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Nataniel Greene
CUNY Kingsborough Community College

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NATANIEL GREENE
Department of Mathematics and Computer Science
Kingsborough Community College, CUNY
2001 Oriental Boulevard, Brooklyn, NY 11235
UNITED STATES

Abstract: A classical method for ranking *n* potential events as sources of error is Bayes' theorem. However, a ranking based on Bayes' theorem lacks a fundamental symmetry: the ranking in terms of blame for error will not be the reverse of the ranking in terms of credit for lack of error. While this is not a flaw in Bayes' theorem, it does lead one to inquire whether there are related methods which have such symmetry. Related methods explored here include the logical version of Bayes' theorem based on probabilities of conditionals, probabilities of biconditionals, and ratios or differences of credit to blame. We find that of all the methods described, probabilities of biconditionals and a corresponding notion of logical correlation coefficients provide a particularly attractive method for ranking blame for error and credit for lack of error which has the symmetry property we are interested in.

Key–Words: Bayes' theorem, error analysis, Probability of an implication, probability of a conditional, probability of a biconditional.

1 Introduction: Bayes' Theorem for Assessing and Ranking Sources of Error

Error is a pervasive part of all computation and industry, and methods for assessing and ranking potential causes of error are always of interest. A classical approach to assessing and ranking potential causes of error utilizes Bayes' theorem. We are concerned here with the general scenario where n mutually exclusive potential sources of error are identified, $S_1, S_2, ...S_n$, with corresponding probabilities of occurrence given by $\mathbf{P}(S_1), \mathbf{P}(S_2), ..., \mathbf{P}(S_n)$. The conditional probabilities $\mathbf{P}(E|S_1), \mathbf{P}(E|S_2), ...\mathbf{P}(E|S_n)$, are also known. In this case one may compute $\mathbf{P}(\overline{E}|S_i) = 1 - \mathbf{P}(E|S_i)$ and all the probabilities can be placed in a tree diagram for n small. Denote by E the event that an error occurred and denote by $\overline{E}$ the event that the error did not occur. According to the law of total probability, the probability the error did occur can be computed using the given data:

$$\mathbf{P}(E) = \sum_{i=1}^{n} \mathbf{P}(E|S_i)\,\mathbf{P}(S_i)$$

and the probability the error did not occur is:

$$\mathbf{P}(\overline{E}) = 1 - \mathbf{P}(E),\ i = 1...n.$$

Now, given that an error did occur, we are interested in the probability that the source of the error was S_i.These posterior probabilities $\mathbf{P}(S_i|E)$ are found using Bayes' theorem, which states:

$$\mathbf{P}(S_i|E) = \mathbf{P}(E|S_i)\,\frac{\mathbf{P}(S_i)}{\mathbf{P}(E)},\ i = 1...n.$$

Once computed, these posterior probabilities can also be used to rank the potential sources of error. A ranking of the potential sources of error based on $\mathbf{P}(S_i|E)$ can be thought of as a way of apportioning *blame* to various potential sources of error. Similarly, a ranking of the potential sources of error based on $\mathbf{P}(S_i|\overline{E})$ can be thought of as a way of apportioning *credit* to the various sources for the error not having occurred. These probabilities can also be computed using Bayes' theorem

$$\mathbf{P}(S_i|\overline{E}) = \mathbf{P}(\overline{E}|S_i)\,\frac{\mathbf{P}(S_i)}{\mathbf{P}(\overline{E})},\ i = 1...n.$$

Thus given that an error occurred, Bayes' theorem can be used to rank the various potential sources of error for possible blame and given that the error did not occur Bayes' theorem can be used to rank the various potential sources of error for possible credit.

We illustrate this with a model problem which we will make use of throughout the paper.

Example 1 *A company manufactures items and three mutually exclusive potential sources of error are identified. 40% of the manufactured items were associated with Source 1, 40% were associated with Source 2 and 20% with Source 3. Of those items associated with Source 1, 95% work properly, of those items associated with Source 2, 90% work properly, and of those items associated with Source 3, 93% work properly.*

Let S_1, S_2, and S_3 denote the events that a manufactured item comes from Source 1, 2, and 3 respectively. Let E denote the event that there was an error in manufacturing, i.e. that the manufactured item was in some way defective.

$\mathbf{P}(S_1) = 40\%$	$\mathbf{P}(\overline{S_1}) = 60\%$
$\mathbf{P}(S_2) = 40\%$	$\mathbf{P}(\overline{S_2}) = 60\%$
$\mathbf{P}(S_3) = 20\%$	$\mathbf{P}(\overline{S_3}) = 80\%$

and

$\mathbf{P}(\overline{E}\vert S_1) = 90\%$	$\mathbf{P}(E\vert S_1) = 10\%$
$\mathbf{P}(\overline{E}\vert S_2) = 95\%$	$\mathbf{P}(E\vert S_2) = 5\%$
$\mathbf{P}(\overline{E}\vert S_3) = 93\%$	$\mathbf{P}(E\vert S_3) = 7\%$

Furthermore we can compute using the law of total probability

$\mathbf{P}(E) = 7\%$	$\mathbf{P}(\overline{E}) = 93\%$

and we can compute using Bayes' theorem

Credit	Rank	Blame	Rank
$\mathbf{P}\left(S_1\vert\overline{E}\right) = 39\%$	2	$\mathbf{P}(S_1\vert E) = 57\%$	1
$\mathbf{P}\left(S_2\vert\overline{E}\right) = 41\%$	1	$\mathbf{P}(S_2\vert E) = 29\%$	2
$\mathbf{P}\left(S_3\vert\overline{E}\right) = 20\%$	3	$\mathbf{P}(S_3\vert E) = 19\%$	3
Total $= 100\%$		Total $=100\%$	

Given the knowledge that the item is working properly we can apportion the credit according to the first column and given that the item is defective we can apportion the blame according to the second column. While the analysis we have described is standard and correct, we note a lack of fundamental symmetry: the ranking in terms of blame for error is not the reverse of the ranking in terms of credit for lack of error. This is not a flaw in Bayes' theorem, but it does lead one to inquire whether there are related methods for assessing and ranking sources of error which do have such symmetry.

The related methods which we will explore here include the logical version of Bayes' theorem based on probabilities of conditionals, probabilities of biconditionals, and ratios or differences of blame to credit. Of all the methods we describe, we find that probabilities of biconditionals and the related notion of logical correlation coefficients have the desired symmetry property.

2 Logical Bayes' Theorem for Ranking Sources of Error

A logical version of Bayes' theorem has been described by authors working in artificial intelligence [5,7,8] as well as in an earlier paper [2]. We review the logical version of Bayes' theorem here. The standard Bayes' theorem is based on the notion of the probability of P *given* Q. The logical version of Bayes' theorem makes use of an analogous, but understudied concept: *the probability of P if Q* The probability of P if Q or if Q then P, is referred to as the probability of a conditional, or the probability of an implication and the statement P if Q is known as material implication.

Definition 1 *Define the event*

$$Q \to P = \overline{Q} \cup P$$

The following theorem describes the probability of a conditional.

Theorem 2

$$\mathbf{P}(Q \to P) = \mathbf{P}\left(\overline{Q}\right) + \mathbf{P}(P \cap Q)$$

Proof.

$$\begin{aligned} \mathbf{P}(Q \to P) &= \mathbf{P}\left(\overline{Q} \cup P\right) \\ &= \mathbf{P}\left(\overline{Q} \cup (P \cap Q) \cup \left(P \cap \overline{Q}\right)\right) \\ &= \mathbf{P}\left(\overline{Q} \cup (P \cap Q)\right) \\ &= \mathbf{P}\left(\overline{Q}\right) + \mathbf{P}(P \cap Q) \end{aligned}$$

■

Logical Bayes' theorem relates the probability of the conditional $Q \to P$ with the probability of the converse $P \to Q$.

Theorem 3 *Logical Analog of Bayes' Theorem (1):*

$$\mathbf{P}(Q \to P) = \mathbf{P}(P \to Q) + \mathbf{P}(P) - \mathbf{P}(Q)$$

Theorem 4 *Logical Analog of Bayes' Theorem (2): Let Q_i, $i = 1...n$ be mutually exclusive events which exhaust the sample space. Then*

$$\mathbf{P}(P \to Q_i) = \mathbf{P}(Q_i \to P) + \mathbf{P}(Q_i) - \mathbf{P}(P)$$

where $\mathbf{P}(P)$ is computed according to the law of total probability.

Conditional probabilities and probabilities of conditionals share the same order relations. Therefore a ranking obtained using the standard version of Bayes' theorem will be the same as a ranking obtained using the logical version of Bayes' theorem.

Theorem 5 *Let P, Q and R be events. Then*

$$\mathbf{P}(P|Q) > \mathbf{P}(R|Q)$$

if and only if

$$\mathbf{P}(Q \to P) > \mathbf{P}(Q \to R).$$

Proof. We have

$$\begin{aligned} \mathbf{P}(Q \to P) &> \mathbf{P}(Q \to R) \\ \mathbf{P}(\overline{Q}) + \mathbf{P}(P|Q)\mathbf{P}(Q) &> \mathbf{P}(\overline{Q}) + \mathbf{P}(R|Q)\mathbf{P}(Q) \\ \mathbf{P}(P|Q) &> \mathbf{P}(R|Q) \end{aligned}$$

and vice versa. ■

We now resume with the previous example, utilizing the logical analog of the Bayes' theorem described above. First we may compute using the initial data:

$\mathbf{P}(S_1 \to \overline{E}) = 96\%$	$\mathbf{P}(S_1 \to E) = 64\%$
$\mathbf{P}(S_2 \to \overline{E}) = 98\%$	$\mathbf{P}(S_2 \to E) = 62\%$
$\mathbf{P}(S_3 \to \overline{E}) = 99\%$	$\mathbf{P}(S_3 \to E) = 81\%$

Next we may compute either directly or using the logical analog of Bayes' theorem the reverse probabilities of conditionals.

Credit	Rank
$\mathbf{P}(\overline{E} \to S_1) = 43\%$	2
$\mathbf{P}(\overline{E} \to S_2) = 45\%$	1
$\mathbf{P}(\overline{E} \to S_3) = 26\%$	3
Blame	Rank
$\mathbf{P}(E \to S_1) = 97\%$	1
$\mathbf{P}(E \to S_2) = 95\%$	2
$\mathbf{P}(E \to S_3) = 94\%$	3

The numerical example illustrates that values of $P(E \to S_i)$ may be less intuitive than the conditional probabilities $\mathbf{P}(S_i|E)$, but the ranking one obtains remains the same. That is, one comes to the same decision as to how to rank the sources in terms of credit and blame whether one uses conditional probabilities or probabilities of conditionals.

Note that when $\mathbf{P}(E) = 0$, the standard version of Bayes' theorem is technically undefined, while the logical version is never undefined. From a computational point of view, the logical version of Bayes' theorem involves only additions and subtractions whereas the standard version involves multiplications and divisions. Nevertheless, the logical version of Bayes' theorem suffers from the same problem as the standard version of Bayes' theorem: the lack of symmetry in the ranking of blame for error and credit for lack of error.

3 Probabilities of Biconditionals for Ranking Sources of Error

3.1 Biconditionals

The probability of the biconditional P if and only if Q, $\mathbf{P}(Q \leftrightarrow P)$, is the probability that P and Q are *either* simultaneously true *or* simultaneously false. It is the probability that two events are *logically equivalent*. The probability of the negated biconditional $P(Q \not\leftrightarrow P) = \mathbf{P}(Q \leftrightarrow \overline{P}) = \mathbf{P}(\overline{Q} \leftrightarrow P)$, is the probability that P and Q are *neither* simultaneously true *nor* simultaneously false. It is the probability that two events are not logically equivalent.

Definition 6 *Define the event*

$$Q \leftrightarrow P = (P \cap Q) \cup (\overline{P} \cap \overline{Q})$$

and the corresponding probability

$$\mathbf{P}(Q \leftrightarrow P) = \mathbf{P}((P \cap Q) \cup (\overline{P} \cap \overline{Q})).$$

The following theorem describes how the probability of a biconditional may be computed in practice in terms of unions and intersections.

Theorem 7

$$\begin{aligned} \mathbf{P}(Q \leftrightarrow P) &= \mathbf{P}(P \cap Q) + \mathbf{P}(\overline{P} \cap \overline{Q}) \\ \mathbf{P}(Q \leftrightarrow P) &= 1 - \mathbf{P}(P \cup Q) + \mathbf{P}(P \cap Q) \\ \mathbf{P}(Q \leftrightarrow P) &= 1 - \mathbf{P}(P) - \mathbf{P}(Q) + 2\mathbf{P}(P \cap Q) \end{aligned}$$

Corollary 8

$$\begin{aligned} \mathbf{P}(P \leftrightarrow P) &= 1 \\ \mathbf{P}(P \leftrightarrow \overline{P}) &= 0 \\ \mathbf{P}(P \leftrightarrow S) &= \mathbf{P}(P) \\ \mathbf{P}(P \leftrightarrow \varnothing) &= \mathbf{P}(\overline{P}) \end{aligned}$$

The first two statements can be interpreted as saying that P is logically equivalent to itself with certainty and P is logically equivalent to its negation not at all. The second two statements say in words that the probability of an event P measures the degrees to which an event is logically equivalent to certainty. The probability of P not occurring measures the degree to which the event is logically equivalent to impossibility.

3.2 Logical Correlation Coefficients

When $\mathbf{P}(P \leftrightarrow Q)$ is near 1 we can say that P and Q are nearly logically equivalent. This corresponds

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to P and Q exhibiting a strong direct logical correlation. When $\mathbf{P}(P \leftrightarrow Q)$ is near 0 we can say that P and $\sim Q$ are nearly logically equivalent, which corresponds to P and Q exhibiting a strong inverse logical correlation. In this case the events are nearly complementary. It follows that $\mathbf{P}(P \leftrightarrow Q) = \frac{1}{2}$ corresponds to P and Q having a logical equivalence of $\frac{1}{2}$ and a logical correlation of zero. We formalize this and define a logical correlation coefficient for two events.

Definition 9 *The logical correlation coefficient for the events P and Q is given by*

$$\boldsymbol{\rho}(P,Q) = 2\mathbf{P}(P \leftrightarrow Q) - 1.$$

It follows that

$$\mathbf{P}(P \leftrightarrow Q) = \frac{\boldsymbol{\rho}(P,Q)+1}{2}$$

and that

$$-1 \leq \boldsymbol{\rho}(P,Q) \leq 1.$$

The next theorem is quite fundamental in that it gives conceptual justification for the preceding definition. It says that the *logical correlation coefficient of two events is the probability that those events logically equivalent minus the probability that those events are not logically equivalent.*

Theorem 10

$$\boldsymbol{\rho}(P,Q) = \mathbf{P}(P \leftrightarrow Q) - \mathbf{P}(P \not\leftrightarrow Q)$$

Proof. By definition

$$\begin{aligned} \boldsymbol{\rho}(P,Q) &= 2\mathbf{P}(P \leftrightarrow Q) - 1 \\ &= \mathbf{P}(P \leftrightarrow Q) + \mathbf{P}(P \leftrightarrow Q) - 1 \\ &= \mathbf{P}(P \leftrightarrow Q) + (1 - \mathbf{P}(P \leftrightarrow Q)) \\ &= \mathbf{P}(P \leftrightarrow Q) - \mathbf{P}(P \not\leftrightarrow Q) \end{aligned}$$

■

The next corollary shows how a logical correlation can be computed in practice.

Corollary 11

$$\boldsymbol{\rho}(P,Q) = 1 - 2\mathbf{P}(P) - 2\mathbf{P}(Q) + 4\mathbf{P}(P \cap Q)$$

The following are some algebraic properties of logical correlation which the reader can verify.

Theorem 12

$$\begin{aligned} \boldsymbol{\rho}(P,Q) &= \boldsymbol{\rho}(Q,P) \\ \boldsymbol{\rho}(\overline{P},Q) &= -\boldsymbol{\rho}(P,Q) \\ \boldsymbol{\rho}(P,P) &= 1 \\ \boldsymbol{\rho}(\overline{P},P) &= -1 \\ \boldsymbol{\rho}(P,S) &= 2\mathbf{P}(P) - 1 \\ \boldsymbol{\rho}(P,\varnothing) &= 1 - 2\mathbf{P}(P) \\ \boldsymbol{\rho}(P,Q) &= 0 \textit{ whenever } \mathbf{P}(P \leftrightarrow Q) = \frac{1}{2} \end{aligned}$$

Continuing with our example, we rank the potential sources of error in terms of credit and blame using the probabilities of biconditionals.

Credit	Rank
$\mathbf{P}(E \not\leftrightarrow S_1) = \mathbf{P}(\overline{E} \leftrightarrow S_1) = 39\%$	2
$\mathbf{P}(E \not\leftrightarrow S_2) = \mathbf{P}(\overline{E} \leftrightarrow S_2) = 43\%$	1
$\mathbf{P}(E \not\leftrightarrow S_3) = \mathbf{P}(\overline{E} \leftrightarrow S_3) = 25\%$	3
Blame	Rank
$\mathbf{P}(E \leftrightarrow S_1) = 61\%$	2
$\mathbf{P}(E \leftrightarrow S_2) = 57\%$	3
$\mathbf{P}(E \leftrightarrow S_3) = 75\%$	1

We note that for biconditionals:

$$\text{credit} + \text{blame} = 100\%$$

or

$$\mathbf{P}(E \not\leftrightarrow S_i) + \mathbf{P}(E \leftrightarrow S_i) = 100\%.$$

We can express these results equivalently in terms of their logical correlation coefficients:

Credit	Rank	Blame	Rank
$\boldsymbol{\rho}(\overline{E},S_1) = -0.22$	2	$\boldsymbol{\rho}(E,S_1) = 0.22$	2
$\boldsymbol{\rho}(\overline{E},S_2) = -0.14$	1	$\boldsymbol{\rho}(E,S_2) = 0.14$	3
$\boldsymbol{\rho}(\overline{E},S_3) = -0.5$	3	$\boldsymbol{\rho}(E,S_3) = 0.5$	1

For logical correlation coefficients, we have the equivalent relation:

$$\text{credit} + \text{blame} = 0$$

or

$$\boldsymbol{\rho}(\overline{E},S_i) + \boldsymbol{\rho}(E,S_i) = 0.$$

The fact that all the logical correlations are negative indicates that for a given source, it is more likely than not that an item was either defective and involved with that source, or working and involved with another source. This does not speak well for the manufacturer. As we expect, the logical correlation coefficients yield the same ranking as the biconditionals. Most importantly, however, we observe that the ranking in terms of credit is the reverse of the ranking in terms of blame which makes biconditionals particularly attractive to us.

4 Credit-Blame Ratios and Differences for Ranking Sources of Error

Given a partition of the sample space into the events Q_i and given an event P, one possible interpretation of the conditional probability $\mathbf{P}(Q_i|P)$ is the degree to which one may *credit the event Q_i* for the event P

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and $\mathbf{P}\left(Q_i|\overline{P}\right)$ is the degree to which one may credit the event Q_i for the nonoccurrence of the event P. In the case where the events $P = \overline{E}$ and $\overline{P} = E$ are interpreted as lack of error and error, $\mathbf{P}(Q_i|E)$ is a means of assigning credit to Q_i for error (blame) and $\mathbf{P}\left(Q_i|\overline{E}\right)$ is a means of assigning credit to Q_i for lack of error (credit). Our goal is to rank the events Q_i in such a way as to maximize the credit (for lack of error) and minimize the blame (for error). One way of doing this involves using the conditional probabilities to compute either a credit-blame ratio

$$\begin{aligned} R_i &= \frac{\text{credit}}{\text{blame}} \\ &= \frac{\mathbf{P}(Q_i|P)}{\mathbf{P}\left(Q_i|\overline{P}\right)} \end{aligned}$$

or a credit-blame difference

$$\begin{aligned} D_i &= \text{credit-blame} \\ &= \mathbf{P}(Q_i|P) - \mathbf{P}\left(Q_i|\overline{P}\right). \end{aligned}$$

The event with the most credit and the least blame will have the highest credit-blame ratio or difference.

Instead of using conditional probability to formulate credit and blame one can use probabilities of conditionals. One may then compute either a ratio

$$\begin{aligned} r_i &= \frac{\text{credit}}{\text{blame}} \\ &= \frac{\mathbf{P}(P \to Q_i)}{\mathbf{P}\left(\overline{P} \to Q_i\right)} \end{aligned}$$

or a difference

$$\begin{aligned} d_i &= \text{credit-blame} \\ &= \mathbf{P}(P \to Q_i) - \mathbf{P}\left(\overline{P} \to Q_i\right). \end{aligned}$$

Whether one uses ratios or differences the ranking one obtains will be the same. However the ranking one obtains using conditional probabilities will not always coincide with the ranking one obtains using probabilities of conditionals. The question then arises: which method of ranking the events is preferred, the one which uses conditional probability or the one which uses probabilities of conditionals? The reader may be tempted to prefer conditional probabilities just because of their greater familiarity. We demonstrate here that there is merit in using a credit-blame difference involving conditionals. *A ranking obtained using conditionals will be the same ranking one obtains using probabilities of biconditionals or logical correlation coefficients.*

Lemma 13 *A credit-blame difference based on conditionals can be expressed in terms of the probability of a biconditional or in terms of the logical correlation coefficient according to the following formulas:*

$$d_i = \mathbf{P}(P \leftrightarrow Q_i) - \mathbf{P}(P)$$

or

$$d_i = \frac{1}{2}\boldsymbol{\rho}(P, Q_i) + \frac{1}{2}\boldsymbol{\rho}(P, \varnothing)$$

Proof.

$$\begin{aligned} d_i &= \mathbf{P}(P \to Q_i) - \mathbf{P}\left(\overline{P} \to Q_i\right) \\ &= \left(\mathbf{P}\left(\overline{P}\right) + \mathbf{P}(P \cap Q_i)\right) - \left(\mathbf{P}(P) + \mathbf{P}\left(\overline{P} \cap Q_i\right)\right) \\ &= \mathbf{P}(P \cap Q_i) + \left(\mathbf{P}\left(\overline{P}\right) - \mathbf{P}\left(\overline{P} \cap Q_i\right)\right) - \mathbf{P}(P) \\ &= \mathbf{P}(P \cap Q_i) + \mathbf{P}\left(\overline{P} \cap \overline{Q_i}\right) - \mathbf{P}(P) \\ &= \mathbf{P}(P \leftrightarrow Q_i) - \mathbf{P}(P) \end{aligned}$$

■

Theorem 14 *The ranking obtained using a credit-blame difference with conditionals is equivalent to the ranking obtained using credit-blame ratios, probabilities of biconditionals or logical correlation coefficients. That is:*

$$d_i < dj$$

whenever

$$r_i < r_j$$

or whenever

$$\mathbf{P}(P \leftrightarrow Q_i) < \mathbf{P}(P \leftrightarrow Q_j)$$

or whenever

$$\boldsymbol{\rho}(P, Q_i) < \boldsymbol{\rho}(P, Q_j).$$

Proof. The result follows directly from the previous lemma. ■

We illustrate these ratios and differences going back to our example problem. First we let $P = \overline{E}$ and $\overline{P} = E$. Employing conditional probability to compute credit and blame yields the following ranking:

Credit-blame Ratio R_i	Rank
$\mathbf{P}(S_1\vert\overline{E})/\mathbf{P}(S_1\vert E) = 68\%$	3
$\mathbf{P}(S_2\vert\overline{E})/\mathbf{P}(S_2\vert E) = 141\%$	1
$\mathbf{P}(S_3\vert\overline{E})/\mathbf{P}(S_3\vert E) = 105\%$	2
Credit-blame Difference D_i	Rank
$\mathbf{P}(S_1\vert\overline{E}) - \mathbf{P}(S_1\vert E) = -18\%$	3
$\mathbf{P}(S_2\vert\overline{E}) - \mathbf{P}(S_2\vert E) = +12\%$	1
$\mathbf{P}(S_3\vert\overline{E}) - \mathbf{P}(S_3\vert E) = +1\%$	2

However, if we employ conditionals to compute credit and blame we obtain a different ranking.

Credit-blame Ratio r_i	Rank
$\mathbf{P}(\overline{E} \to S_1)/\mathbf{P}(E \to S_1) = 44\%$	2
$\mathbf{P}(\overline{E} \to S_2)/\mathbf{P}(E \to S_2) = 47\%$	1
$\mathbf{P}(\overline{E} \to S_3)/\mathbf{P}(E \to S_3) = 28\%$	3
Credit-blame Difference d_i	Rank
$\mathbf{P}(\overline{E} \to S_1) - \mathbf{P}(E \to S_1) = -54\%$	2
$\mathbf{P}(\overline{E} \to S_2) - \mathbf{P}(E \to S_2) = -50\%$	1
$\mathbf{P}(\overline{E} \to S_3) - \mathbf{P}(E \to S_3) = -68\%$	3

Observe that the above credit-blame ratios or differences computed using conditionals give the same ranking as the ranking based on $\mathbf{P}(\overline{E} \leftrightarrow S_i)$. Theorem 14 and the previous computations lead us to recommend computing probabilities of biconditionals or logical correlation coefficients and to dispense with credit-blame ratios and differences in actual practice. Probabilities of biconditionals have nice algebraic properties, such as

$$\begin{aligned} \mathbf{P}\left(P \not\leftrightarrow Q_i\right) &= \mathbf{P}\left(\overline{P} \leftrightarrow Q_i\right) \\ &= \mathbf{P}\left(P \leftrightarrow \overline{Q}_i\right) \\ &= 1 - \mathbf{P}\left(P \leftrightarrow Q_i\right) \end{aligned}$$

and the corresponding logical correlation coefficients have the equivalent properties

$$\boldsymbol{\rho}\left(\overline{P}, Q_i\right) = \boldsymbol{\rho}\left(P, \overline{Q}_i\right) = -\boldsymbol{\rho}\left(P, Q_i\right)$$

which make them useful for computing an inverse ranking. The ranking one obtains involving blame will always be the reverse of the ranking involving credit which makes biconditionals particularly attractive.

5 Conclusion

Our conclusions are that probabilities of conditionals and the logical version of Bayes' theorem provide the same ranking in terms of blame for error and credit for lack of error as do conditional probabilities and the standard version of Bayes' theorem. However both of these methods suffer from a lack of symmetry: the ranking in terms of blame for error will not be the reverse of the ranking in terms of credit for lack of error.

Probabilities of biconditionals provide an alternative method for ranking blame for error and credit for lack of error which has the desired symmetry property. Furthermore, a ranking based on biconditionals will always coincide with a ranking based on a credit-blame ratio or difference which utilizes conditionals. Thus we conclude that probabilities of biconditionals and the corresponding logical correlation coefficients provide an attractive alternative to the classical Bayes' theorem approach.

References

[1] I. M. Copi, *Symbolic Logic*, Macmillan Publishing Co., Inc., New York, 1979.

[2] M. Doctorow and O. Doctorow, On the Nature of Causation, *Proceedings of the Philosophy of Education*, Chapter Eight, 1983.

[3] O. Doctorow, Usenet postings, available at: http://groups.google.com.

[4] N. Greene, An Overview of Conditionals and Biconditionals in Probability, *Recent Advances on Applied Mathematics, Proceedings of the American Conference on Applied Mathematics (MATH '08),* Cambridge, Massachusetts, March 24-26, 2008. Available at:

[5] M. Mukaidono and Y. Yamauchi, Logical Version of Bayes Theorem, *Proceedings of the Second Vietnam-Japan Bilateral Symposium of Fuzzy Systems and Applications VJFUZZY'2001*, Hanoi, Vietnam, December 7-8, 2001, pp. 17-21.

[6] N. J. Nilsson, "Probabilistic Logic," Artificial Intelligence, Vol. 28, No. 1, pp. 71-78.

[7] H. T. Nguyen, M. Mukaidono, and V. Kreinovich, Probability of Implication, Logical Version of Bayes' Theorem, and Fuzzy Logic Operations, *Proceedings IEEE-FUZZ*,Volume 1, 2000, pp. 530-535.

[8] Y. Yamauchi and M. Mukaidono, Probabilistic Inference and Bayesian theorem based on logical implication, in: N. Zong, A. Skowron, and S. Ohsuga (eds.), *New Directions in Rough Sets, Data Mining, and Granular Soft Computing*, Springer-Verlag Lecture Notes in Artificial Intelligence, Vol. 1711, 1999, pp. 334-342.