics, goals, assists and shots, Robitaille had, unambiguously, the best season.

The correlations between pairs of the three offense statistics above are all over 0.9. The high correlations between goals and shots (0.91) and assists and shots (0.95) are particularly interesting because the latter is persistently higher, suggesting that if a player shoots at the goal he is more likely to get an assist than a goal. While this fact does not seem to be widely known, it will not surprise Ray Bourque fans because the stellar defenseman often remarks that he shoots at the net expecting *rebounds*, not goals!

That goals, assists and shots are all very closely related is not surprising, because they all measure related aspects of offensive play. But it is exactly this type of strong relationship that can be exploited to reduce the many available statistics into useful performance indices. The following simple example illustrates this.

In Fig. 7.1, the high correlation between goals and assists suggests that the orthogonal axes be rotated so that one of the axes goes through the data points in the direction of the maximum spread of the points. The direction of this line then has maximum variability among the players and so has maximum discriminating ability among them. Distance, call it $P$, along this new axis (not shown) gives almost complete information on both goals and assists and so, in this sense, this direction (called the first principal component) may reasonably be described as the *single* piece of information contained in Fig. 7.1.

The equation of the new axis is a linear function of both goals and assists. Since $P$ increases with an increase in goals and assists, which are both positive
measures of performance, we can interpret larger \( P \) values as better on-ice performances. In this sense, \( P \) is an index of player performance which reflects a player's offensive production in both goals and assists.

The percentage variability associated with each of the axes can be measured; in this illustrative example, the first component explains 95% of the total variability. So in the event that we were really restricted to just goals and assists, the single dimension \( P \) would capture nearly all of the potential to distinguish among the players.

Any believable index should be strongly related to the input statistics which it purports to replace. In this example, the correlations of \( P \) with goals and assists are high, 0.821 and 0.896, respectively. So relatively little is lost by replacing the two statistics, goals and assists, by the single statistic, \( P \). Furthermore, statistical theory shows that the weighting of goals and assists in \( P \) is proportional to these correlation coefficients, which means that assists are weighted slightly more heavily in \( P \) than goals.

Finally, if \( P \) is consistent with the fact that Robitaille was first in both goals and assists, Robitaille should be highest on the \( P \) index. He is. Kurri was second in assists and fifth in goals and Gretzky was ninth and third. How does \( P \) rank these players? The first six players, ranked by \( P \), are in order: Robitaille, Kurri, Granato, Carson, Donnelly and Gretzky. Since \( P \) weights goals and assists almost equally, it is not surprising that the \( P \) ranking is the same as the ranking of players by total season points, goals plus assists.

The principal component procedure can be applied no matter how many different, original input variables exist at the outset. This is very important because it aids us in determining which variables are most important and enables us not to be seriously impeded by the complexity caused by including many on-ice statistics. With the use of many input variables, additional principal components are determined. Each component adds more information, but less than the preceding components. Formally complete descriptions of principal component analysis can be found in many statistics books, e.g. Johnson and Wichern (1992) and Searle (1982).

### 7.7 The complete statistical ice hockey player

#### 7.7.1 Total statistics

The basic hockey statistics used to assess players' performances are: penalty minutes, plus/minus, and shots-on-goal along with goals and assists (which were both classified by whether they were obtained while the team was playing at even strength, with a power play advantage, or short-handed). Using these data, the principal components (as illustrated in the previous section) were calculated. The first principal component accounts for 67.7% of the total variability in the base statistics and the second component accounts for 17.6%, for a comfortable total of 85.4%. The remaining 14.6% of variability was spread over the remaining principal components. This effectively reduces the dimensionality of the data to two and, as we shall show, both of these components have very appealing hockey interpretations.

The highest correlations of the base statistics with the first principal component are goals (0.946), assists (0.954), shots (0.945) and power play goals (0.879).
Since the weight of each variable is proportional to these correlations, we know that the first principal component involves these variables with the heaviest weights. Each of these heavily weighted variables is a measure of offense. Consequently, we interpret the first principal component as an Offensive Performance Index (OPI).

As designed into the study, the better performing players will always appear towards the upper right-hand corner of the graphs. Not surprisingly then, in a normal probability plot of OPI (Fig. 7.2), Robitaille has the highest rating on offense, Kurri is third, but Gretzky’s OPI rating places him 11th! Does this ‘low’ ranking of Gretzky counter reality, or did the Great One really have an off-year? For Gretzky fans, this result demands further study.¹

Figure 7.3 shows a normal probability plot of the second principal component. In this dimension, Gretzky has the top score which raises the question of what aspect of the game is reflected in it. Why does Gretzky rank so high? The second principal component is most highly correlated with (negated) penalty minutes (0.894) and ‘plus/minus’ (0.488), which means that these are the variables most heavily weighted. Other variables receive much smaller weights. This component is an efficiency index which rates players highly whose playing style does not yield many points or scoring opportunities to

¹ A Normal probability plot for a set of values plots each value against the standard Normal quantile for its rank in the set. Points in a straight line indicate that the set of values follows a Normal distribution. Normal probability plots are not necessary simply to rank the players, but the resultant distributional observations are often very informative for making year-to-year and team-to-team comparisons. For that reason, the authors use them routinely.
the opposing team. As a result, we have labeled this dimension, EPI, for Efficiency Performance Index. Gretzky was the clear team leader.

Figure 7.4 displays the offense index, OPI plotted against the efficiency index, EPI. (Keep in mind that OPI and EPI are uncorrelated.) Since the better performing players will have higher OPI indices and/or higher EPI indices, their plot points are towards the upper right corner of the figure. Gretzky is certainly in the upper right corner, although he is not rated highest in both dimensions. So, judged on the season as a whole, Gretzky was not the top Kings' offensive player, although he did have value as their player with the highest efficiency.

7.7.2 Per game statistics

The analysis so far compares the Kings players on the basis of total season statistics. So considering that Gretzky played in only 45 games, barely over half the season, the fact that Gretzky is near the top at all should be considered a big plus for him. But this does re-raise the question, how would the analysis turn out if repeated for the same variables, but rather on a per game basis? It seems reasonable to expect that the strength of Gretzky's known offensive ability will be apparent in such an analysis and will be reflected in a higher OPI rating. Simultaneously, there is no obvious reason to suspect that a per game analysis would change his top EPI rating.

Each of the input variables was divided by the number of games played by each player and the principal components recalculated. The results show that Gretzky still has the top efficiency rating, but counter to our earlier intuition.
7.8 Did Gretzky have a bad year?

If a 'bad' year for Gretzky is not leading the team in offense, then Gretzky's offense did not lead the Kings' team in 1992/93, Robitaille's did. But in spite of missing 39 games, Gretzky's efficiency was the highest on the team; and he produced points without the damaging side effects of penalties and allowing goals by the other team – not a trivial accomplishment.
Even though Gretzky’s OPI rating was hurt badly by the anomaly that he scored no power play goals in 1992/93, the analysis strongly suggests that his ability was still there. Historically, we now know that Gretzky eventually also came to this conclusion and returned the following season to his accustomed position, leading the entire National Hockey League in total points.

Gretzky’s play during the 1992/93 season has been compared to the other players on the Kings team. We have not analyzed whether Gretzky’s ability is leaving him by comparing his 1992/93 season with his earlier years. Certainly, he did not score the phenomenal number of points that he did during his earlier years in Edmonton, but then he was no longer surrounded by the great Oiler supporting cast. Furthermore, comparative team analyses are much more complex (e.g. Williams and Williams, 1996).

7.9 Conclusions

This chapter has described several research areas in statistics for ice hockey. However, its main goal has been the description of standard statistical techniques that can be applied to develop performance indices. These techniques have three important advantages.

1. All of the available input statistics may be included. This is an important property that can be used to eliminate any arguments that particular statistics were, or were not, included in an analysis.

2. The indices do not require subjective, arbitrary weightings of the base statistics. The methodology determines the importance of every variable, and does it in such a way to maximize the numerical discrimination among players.

3. The derived indices are highly related to the base variables and so are intuitively very satisfying.

Finally, it is important to remark that this is a numerical analysis. Hockey is a physically intense team game that depends on many factors that are not measurable at all, let alone without ambiguity. Nonetheless, the application of these techniques in NHL salary arbitration cases demonstrates their usefulness and the need for further research in this area.

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