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***PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies,* Accepted Manuscript**

Multiple Problem Solving Strategies Provide Insight into Students' Understanding of Open-Ended Linear Programming Problems

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Author Note

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Abstract: Open-ended questions that can be solved using different strategies help students learn and integrate content, and provide teachers with greater insights into students' unique capabilities and levels of understanding. This article provides a problem that was modified to allow for multiple approaches. Students tended to employ high-powered, complex, familiar solution strategies rather than simpler, more intuitive strategies, which suggests that students might need more experience working with informal solution methods. During the semester, by incorporating open-ended questions, I gained valuable feedback, was able to better model real-world problems, challenge students with different abilities, and strengthen students' problem solving skills.

Word Count: 99

Keywords: Assessment, linear programming, open-ended questions, problem solving, solution strategies

Open-Ended Linear Programming Problems: Multiple Problem Solving Strategies Provide Insight into Students' Understanding

1. INTRODUCTION

Despite decades of educational reform that have changed the way mathematics is taught, there have been few changes in the way educators and mathematics textbooks approach teaching linear programming problems. In textbooks, different approaches to solve linear programming problems are presented as separate ideas [20]. Solution methods to linear programming problems are described as a series of steps or rules to be followed in a clearly defined order [31], and most often closed-ended questions are used [7]. The topic seems to be approached in a manner that limits the audience to students who are in a few select fields [7]. One significant change in the teaching of linear programing has been the introduction of computer programs that can efficiently solve complex problems [1]. However, this one change may not improve problem solving skills or make these types of problems more approachable to students who have different learning styles or a broad range of backgrounds.

In general, there is a concern that undergraduate mathematics students are not developing an in-depth level of understanding of the discipline, but are instead reproducing familiar strategies to solve problems [15]. Given that the way in which linear programming has been taught has not significantly changed, this article explores whether select research-based recommendations that were designed primarily to improve the teaching of mathematics at the K-12 level could be applied to change the way finite mathematics is taught. Incorporating these changes potentially could increase the range of approaches used to solve problems, strengthen problem-solving skills, and increase the likelihood that these traditional mathematics topics appeal to a broader audience.

This article proposes that closed-ended traditional exercises can be modified to allow for multiple strategies, to challenge students who have different abilities, to foster persistence, to strengthen problem-solving skills, and to provide teachers with greater insights. Requiring students to develop and compare different procedures that are used to solve a problem can create a rich learning environment.

The paper, which reflects on my experience teaching open-ended questions, is divided into five sections. Because the decision to modify the approach to teaching Finite Mathematics was based on a subset of recommendations intended to reform K-12 mathematics education, the first section discusses these recommendations to provide a context for the changes made. This section discusses the benefits of using open-ended questions and teachers' concerns and reluctance to include real-world open-ended questions that can be solved using multiple approaches. The second section provides an example of an open-ended linear programming problem that could be solved by at least four distinctly different approaches, which are presented. Difficulties emerged as students easily employed two high-powered complex strategies, but overlooked simpler strategies that most likely would be the way the problem would be approached in the real-world. Students' attempts to find unique strategies are discussed in the third section. The fourth section reflects on the benefits of this teaching experience and reiterates my belief that offering students opportunities to apply distinctly different procedures to solve problems is beneficial, despite the difficulty of designing questions, implementing lessons that contain open-ended exercises, and assessing the work produced.

2. CONTEXT FOR PEDAGOGICAL CHANGE

The decision to assign an open-ended linear programming exercise that could be approached in different ways was based on research and recommendations intended mainly to reform mathematics teaching at the precollege level. For decades, there has been a call to reform K-12 mathematics education instruction and assessment practices [12, 17, 18]. As part of that call, problem solving and communication were stressed [12]. In addition to proposing changes to the curriculum, mathematics educators advocated that pedagogical techniques needed to change to promote students' ability to be flexible, independent, resourceful, and engaged problem solvers who value mathematics [17]. In recognition of the fact that students differed in their ways of thinking about mathematics problems and demonstrating what they know and can do, the *Principles and Standards for School Mathematics* proposed that students have the opportunity to approach the same problem in several different ways [17].

In 2010, the Common Core State Standards for Mathematics (CCSSM) were released, which were designed to raise the level of mathematics understanding and achievement and to help prepare students for college or a career upon graduation. A key tenet of CCSSM was that mathematical proficiency was aligned with the ability to create a path to a solution and to persevere to find the solution [18]. Exposing students to different perspectives and problem-solving strategies helps them progress [17]. Moreover, one skill that students who are deemed to be mathematically proficient should possess is the ability to apply different methods to validate potential solutions [18].

As the curriculum and pedagogical approaches changed, there emerged a need to broaden the modes of assessment to allow for different approaches, which creates a more complete picture and gives students the opportunity to play to their strengths. One way to elicit a more varied range of responses and, as a result, provide a more accurate and complete picture of students' understanding and thinking is to use open-ended questions [16]. Whereas closed-ended questions require the application of a specific algorithm to solve a problem that has one solution, open-ended questions afford students the opportunity to use multiple approaches to solve a problem [4, 10, 14, 29]. In some cases, these can have a range of correct solutions [10, 29]. For example, consider the question, "What is the best way to get from location A to location B?" Even if everyone working on the problem interpreted the word "best" to mean the shortest route, there could be several distinctly different correct answers. In open-ended questions, the only constraints placed on the approach employed would be a students' prior knowledge and level of mathematical sophistication. When the choice of problem-solving strategies is not constrained, more diverse approaches are used [10, 29].

Additionally, because students learn and integrate knowledge in different ways, multiple problem-solving strategies allow students to perform to the best of their abilities by affording them the opportunity to play to their strengths to demonstrate their understanding. Open-ended problems can also be helpful in revealing students' misconceptions because they lack the clues that are often found in textbook exercises that essentially tell students how to proceed [4]. Researchers have suggested that open-ended questions should be an integral part of teachers' instruction and assessment practices [3, 8, 10, 14, 17, 22, 26], and that one goal of problem-solving activities beyond finding a correct solution should be to search for multiple approaches to reach the solution [23].

2.1 **WHY INCORPORATE MULTIPLE SOLUTION STRATEGIES?**

Using open-ended questions is beneficial. These types of questions, which can be solved by different approaches, provide problem solvers the opportunity to be more creative in their responses [8, 19, 26]. Multiple approaches to a problem promotes learning [9, 10, 29] and produces more thoughtful explorations [29]. The experience of seeing different strategies and discussing various approaches makes students aware of alternative strategies and helps increase their understanding of different methods, which has the potential to make students more flexible problem solvers in the future [26]. Seeing the diverse range of approaches their peers had taken to solve problems motivates students to look for alternative approaches [4].

Another benefit is that encouraging the production of multiple solutions or offering the opportunity to compare different solutions positively impacts achievement [9, 30] and interest [24]. Middle school students who had the opportunity to compare alternative methods made greater gains with respect to their knowledge of procedures and ability to be flexible problem solvers compared to students who reflected on only one solution [21].

The experience of being exposed to a variety of problem-solving strategies can assist with future problems to be solved and can facilitate connections between the present problem and present knowledge base [26]. Fifth graders who were given the opportunity to compare different strategies used to solve a problem were more flexible problem solvers than their peers who reflected on a single strategy [30]. Additionally, those students who already knew some strategies before the exercise started had greater conceptual knowledge after completing the exercise if they compared strategies [30].

As a tool for assessment, the benefits of open-ended questions are twofold: teachers can more easily assess students' understanding, and this more thorough assessment communicates to students that the process by which problems are solved, and not just the final answer, is valued [10]. As these types of problems frequently require that the thought process be explained, gaps in students' understanding are readily revealed [16]. If strategies are overlooked, this might not be just a preference; this might indicate that a particular problem-solving strategy was not fully understood. As a result, these types of problems can be effective in revealing students' misconceptions [4].

One goal of mathematics is to develop the ability to solve real-world problems which come without instructions specifying how to proceed, as these are the types of problems students will most likely encounter in their personal and professional lives. The value of open-ended questions is that they prepare students for more than classroom tests, they prepare them for the types of problems they are likely to encounter outside of school [4, 22, 29].

2.2 OPEN-ENDED QUESTIONS: TEACHERS' RELUCTANCE AND CONCERNS

Despite overwhelming support from mathematics educators and mathematics education organizations for using open-ended questions and examining multiple strategies for a single problem [3, 8, 17, 18, 22, 23, 26, 27, 29, 32, 33], some teachers are reluctant to incorporate these types of questions into their classroom practices [26] or modes of assessment [25]. Although some teachers do not see the value in having students produce different solutions for a single mathematical problem [2], even those who do see value have reported concerns about the feasibility of incorporating these types of questions [5, 6, 9, 11, 25, 26].

One potential barrier to employing open-ended questions that allow for multiple strategies is that these types of questions are difficult to create [5] and take longer to design [25] and grade [6, 25]. Perhaps that is why, upon examining test questions used in five high schools, on average only 2% of the questions were found to be open ended [25]. Additionally, because more time is needed to respond to such questions [6], K-12 teachers may be unable or unwilling to devote the instruction time needed to work on open-ended questions, especially given that these types of problems are not well aligned with the types of questions that are on state-mandated high-stakes standardized tests.

Another pedagogical concern was that the methods used solve to open-ended questions are unpredictable [5, 11]. Due to the unanticipated range of different responses, pre-service teachers have been found to have had difficulty responding in meaningful ways to open-ended questions [11]. These issues could also arise in more experienced teachers' classes.

As another example, some teachers questioned whether it is advisable to attempt to teach students with a range of abilities multiple ways to solve a problem: and they expressed concern that students who have less mathematical ability would be confused and that students with greater ability would be bored [26]. The teachers perceived the need for students to be able to comprehend and integrate different representations as a challenge [9], which could deter some from incorporating multiple solution methods into their lessons.

Given all of these obstacles, it is not surprising that open-ended questions and multiple approaches to a single problem are rarely part of teachers' lessons or tests. However, in light of the fact that there have been few changes in the way linear programming has been taught, I was interested in making some questions open-ended to potentially increase students' interest in the subject, to strengthen their problem-solving skills, and to provide me with greater insights into students' level of understanding.

3. THE ASSIGNMENT: MAXIMIZING THE PROFITS OF A BUSINESS VENTURE

The exercise described here was given to thirty college students enrolled in an introductory Finite Mathematics course. The students were approximately between 18 and 25 years old. All of them had previously taken College Algebra or College Algebra with Trigonometry. Typically, at least 25% of the students who enrolled in Finite Mathematics had taken Pre-Calculus or Calculus prior to taking Finite Mathematics and planned to major in Computer Science or Business. Hence, in Finite Mathematics, there were students with different levels of mathematical knowledge.

Students were given a real-world linear programming problem that had two decision variables and two constraints to solve. Unlike typical textbook exercises which are often already written in the language of mathematics [13], the relationships among variables needed to be set up in this assignment. This was done because the number of approaches may be limited when problems are already written in the language of mathematics [28]. Students were told that the problem could be solved by at least four unique methods. They were given two weeks to produce a solution obtained by applying three distinctly different methods. Although students worked on the exercise primarily outside of class time, I met with students periodically to offer feedback on potential strategies. Students were told to try to find particularly well-designed or interesting solutions that could be easily understood by someone who did not have a strong background in mathematics. Here is the problem that the students were given.

> Alison, a recent college graduate with a degree in Fine Arts, wants to start a business. She is interested in selling hand-painted children's tables and toy chests with matching designs. She raised $25,000 from angel investors to start her business, but will need to reserve $5,000 to send rewards to backers. The toy chests and tables cost $10 and $8 each, respectively, to make. Based on the money Alison saved from her past job, she will be able to devote 5,000 hours to this new business to see if it is profitable or if she should look for another full-time position. It will take her 6 hours to paint a toy chest and 4 hours to paint a table. She will make a profit of $40 for each toy chest sold and $50 for each table sold. Find the number of toy chests and tables Alison should make to maximize the profit and find the maximum profit.

Unlike typical textbook exercises, the students were also asked to return to the real-world context and consider if the solution found would be the best strategy. After the assignments were submitted, the following four unique strategies were discussed in class.

3.1 The First Strategy: A Common Sense Approach

Comparing the two items, the tables cost less, take less time to produce, and have a higher profit margin; see Table 1. Therefore, to maximize the profit, Alison should make only tables. The number of tables she can make is the minimum of \$20,000/(\$8 per table) = 2,500 tables and 5,000 hours/(4 hours per table) = 1,250 tables. If Alison sells 1,250 tables, her profit will be 1,250 tables × \$50 per table = \$62,500.

INSERT TABLE 1 ABOUT HERE

3.2 The Second Strategy: A Graphical Approach

To solve the problem graphically, let x = the number of toy chests produced and y = the number of tables produced. Based on the financial limitations and restrictions on the number of hours Alison can devote to this business venture, the situation could be modeled by the following system of linear inequalities.

$$10x + 8y \leq 20{,}000,$$

$$6x + 4y \leq 5{,}000,$$

$$x \geq 0,$$

$$y \geq 0.$$

The graph of the system of linear inequalities is shown in Figure 1. The region that satisfies all of the inequalities is shaded. The corner points of the shaded region are (0, 0), (0, 1250) and (833, 0).

INSERT FIGURE 1 ABOUT HERE

To maximize the profit, all three corner points are substituted into the equation for the profit, $P = \$40x + \$50y$; see Table 2.

INSERT TABLE 2 ABOUT HERE

Alison will be able to maximize the profit from this business if she makes 1,250 tables and 0 toy chests. Under these conditions, the profit from this business venture will be $62,500.

3.3 The Third Strategy: The Simplex Method

Let x_1 and x_2 represent the number of toy chests and tables, respectively, that are to be produced. Alison's goal is to maximize the profit,

$$P = 40x_1 + 50x_2,$$

subject to the following constraints

$$10x_1 + 8x_2 \leq 20{,}000,$$

$$6x_1 + 4x_2 \leq 5{,}000,$$

$$x_1 \geq 0 \quad x_2 \geq 0, \qquad s_1 \geq 0 \quad s_2 \geq 0.$$

To set up the initial simplex tableau, the objective function is written in the form

$$P - 40x_1 - 50x_2 = 0.$$

After the slack variables s_1 and s_2 are introduced, the constraints are

$$10x_1 + 8x_2 + s_1 = 20{,}000,$$

$$6x_1 + 4x_2 + s_2 = 5{,}000.$$

The initial simplex tableau is

BV	P	x_1	x_2	s_1	s_2	RHS
S_1	0	10	8	1	0	20,000
S_2	0	6	**4**	0	1	5,000
P	1	-40	-50	0	0	0

The pivot element in the initial simplex tableau is 4. After pivoting the matrix by first applying the row operation to the second row and then to the other two rows, the new tableau is

BV	P	x_1	x_2	s_1	s_2	RHS	
S_1	0	-2	0	1	-2	10,000	$R_1 = -8r_2 + r_1$
x_2	0	3/2	1	0	1/4	1,250	$R_2 = r_2/4$
P	1	35	0	0	25/2	62,500	$R_3 = 50r_2 + r_3$

Since the smallest entry in the objective row is 0, this is the final tableau. The equation from the objective row is

$$P = -35x_1 - 25s_2/2 + 62{,}500.$$

Since $x_1 \geq 0$ and $s_2 \geq 0$, any positive value of x_1 or s_2 would lower the profit. Therefore, by choosing $x_1 = 0$ and $s_2 = 0$, the largest profit is obtained, which is \$62,500. Writing the equations from the second and third row and then replacing x_1 and s_2 with 0 gives us

$s_1 = 2x_1 + 2s_2 + 10{,}000,$	$s_1 = 10{,}000,$
$x_2 = -3/2x_1 - 1/4s_2 + 1{,}250,$	$x_2 = 1{,}250.$

The maximum profit that Alison can expect to make is $62,500 if she produces and sells 1,250 tables and 0 toy chests.

3.4 The Fourth Strategy: Successive Approximations

To investigate whether it is possible to make 50 tables and 50 toy chests, the cost and time are computed. Both the cost $10 per toy chest × 50 toy chests + $8 per table × 50 tables = $900 and the time 6 hours per toy chest × 50 toy chests + 4 hours per table × 50 tables = 500 hours are well within the range of acceptable numbers. The profit from selling 50 tables and 50 toy chests would be $40 per toy chest × 50 toy chests + $50 per table × 50 tables = $4,500. To decide whether it would be more profitable to produce more toy chests or more tables, different numbers are tested. The results are shown in Table 3.

INSERT TABLE 3 ABOUT HERE

The profit was increased by producing more tables, which also kept the cost down and used fewer hours. The solution could be found by successively increasing the number of toy chests and lowering the number of tables.

An additional open-ended question using a system of equations that could be solved by several distinctly different methods appears in appendix A.

4. CHOICE OF STRATEGY AND IMPLICATIONS

I identified four unique methods that could be used to solve a maximization problem in the last section. Although the real-world situation was readily understood, only one student was able to use the simpler, more intuitive solution method successfully, which had the advantage that it could be more easily explained to someone who did not have a strong background in mathematics. All of the students used some version of the simplex method and the graphical approach.

After being shown the first strategy, almost all of the students expressed that finding a third novel approach was a formidable task and therefore many had assumed that another unique strategy might require more advanced mathematics. No student tried using successive approximations to solve the problem, which is a distinctly different method, albeit a cumbersome one.

The students were not assigned to work in groups. The majority of them worked individually at first, and then worked collaboratively to compare strategies, validate their answers, and search for a third unique or more elegant solution. The students were not deterred by the challenge of finding another method. Several students who were studying computer science proposed writing a computer program. Prompting students to find different solution methods motivated them to explore connections to other disciplines.

Some students suggested that the graphical approach could be altered by using software or a calculator to graph the region and find the points of intersection. A few students mentioned that there would have been several ways to find the set of feasible points to be substituted into the objective equation if the lines had intersected. A few students suggested using calculators or Excel to perform the matrix row operations for the tableau. Even though most of the students recognized that these methods were not distinctly different, they were encouraged to keep proposing potential alternative approaches.

In response to the question I had posed regarding a solution being the best strategy, many students struggled to understand why Alison would consider producing any toy chests, given that the tables were more profitable. My students had rarely had the opportunity to delve deeper into open-ended questions that ask solvers to reconsider the applicability of a mathematically correct solution. After being reminded that the tables and toy chests came in matching designs, students were asked to consider if displaying both products might increase sales. There was a risk of being left with unsold inventory, given the time restrictions. Additionally, given the time constraints, it might be beneficial to produce two different pieces of furniture as there might be a higher demand for one. Although an optimal solution could not be found taking these variables into account, once in business, data on sales could be collected and assessed to better predict the demand for each item. This demonstrates that it is important to connect potential solutions to the real-world situation to confirm that the solutions make sense in this context.

As expected, the work took longer to grade. Because I could not evaluate computer programs designed to pivot the matrix, students were discouraged from further exploring and submitting this work. As seen by the range of possible solution methods, it is important that teachers who allow students to use multiple methods to solve problems have an in-depth understanding of the context, be aware of the technology that could be used, and be knowledgeable of potential unique strategies and connections to other disciplines. Given the time required to explore a broad range of strategies, it would have been difficult to complete this lesson in class. The students first attempted to produce a diverse range of strategies before discussing alternative routes, which led to meaningful connections and helped facilitate the class discussion.

5. TEACHING REFLECTION

Reflecting on the teaching experience, there were several benefits to incorporating open-ended questions and requiring distinctly different approaches be used to solve the problem. I was able to simultaneously challenge students with different strengths, backgrounds, and at different levels of ability. Students were able to play to their strengths and the members of the class that were thought to be the strongest mathematically did not always have an advantage.

Another benefit was that students' attitude improved. A few students remarked after having been shown the four approaches that the two methods, which had eluded them, were "simple" and should have been "obvious." Most had expected that more advanced mathematics was needed for some of the solution strategies. This lesson reinforced the idea that just because an approach to solve a problem did not immediately appear, a new solution strategy might emerge if they kept working on it. As a result, students became more persistent and resistant to setbacks. This was a powerful lesson.

These types of problems seemed to be seen by students as puzzles, and they become more willing to investigate potentially new paths to a solution. "Unique" solutions went beyond merely replicating techniques previously seen in class or read in their textbook. New strategies had to be invented. Students became more willing to propose methods not covered in class, knowing that there might be an intuitive or simple solution strategy. It was extremely valuable for students to realize that they could create new problem-solving strategies and that mathematics exercises did not always just involve reproducing solution methods they had previously been shown.

Assigning open-ended questions and requesting multiple approaches provided me with valuable feedback. Since this problem required multiple approaches, I assumed that all of the approaches previously covered in class, which were familiar, would be used. Therefore, these types of problems readily revealed gaps in students' understanding, as strategies that were not used were usually not understood.

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Item	Cost to Produce ($)	Time to Produce (hours)	Profit ($)
Toy chests	10	6	40
Tables	8	4	50

Table 1: Unit cost, time and profit

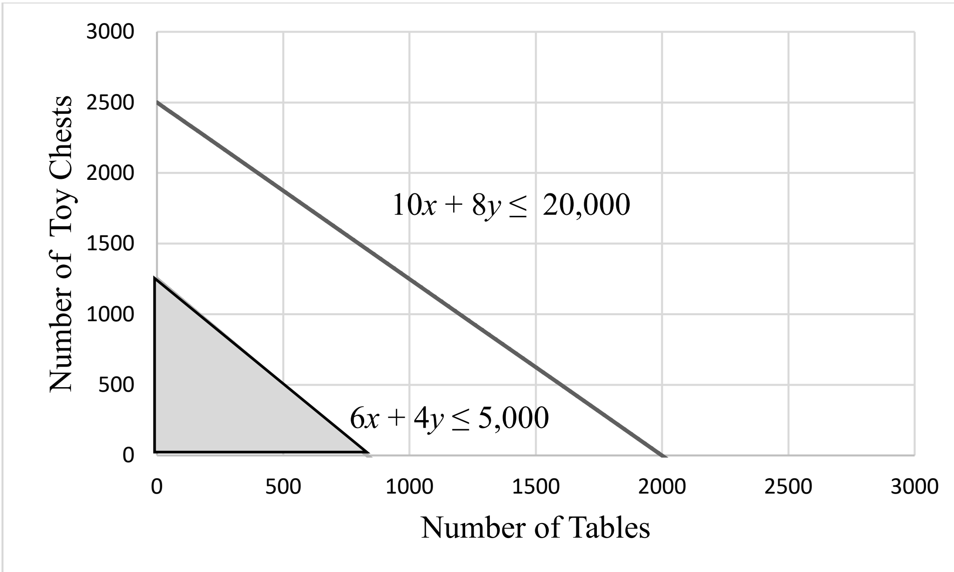

Figure 1**:** Graph of feasible points for toy chests and tables.

Corner point (x, y)	Value of objective function P = \$40x + \$50y
(0, 0)	P = \$40 × 0 + \$50 × 0 = \$0
(0, 1250)	P = \$40 × 0 + \$50 × 1,250 = \$62,500
(833, 0)	P = \$40 × 833 + \$50 × 0 = \$33,320

Table 2: Corner points and the profit

Number of Toy Chests	Number of Tables	Cost to Produce ($)	Time to Produce hours	Profit ($)
100	50	1,400	800	6,500
200	50	2,400	1,400	10,500
50	100	1,300	700	7000
50	200	2,100	1,100	12,000

Table 3: Cost, production time, and profit from the business venture

Appendix A

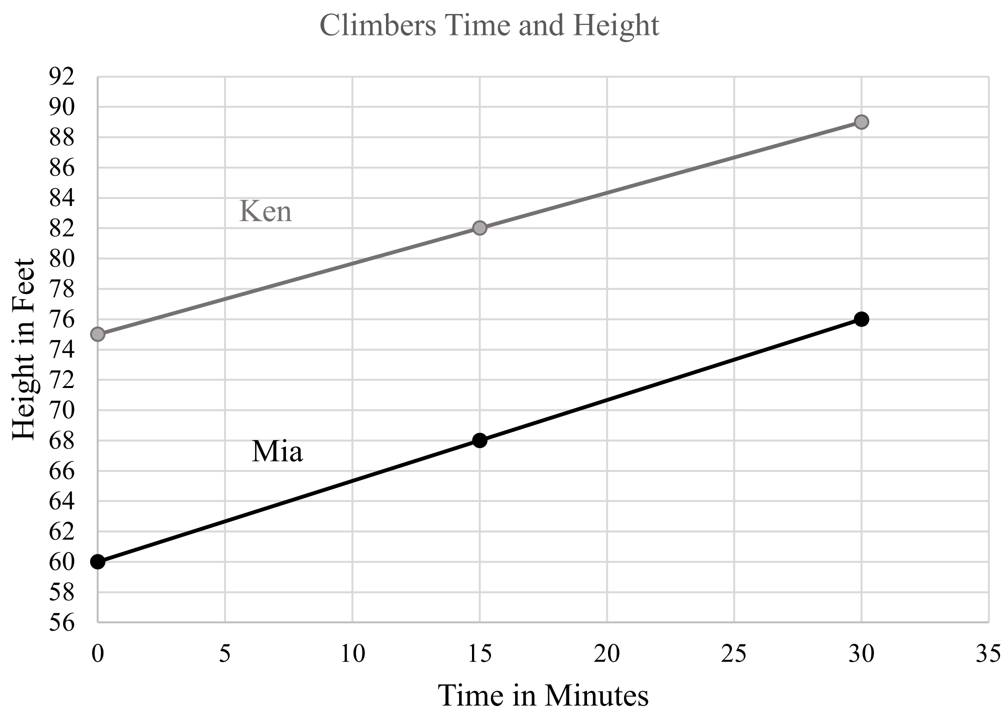

Figure 2: Progress climbing up a mountain.

Ken and Mia, starting from different points, were climbing up a mountain along the same path, but at different constant rates. Their progress is shown in Figure 2. Determine whether or not they will meet. If they will cross paths, find the time and height that this will occur. This problem can be solved by at least four distinctly different methods. Find at least one solution that could be explained to someone who does not have a strong background in mathematics. Here are potential solution methods without having found the equations of the two lines.

- By hand, count the distance between the two climbers at selected points or use the distance formula to determine the distance between the two climbers at a series of points. As time passes, the distance narrows and, therefore, given the constant rate of climbing, would approach zero.

- Count the height each climber gained at set increments of time. After 15 minutes, Mia climbed 8 feet, whereas Ken climbed 7 feet; and after 30 minutes, Mia climbed 16 feet, whereas Ken climbed 14 feet. Therefore, as time passes, Mia would get closer to Ken, and they should meet eventually.

Here are potential solution methods after having found the equations of the two lines.

- Solve the system of equations by using substitution or addition.
- Find the slopes of the two lines, which indicate that the lines would intersect because the slopes are different. By hand or with technology, graph the lines and find the point of intersection.
- Write the augmented matrix for the system of linear equations and perform row operations on the matrix to find the solution.