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$\bar{K}pp$ and $\bar{K}\bar{K}p$ Clusters

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Abstract. The three-body kaonic $\bar{K}pp$ and $\bar{K}\bar{K}p$ clusters are studied using the method of hyperspherical harmonics (HH) in momentum representation and differential Faddeev equations (DFE). We use different NN and $\bar{K}N$ interactions. Results obtained by the methods of HH and DFE are in reasonable agreement. The binding energy and the width show very strong dependence on the $\bar{K}N$ potential. We have two different classes of results: the results based on phenomenological strong $\bar{K}N$ potential, and the results obtained with much weaker chiral SU(3)-based $\bar{K}N$ potential.

1 Introduction

The light kaonic $\bar{K}pp$ and $\bar{K}\bar{K}p$ clusters represent three-body systems and have been treated in the framework of various theoretical approaches such as variational methods [1–5], the method of Faddeev equations [6–9], and the method of hyperspherical harmonics (HH) [10], [11]. All the aforementioned approaches predict the existence of a bound state for the $\bar{K}NN$ system. The different predictions for the binding energy and the width are in considerable disagreement, varying within the ranges 9–95 MeV and 34–110 MeV, respectively [12]. To resolve this controversial situation that perhaps stems from the ambiguity of the $\bar{K}N$ interaction, the importance of the NN interaction or methods of three-body calculations, the further theoretical investigations are apparently needed. We present three-body nonrelativistic calculations using a potential model for the kaonic $\bar{K}pp$ and $\bar{K}\bar{K}p$ systems and applying two methods: the method of HH in momentum representation and the Faddeev equations in configuration space. Calculations for a binding energy and width of the kaonic $\bar{K}pp$ system are performed using three different potentials for the NN interaction, as well as two different potentials for the description of the kaon-nucleon interaction. The latter we also use for study of the $\bar{K}\bar{K}p$ system. Such approach allows one to understand the dependence of the bound state and the width of the kaonic three-body system on the method of calculations, the importance of nucleon-nucleon interaction, and the key role of the kaon-nucleon interaction.

2 Theoretical framework

The binding energy and the wave function of the three nonrelativistic particle can be obtained by solving the Schrödinger equation

$$(\widehat{T} + V_{123})\Psi = E\Psi, \tag{1}$$

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Table 1. The binding energy B and the width for the $\bar{K}pp$ system for different NN and $\bar{K}N$ interactions.

	MeV	AV14+AY	MT+AY	T+AY	AV14+HW	MT+HW	T+HW
HH	B	46.2	46.5	46.3	–	20.5	20.6
	Γ	66.7	84.3	74.5	–	48.1	49.5
DFE	B	47.3	46.0	46.3	21.6	20.4	20.6

where $\widehat{T}$ is the operator of the kinetic energy and the potential energy $V_{123} = V_{NN} + V_{\bar{K}N_1} + V_{\bar{K}N_2}$ for the $\bar{K}NN$ system and $V_{123} = V_{\bar{K}\bar{K}} + V_{\bar{K}p} + V_{\bar{K}p}$ for the $\bar{K}\bar{K}p$ system. Here V_{NN} is the nucleon-nucleon potential, $V_{\bar{K}N_1} + V_{\bar{K}N_2}$ is the sum of a pairwise effective antikaon interaction with the first and second nucleon, respectively, and $V_{\bar{K}\bar{K}}$ is the $\bar{K}\bar{K}$ interaction. The effective interactions of the $\bar{K}N$ and $\bar{K}\bar{K}$ two-body subsystems are discussed in detail in Refs. [1], [2], [5], [13]. Below, we use two effective $\bar{K}N$ interactions. One is the energy-independent phenomenological $\bar{K}N$ potential from Refs. [1] (AY potential), and the other is the energy-dependent effective $\bar{K}N$ interaction [13] derived within the chiral SU(3) effective field theory that we refer to as the HW potential. We use three NN potentials [14]: the realistic Argonne V14 (AV14), the semi-realistic Malfliet and Tjon MT-I-III (MT), and the Tamagaki G3RS that we refer to as the T potential. We solve Eq. (1) using two different methods: the method of Faddeev equations in configuration space and the method of hyperspherical functions in momentum representation. In the framework of both methods we introduce the trees of the Jacobi coordinates for three particles in configuration space and the conjugated set of momenta in the momentum space. In the Faddeev method the appropriate division of the 3-body wave function in the configuration space into the Faddeev components leads to the system of differential equations in configuration space known as the differential Faddeev equations (DFE) [16]. For the $\bar{K}pp$ cluster the total wave function is decomposed into the sum of the Faddeev components U and W corresponding to the $(pp)\bar{K}$ and $(\bar{K}p)p$ types of rearrangements: $\Psi = U + W - PW$, where P is the permutation operator for two identical fermions. For the $\bar{K}\bar{K}p$ system the decomposition has the form $\Psi = U + W + PW$, where P is the permutation operator for two identical bosons and the Faddeev components U and W correspond to the $(\bar{K}\bar{K})p$ and $(\bar{K}p)\bar{K}$ types of rearrangements. For the both kaonic systems the set of the Faddeev equations can be reduced to the system of two equations for the components U and W [17], which allows one to find the total wave functions and binding energies for the $\bar{K}pp$ and $\bar{K}\bar{K}p$ systems.

To use the method of HH in momentum representation one can write the Schrödinger equation (1) in an integral form $\Psi(\mathbf{p},\mathbf{q}) = -\frac{1}{(2\pi)^6}\int G(\mathbf{p},\mathbf{q}) < \mathbf{p}'\mathbf{q}'|V_{123}|\mathbf{pq} > \Psi(\mathbf{p}',\mathbf{q}')d\mathbf{p}'d\mathbf{q}'$, where $< \mathbf{p}'\mathbf{q}'|V_{123}|\mathbf{pq} > = \frac{1}{(2\pi)^6}\int V_{123}\exp\left[i(\mathbf{q}-\mathbf{q}')\mathbf{x}+i(\mathbf{p}-\mathbf{p}')\mathbf{y}\right]d\mathbf{x}d\mathbf{y}$ is the Fourier transformation of the V_{123} interactions, which are defined above. At the next step we introduce the hyperradii ϱ and $\varkappa$, and two sets of five angles denoted by Ω_ρ and $\Omega_\varkappa$ which define the direction of the vector ρ and vector $\varkappa$, in the six dimensional configuration and momentum spaces, respectively [15]. Following Refs. [15] and [11] we expand the wave function of three bound particles in terms of the symmetrized hyperspherical harmonics $\Phi_\mu^{l_p l_q}(\Omega_\varkappa,\sigma,\tau)$ in momentum representation: $\Psi(\mathbf{p},\mathbf{q}) = \sum_{\mu l_p l_q} u_\mu^{l_p l_q L}(\varkappa)\Phi_\mu^{l_p l_q L}(\Omega_\varkappa,\sigma,\tau)$, where μ is the grand angular momentum, L is the total orbital momentum, and l_p and l_q are the angular momenta corresponding to the Jacobi momenta p and q that are conjugated to the standard Jacobi coordinates for three particles. Here $\Phi_\mu^{l_p l_q}(\Omega_\varkappa,\sigma,\tau)$ are written as a sum of products of spin and isospin functions and HH, using the Raynal-Revai coefficiants. For the system $\bar{K}pp$ the wave function is antisymmetrized with respect to two protons, while for the $\bar{K}\bar{K}p$ system it is symmetrized with respect to two kaons. For the hyperradial functions $u_\mu^{l_q l_p L}(\varkappa)$ we obtain the coupled integral equations. By

Table 2. The bound state energies of $\bar{K}p$ (E_2) and $\bar{K}\bar{K}p$ (B) systems, and the root-mean-square radius of the $\bar{K}$ distribution. ΔE is the binding energy measured from the two-body threshold.

	$\bar{K}\bar{K}$	$\bar{K}p$	$< r^2 >^{1/2}$, fm	E_2, MeV	B, MeV	ΔE, MeV
DFE	AY	AY	1.35	30.3	31.7	1.4
	$V_{\bar{K}\bar{K}} = 0$	AY			35.2	4.9
HH	AY	AY	1.36	30.0	31.7	1.7
	$V_{\bar{K}\bar{K}} = 0$	AY			35.3	5.3
DFE	HW	HW	1.94	11.16	unbound	
	$V_{\bar{K}\bar{K}} = 0$	HW			12.17	1.01
HH	HW	HW	1.96	11.42	11.43	0.01
	$V_{\bar{K}\bar{K}} = 0$	HW			12.21	0.79

solving the coupled integral equations one can find the hyperradial functions $u_{\mu}^{l_q l_p L}(\varkappa)$ for a given L and the binding energies for the $\bar{K}pp$ and $\bar{K}\bar{K}p$ systems.

3 Results and discussion

$\bar{K}pp$ system. Results of our calculations for the $\bar{K}pp$ cluster are presented in table 1. For the calculations of the binding energy and the width with the methods of HH and DFE we use MT, T, and AV14 potentials for the NN interaction, while for the $\bar{K}N$ interaction we use the energy-dependent effective HW and the phenomenological AY potentials with the range parameter b=0.47 fm and b=0.66 fm, respectively. Such an approach allowed us to examine how the $\bar{K}pp$ cluster's structure depends on different choices of the $\bar{K}N$ interactions for the same NN potential, as well as its dependence on different choices of the NN interaction for the same $\bar{K}N$ interaction, and to understand the sensitivity of the system to the input interactions. The analysis of the results presented in table 1 shows that the values for the binding energy for the $\bar{K}pp$ system obtained by both methods are in reasonable agreement, and that the ground state energy is not sensitive to the NN interaction. However, there is a very strong dependence on the kaon-nucleon interaction. The energy of the bound state, as well as the width calculated for the AY potential [1] are more than twice as big as for the energy-dependent chiral $\bar{K}N$ potential [13]. Therefore, the highest binding energies are those that are obtained based on the phenomenological AY potential. Discrepancies obtained for the binding energy using the same potentials but different methods are mostly related to a problem of an equivalent representation of the potentials in momentum and configuration spaces.

$\bar{K}\bar{K}p$ system. We study a possible bound state of the $\bar{K}\bar{K}p$ cluster with $S = -2$, $I = 1/2$, $J^+ = 1/2^+$ using the effective $s-$wave AY and HW potentials assuming that this state is formed due to the strong $\bar{K}p$ attraction. The strength of the $s-$wave $\bar{K}\bar{K}$ interaction for the isospin $I = 0$ is zero due to Bose statistics, and we consider a weak repulsion for the isospin $I = 1$ that reproduces the scattering lengths $a_{\bar{K}\bar{K}}$=-0.14 fm for the range parameter b=0.66 fm (AY potential) and b=0.47 fm (HW potential). The results of calculations for the binding energies for the $\bar{K}p$ and $\bar{K}\bar{K}p$, the bound $\bar{K}\bar{K}p$ state without $\bar{K}\bar{K}$ interaction, and the root-mean-square radius of the $\bar{K}$ distribution are presented in table 2. For the AY potential, the $\bar{K}\bar{K}p$ system is still bound even with a much stronger $\bar{K}\bar{K}$ repulsion, while for the HW potential there is no bound state found within the DFE calculation and there is a bound state with energy 0.01 MeV relative to the $\bar{K}p + \bar{K}$ threshold for the HH calculation. Thus, although the $\bar{K}p$ with $I = 1$ is attractive, the attraction is not enough to overcome the $\bar{K}\bar{K}$ repulsion. For the width within the method of HH we obtain 58.6 MeV and 20.8 MeV with the AY and HW potentials, respectively.

The results for the binding energy of the $\bar{K}\bar{K}p$ system obtained by the methods of HH and DFE are in good agreement, and there is reasonable agreement between our calculations and results obtained using a variational method [5] and the Faddeev calculations [9].

4 Conclusion

The calculations show that we have two different classes of results: the results based on the phenomenological strong $\bar{K}N$ potential predict a strong bound state of the $\bar{K}pp$ system, and the results obtained with much weaker HW potential indicate a shallow bound state for this cluster. Therefore, the binding energy of the $\bar{K}pp$ system depends entirely on the ansatz for the $\bar{K}N$ interaction and substantially changes when we use the AY and HW potentials. On the other hand, the binding energy of the $\bar{K}pp$ and $\bar{K}\bar{K}p$ systems obtained by the methods of HH and DFE are in reasonable agreement. Thus, results are not sensitive to the method of calculations. The sensitivity of the bound state of the $\bar{K}pp$ system to details of the NN interaction is marginal for the s-wave MT and T potentials and is slightly more pronounced for the l-dependent AV14 potential. Our calculations, as well as calculations within other theoretical approaches [1–3], [6–11], show that the width is more than twice as much as the binding energy. Thus, we are facing a situation in which the states have a much larger width than the binding energy. That makes the experimental observation of such states challenging and it may be hard to identify the resonances. However, the continuation of the experimental search for the quasi-bound kaonic clusters still remains important.

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