2023

MTH 125 - Modeling with Exponential Functions

Stivi Manoku
CUNY College of Staten Island, stivi.manoku@csi.cuny.edu

Modeling with Exponential Functions

Exponential Growth (Doubling Time) $n(t) = n_0 \cdot 2^{t/a}$

1. A bacteria population doubles every 3 hours. Initially there are 1000 bacteria in the colony.
   (a) Find a model for the bacteria population after *t* hours.
   (b) How many bacteria are in the colony after 15 hours.
   (c) After how many hours will the bacteria count reach 100000?

2. A breed of fox was introduced onto a small island about 8 months ago to control the rabbit population. The current fox population on the island is estimated to be 4100 and doubling every 3 months.
   (a) What was the initial size of the fox population?
   (b) Estimate the fox population 1 year after they were introduced to the island.
   (c) Sketch a graph of the fox population.

Exponential Growth (Relative Growth Rate) $n(t) = n_0 \cdot e^{r \cdot t}$

3. The initial bacterium count in a culture is 500. A biologist later makes a sample count of bacteria in the culture and finds that the relative rate of growth is 40% per hour.
   (a) Find a function that models the number of bacteria after *t* hours.
   (b) What is the estimated count after 10 hours?
   (c) After how many hours will the bacteria count reach 80,000?
   (d) Sketch a graph of the function n(t).

4. The population of a certain species of fish has a relative growth rate of 1.2% per year. It is estimated that the population in 2010 was 14 million.
   (a) Find an exponential model for the population (in millions) *t* years after 2010.
   (b) Estimate the fish population in the year 2015.
   (c) After how many years will the fish population reach 17 million?

Radioactive Decay $m(t) = m_0 \cdot 2^{-t/h}$; $m(t) = m_0 \cdot e^{-r \cdot t}$; $r = \frac{\ln 2}{h}$

5. Polonium-210 (^{210}Po) has a half-life of 140 days. Suppose a sample of this substance has a mass of 300 mg.
   (a) Find a function $m(t) = m_0 \cdot 2^{-t/h}$ that models the mass remaining after *t* days.
   (b) Find a function $m(t) = m_0 \cdot e^{-r \cdot t}$ that models the mass remaining after *t* days.
   (c) Find the mass remaining after one year.
   (d) How long will it take for the sample to decay to a mass of 200 mg?
   (e) Draw a graph of the sample mass as a function of time.

6. The half-life of strontium-90 is 28 years. How long will it take a 40-mg sample to decay to a mass of 32 mg? (Round your answer to the nearest whole number.)

7. After 3 days a sample of radon-222 has decayed to 58% of its original amount.
   (a) What is the half-life of radon-222? (Round your answer to two decimal places.)
   (b) How long will it take the sample to decay to 10% of its original amount?