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Study of the $KK\bar{K}$ system in hyperspherical formalism

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Abstract. The three-body kaonic system $KK\bar{K}$ is studied in the framework of a single-channel potential nonrelativistic model using the method of hyperspherical harmonics in momentum representation. Calculations are performed with two sets of KK and $K\bar{K}$ potentials. A quasibound state for the $KK\bar{K}$ system with spin-parity 0^- and isospin 1/2 is found below the three-kaon threshold.

Recently, there has been special interest in few-body systems constituted of two or more kaons. Particularly noteworthy is the possibility of formation of quasibound states in a $KK\bar{K}$ system. We study the $KK\bar{K}$ system using a nonrelativistic potential model within the framework of the hyperspherical harmonics (HH) method in momentum representation and consider the $KK\bar{K}$ system as three interacting kaons. Once the two-body interactions for the $K\bar{K}$ and KK subsystems are defined one can determine the wave function of the $KK\bar{K}$ system by solving the Schrödinger equation $H\Psi = E\Psi$ for the Hamiltonian $H = \widehat{T} + V_{\bar{K}K}(r_{12}) + V_{KK}(r_{13}) + V_{\bar{K}K}(r_{23})$. Here $\widehat{T}$ is the operator of the kinetic energy and the potentials of the effective KK and $K\bar{K}$ interactions are $V_\alpha(r_{ij})$, where the index α indicates the type of interaction: $\alpha \in KK, K\bar{K}$. For the description of the effective kaon-kaon interactions we use the local potentials from Refs. [1] and [2] that can be written in one-range Gaussian form as $V_\alpha(r) = \sum_{I=0,1} U_\alpha^I \exp\left[-(r/b)^2\right] P_\alpha$, where b is the range parameter and P_α is the isospin projection operator. The set of values of the potential depth for $K\bar{K}$ interaction and two optimized values for the range parameter (set A and set B) are given in Table 1. In Ref. [2] the $K\bar{K}$ interaction is derived under the assumption that $K\bar{K}$ forms the quasibound states $f_0(980)$ and $a_0(980)$ in the $I = 0$ and $I = 1$ channels, respectively, and reproduces the masses and widths of these resonances. Table 1 also presents sets of parameters (A1 and B1) and (A2 and B2) for the effective KK interaction that reproduce lattice QCD calculations [3] for the scattering lengths $a_{\bar{K}\bar{K}} = -0.14$ fm and $a_{\bar{K}\bar{K}} = -0.10$ fm, respectively. Starting with the Hamiltonian for the $KK\bar{K}$ system one can write the integral Schrödinger equation in the momentum representation. Then, following Ref. [4], we expand the wave function Ψ of the $KK\bar{K}$ system in terms of the symmetrized HH in momentum space and substitute this expansion into the integral Schrödinger equation. As a result, we obtain a system of coupled integral equations for the hyperradial functions. The solution of this system of equations allows us to construct the wave function Ψ for the $KK\bar{K}$ system and determine the binding energy B. The convergences of binding energy calculations for the ground state of the $KK\bar{K}$ system as a function of the grand angular momentum for the sets of parameters A2 and B1 are shown in Table 2. A reasonable convergence for the ground state energy is reached for the grand angular momentum 10, and we limit our considerations to this

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Table 1. Sets of parameters for the $K\bar{K}$ and KK interactions.

	A (b = 0.66 fm)		B (b = 0.47 fm)	
Interaction	$U^{I=0}$, MeV	$U^{I=1}$, MeV	$U^{I=0}$, MeV	$U^{I=1}$, MeV
$K\bar{K}$	$-630-210i$	$-630-210i$	$-1155-283i$	$-1155-283i$
KK ($a_{\bar{K}\bar{K}}=-0.14$ fm)	0	104	0	313
KK ($a_{\bar{K}\bar{K}}=-0.10$ fm)	0	70	0	205

Table 2. The convergence of the ground state energies of the $KK\bar{K}$ system for sets of parameters A1 and B2.

	μ =0	μ =2	μ =4	μ =6	μ =8	μ =10
B, MeV, for A2	20.6	22.4	23.3	23.8	23.9	24.2
B, MeV, for B1	15.6	16.9	17.7	18.1	18.3	18.6

Table 3. Results of calculations and comparison with the Faddeev and variational calculations from Ref. [5].

	Faddeev [5]	Variational [5] with A1	A1	A2	B1	B2
Mass, MeV	1420	1467	1464.1	1463.8	1469.4	1468.2
Width, MeV	50	110	48.4	49.1	42.0	41.1
B, MeV		21	23.9	24.2	18.6	19.8
$\sqrt{< r^2 >}$, fm		1.6	1.61	1.56	1.67	1.63

value. The width of the bound state is evaluated from the imaginary part of the $K\bar{K}$ interactions as $\Gamma = -2\left\langle \Psi \left| \mathrm{Im}\left(V_{\bar{K}K}(r_{12}) + V_{\bar{K}K}(r_{23})\right)\right| \Psi \right\rangle$. The results of our calculations for the binding energy and the width for the $KK\bar{K}$ system are presented in Table 3. To compare our results with those obtained in Ref. [5] we consider the same K meson mass m_K = 496 MeV. The total mass of the $KK\bar{K}$ system ranges from 1463.8 to 1469.4 MeV. The width falls into the 41–49 MeV range for all sets of the $K\bar{K}$ and KK interactions. The quasibound state for the $KK\bar{K}$ with spin-parity 0^- and total isospin 1/2 is found to be below the three-kaon threshold. The comparison of our results with those obtained with the variational method [5] shows that, while the binding energies calculated within the HH and variational methods are close enough, the difference for the widths are more than 50%. A different scenario is observed for the HH and the Faddeev calculations in the momentum representation: the difference in masses is more than 40 MeV.

Thus, our calculations using the three body nonrelativistic potential model predict a quasibound state for the $KK\bar{K}$ system with a mass around 1460 MeV that can be associated with the $K(1460)$ resonance. Our results support the conclusion obtained through the variational calculations and a coupled-channel approach using the Faddeev equations that $K(1460)$ could be considered a dynamically generated resonance.

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