Creating Art Patterns with Math and Code
The MAA MathFest 2016, Columbus, OH

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Introduction

The goal of this talk is to showcase some visualization projects that we developed for a 3-day Code in R summer program, designed to inspire the creative side of our STEM students by engaging them with computational projects that we developed with the purpose of mixing calculus level math and code to create complex patterns. One of the goals of this program was to attract more minority and female students into applied math and computer science. The projects are designed to be implemented using the high-level, open-source and free computational environment R, a popular software in industry for data analysis and visualizations. Our hope is that familiarity with R could improve our students’ chances of getting internships and full-time jobs.

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A 3-day summer program for STEM students from City Tech and BMCC (CUNY) on the foundations of scientific programming, simulations and visualizations using R, and how to create project reports in RStudio, using Rmarkdown.

Code in R Program Highlights:

- Held at City Tech on May 26-31, 2016.
- 74 students applied (35 minority, 25 female, of which 13 minority), 34 from City Tech, 37 from BMCC, and 3 high school students.
- Student majors: applied math, math education, computer science, computer network and engineering technology, bioinformatics, business, liberal arts.
- The first day of the program was a 5-hour introduction to R that we gave, based on instructional materials that we developed.
We had 46 students who signed up for the Code in R competition on day two. We formed 17 teams, divided into 2 groups, new and not new to R:

- 10 teams (29 students) were in the group of students who were new to R.
- 7 teams (17 students) were in the group of students who knew some R.

Gender and ethnicity of students in the Code in R competition:

- 17 female students (12 in group new to R), 6 minority.
- 29 male students (17 in group new to R), 13 minority.

The winning team in the group of students new to R was made up of 3 high school students.
Visualization Projects Using R

We next present several computational and visualization projects, **scaffolded** and designed to mix **programming**, **experimentation** and **calculus level math**, aimed at our STEM students interested in combining math and code:

1. Visualizing Finite Sums of Complex Exponentials.
2. Visualizing Spiral Galaxies in 2D and 3D Using Logarithmic Spirals.
3. Art Patterns as Contour Projections of Surfaces on Heat Maps.
5. The Sierpinski Carpet and Triangle, and the Heighway Dragon.
6. A Glimpse into Interactive 3D Visualizations.
Visualizing Sums of Complex Exponentials

The *mystery curve* of Frank Farris:

\[ f(t) = e^{it} + \frac{1}{2} e^{6it} + \frac{i}{3} e^{-14it}, \quad t \in (0, 2\pi) \]

- The vectorized R code needed to implement and visualize \( f(t) \) is compact:

```r
n<-1000 # points
t<-seq(0,2*pi,len=n) # a vector of t values
f<-exp(1i*t)+1/2*exp(6*1i*t)+1i/3*exp(-14*1i*t)
plot(f,pch=20,cex=0.7,col=rainbow(n),axes=F,
     xlab="",ylab="") # complex plot of f(t)
```

A Plot of $f(t)$ on the Complex Plane
Visualizing Spiral Galaxies

Objective: Visualize spiral galactic arms by plotting points from two out of phase logarithmic spirals. We plot dots on the plane with Cartesian coordinates \((x, y)\) according to the parametric equations of a logarithmic spiral:

\[
\begin{align*}
x(\theta) &= r(\theta) \cos(\theta) = e^{\theta \tan \phi} \cos(\theta) \\
y(\theta) &= r(\theta) \sin(\theta) = e^{\theta \tan \phi} \sin(\theta)
\end{align*}
\]

where \(\theta \in (0, 9\pi)\), \(\phi \approx 0.21\) rad \((12^\circ)\); \(\phi\) controls the curvature of the spiral.

The second out of phase spiral has the parametric equation in polar coordinates:

\[
r(\theta) = e^{(\theta + \pi) \tan \phi}
\]

It was confirmed in 2013 that the Milky Way Galaxy actually has four spiral arms.
Two Out of Phase Logarithmic Spirals
To create a more realistic galactic distribution of stars we create *fuzzy spirals*. The fuzzy spirals are generated by adding *random jitter* to the $x$ and $y$ coordinates of the two out of phase spirals, using some random amounts sampled from the uniform distribution $U(-a, a)$, for $a \approx 10$. We create the scatterplot of the two fuzzy spirals by plotting the points:

$$(x(\theta) + u_x, y(\theta) + u_y), u_x, u_y \sim U(-a, a)$$

where $x$ and $y$ are generated from the parametric equations of the two out of phase spirals, and $u_x, u_y$ are independent samples generated from $U(-a, a)$. 
Visualizing Spiral Galaxies in 2D
Visualizing Spiral Galaxies in 3D

We extend our fuzzy 2D spirals to 3D spirals by generating random $z$ coords. sampled from the Normal r.v. $N(\mu = 0, \sigma = 0.5)$. Here is a static 3D scatterplot:
Available in the html version.
‘Native American’ Art as Contour Projections

\[ z = \cos(x^2 + y^2)e^{-\frac{1}{7}\sqrt{x^2+y^2}} \]
More Math to Art Creations

$$z = \tan(x^2 + y^2) e^{-\frac{1}{r} \sqrt{x^2 + y^2}}$$
Butterfly Curves and Geometric Transformations

\[ r = e^{\cos(\theta)} - 2\cos(4\theta) + \sin^5\left(\frac{\theta}{12}\right) \]
The Sierpinski Carpet & the Heighway Dragon
Interactive 3D Lissajous Figures

Available in the html version.
Figure 1: Thank you!