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Models for Decision-Making - Second Edition

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Second Edition

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About This Document

Decision-Making often refers to a multi-stage process that starts with some form of introspection or reflection about a situation in which a person or group of people find themselves. For example, the owner of a business may wonder whether profits are as high as they could be, a family may question whether it is time to move to another neighborhood or to another state, a high school student about to graduate may think about whether to go to college, or a government may want to address complaints about the ineffectiveness or inefficiency of some municipal service. These ruminations usually lead to series of questions that need to be answered, or to a set of data that needs to be collected and analyzed, or to some calculations that need to be performed before someone can be in a position to make informed decisions and take appropriate actions.

In this document, we focus on the aspects of decision-making that are *quantitative* in nature. These are questions that take on forms like, "How much?", or "How often?", or "For how long?", i.e., that focus on the values or quantities in a situation, rather than the more emotional or qualitative aspects. Part I provides some simple examples of Quantitative Models, which represent a variety of tools and techniques used to identify, organize, and analyze the quantitative information found in a decision-making situation. In this section, we focus on the use of algebraic equations to model the relationships between quantities. A common template, which we call the "Cost-Volume-Profit" model, seeks to measure the potential profitability of a product as a function of the customer demand for it. In Part II, we discuss models to represent uncertainty in a situation by quantifying the relative likelihood of potential random events, e.g., using probability distributions. We show how expected values, utilities, and measures for risk help us make decisions in the presence of such uncertainty. In Part III, we combine the approaches from the prior sections to help a decision-maker evaluate and compare alternatives using a "Payoff Table" model. Thus they have the means to justify their selection of one of these alternatives. Part IV describes a generalization called a "Decision Tree" model, which is used to represent situations involving a sequence of decisions over time. This also covers situations in which some decision cannot be made until the outcome of some random event is realized. Concepts are illustrated with a large set of examples that can be presented during classroom instruction and can be practiced by the students, either individually or in groups, through homework or lab exercises.

Part I: Introduction to Quantitative Modeling

Atlas Bookbinders

It costs Atlas Bookbinders about $80,000 to prepare for production the new edition of their popular Business Math textbook. This includes the payment to the authors and editors, the costs of setting the presses, adding the book to their catalog, and designing the book's cover. These are one-time "fixed" costs that are incurred, even if no copy of the textbook is printed. Materials, marketing, pressing and binding cost a total of about $44.00 for each book. These are called "variable" costs because the total depends on the number of units that will be printed. Any number of books can be produced at that price. Copies of the book are sold to campus bookstores for $60 each. The total demand for this book is not known exactly, but some reasonable estimates are available.

Question: Would it be a good idea for Atlas to spend the $80,000 to initiate production of the textbook? If so, how many books should Atlas make in its first production run?

To keep its operations running smoothly and profitably, an organization, like Atlas Bookbinders, needs to have managers who make intelligent decisions about how to utilize resources, design facilities, schedule activities, produce items, deploy personnel, and invest capital. It is the role of these managers to establish the procedures for the day-to-day operations of the business, to determine how resources like time, money, materials, and people, are allocated to the different business activities, and to make decisions that address new situations as they arise. We will henceforth refer to these people using the more general term "decision-makers" because at different times anybody may have to take on such a role. The decision-making activities are performed so that the business can achieve objectives like maximizing profits, minimizing financial risk, minimizing its carbon footprint, having effective resource utilization, and/or maintaining a lean manufacturing process that keeps idle inventories levels low.

As illustrated with Atlas Bookbinders, the decision-maker encounters a situation that gives rise to a question that needs to be answered or a problem that needs to be solved. The decision-maker in this situation, most likely a publisher, would weigh the consequences associated with the available alternatives: whether to initiate production of the textbook; if so, how many to produce in the first pressing; whether a second pressing might be necessary. The objective in this case would be to make enough in total revenue to offset the fixed and variable costs of production and hence make a profit. Common sense dictates that if the potential demand for the book is high, then the product line will be profitable; otherwise, it is better not to initiate production of the book.

Examples of similar decision-making scenarios abound. A retailer's inventory manager must determine which items to carry, how many of each to keep on hand, and how often to replenish the supplies. The objective of the decision-maker in this case is to minimize the total costs due to storage, unsold items, and shipping. An investment analyst must determine if a mutual fund or portfolio of stocks is likely to generate returns, and if so, how much money to invest in it. The objective here is to maximize the total return on investments over some time horizon. A taxi dispatch system like Uber or Lyft must determine how to assign the available drivers to calls as they arise. The objective is to maximize the total fares and to fairly distribute the workload among the drivers. When a company builds a new factory, the operations manager must determine how the facility is to be laid out. The objective is to maximize the production capacity while minimizing the construction cost. Drawing from experience, expertise, intuition, and computerized decision-support tools, decision-makers hope to find the best course of action and then see to it that their decisions are implemented.

It is necessary for decision-makers to continually examine their operations to make sure that: a) the right activities are being performed, (i.e., they are *effective*); and b) the activities are being performed without wasting time or money, (i.e., they are *efficient*). For example, the manager of Atlas Bookbinders must be assured that the production process is well designed, and that the resources are being allocated to the most profitable projects. What makes the job of a decision-maker particularly difficult is that the situations they encounter usually have both *quantitative* and *qualitative* components. The *quantitative* components take the form of questions that include phrases like: "how many?" or "how often?" or “at what distance?” or "at what price?” or "for how long?" Such questions can usually be answered using analytical methods like those provided in Mathematics, Engineering, Statistics, Management Science, and Computer Science. Like many situations described in a Mathematics class, the situation described for Atlas Books has been presented to emphasize the quantitative aspects.

The *qualitative* components of a situation, on the other hand, often defy direct measurement. They include issues like public relations, employee morale, image, politics, corporate culture, government regulation, and the perceptions of customers. Such issues may not be easily represented with numbers. They are usually addressed using intuition, relevant past experiences, political savvy, and common sense. Fields of study like Political Science, Psychology, Sociology, and Business Administration help to prepare a decision-maker to determine policies for predominantly qualitative situations. It often turns out that such qualitative factors often compel a decision-maker to select a course of action that is not necessarily an optimal one in terms of costs or profits. For example, the publisher at Atlas Books might be more likely to print the Business Math textbook if she values her relationship with the authors, even if the sales might be lower than she would hope for.

Illustration: *Suppose Barbara and Richard Starr and their family have just relocated to a new state and must decide between renting an apartment in the city or buying a house in the suburbs. What are the quantitative and qualitative factors that affect their decision? Give some reasons why the Starr family would decide to buy the house, even if it is not the cheaper alternative.*

You can probably see that the quantitative components in the Starr’s situation are not expected to be the sole factors involved in making the decision. If you have ever decided to buy a house, you will know that saving money is not the reason! Qualitative issues like the desire for personal space, the availability of parking, and the reputation of the school district play as much a part in the decision as do measurable factors like rent, inflation, interest rates, taxes, depreciation, and the cost of houses. Quantitative analysis serves mainly to identify the more economical courses of action and to measure the relative costs and benefits associated with making a choice.

Basically, the decision-making process requires organizing and manipulating the relevant data so that the most reasonable alternatives can be evaluated and compared using quantitative methods. Important qualitative factors are then considered in conjunction with these evaluations before a final decision is made.

A key component in the decision-making process is a *quantitative model*, which is an abstract representation of the quantitative components of a decision. Quantitative models identify the key quantities and their inter-relationships so that they can be measured and analyzed. Building a model for a situation can include such simple tasks as drawing a picture, or jotting down numbers on a sheet of paper, or making charts and graphs. In an *algebraic model*, a set of simple functions or expressions might best represent the situation at hand. A more sophisticated model may include spreadsheets, computer graphics, probability distributions, or computer simulation programs to represent the situation. A good model can make the job of the decision-maker easier because it demonstrates, in an organized fashion, the important quantitative factors that affect a decision.

Quantitative models focus on measurable entities like amounts of money, lengths of time, costs, prices, distances, resource levels, customer demands, production rates and capacities. They enable efficient and effective decision making by organizing the relevant information in a situation, measuring the consequences of the available decision alternatives, and establishing a documented rationale to evaluate or justify the decisions that are made.

The task of developing a good model to represent a situation may be quite difficult and could require a fair amount of mathematical sophistication and modeling experience. The decision-maker might enlist the services of some experienced expert model builder to perform this task. In other situations, the model builder may be aware of an appropriate existing *template* model, which has already been developed and studied and, with some modifications, can be used to support the decision-making process for the situation at hand.

As we shall see, for many commonly occurring decision-making situations there already are appropriate template models. It is not unusual to find that different organizations in different industries are faced with the same basic quantitative problems, e.g., maximize profit out of limited resources, ship goods between locations as cheaply and reliably as possible, or find the best location for a new warehouse given the locations of the terminals. Hence, a pair of seemingly different situations might use the same template model! The only differences would be in the *parameters*, i.e., size of the model and the specific data values involved.

Algebraic Models

In an algebraic model, some relationship between quantities is represented using a mathematical expression, like an equation or inequality. Expressions can contain one or more *variables*, which are values that have yet to be determined or specified, so must be represented as a letter, symbol, or word. For example, if the manager of Atlas Bookbinder has yet to decide on the number of units of the Business Math textbook to produce, she could represent this value using the variable x. Recall that it costs Atlas Bookbinders $80,000 in fixed costs to prepare the textbook and the variable cost is $44.00 per book. Even though x is not yet determined, we know that if the book is printed, the total cost of production is $C(x) = 80{,}000 + 44 * x$. We would say that “the production cost is represented as a function of the number of units produced”. Each copy of the book generates revenue of $60, so the total revenue as a function of the number of units would be $R(x) = 60 * x$.

Illustration: *A car rental company offers customers two rate plans. With plan A, the customer pays $25 per day plus $0.40 per mile driven. With plan B the customer pays $50 per day plus $0.10 per mile. To allow for comparison, represent these costs using an algebraic model.*

We create variables for the number of days (d), and the number of miles (m), which have yet to be specified. The costs for each plan can be represented as follows:

Cost of Plan A: $CA(d, m) = 25 * d + 0.4 * m$

Cost of Plan B: $CB(d, m) = 50 * d + 0.1 * m$

Suppose you need a car for 4 days and expect to drive a total of 750 miles. Which plan would you use?

Having functions makes the costs easier to determine:

$CA(4, 750) = 25 * 4 + 0.4 * 750 = \$400$

$CB(4, 750) = 50 * 4 + 0.1 * 750 = \$475$

So, plan A would be cheaper in this case.

Having an algebraic model for a situation allows the decision-maker to use the techniques of algebra to answer a variety of questions regarding a situation.

Suppose you need a car for 2 days. What is the limit on the number of miles you can drive using Plan A before it becomes the more expensive alternative?

In this case, we know the value of d is 2. Using algebra, we can determine the value of m where both costs are equal. This value of m is called a breakpoint.

Cost of Plan A: $CA(2, m) = 50 + 0.4*m$

Cost of Plan B: $CB(2, m) = 100 + 0.1*m$

The two costs are the same when m satisfies: $50 + 0.4*m = 100 + 0.1*m$

This happens at the breakpoint, where $0.3*m = 50$ or $m = 166.6667$ miles.

So, if you plan to drive up to 166 miles, then Plan A would be cheaper. If you plan to drive more than 167 miles, then Plan A would be more expensive.

Constraints in a model are rules that are placed into an algebraic model to make it more realistic. Constraints prevent the model from suggesting variable values that represent impossible or unrealistic decisions. For example, the value of x for Atlas must satisfy the inequality, $x \geq 0$ because it is impossible to produce a negative number of books. Similarly, variables representing time spans, distances values, masses and volumes can never be negative. Constraints can also be used to represent limitations on the amount of resources, (like time, capital, personnel or raw materials), available.

The Cost-Volume-Profit (CVP) Model

The algebraic model for Atlas Bookbinders would consist of functions that represent the total cost of production and total revenues obtained from sales. With data regarding the expected demand for the textbook, it would be possible to quantify the profit and determine whether it is worthwhile to go into production. Together they comprise the template that we call the *Cost-Volume-Profit Model.*

We use the variable x to represent the number of units of the textbook that Atlas will sell, should they decide to print the book. We call this the *demand* for the book. This value may not be known precisely, but there may be some information available based on experience with similar products. To make the model easier to use, we assume that the level of production can easily be adjusted to match the demand, so that x, the value of the demand, is the only variable needed in the model. The CVP model will help us decide whether to print the textbook, based on the value of this variable.

In the CVP model, the parameters are the values given for the revenue and costs associated with the product. These are the coefficients in the mathematical expressions. (They are not variables because we already know these values and cannot make any decisions that would change them). The letter F is a constant that represents the fixed cost in dollars, which is 80,000 for Atlas. The letter v represents the unit variable cost in dollars, which is $44 in this situation. The letter r represents the sales revenue in dollars per unit sold, which is $60 for Atlas. Letters given to these quantities also allow for flexibility in the analysis. For example, if one parameter value changes, then most of the work performed can still be utilized, without having to start from the beginning. The difference between Atlas and other companies having the same basic decision problem is that the specific values of F, v and r will differ. The total gain or loss from the operations can be represented as a function of the variable x. The total profit is equal to the revenue from sales less the costs incurred. This relationship can be represented algebraically as follows:

Revenue from selling x units = $R(x) = r * x$
Cost of producing x units = $C(x) = F + v * x$

Profit is defined as the difference between revenue and cost, so:
Profit from selling x units = $P(x) = R(x) - C(x) = (r - v)\, x - F$

The *break-even point* is the number of units for which total costs equals total revenue; i.e., the value of x satisfying $P(x) = 0$. Clearly, one should go into production only if one expects to make a positive profit. So, if the company is likely to sell more than the break-even quantity, represented by x_{BEP}, then production would be profitable.

In terms of the parameters from the model, we have the following formula.
Break-even point = $x_{BEP} = F / (r - v)$

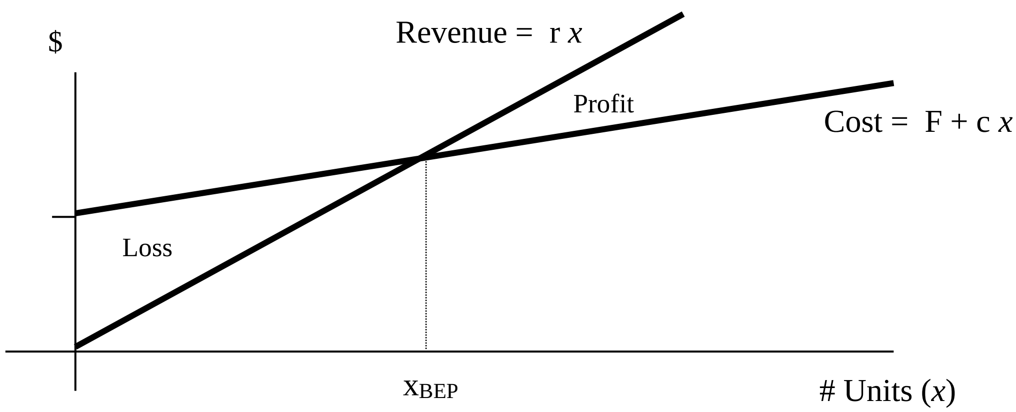
So, the number of textbooks that need to be sold in order for Atlas Bookbinding to break even is:

$$x_{BEP} = F / (r - v) = 80{,}000 / (60\text{-}44) = 5{,}000 \text{ units.}$$

The profit function also tells us that if we want to know the value of *x* that would result in some desired profit level P, the formula is:

$$\text{Number of units to sell to obtain P dollars of profit} = x_P = (F + P) / (r - v)$$

Some decision-makers may not decide to go into production unless they are likely to make at least some positive profit level. This formula is helpful to them. The following graph provides a picture-based model for the situation. It provides a clear illustration of how the profit or loss depends on the quantity that will be sold.



The algebraic model provides the decision-maker with clear information about the potential profit (or loss) as a function of the demand. If demand is likely to be high, (i.e., larger than the break-even point), then the product line would be profitable.

When the publisher at Atlas is confronted with the decision of whether to produce the textbook, she may decide to use some *probabilistic* decision criterion. For example, she may market the item only if the probability of breaking even is greater than some desired percentage. In order to do this, probability information regarding the demand for the product is required.

Such information may take on the following form:

Event	**Probability**
Demand Exceeds 1000 units	0.95
Demand Exceeds 2500 units	0.75
Demand Exceeds 5000 units	0.60
Demand Exceeds 10,000 units	0.10

This shows that the company is very likely to sell over 2500 books, but not very likely to sell more than

10,000. With the parameter values and the quantitative model available, it is possible to perform experiments and analyses that help the decision-maker improve his or her understanding of the situation and make an informed decision about whether to go into production. For Atlas the profit function is

$$P(x) = (60 - 44) * x - 80{,}000 = 16x - 80{,}000$$

This gives the profit or loss that would result from total sales of *x* units. The break-even point is

$$x_{BEP} = 80{,}000 / (60 - 44) = 5{,}000 \text{ units.}$$

This implies that if the sales are expected to exceed 5,000, which is somewhat likely since the event has a probability of 60%, then it would be profitable to produce the textbook.

Notice that for different values of F, r and v, the location of the break-even point would change. For example, if the per-unit revenue from the album were to increase from $60 to $65, then the break-even point would move down to 80,000 / (65 – 44) = 3809.52 or, more realistically, 3810 units. However, demand would likely be smaller at that new price. The publisher could use the model to help determine a price that would provide a reasonable level of overall profitability.

If the fixed costs in the process were to increase from $80,000 to $85,000, the break-even point would have to move up to 85,000 / (60 – 44) = 5312.5 or 5513 units. Notice that when we measure the impact of a change to the data, we revert to the original values of the other parameters. So, the model also shows the relationship between production costs and profitability.

With the experimentation on the model completed, the decision-maker is poised to make intelligent decisions. For example, after having determined the relative likelihood of achieving various profit levels, the publisher at Atlas Bookbinding Company can finally determine whether the textbook should be printed. Suppose she has decided that she will produce the album only if the chances of at least breaking even are at least 75%. Then the book would not be produced since probability that demand exceeds the break-even point of 5000 is only 60%. This particular decision criterion is somewhat subjective (qualitative); a different decision-maker might be less conservative, so would be willing to print the book if the chance of breaking even is some lower value, like 50%. In that case he or she would decide to print the textbook.

The Cost-Volume-Profit model and the analysis performed for Atlas Bookbinding can be reused in a variety of similar situations in which one is trying to determine whether the revenue generated by an activity is enough to offset the costs. It could also be used in situations where a decision-maker must select a production facility, where each option has a different fixed and variable cost structure.

Illustration: *Suppose that the selling price of a small lamp is $7.50, its variable cost is $4.50, and the fixed cost for production is $15,000. Suppose we are given the following information regarding demand:*

Event	Probability
Demand Exceeds 2500 units	0.90
Demand Exceeds 5000 units	0.80
Demand Exceeds 7500 units	0.60
Demand Exceeds 10,000 units	0.20

What is the break-even point? Is it likely make a profit?

The break-even point based on these values is $x_{BEP} = \frac{15000}{7.5 - 4.5}$ = 5000 units. The probability of at least breaking even is 80%.

How likely is it to achieve a profit of $10,000 or more?

The required sales volume for a profit of $10,000 is $x = \frac{15,000+10,000}{7.5-4.5}$ = 8333 units. The likelihood of achieving this number of sales is somewhere between 20% and 60%, (maybe about 50%).

Suppose you are the manager. Would you decide to produce lamps? Why or why not?

Since it is somewhat likely to obtain a reasonable profit from the investment, many managers would see benefit in producing the lamps. However, reasonable people may disagree, so there is no wrong answer.

Illustration: *Alpha Cosmetics company has developed a new perfume that they want to market. If Alpha goes into small-scale production, its annual fixed costs will be $400,000 and it can produce for a variable cost of $5 per unit. If it goes into large-scale production, its annual fixed costs are $1,050,000 and it can produce for a variable cost of $3 per unit. The selling price will be $10 per unit regardless of the scale of production.*

Determine the break-even points for small-scale and large-scale production.

The break-even point for small-scale production is $\frac{400{,}000}{10-5}$ = 80,000 units.

The break-even point for large-scale production is $\frac{1{,}050{,}000}{10-3}$ = 150,000 units.

Suppose the demand is expected to be 150,000 units. What profit can be expected?

To determine the profits, we can use the formula P(x) = (r - v) * x – F. So the profit using small-scale production is given by P(150,000) = (10 – 5)*150,000 – 400,000 = 350,000. For large-scale production, 150,000 units is the break-even point so the profit would be $0.

Suppose the expected demand is for 500,000 units. Which production method would be more profitable?

The profit for small scale is given by P(500,000) = (10 – 5) * 500,000 – 400,000 = 2,100,000. The profit for large scale is given by P(500,000) = (10 – 3) * 500,000 – 1,050,000 = 2,450,000. So, in this case, large scale is more profitable.

Find the number of units for which one would be indifferent between using small-scale and large-scale production, i.e., the number of units for which the costs are equal.

One would be indifferent if the total costs are the same, so this is the point where

$$400{,}000 + 5x = 1{,}050{,}000 + 3x.$$

This happens when x = $\frac{1{,}050{,}000-400{,}000}{5-3}$ = 325,000 units. This implies that demand must exceed 325,000 units for large-scale production to be more profitable. If the demand is between 80,000 and 325,000, then small-scale production is most profitable. If the demand is less than 80,000, then the perfume is not profitable.

Exercises

1. You need a car for 5 days. A car rental company offers customers two rate plans. With Plan A, the customer pays $25 per day plus $0.15 per mile driven. With Plan B the customer pays $150 for the week, even if the car is returned early, with no per-mile charge. Which payment plan would you select if you plan to drive a total of 100 miles?

2. The Reclamation Machining Company (RMC) makes nuts and bolts from scrap material supplied from two firms, Company A and Company B. For each 100 pounds of material provided by Company A, RMC makes 10 cases of nuts and 4 cases of bolts. Each 100 pounds from Company B can be converted to 6 cases of nuts and 8 cases of bolts.

   Let variable x represent the number of hundreds of pounds of scrap from Company A and variable y represent the number of hundreds of pounds of scrap from Company B.

   a) Write the algebraic expression for the function N(x, y) which represents the total number of nuts produced by RMC.
   b) Write the algebraic expression for the function B(x, y) which represents the total number of bolts produced by RMC.
   c) How many nuts and bolts would be produced if Company A provides 1200 pounds and Company B provides 1150 pounds?

3. A cosmetic company has developed a new after-shave lotion. If the company goes into small-scale production, its fixed costs will be $200,000 and it can produce the product for a variable cost of $3 per unit. If the company goes into large-scale production, its fixed costs will be $400,000 and it can produce the product for a variable cost of $1.25 per unit. Assume that the selling price is $7 per unit.

   a) Determine the break-even points for small scale and large-scale production.
   b) If the demand is expected to be 500,000 units, which production process is more profitable: small-scale or large-scale? Determine profits for both cases.
   c) Find the number of units for which one would be indifferent between small-scale and large-scale production.

4. RMC Training is trying to decide if it should sell a new type of training product. Fixed costs associated to the production of the product are estimated to be $300,000. The product will sell for $30 with variable cost of $15.

   a) What is the breakeven point?
   b) What is the profit or loss if 60,000 units are sold?
   c) How many units must be sold to earn a profit of $150,000?

Part II: Modeling Uncertainty and Measuring Risk

Acme Subscription Services

Acme Streams allows you to rent any movie from its exceptionally wide collection for the low, low price of $3.99 per movie! Acme also offers a subscription service that allows you to rent as many movies as you want for the unbelievably low price of $15.99 per month! All you need to do to receive this soon-to-be-gone offer is commit to a two-year contract.

Question: Based on your expected movie-watching activity, is it worth subscribing to the service?

At first glance, the problem above seems relatively easy to solve: if you are certain that you will watch more than four movies in a month then the subscription is a better deal. However, there are some months when you get so busy at work or school that you can't sit for any movies, let alone four! In other months, you may find the time to watch a movie practically every day for a week straight. The terms of the subscription say that you can't opt in only during those months. It is this "uncertainty" about the future that makes this a difficult decision to make.

In general, uncertainty in a decision-making situation occurs because some essential information, e.g., about the values of some problem parameters, is missing. For example, you may want to visit a friend for dinner in her new apartment across town but find it difficult to determine the best time to leave because you don't know how long it typically takes to get there. In other situations, some relevant data value might fluctuate so widely over time that it is hard to make the decision when it is needed. For example, suppose you want to buy stock in a company like Amazon today. You won't know precisely how many shares you will get from the money you budgeted because the price of the shares keeps changing. Finally, since the consequences of many decisions we make cannot be calculated until after some event in the future, we cannot know, in advance, which decision is the "best". If you decide to place a wager on the next playoff game, you have no way of knowing which team will make you a winner. We have to make some decision in these situations, despite the uncertainty. We just need to work with whatever information is currently available.

The difficulty in deciding about whether to subscribe to Acme's service comes from not knowing precisely how many movies you will have time to watch in the next 24 months unless you have solid plans about your future movie-viewing schedule. If you don't, you can look at your rental bills from the last couple of years to get a better handle on your viewing habits. With this, you can get a sense of how likely you are to benefit from the subscription.

Illustration: *Suppose that you found the following data regarding your past movie rentals:*

January	5	July	6	January	7	July	5
February	6	August	5	February	7	August	5
March	3	September	3	March	2	September	4
April	1	October	2	April	0	October	3
May	1	November	1	May	1	November	1
June	5	December	3	June	4	December	5

Notice that in 13 out of 24 months, (about 54% of the months), you viewed less than four movies. The total number of movies viewed during the last two years is 85. The average number of movies per month is 85 / 24 = 3.54.

If we had purchased these rentals from Acme, we would have spent about 3.54 * $3.99 = $14.12 per month on movies. This is less than Acme's monthly subscription fee of $15.99.

Do you subscribe to the monthly service?

If we assume that the past is reflective of the future, then the cheaper alternative is to not subscribe to the service. We say that this decision is based on the "empirical" evidence.

Your final decision, however, may differ from the analytical solution found above, especially since the difference between the values is somewhat small. You may decide to subscribe to the service, based on some other factors that were not considered above. For example, some people would watch more movies if they were essentially free, so they would pay (the extra $1.77 per month) for the extra movies allowed by the subscription. Some people would subscribe because they like the stability of paying the same total amount every month. Some people may decide to subscribe because they worry that if they don't make the commitment now, the price of the rentals or the subscription might go up soon.

Probability theory provides some of the best tools to exploit the quantitative data available in a situation to help us make decisions despite the sources of uncertainty. When we use these tools, in conjunction with our own common sense about what information is relevant and what data can be reasonably obtained, we can make decisions in which we can be confident and we can justify, even when the future provides its set of unexpected outcomes.

Probability Distributions

We use the term "random experiment" to describe a situation that we can observe, for which there are some set of potential results or "outcomes", each having some likelihood of occurring. For example, before a baseball game is played, we don't know the results, but the outcomes are: "Home-team wins", "Visiting-team wins", "Game is postponed". We use the term "result" to describe which outcome occurred at the end of the experiment. If you flip a coin, the outcomes are: "Heads", "Tails", "Side", (which is very unlikely, so might be excluded from consideration). If you arrive at a bus stop, the outcome - your total waiting time - may range from 1 second to two hours, (after which you will likely decide to walk). Next month, you may watch a total of 0, or 1, or 2, … or 50 Acme movies.

Once we have a complete list of the outcomes from a random experiment, we can focus attention on which of the outcomes are more likely and which are less likely. We use a "probability" value, which is a proportion used to measure and compare the relative likelihoods of the outcomes. To do this we say that "the sum of the probabilities of all possible outcomes is 1.0 or 100%". Then we can *distribute* the 100% among the potential outcomes.

We let *Prob(o)* represent the probability that outcome o will be the result of the experiment. If E represents some "event", which is a set of outcomes satisfying some description, then *Prob(E)* represent the total probability of the event, i.e., that the result is among the outcomes in E.

For example, if we were to roll a die, the outcomes are: 1, or 2, or 3, or 4, or 5, or 6. If the die is "fair" then each outcome is equally likely, so Prob(1) = 1/6, and Prob(2) = 1/6. and …, and Prob(6) = 1/6. Suppose we define E to be the event that the roll is an odd number. Then Prob(E) = Prob(1 or 3 or 5) = 3/6 or 50%.

There are a couple of different methods that decision-makers use to distribute the probabilities among the outcomes, but whichever method is used must adhere to the following principles:

- The probability of an outcome *Prob(o)* must be non-negative.
- The sum of the probabilities of all possible outcomes must equal to 1 or 100%.
- Impossible outcomes must have probability 0.
- If Event A is deemed more likely than Event B, then the probabilities assigned to the outcomes must be such that Prob(A) $\geq$ Prob (B).

Many courses in Probability and Statistics focus on "classical" approaches to assigning probabilities to outcome. They look at experiments involving coin flips, dice rolls, random selection of balls from an urn or cards from a standard deck, etc., where each outcome is equally likely. They often ask the student to calculate the probabilities associated with complex events, sometime involving multiple inter-dependent classical random experiments. These exercises are extremely important to help build a good sense of relative likelihood and to develop rational decision-making skills.

"Empirical" probabilities are assigned to outcomes based on data, e.g., from past experiences. This method is used in situations where certain outcomes are more likely than others for reasons that cannot be explained by pure randomness. For example, if you were to go to the track and observe that the next race has 9 horses running, you shouldn't assume that each horse has a 1/9 probability of winning. Some of the horses are more comfortable with the current weather conditions. Others have shown to be particularly fast in past races, while others have trainers or owners with a record of consistent wins. Thus, each horse is given its own "odds" of winning that can be translated to a distribution of probabilities.

Illustration: *Suppose that you ask 30 friends how many siblings they have, and their answers are as follows:*

0	2	3	4	1	1
4	1	1	1	0	3
2	0	3	2	1	2
1	2	4	1	2	2
2	1	0	1	6	1

Which of these outcomes appears to be most common?

It appears that the values 1 and 2 are the most common. This is not surprising because, from our experience, we know that smaller families are more common than larger families.

Consider the following experiment: You choose one of these 30 friends at random. What is the probability that the person you select will have 0 siblings, (i.e., that the result is 0)?

Four out of the 30 friends responded with a 0, so the probability is 4/30.

Give the probability distribution for the outcomes of this experiment.

Outcome	**Probability**
0	4/30
1	11/30
2	8/30
3	3/30
4	3/30
5	0/30
6	1/30

In decision-making situations, each outcome may be associated with some value called a "payout", which measures the benefit (or "cost", which is a negative benefit), that would be realized should that outcome be the result. A payout can be some monetary value, like the gain or loss from some wager or investment, or it could be some "utility" or number of points that we assign to good outcomes (a positive number) or bad outcome (a negative number). This information should be available with the probability information so that the potential impacts of the experiment can be articulated and measured.

Illustration: *Suppose you planned a picnic with your friends for tomorrow but found out that there is a 30% probability that it will rain and a 25% chance that it will be cloudy. Give the probability distribution with utility values to describe the situation.*

Outcome	**Probability**	**Utility**
Sunny	45%	+100
Rainy	30%	-125
Cloudy	25%	+75

The utility associated with a sunny day is +100 points because we will have a good time. If it rains, we will have to cancel the picnic and waste the food that we bought, so we give that a negative value. If it is cloudy, we will still have a good time, but not as good as on a sunny day.

The choice of values for the utility points is up to the individual decision-maker, though there are some rules to keep the process "rational". For example, more desirable outcomes should be assigned higher utility values. Different people are likely to assign different point values to the outcomes, which is why, in general, different people in the same situation might make different decisions.

Illustration: *Suppose you play a gambling game where you roll a die. If it lands on 3 or 6 you win $3. If it lands on 1 you lose $5. Give the probability distribution with payout values to describe the situation.*

Outcome	**Probability**	**Payout**
3 or 6	2/6	+$3
1	1/6	-$5
2, 4, or 5	3/6	0

Expected Value

When the payouts or utilities are placed in the same table with the associated probabilities it is easier to evaluate a situation. In both illustrations above, there were some positive outcomes and some negative outcomes within the same situation. It would be our job to make an overall evaluation. Depending on the decision-maker's outlook, he or she may focus only on the negative outcomes or only on the positive ones. For some people, the prospect of losing $5 in the dice game above may keep them from playing the game, even though it is more likely that they will win money. Thus, a more rational approach might consider *all* possible outcomes and their relative likelihoods.

One of the most common ways to evaluate a random experiment with payouts or utility values is to calculate a "weighted average value, using probabilities as the weights" or the "Expected Value" (EV). Thus, more likely outcomes are given more weight in the overall evaluation. For a valid probability distribution table with the form:

Outcome	Probability	Value
O1	P1	V1
O2	P2	V2
O3	P3	V3
ON	PN	VN

The Expected Value (EV) is equal to (P1*V1) + (P2*V2) + (P3*V3) + … + (PN*VN).

Illustration: *Suppose you have the following probability distribution with payout values. What is the overall value of the game?*

Outcome	Probability	Payout
3 or 6	2/6	+$3
1	1/6	-$5
2, 4, or 5	3/6	0

The expected payout is (2/6 * 3) + (1/6 * $^{-}$5) + (3/6 * 0) = 1/6 = $0.17. This means that, overall, the positive outcomes outweigh the negative. When you play this game, you achieve a benefit that is worth 17 cents. It would be profitable to play this game as many times as you can, even if you might lose some money at first.

A “fair game” would be one where the expected value is $0. Neither the player nor the opponent would have a financial advantage. You would expect this when playing for fun with your friends. As you can see, the expected benefit (or cost) can help you with making decisions about whether to pursue some course of action. If the overall value of the dice game were negative, you might not want to play unless you find it fun to gamble away your money! (Note that every game in a Las Vegas casino has a negative expected value, but people enjoy going there to gamble anyway).

In the Cost-Volume-Profit model described in a previous section, we observed that having probability information regarding the demand for the product, which is uncertain, will provide the decision-maker with a better idea about how much profit (or loss) is likely, and whether to go into production. The expected profit (or loss) will go far in helping make this decision.

Illustration: *It costs Atlas Bookbinders about $80,000 in fixed costs to prepare for production the new edition of their popular Business Math textbook. Variable costs for materials, marketing, pressing and binding cost a total of about $44.00 for each book. Copies of the book are sold to campus bookstores for $60 each. Information about the demand takes the following form*:

Event	Probability
Demand Exceeds 1000 units	0.95
Demand Exceeds 2500 units	0.75
Demand Exceeds 5000 units	0.60
Demand Exceeds 10,000 units	0.10

What is the expected profit (or loss) if Atlas decides to produce the textbook?

The information provided must first be converted to the form of a probability distribution with payouts. We can translate the first row to say that the probability that the demand is between 0 and 1000 units is 5%. We will represent this as an outcome of 500 units, i.e., the average of 0 and 1000. The second row says that the probability that the demand is between 1000 and 2500, (which averages to (1000+2500) / 2 = 1750), is 20%. Using this approach, the table is as follows:

Outcome	Probability	Payout
Demand is about 500 units	0.05	500*16 – 80,000 = -$72,000
Demand is about 1750 units	0.2	1750*16 – 80,000 = -$52,000
Demand is about 3750 units	0.15	3750*16 – 80,000 = -$20,000
Demand is about 7500 units	0.50	7500*16 – 80,000 = +$40,000
Demand is about 12,500 units	0.1	12,500*16 – 80,000 = +$120,000

So, the expected payout in ($000) is: (0.05 * -72) + (0.2 * -52) + (0.15 * -20) + (0.5 * 40) + (0.1 * 120) = +15 or $15,000

When the expected profit is positive, then the overall benefit to going into production is positive. This information helps in making the final decision. In the above case, the overall value from printing the books is +$15,000. Relative to the $80,000 in fixed costs required to start production, this is a reasonably good profit margin, so many managers would decide to produce the book.

Illustration: *A life insurance company offers you a one-year term product that will pay your beneficiaries $100,000 if you die within the next 365 days. It costs $200. Do you buy the policy?*

The probability that someone in their middle 20's will die within a year is about 0.0015, so if you buy the insurance the distribution table looks like:

Outcome	Probability	Payout
Live	0.9985	-$200
Die	0.0015	+$99800

The expected payout from having the policy is (0.9985 * -200) + (0.0015 * 99800) = -$50. Though it is most likely that you will lose the $200, the potential for a claim makes for an overall loss about $50 for buying the policy.

Summary: The overall value of having the insurance is worth $150, but you will have to pay $200 if you want this protection. If you base your decision on expected values, you would *not* purchase the insurance product.

There are many people who would decide to buy the insurance anyway! Can you think of some reasons why?

In situations like the one above, there may be more than money to consider. Some people may believe that the overall benefit of having the protection of insurance greatly outweighs the negative utility of having to overpay for it, especially when they take into consideration that their death might have severe emotional, as well as financial, costs. In such cases, we suggest replacing the dollar payout values in the chart with utility points before calculating the expected value. Then, a broader evaluation of the situation might be possible.

The revised distribution table looks like:

Outcome	Probability	Utility Value
Pay $200 for insurance that is not needed	0.9985	-1
Family will receive money	0.0015	+1000

The expected utility associated with the policy is (0.9985 * -1) + (0.0015 * 1000) = 1.0015. The overall benefit from having the insurance is positive, so it is worth the cost of the protection.

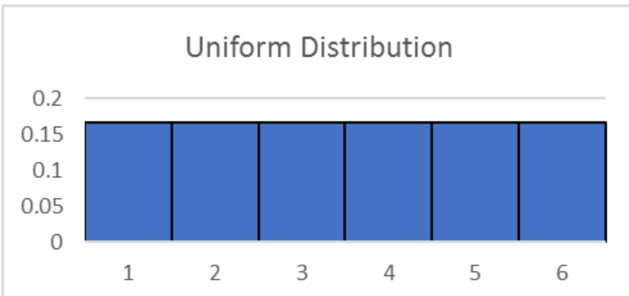
Uniform and Bell-Shaped Probability Distributions

A lot of information regarding a probability distribution can be obtained by drawing a graph to visually represent the relative likelihoods of the potential outcomes or events. With this information, we can describe the "shape" of the distribution and discuss characteristics, like "symmetry" and "spread".

For example, if we were to roll a single die, the number of spots follows this probability distribution:

Outcome	Probability
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

Which can be drawn as follows:



We call this a uniform distribution because the probability is distributed evenly across the outcomes. The distribution is also described as being "symmetrical" because the shape at the left of center is a mirror image of the shape to the right of center. Because of this, we can tell that the expected value is at the center of the distribution, i.e.,

EV = (Lowest Value + Highest Value) / 2 = (1 + 6) / 2 = 3.5

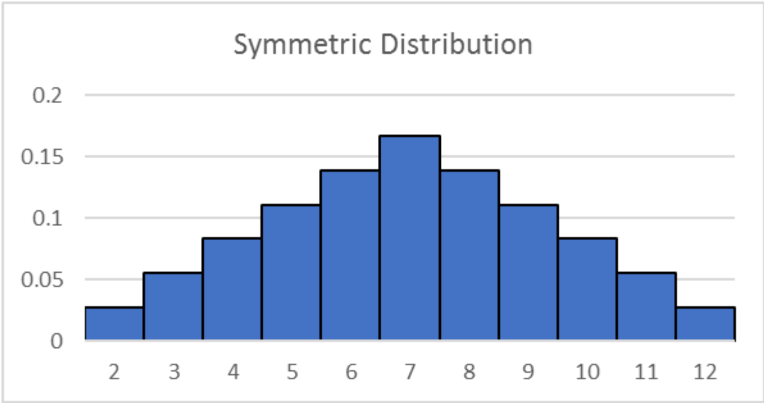
When the highest value and the lowest value in a uniform distribution are further apart, we say that the distribution has more "spread". Other terms used to describe spread in a distribution include "dispersion", "deviation" and "variability".

Suppose we were to roll two dice. The sum from the two dice follows this probability distribution:

Event: Sum of Dice	Outcomes	Probability
2	(1,1)	1/36
3	(1,2) or (2,1)	2/36
4	(1,3), (2,2), or (3,1)	3/36
5	(1,4), (2,3), (3,2), or (4,1)	4/36
6	(1,5), (2,4), (3,3), (4,2), or (5,1)	5/36
7	(1,6), (2,5), (3,4), (4,3), (5,2), or (6,1)	6/36
8	(2,6), (3,5), (4,4), (5,3), or (6,2)	5/36

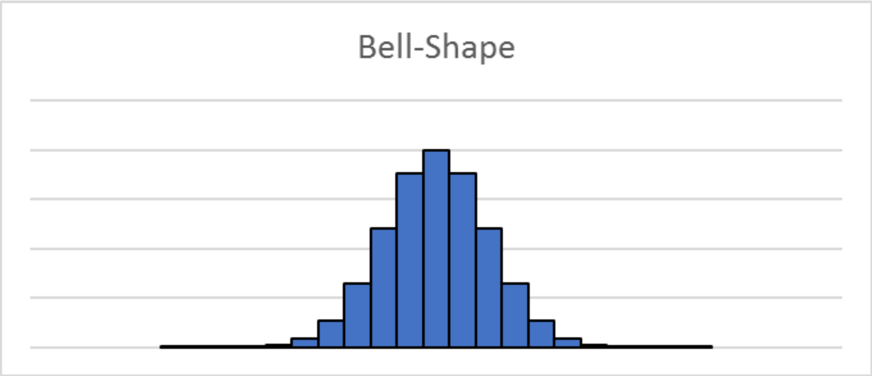
9	(3,6), (4,5), (5,4), or (6,3)	4/36
10	(4,6), (5,5), or (6,4)	3/36
11	(5,6) or (6,5)	2/36
12	(6,6)	1/36

Which can be drawn as follows:



This distribution is also symmetric. We say that the distribution has "central tendency" because values in the center have greater likelihood than the more extreme values. The expected value is equal to 7, which, like the uniform distribution, is at the center.

If we were to toss many dice, then the central tendency of the distribution representing the total would be more pronounced than in the picture above. The central values would be much more likely, and the extreme values would be rarer. We say that the distribution would take on a "bell shape", which represents a wide variety of decision-making situations. This is illustrated with the following picture:



The "Normal Distribution" was created to represent this shape mathematically. Once the desired expected value ("mean") and spread value ("standard deviation") for the outcomes are specified it is possible to use a "Standard Normal Distribution" table to find the probability that the outcome will be within some range of values.

Illustration: *It costs Atlas Bookbinders about $80,000 in fixed costs to prepare for the production of the new edition of their popular Business Math textbook. Variable costs for materials, marketing, pressing, and binding cost a total of about $44.00 for each book. Copies of the book are sold to campus bookstores for $60 each. The potential demand can be represented using a Normal Distribution with mean 6000 units and standard deviation 1200 units.*

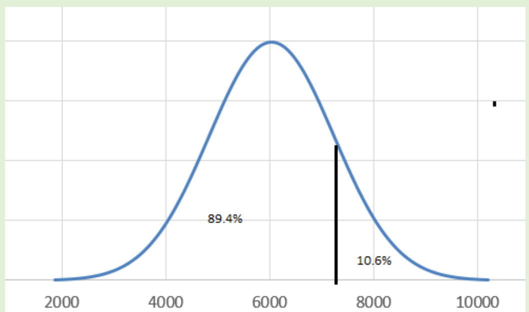
What is the expected profit (or loss) if Atlas decides to produce the textbook?

Since the mean demand is 6000, the expected profit is 6000*16 – 80,000 = $16,000

What is the probability that the profit from the book will be at least $40,000?

In order to achieve a profit of $40,000 the demand for the book must be at least:

(80,000 + 40,000) / 16 = 7500 units



This value is 1500 units greater than the mean, which can be expressed as a "z-score" of 1500/1200 = +1.25, i.e., 1.25 standard deviations above the mean. According to the Standard Normal Distribution table, the probability that a z-value exceeds +1.25 is 10.6%.

Risk

Taking risks means selecting a course of action that exposes the decision-maker to some (potentially severe) negative consequences (or costs), in the pursuit of great benefits. *Averting risk* is just the opposite. It means being willing to pay money or to forego some potential benefits by selecting an action that has a more predictable and less severe set of potential consequences. We often observe risky behavior in our friends and ourselves – consider how many smoke cigarettes, drive in cars, go skiing, cross the street, play sports, eat sugary snacks, etc. The consequences of these actions are ignored or are considered "acceptable" risks. However, we know that most people would reject a dare to walk a tightrope between tall buildings, even if they were offered a lot of money. The final decisions made about these activities is based on more than just expected payout or utility; it involves a deeper assessment of the downsides in terms of their likelihoods (probabilities) and their severity, (the magnitude of the negative payout or utility value). The fun and health benefits of playing tennis make it a popular sport, for some, and the possibility of twisting an ankle during play does not appear to keep them off the court. However, these same people would likely refuse to participate in a drag-race (with cars) because they are likely to crash and get terribly hurt as a result.

Every situation in which there are multiple outcomes having different payouts has inherent risks. It is practically impossible to completely avoid risk, so it is up to the decision-maker to include risk as a consideration before selecting a course of action. Those who do not are considered *risk neutral*. By ignoring risk, they may rely solely on expected payouts to help them decide. Most rational decision-makers, however, are *risk-averse*. They treat risk as a cost, i.e., given two situations with identical expected payouts, they consider the one with smaller risk to be more preferable, hence more valuable.

Risk is often measured by considering the *spread* in the potential payouts, i.e., the differences in value between the positive payouts and the potential costs. When reliable probability information about the potential payouts is available, the *standard deviation* of the distribution is a commonly used value to represent spread. This is especially appropriate when the outcomes follow a symmetrical distribution, like the uniform or normal distributions, because the standard deviation reflects both the severity of the risky events as well as their likelihood.

Illustration: *Consider the following games where a single coin is flipped:*

Game A	Probability	Payout
Heads	0.5	-$5
Tails	0.5	+$5

Game B	Probability	Payout
Heads	0.5	-$500
Tails	0.5	+$500

Both games are considered fair because they both have an expected value of $0. However, it should be clear that Game B, which has a larger spread in payout values, thus has a larger amount of risk. A gambler (i.e., a risk-taker) would prefer game B, whereas someone more risk-averse would prefer Game A.

Another approach to decision-making that considers risk would be to replace all payout values with utility values before calculating the expected value. These utilities better reflect the nuanced and emotional consequences of an outcome, so can represent the specific level of risk-aversion of a given decision-maker.

Illustration: *Consider the following games where a single coin is flipped:*

Game A	Probability	Outcome	Utility Value
Heads	0.5	Lose $5	-5
Tails	0.5	Win $5	+5

Game B	Probability	Payout	Utility Value
Heads	0.5	Lose $500	-550
Tails	0.5	Win $500	+500

The utility values in the tables represent someone who places more importance on losing than on winning, especially when the stakes are high, (i.e., someone who is risk-averse). Thus, Game B has a negative expected utility so would be rejected by this decision-maker.

Exercises

1. Megley Cheese Company is a small manufacturer of several cheese products. One of the products is a cheese spread that is sold to retail outlets. Jason Megley must decide how many cases of cheese spread to manufacture each month. The probability that the demand will be six cases is .1, seven cases .3, eight cases .3, and nine cases .3. The cost of every case is $45, and the price that Jason gets for each case is $95. Cases not sold by the end of the month are of no value due to spoilage. Construct a payoff table for this situation. Obviously, the amount sold cannot be larger than the amount manufactured in a given month.

2. A T-shirt company has developed a new T-shirt that they want to market. The company has determined that there is a 40% probability that the annual sales will be roughly 500,000 units, a 35% chance that the annual sales will be roughly 300,000 units, and a 25% chance that the annual sales will be roughly 50,000. If the company decides to subcontract to carry out production, its annual fixed costs will be $0 and it can produce for a variable cost of $6 per unit. If the company decides to produce itself, its annual fixed costs are $1,000,000 and it can produce for a variable cost of $3.00 per unit. The selling price is $14 per unit regardless of how the item is produced. Construct a payoff table to analyze the production decision.

3. A bookstore sells Hardtimes magazine for $11 each, buys them for $4 each, and receives a $2 credit for every unsold copy. Over the past 120 weeks it has experienced the following demand.

Demand	100	200	350	500
Number of Weeks	25	30	40	25

a) Construct a payoff table for the decision of determining how many magazines to order. Use the four alternatives of ordering 100, 200, 350 or 500.
b) Estimate the probability distribution of the demand.
c) Determine the payoffs when 300 magazines are ordered. Assume that the states are still 100, 200, 350 and 500.

4. Infinity Computer Company has developed a new computer product and has obtained the following forecast of sales.

Sales	1,000	2,500	5,000
Probability	.45	.35	.20

The company is trying to decide how to produce the product and has identified three alternatives: subcontract, build a small production line, or build a large production line. If they subcontract there is no fixed cost and variable cost is $80 per unit. If they build a small production line, the fixed cost is $35,000 and the variable cost is $40. If they build a large production line, the fixed cost is $65,000 and the variable cost is $25. The product will be sold for $120 regardless of how it is made.

Fill in the payoff table given below for the decision of determining how to produce the product.

Sales

Alternatives	**1,000**	**2,500**	**5,000**
Subcontract			
Build Small			
Build Large			
Probability			

Part III: Selecting an Alternative

Carter Coffee Company

Carter Coffee is considering a number of different ways to increase the production capacity at its Capital Hills factory. They may either: 1) build a new, larger facility, 2) expand their present facility, or 3) lease a subcontractor's facility. Based on the information provided in a recent marketing survey, the following table lists the potential increase or decrease in profits (in $1000s) for each decision alternative, depending on the amount of demand that might arise next year.

	Potential Outcomes		
	High Demand	Moderate Demand	Low Demand
	Probabilities		
	(0.3)	(0.5)	(0.2)
Alternatives:	**Payouts**		
Build	500	200	-150
Expand	400	250	-50
Lease	250	200	150

Question: Which expansion approach should Carter Coffee implement?

The chart that Carter Coffee built to describe their decision-making situation is a model that is often called a *payoff table*. It lists the set of available decision alternatives (choices) as rows in the chart. The columns correspond to the exhaustive list of the possible outcomes that might occur, with associated probability information. The values in the table tell how much profit or loss might be experienced for each alternative/outcome. Thus, the rows represent a payout distribution for each decision alternative. As we have seen, the table lists payouts, (which are the dollar-values of profits or losses), although a point-system called a *utility* might be used instead. The objective of the decision-maker is to select the alternative that has the most desirable payout distribution. Often, each alternative is evaluated in a way that is reflective of the values of the payoffs, the relative likelihood of the outcomes, and the preferences of the decision-maker. The alternative with the highest value would be selected.

Illustration: *Suppose that the managers at Carter Coffee are risk-neutral, that is, they are not especially afraid of experiencing a loss in the pursuit of profits. Which of the expansion alternatives would be most attractive?*

If we knew in advance that the demand will turn out to be high, then "Build" would be most profitable alternative, but if the demand turns out to be low, then this alternative results in the largest loss. Thus, the "Build" alternative has the most risk. The "Lease" option has the least risk because the payouts are about the same, regardless of the outcome regarding demand, but the payouts are relatively low.

When the decision-maker is risk-neutral, the most common way to evaluate a payout distribution is to calculate its expected value (EV). This is defined as the weighted average of the payouts, using the relative likelihoods (probabilities) of the outcomes as the weights.

The EV for the "Build" alternative is (0.3)(500) + (0.5)(200) + (0.2)(-150) = 220

The EV for the "Expand" alternative is (0.3)(400) + (0.5)(250) + (0.2)(-50) = 235

The EV for the "Lease" alternative is (0.3)(250) + (0.5)(200) + (0.2)(150) = 205

The largest overall payout would come from selecting the "Expand" option.

In order to build a payoff table for a given situation, it is necessary to identify all of the outcomes. These are the future events – whose likelihoods are out of control of the decision-maker - that would affect the potential payoffs in a scenario. Then, any list of reasonable decision alternatives could then be placed into the table for evaluation and comparison.

Illustration: *A bookstore sells a magazine for $8 each and buys them for $5. Unsold copies are sold for recycling for $1. Over the past 150 weeks the store has experienced the following demand.*

Demand (# Issues):	10	20	40	60
Frequency (# Weeks):	24	42	48	36

Develop a payoff table to model the decision of determining how many magazines to stock each week.

Its reasonable for the owner of the bookstore to choose to stock between 10 and 60 magazines. Clearly, by stocking less than 10 issues, she would forego certain profits; by stocking more than 60, she would be wasting money on issues that will never be sold. (You might be able to see, by using similar logic, that it would never be appropriate to stock a number between say 10 and 20, or between 20 and 40 – intermediate values would always be worse that one of the endpoints.) So, the list of available decision alternatives is: "Stock 10", "Stock 20", "Stock 40" or "Stock 60".

We can use the past experience to develop a probability distribution for the potential demand for the magazine. For each alternative, we calculate the payout distribution over the set of potential outcomes. The payoff table for the situation looks like the following:

	Potential Outcomes			
	Demand is for 10	Demand is for 20	Demand is for 40	Demand is for 60
	Probabilities			
	24/150 = (0.16)	42/150 = (0.28)	48/150 = (0.32)	36/150 = (0.24)
Alternatives:	**Payouts**			
Stock 10	30	30	30	30
Stock 20	-10	60	60	60
Stock 40	-90	-20	120	120
Stock 60	-170	-100	40	180

For example, if the bookstore stocks 60 magazines and the demand is only for 40, then the bookstore incurs a cost of (60)($5) = $300, and makes (40)($8) = $320 revenue for the magazines sold to customers plus (20)($1) = $20 for the magazines recycled. So, the profit listed in the corresponding cell of the chart is $40. If the bookstore stocks 40 magazines and the demand is for 60, then the bookstore could only sell 40 issues, so the corresponding total profit of (40)($3) = $120 is listed in the chart. In general, we can find payoffs in this table by using the formula:

Payoff = 8*(number sold) – 5*(number stocked) + 1*(number unsold).

Once the important information in a decision-making situation, i.e., selection alternatives, outcomes, probabilities, and payouts, are identified and organized into a table, it would be somewhat easier to compare alternatives and select the most appropriate one.

Illustration: *Alpha Cosmetics has developed a new perfume that they want to market. Their analysts have determined that there is a 30% probability that the first year's sales will be about 750,000 units, a 20% chance that the sales will be closer to 500,000 units, a 20% chance that the annual sales will be about 300,000 units, and a 30% chance that they will sell only 100,000 units. If Alpha goes into small-scale production, its annual fixed costs will be $400,000 and it can produce for a variable cost of $8 per unit. If the company goes into large-scale production, its annual fixed costs are $1,000,000 and it can produce for a variable cost of $6.50 per unit. The selling price will be $10 per unit regardless of the scale of production. Construct a payoff table to model the situation.*

To calculate the payoff values for the table, we use the formula from the *Cost-Volume-Profit* model: Profit = (r – v) * x – F, where x is the number sold, r is the revenue per unit, which has the value of 10, and F, the fixed production cost and v the variable cost per unit depend on which alternative we are considering.

If Alpha decides to go into small scale production, then the profit function is

$$P_{small}(x) = (10 - 8) * x - 400{,}00 = 2x - 400{,}000.$$

If x = 300,000 units are sold, then the profit for small scale production is given by

$$P_{small}(300{,}000) = 2 * 300{,}000 - 400{,}000 = \$200{,}000.$$

If Alpha decides to go into large scale production, then the profit function is

$$P_{large}(x) = (10 - 6.50) * x - 1{,}000{,}00 = 3.50x - 1{,}000{,}000.$$

If x = 300,000 units are sold, then the profit for large scale production is

$$P_{large}(300{,}000) = 3.50*(300{,}000) - 1{,}000{,}000 = 50{,}000.$$

Each payoff value in the table represents the revenue generated from sales minus the fixed and variable costs associated with each production process, (in thousands of dollars). Recall that with the CVP model there is a "Do Nothing" option, where the decision-maker chooses not to go into production.

	Potential Outcomes			
	Sell 750K units	Sell 500K units	Sell 300K units	Sell 100K units
	(0.3)	(0.2)	(0.2)	(0.3)
Alternatives:	**Payouts**			
Don't Produce	0	0	0	0
Small Scale	1100K	600K	200K	-200K
Large Scale	1625K	750K	50K	-650K

The table shows that the "Large Scale" decision alternative has the highest profit potential, especially if the demand level is high, but it is also the riskiest option because there is a reasonable likelihood that the decision will result in a considerable loss. The option of not producing is, of course, the least risky. Since it is obvious that this decision will have no payoff, it is usually omitted from the payoff table model, though the “Do Nothing” option should always be considered.

Selection Criteria

When there are no estimates available concerning the relative likelihood of the outcomes, the selection of an alternative is based solely on the values of the payoffs. Because of the uncertainty about which outcome will occur, each alternative comes with some risk that an undesirable payoff will be realized. In general, the selection process involves calculating a summary evaluation for each decision alternative based on some criterion, and then selecting the alternative having the best value.

When reliable probability information regarding the outcomes is available, a reasonable approach would be to select the alternative having payouts (or utility values) with the highest expected value (EV).

If alternative A has payoff values $x_1, \ldots, x_n$, corresponding with the *n* outcomes, then the *expected payoff from A*, denoted EV(A), is a weighted average of the payoffs, that uses the probabilities as weights, i.e., $EV(A) = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n$. Given probability information, the expected payoff represents a better measurement than the mean payoff calculated earlier because it takes into consideration the relative likelihood of the outcomes. This approach is considered *risk-neutral* because the decision-maker would be indifferent between two alternatives having the same expected value, but vastly different risk profiles.

Illustration: Recall that Alpha Cosmetics constructed the following table for their production decision:

	Potential Outcomes			
	Sell 750K units	Sell 500K units	Sell 300K units	Sell 100K units
	(0.3)	(0.2)	(0.2)	(0.3)
Alternatives:	**Payouts**			
Don’t Produce	0	0	0	0
Small Scale	1100K	600K	200K	-200K
Large Scale	1625K	750K	50K	-650K

EV(Don’t Produce) = 0
EV(Small Scale) = (0.3)(1100) + (0.2)(600) + (0.2)(200) +(0.3)(-200) = 430K
EV(Large Scale) = (0.3)(1625) + (0.2)(750) + (0.2)(50) +(0.3)(-650) = 442.5K

So, building a large-scale facility would provide the greatest expected payout.

Many decision-makers are "risk-averse" and might want to choose an alternative with fewer severe negative consequences, even if it has a lower expected dollar payout. As we have seen, this can be reflected by replacing the payout values in the table with utility values that better reflect the emotional value of an outcome than money alone. In the above case, the outcome where mangers build a large facility, but the demand does not materialize, represented by a loss of $650,000, might be accompanied by a bankruptcy of the company or a firing of its managers. Perhaps it would be appropriate to place into the table a utility value that represents an even bigger loss, like -800K. The EV of the utility values would then indicate that building the Small-Scale would be preferable.

An alternate approach would be to apply a selection criterion that treats risk as a cost. A wide range of payout values between the best-case and worst-case payoffs indicates a high level of risk. Measurements that incorporate probabilities are often used as values to represent risk. They can be incorporated into decision criteria to allow the decision maker to select an alternative that provides a favorable return at an acceptable level of risk. Such criteria should recognize that some decision makers are willing to accept risk for a higher level of return, while others are not.

We re-acquaint readers with formulas for the "Variance" and "Standard-Deviation" of a probability distribution, which is a common measure for risk. Suppose each outcome *i* is associated with a payout of x_i and a probability of p_i. Then the variance is calculated as follows:

$$\text{Var(A)} = [(x_1)^2 p_1 + (x_2)^2 p_2 + \dots + (x_n)^2 p_n] - [(x_1) p_1 + (x_2) p_2 + \dots + (x_n) p_n]^2$$

The standard deviation, $\text{SD(A)} = \sqrt{\text{Var(A)}}$

If two alternatives have similar expected payouts, then we can infer that the alternative with the smaller standard deviation is less risky. Since reasonable investors do not accept risk for free. A rational decision-maker would not select an alternative having a high level of risk unless there is a commensurately high level of expected payoff.

Henceforth, we use R(A) to represent the value of the relative risk measured for alternative A. There are three basic decision-making approaches that can be used to select an alternative. With a "Risk-Constrained Expected Payoff" criterion, some value R is selected as the maximum acceptable risk level. The alternative having the largest value of EV(A), considering only those with $R(A) < R$ is selected. (If no such alternative exists, then R must be increased or the "Do Nothing" alternative may be worth considering).

With a "Payoff-Constrained Minimum Risk" criterion, some value V is selected as the minimum acceptable payout level. The alternative having the smallest value of R(A), considering only those with $EV(A) > V$ is considered. (If no such alternative exists, then V must be decreased).

With a "Risk-Adjusted Payout" criterion, some non-negative value of α is selected so that the utility values of the payouts are the same as the point values of the risk. The alternative having the largest value of $EV(A) - \alpha\, R(A)$ is selected. The value of α must be selected carefully. Larger values would be used for more risk-averse decision-makers. A value of 0 would be used for someone who is risk neutral.

The above criteria provide different ways to select an alternative that is a compromise between risk and return. All are reasonable approaches that are used for risk management in investing. For the remainder of our discussion, we will focus on the Risk-Adjusted Payout criterion.

Illustration: Consider the following payoff table for their production decision:

	Outcome 1	Outcome 2	Outcome 3
Alt A	400	200	100
Alt B	500	250	-150
Alt C	750	375	-400
Alt D	350	300	100
Alt E	300	300	200
Probability:	(0.5)	(0.3)	(0.2)

The values of the expected payoff and the standard deviation for each alternative are:

	EV	**R**	**EV – 0.3 R**
Alt A	280	125	242.5
Alt B	295	247	220.9
Alt C	408	435	277.5
Alt D	285	95	256.5
Alt E	280	40	268.0

It should be clear that a rational decision-maker would never select alternative A because E provides the same expected payout with much less risk. Alternative C has the highest value in this case, but E would be selected by a decision-maker who is more risk-averse and would use a larger value for α, like 0.5.

Illustration: *Suppose a mutual fund company offers a variety of products in which an investor can choose to invest. For one of the products, say product A, past history indicates that yearly returns are normally distributed with a mean of 6% and a standard deviation of 3%. Another product, say product B, has returns that are uniformly distributed between –2% and 20%, in other words, every value between –2% and 20% is equally likely to occur.*

Which product is better using the Risk-Adjusted Payout criterion with $\alpha = 0.5$?

Applying the criterion, the Adjusted Payout for A = EV – αR = 6% – (.5)(3%) = 4.5%.

To evaluate product B, we first note that from probability theory that a uniform distribution with lower bound a, and upper bound b, has a mean of (a + b)/2 and the standard deviation is $(b - a)/\sqrt{12}$. For product B, a = –2% and b = 20%. Thus, product B yields an expected return of (–2% + 20%) = 9% with a standard deviation of $(20\% - (-2\%))/\sqrt{12} = 22\% \div \sqrt{12} \approx 6.4\%$.

This gives us an Adjusted Payout for B = 9% – (.5)(6.4%) = 5.8%,

So, product B would be selected since it has a larger adjusted payoff.

Exercises

1. Infinity Computer Company has developed a new computer product and has obtained the following forecast of sales.

Sales	1,000	2,500	5,000
Probability	0.45	0.35	0.20

The company is trying to decide how to produce the product and has identified three alternatives: subcontract, build a small production line, or build a large production line. If they subcontract, there is no fixed cost and variable cost is $80 per unit. If they build a small production line, the fixed cost is $35,000 and the variable cost is $40. If they build a large production line, the fixed cost is $65,000 and the variable cost is $25. The product will be sold for $120 regardless of how it is made.

Fill in the payoff table given below for the decision of determining how to produce the product.

	1,000 are sold	**2,500 are sold**	**5,000 are sold**
Subcontract			
Build Small			
Build Large			

2. Suppose that a convenience store manager has determined the following distribution for the monthly demand of Big Ski magazine.

Copies Demanded	Probability
20	0.3
21	0.4
22	0.3

The magazine is purchased directly from the local distributor for a price of $2.00 and sells for $4.00 per copy. Unsold copies are returned at the end of the month for a $0.75 credit. Construct a payoff table for the decision of determining how many issues to order.

3. Construct the payoff table for the following investment decision:

(1) Invest $10,000 in commercial property and $5,000 in bonds.
(2) Invest $8,000 in commercial property and $7,000 in bonds.
(3) Invest $4,000 in commercial property and $11,000 in bonds.

Commercial property can increase by 10% with probability .6 or decrease by 5% with probability .4. Bonds can increase by 8% with probability .8 or decrease by 3% with probability .2. Assume that the performance of bonds is independent of the performance of commercial property are independent.

4. An investor has the following options. Invest $10,000 in Stock A and $20,000 in Stock B, or invest $5,000 in Stock A and $25,000 in Stock B. Stock A will either increase by 10% with probability 0.4, or decrease by 5% with probability 0.6. Stock B will either increase by 5% with probability 0.3, or decrease by 5% with probability 0.7. Assume the performance of Stock A and Stock B are independent of each other.

a) Construct a payoff table for this decision.
b) Give the probability distribution of the outcomes.
c) Find the expected payoff for each alternative.

5. A company is trying to decide how to increase its production capacity. The decision data has been organized in the following payoff table.

	Outcomes		
Alternatives	High Demand	Moderate Demand	Low Demand
Expand	35	80	20
Build New Facility	50	60	25
Subcontract	30	100	-20

a) Suppose that the probabilities of the outcomes are 0.5, 0.4, and 0.1. Find the best alternative under the expected payoff criterion.

b) Calculate the standard deviation for each alternative and determine the best decision alternative using the Risk Adjusted Payoff criterion with $\alpha = 0.2$.

6. A corporate committee is studying four different marketing alternatives A_1, A_2, A_3, A_4. The outcomes associated with the alternatives are not the same. The following tables indicate the appropriate payoff values:

Table 1

Alternatives			
A_1	10000	5000	–1000
A_2	8000	9000	–2000
Probability:	0.1	0.5	0.4

Table 2

Alternatives		
A_3	15000	–5000
A_4	12000	2000
Probability:	0.3	0.7

a) Which alternative is the least risky? Which has the most risk?
b) Which alternative would be selected using the expected payoff criterion?
c) Calculate the SD for each alternative. Which alternative would be selected using the Payoff-at-risk criterion with $\alpha = 0.25$?

7. In the game of chuck-a-luck with three dice you pick a number from 1 to 6 and the operator rolls three dice. If the number you pick comes up on all three dice, he pays you \$3; if it comes up on two of the three dice, he pays you \$2; and if it comes up on just one of the three dice, he pays you \$1. Only if the number you picked does not show up at all do you pay him exactly \$1. Chuck-a-luck with four dice is played the same way with a chance to win either: \$4, \$3, \$2, or \$1, but in this game if the number picked does not show up at all the players must pay the operator \$2.50. Use an expected criterion to determine which game is better for the player.

8. An advertising agency is considering three alternative advertising media to promote a client's product: television, radio, and newspaper. The payoffs will depend on the estimated size of the populations reached. The populations are assumed to be either 100,000, 200,000, or 300,000. For a population of 100,000 the payoffs are estimated to be 2,500 for television, 5,000 for radio, and 4,000 for newspaper. For a population of 200,000 the payoffs are estimated to be 3,500 for television, 1,500 for radio, and 5,500 for newspaper. For a population of 300,000 the payoffs are estimated to be 7,000 for television, 2,500 for radio, and 6,000 for newspaper.

a) Construct the payoff table that gives the number of payoffs for each alternative.
b) Which alternative has the largest expected value?

9. A manufacturer produces and sells chilled ready-to-eat pasta salad in lots of 50 serving units each. These items have a limited shelf life; therefore, if items are made but not sold, they have no value. Regular production runs are made on Friday of each week for sales the following week, however, if demand exceeds supply during the week, an extra production run can be made during the week. The cost per unit for a regular run is \$5 per unit, whereas the cost of a production run during the week is \$7 per unit. All items are sold for \$10 per unit regardless of production cost. Historically, demand has

been for 50, 100, or 150 units each week, so the company is trying to decide how many units should be made on Friday: 50, 100, or 150.

a) Prepare a payoff table showing profits for each of the production lot sizes.
b) If probability of demand for 50 units is .40, the probability of demand for 100 units is .50, and the probability of demand for 150 units is .10, what is the expected profit associated with each alternative lot size?
c) Using the SD calculation, determine which alternative has the smallest risk. Which alternative is selected using the Payoff-at-risk criterion with α = .3?

10. An investment analyst is studying three different portfolios given below.

Portfolio 1		**Portfolio 2**		**Portfolio 3**	
Payout	**Probability**	**Payout**	**Probability**	**Payout**	**Probability**
10,000	.6	25,000	.2	6,000	.45
−2,000	.4	10,000	.5	4,000	.55
		−2,000	.15		
		−10,000	.15		

a) Determine the expected payoff for each portfolio.
b) Determine the risk as measured by the standard deviation for each portfolio. Which portfolio has the smallest risk as measured by the standard deviation?
c) Among all portfolios with expected payoff above 5,000, which one involves the least risk?

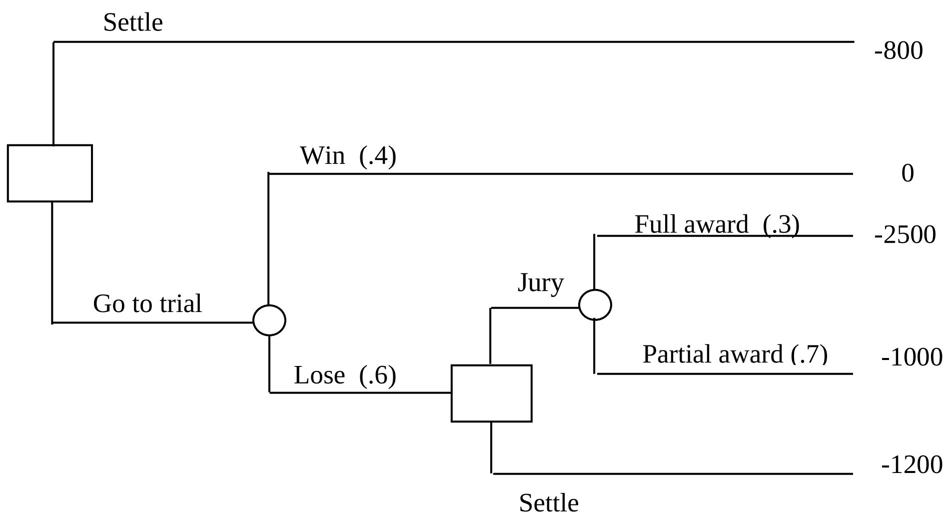
Part IV: Decision Trees for Sequences of Decisions

Cookie Cutter Promotions

The owner of Cookie Cutter Promotions is being sued as a result of an injury suffered by a fan at a music festival in the park. The plaintiff was sitting on the grass when an enthusiastic crowd of people charged for the stage and trampled her. In her suit, the plaintiff accuses the promoter of being negligent and is asking for an award of $2.5 million in damages.

The promoter has been advised by his lawyers that there is a 60% chance he will be found negligent. However, even with a negative verdict, there is only a 30% chance that the jury will award the full $2.5 million. There is a 70% chance that the award will be for only $1 million. The lawyers also indicated that he could settle with the plaintiff for $800,000 before the trial. If the promoter goes to trial and is found negligent, he could still settle the suit for $1.2 million before the jury determines an award.

Suppose that a decision-maker is faced with a *sequence* of inter-related decisions. Each individual decision and the subsequent outcome have an impact on the next set of alternatives and their consequences. Since there are multiple interconnected decisions to evaluate, a payoff table cannot be used. A *decision tree* is a more general model that enumerates the consequences from sequences of decisions and lists their ultimate payoffs. It is a visual model that describes in hierarchical form the expanding set of possibilities that may occur over time. Each of the outcomes that may follow a decision alternative is branched off to enumerate all of the potential final outcomes. The tree gives the payoff (in dollar or utility values) for each of these potential outcomes. The decision tree also represents the probability distribution at each random event.



To represent the passage of time, the decision tree model is interpreted from left to right in chronological order. As the above decision tree shows, the promoter must first decide whether to settle the suit right away or to try the case in court. (This decision is denoted by a box). If he decides to go to court, he may or may not win the case. (The possible outcomes are described as lines emanating from a circular node). The promoter has no control over which outcome will occur but is assumed to have some information about the relative likelihood, (which is represented by the discrete probability values on the lines). If he loses the trial, he then has to decide whether to settle with the plaintiff before the jury determination, (over which, again, he has no control either).

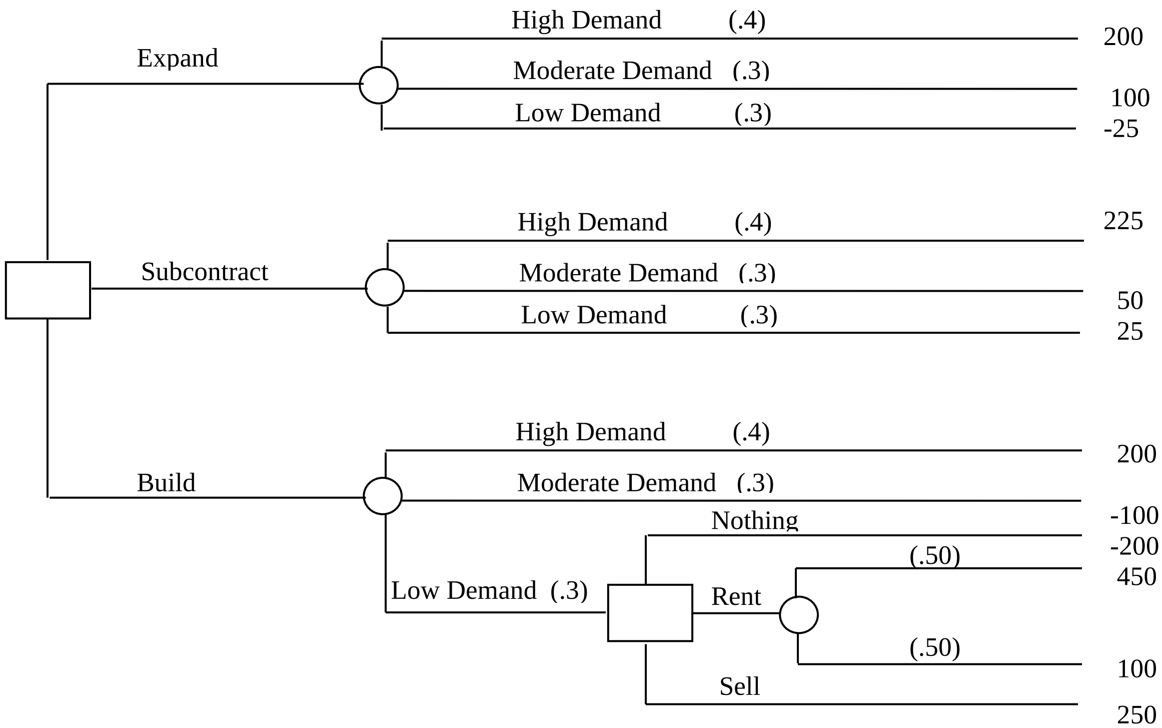
As we shall see, the decision tree model representing the promoter's options will help him establish an appropriate course of action. The terminal points at the right end of the decision tree provide the ultimate payoff value (or cost if the value is negative) that represents the net financial consequence of a sequence of alternatives selected and outcomes occurring. In this situation the best case the promoter can hope for is to break even; in the worst case, he could lose as much as $2.5 million.

Illustration: A manager in a manufacturing company is trying to decide if they should expand their present production facility, or subcontract to increase production, or build a new production facility. Payoffs have been estimated and placed in the following table (in thousands).

		High Demand	Moderate Demand	Low Demand
	Expand	200	100	-25
Alternatives	Subcontract	225	50	25
	Build	200	–100	*
	Probability	.4	.3	.3

* If the manager of the company decides to Build and the demand turns out to be Low, then he has three options. He can do nothing which would result in a loss of $200,000. He can rent out the new facility which could create a profit of $450,000 with a 50% probability or a profit of $100,000 with a 50% probability. Or he can decide to sell the facility to earn a profit of $250,000.

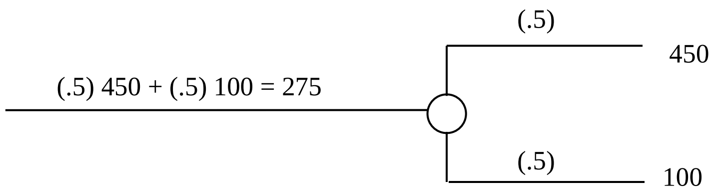
The decision-tree would look as follows:



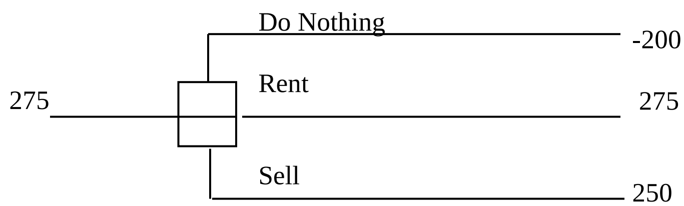
Analyzing a Decision Tree

After the decision tree is constructed, it is analyzed so that the sequence of decision alternatives that maximizes the expected payoff can be found. The following operations are performed, working from the right side of the tree back to the left side:

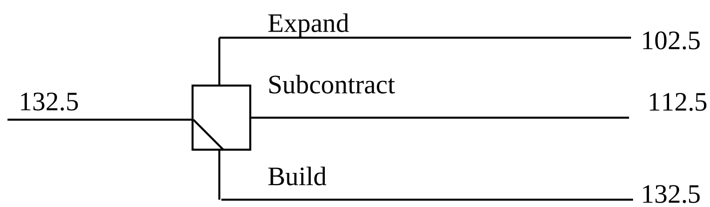
- For a circular node that represents a set of potential outcomes with a probability distribution, calculate the *expected value* of the payoffs on the branches. For example, the branch of the tree in Example 1 that represents the possible outcomes from renting the facility is evaluated as follows. The value associated with this alternative is $275,000.



- When there is a rectangular node representing a decision point, choose the alternative with the *largest payoff*. For example, the branch of the tree in Example 9 where the decision maker must "Do Nothing" or "Rent" or "Sell" is evaluated as follows. The best alternative is to "Rent".



Following this process through to the left side of the tree, the initial decision is evaluated as follows.



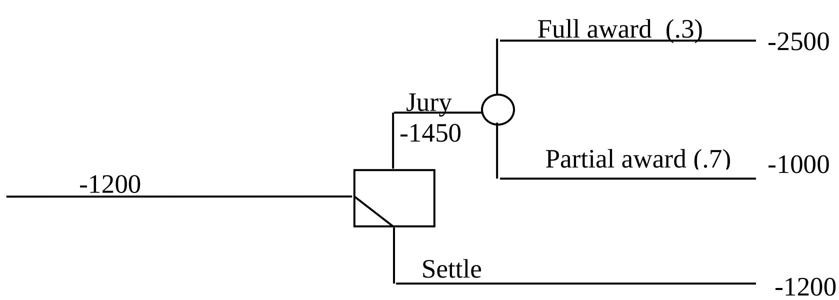
The course of action that has the maximum the total expected payoff ($132,500) is to build a new production facility, if the demand turns out to be low then rent out the facility.

In summary, to analyze a decision tree one works left to right and performs the following two steps.

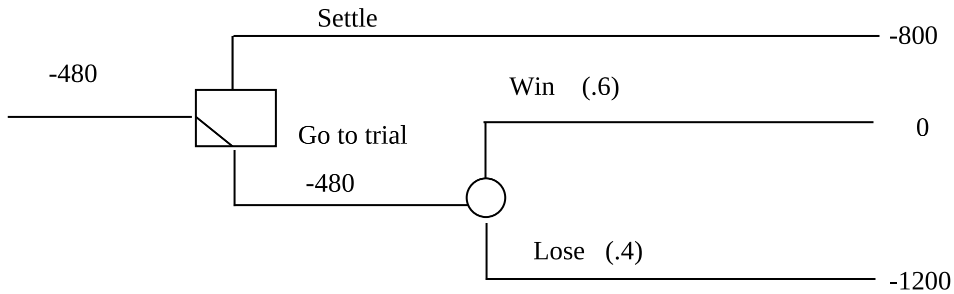
At every random event (indicated by a circle) perform an expected payoff or an expected cost calculation.

At every decision point (indicated by a rectangle) choose the best alternative and ignore the remaining alternatives. Use the payoff from the best alternative in any subsequent expected payoff (cost) calculations.

The evaluation process can be applied to the decision tree for the concert promoter, to help him find the best course of action. The decision about whether to let the jury determine the award if the court case is lost is evaluated as follows:



Now the decision about whether to go to trial can be evaluated as follows:

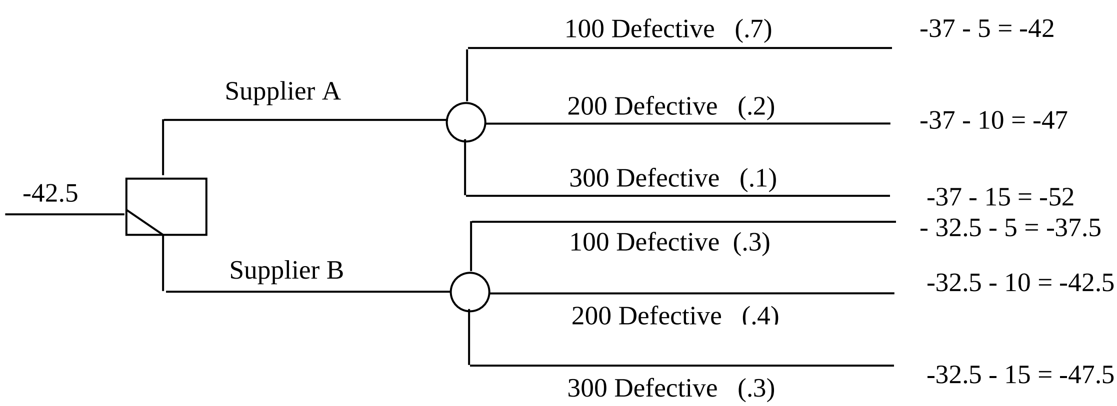


So, the best course of action is for the promoter to go to trial, if he is found negligible then he should settle with the plaintiff before letting the jury determine the award. The expected cost of this strategy is $480,000.

Illustration Quality Components needs to purchase on-off switches, which are available from two different suppliers. These switches are purchased in batches of size 1,000. The quality of any batch from the suppliers is described in the table below:

Number Defective	**Prob. for Supplier A**	**Prob. for Supplier B**
100	.70	.30
200	.20	.40
300	.10	.30

Defectives switches are repaired at a cost of $.05 each. The price of an order from Supplier A is $37 per batch; the price from Supplier B is $32.50. Build a decision tree to determine which supplier should be selected?



Since supplier B has a smaller expected cost per batch, it is selected as the vendor.

Illustration Jerry Johnson, owner of Johnson Motors, is trying to decide what insurance policy to buy to cover hail damage on his inventory of more than 150 cars and trucks. The store is located in an area where storms occur frequently and they sometimes produce large balls of hail that can damage new vehicles. Jerry has obtained the following estimates on the potential damage from hail during a given year.

Hail Damage (in $1,000)	0	15	30	45	60	75	90	105
Probability	.35	.08	.10	.12	.15	.12	.05	.03

Jerry is considering one of the following three policy alternatives for managing his risk:

1. Buy an annual insurance policy for $50,000 covering 100% of any losses.
2. Buy an annual insurance policy for $25,000 that would cover all losses in excess of $35,000 (i.e. a $35,000 deductible).
3. He can self-insure and not purchase any insurance policy.

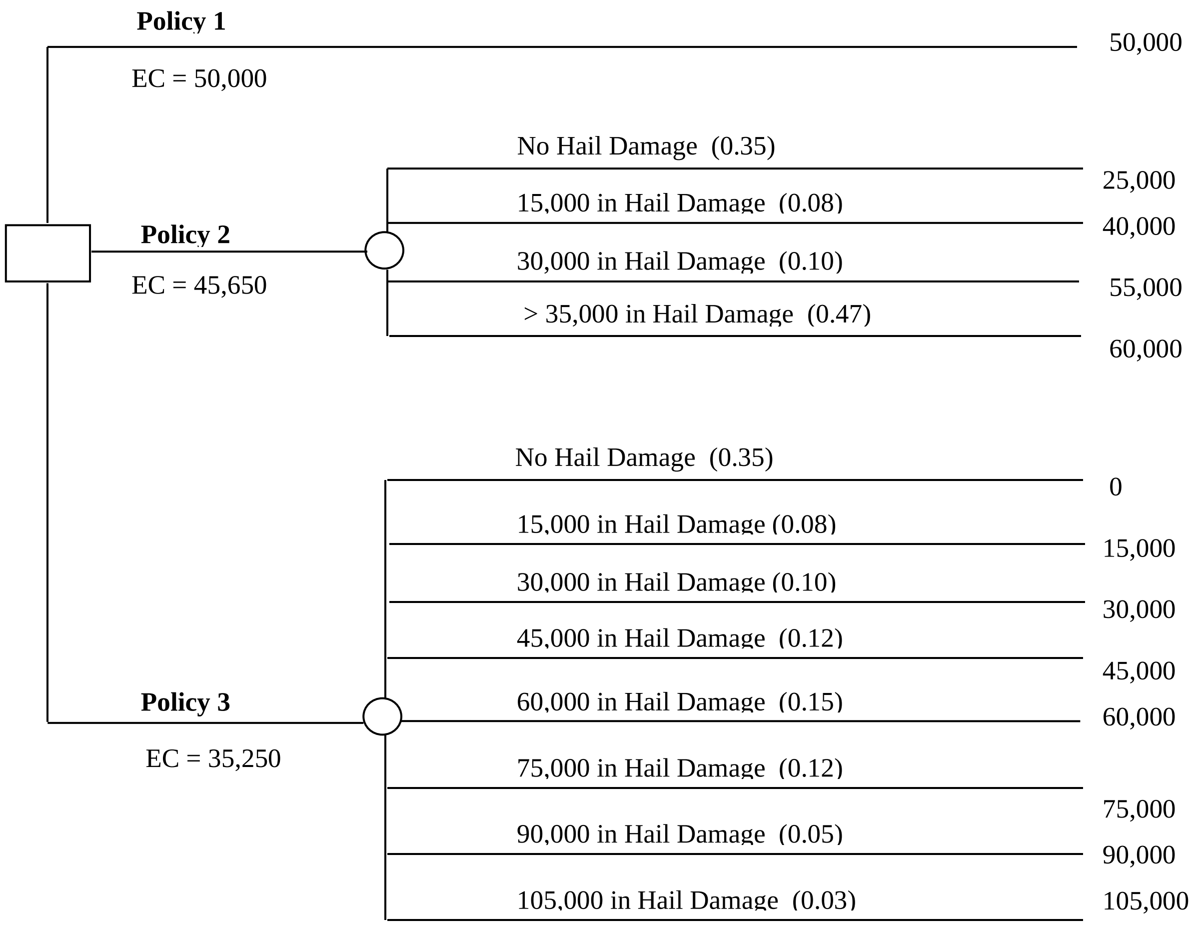
Construct a decision tree and determine which policy will minimize the expected cost.

To construct a good decision tree, it is helpful to group together some of the outcomes based on what the cost will be if alternative 2 is chosen. In particular, if hail damage is more than 35, the total cost to Jerry would be $25,000 for the policy plus $35,000 in damages. The insurance company would pay for any amount of damages over $35,000. So the cost is $55,000 in these situations. The expected cost for Policy 2 can then be calculated using the following cost table.

Hail Damage (in $1,000)	0	15	30	More than 35
Cost with Policy 2(in $1,000)	25	40	55	60
Probability	.35	.08	.10	.47

Expected cost = (25,000)(.35) + (40,000)(.08) + (55,000)(.10) + (60,000)(.47) = $45,650.

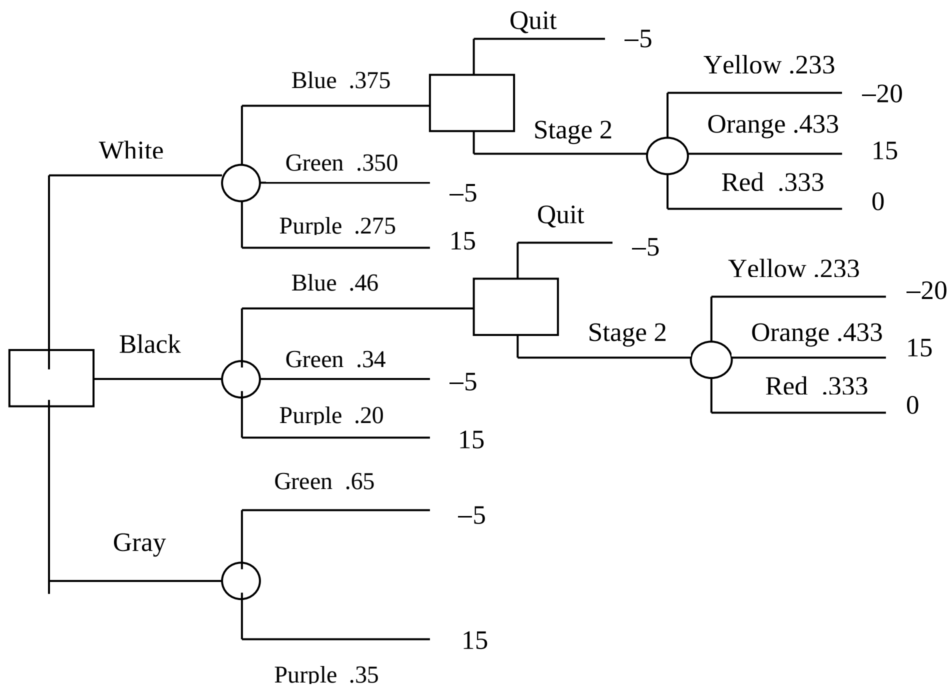
Using this distribution for Policy 2 and the expected cost of $45,650. The expected cost for Policy 3 is found using the given distribution for hail damages. The value is 35,250. The decision tree is given in figure below.



The final decision clearly depends on the objective. Policy 3 will minimize expected cost, but carries the most risk. Policy 1 is just the opposite, it carries the highest expected cost, but its SD is 0, and its worst case is a loss of $50,000 which is the cost of the premium. Policy 2 is between these two, with expected cost and risk level somewhere in between the extremes.

Suppose a gambler has an opportunity to play the following two-stage game. At stage 1 he pays $5 and must choose between a white box, a black box, and a gray box. The white box contains 15 blue cards, 14 green cards, and 11 purple cards. The black box contains 23 blue, 17 green, and 10 purple cards. The gray box contains no blue, 26 green, and 14 purple cards. The cards are identical except for color. If a green card is drawn the player has lost and the game is over. If a purple card is drawn the house pays $20. If a blue card is drawn, the player may now quit or move on to stage 2 for an additional cost of $15. In stage 2, the player draws a card at random from a box that contains 7 yellow, 13 orange, and 10 red cards. If in stage 2 the player draws an orange card, the house pays $35. If he draws a yellow card, the house pays $0. If he draws a red card, the house pays $20.

The decision tree is given below, followed by analysis of the tree.



Expected payoff calculations

Expected payoff of Stage 2 = (.233)(–20) + (.433)(15) + (.333)(0) = 1.83

Expected payoff of the White Box = (.375)(1.83) + (.350)(–5) + (.275)(15) = 3.06

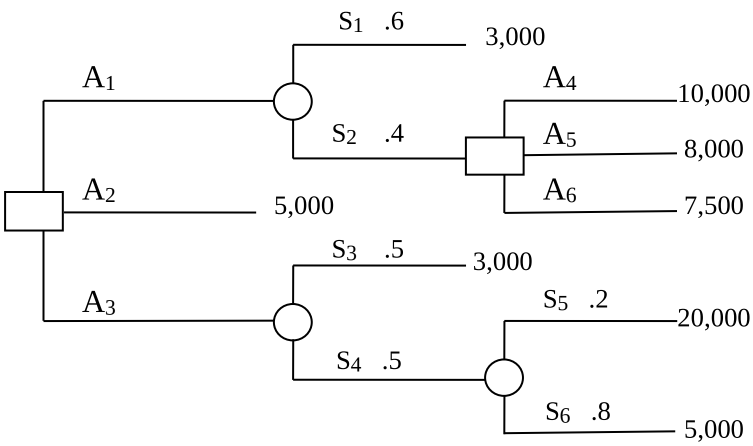
Expected payoff of the Black Box = (.46)(1.83) + (.34)(–5) + (.20)(15) = 2.14

Expected payoff of the Gray Box = (.65)(–5) + (.35)(15) = 2.00

At the decision points on the White Box and Black Box branch, choose to play Stage 2.

Conclusion Choose the White box to maximize expected payoff. If a blue card is chosen, then go on to Stage 2.

Class Exercise A decision tree is given below. Determine the best course of action using an expected payoff criterion.



Solution To begin analyzing the decision tree we must choose between A_4, A_5 and A_6. We select A_4 since the maximum among the payoffs occurs with A_4. Next we must make three expected payoff calculations as follows.

Expected payoff on A_1 branch = (.6)(3,000) + (.4)(10,000) = 5,800

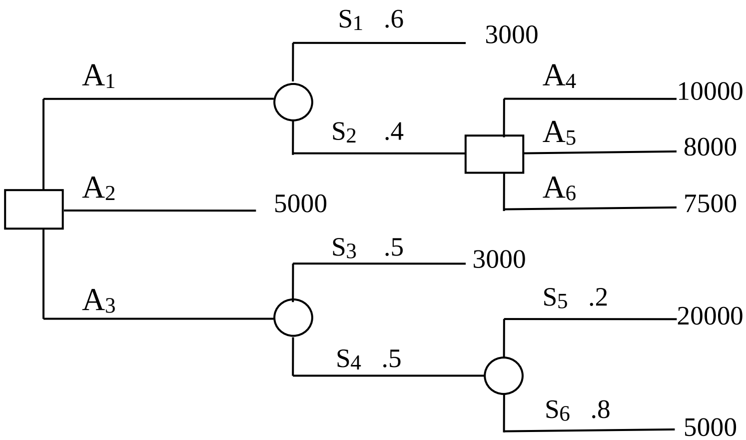
First expected payoff on A_3 branch = (.2)(20,000) + (.8)(5,000) = 8,000

Second expected payoff on A_3 branch = (.5)(3,000) + (.5)(8,000) = 5,500

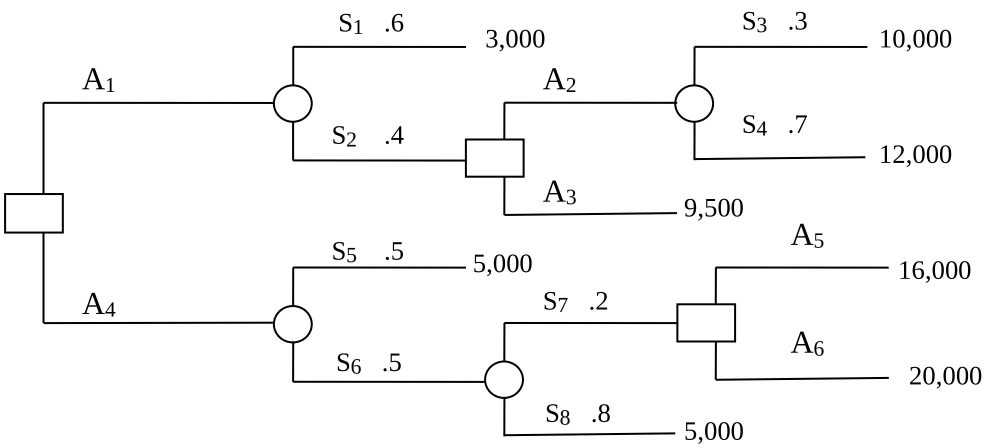
Conclusion Choose A_1, if S2 occurs, then select A_4.

Exercises

1. Determine the best course of action using an expected payoff criterion.



2. A decision tree is given below. Determine the best course of action using an expected payoff criterion.



3. A firm must decide whether to construct a small, medium, or large plant. A consultant's report indicates a 25% chance that demand will be low, and a 75% chance it will be high. If the firm builds a small facility and demand turns out to be low, the payoff will be $22 million. If demand turns out to be high, then the firm may subcontract and realize a profit of $45 million, or it could expand and obtain a profit of $48 million. A medium facility could be built as a hedge: if demand turns out to be low, its payoff will be $42 million; if demand is high the firm can do nothing and obtain a payoff of $46 million, or it could expand for a $50 million profit. If the firm builds a large facility and demand is low, the will be a loss of $20 million. Whereas high demand results in a $72 million profit.

   Illustrate the decision of selecting a plant size using a decision tree. Label the decision alternatives, the outcomes of nature, the probabilities and payoffs appropriately. Be sure to distinguish between square nodes and round nodes where appropriate. Analyze the tree to determine which size is best.

4. A gas company can either buy its supply this year at a cost of $600,000 or spend $1,000,000 to drill for natural gas. If it hits a gusher (20 % chance) the company will bring in $2,700,000 of revenue, but if the deposit is moderate (30% chance) it will bring in $1,400,000. In either case, they will have enough supply. If the well comes up dry, then the company will have to buy its supply at new market prices. For the new market, there is a 20% chance the price will stay the same, a 10% chance it will drop by 5%, a 20% chance it will be 5% higher, a 25% chance it will be 10% higher, and otherwise it will be 15% over the current price. If the company buys its supply, then it believes that a revenue of $900,000 will be produced. Construct a decision tree for this situation and determine the best strategy

5. A company is trying to decide if it should expand its present production facility, subcontract to increase production, or build a new production facility. Payoffs have been estimated as follows.

		High	Moderate	Low
	Expand	300,000	200,000	100,000
Alternatives	Subcontract	300,000	300,000	50,000
	Build	500,000	250,000	-100,000

a) Construct a decision tree which illustrates this decision. Analyze the decision tree using the expected profit criterion where the probabilities for the outcomes are .2, .5, and .3.

b) Suppose that in addition to the above data, if the company decided to build and demand is low, then it has two options. The company can either do nothing which results in a loss of $100,000 or it can rent out the new facility which would create a profit of $75,000. Construct a decision tree which illustrates the above decision including this new information. Analyze the decision tree using the expected profit criterion.

6. Electric sensors are bought from two different suppliers Supplier A and Supplier B. The quality of these sensors is given in the following tables:

Supplier A

Percent Defective	**Probability**
1	.30
2	.50
5	.20

Supplier B

Percent Defective	**Probability**
1	.7
3	.2
4	.1

Orders are always placed for a quantity of 1,000 sensors. Defectives from Supplier A are all repaired at a cost of $3.00 each. Defectives from Supplier B are all repaired at a cost of $5.00 each.

a) What is the expected cost of repair of defective sensors when buying from Supplier A?
b) What is the expected cost of repair of defective sensors when buying from Supplier B?
c) Illustrate the decision with a decision tree and determine what will minimize expected cost.

7. A gambler has an opportunity to play the following two-stage game. Initially the gambler must pay $5 and must choose between a white box and a black box. The white box contains 5 blue cards, 4 green cards, and 6 purple cards. The black box contains 3 blue cards, 5 green cards, and 12 purple cards. The cards are all identical except for color. If a green card is drawn, the player has lost and the game is over. If a purple card is drawn, the house pays $15. If a blue card is drawn, the player may now quit, or move on to stage 2 for an additional cost of $10. In stage 2 the player draws a card at random from a box that contains 3 yellow and 7 orange cards. If in stage 2 the player draws an orange card, the house pays $35. If a yellow card is selected, the house pays $0.

   Construct a decision tree and determine the best strategy based on maximizing expected payoff.

8. An investor can invest his money in one of three different investment plans over an 18-month period. The return on his investment depends on the type of investment plan chosen and the future outcome of the economy. The three plans consist of buying convertible bonds (CB), purchasing government bonds (GB), or investing in money market funds (MMF). In particular, he can buy CB for $10,000, invest $8,000 in MMF, or buy $15,000 worth of GB. The economy has been forecasted to be gloomy with a probability of .30, stable with a probability of .45, or rosy with a probability of .25. The total amount collected, including the initial investment, for the GB is $16,000 for a rosy economy, $15,900 for a stable economy, and $14,500 for a gloomy economy. The amount collected for the MMF investment is $9,000 for rosy, $8,900 for stable economies. However, when the economy is gloomy, the investor can pay a fee of $350 and sell his MMF prematurely in which case he collects $8,900. Otherwise, he may wish to do nothing and collect $8,700. The CB investment will result in collecting $11,000 in a rosy economy. Under a stable economy, the investor can sell the CB prior to maturity for a fee of $200 and collect $11,100, or wait until the end of the 18 months and collect $10,500. When the economy is gloomy, he can sell the CB prematurely and invest in real estate bonds at a cost of $500 in which case he will collect $10,500, or he can do nothing and collect $9,800.

   a) Construct a decision tree that represents the investment plans.
   b) Determine the optimal investment plan which will maximize his expected profit.

9. A manager is considering three options concerning one of his production line machines. He can purchase a new one for $400 and it will easily last three years. He is also considering purchasing a used machine or repairing the current machine. If he repairs the current one, he estimates a repair cost of $150, but also believes that there is only a 30% chance that it will not last a full three years, and he will end up purchasing a new one anyway. If he buys a used machine for $200, he estimates a 60% chance it will last the three years. If it breaks down, he will have the option of repairing it for $150 or buying a new one. Construct a decision tree for this situation and determine the best strategy.