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Numeracy Infusion Course for Higher Education (NICHE), 1: Teaching Faculty How to Improve Students' Quantitative Reasoning Skills through Cognitive Illusions

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Numeracy Infusion Course for Higher Education (NICHE), 1: Teaching Faculty How to Improve Students' Quantitative Reasoning Skills through Cognitive Illusions

Abstract

We describe one of the eight units of a professional development program, the Numeracy Infusion Course for Higher Education (NICHE), which introduces research on cognition, including dual-processing theories, to university faculty. Under the dual-processing framework, System 1 (intuition) quickly proposes intuitive answers to judgment problems as they arise, while System 2 (deliberation) monitors the quality of these proposals, which it may endorse, correct, or override. We present several classic questions that demonstrate the pitfalls of overreliance on intuition without analytical thinking, then describe faculty participants' responses to these questions and their ideas on how to apply cognitive illusion research to quantitative reasoning instruction. The unit has helped generate excellent ideas for quantitative reasoning instruction. A persistent concern shared by many participants, however, is that weakness in basic mathematics and language comprehension among urban public university students might present a challenge in implementing these ideas.

Keywords

cognitive illusion, dual-processing theory, medical diagnosis problem, faculty development program

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Cover Page Footnote

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Introduction

This is the first of two papers on the Numeracy Infusion Course for Higher Education (NICHE). The project is sponsored by the National Science Foundation (NSF TUES 1121844) to promote quantitative reasoning (QR) at the City University of New York (CUNY), the nation's largest urban public university system. NICHE is a predominantly online course that increases faculty interest and comfort in teaching QR while strengthening the faculty's own QR skills. It consists of eight units: (1) QR and Making Numbers Meaningful; (2) QR Learning Objectives; (3) The Brain, Cognition and QR; (4) QR and Writing; (5) Discovery Methods; (6) Representations of Data; (7) QR Assessment; and (8) Math and QR Anxiety.

As of 2014, NICHE has trained two cohorts of faculty participants from 15 CUNY campuses across a wide range of disciplines including African American studies, art, biology, economics, English, mathematics, psychology, and sociology. To reach out to a larger community, NICHE particularly welcomes educators who consider themselves uncomfortable and inexperienced in QR instruction. Additional information about the course can be found at the project website¹ and in several related studies (Wilder 2012; Wilder et al. 2013, 2014).

This paper focuses on the development of Unit 3 (The Brain, Cognition and QR), which introduces faculty participants to psychological research on numerical judgment and guides them to apply cognitive science to QR instruction. We will first recount the events that inspired our interest in cognitive illusions (illusions of thoughts). We will then outline the unit content, which summarizes and simplifies an immense amount of literature to provide an overview for faculty from non-psychological fields. We include several classic questions from the literature on heuristics and biases (cf., Tversky and Kahneman 1974) to stimulate discussions, and we will highlight some faculty participants' responses to various learning activities collected from Blackboard (a content management system) in 2013 and 2014. In the second paper, we will present our effort to incorporate the NICHE material into an elementary statistics course and to assess the impact of QR instruction on students' learning.

Motivation

In 2008, Richard Larrick and Jack Soll of Duke University's Fuqua School of Business published "The MPG Illusion" in *Science*. They found that there is a

¹ See <http://serc.carleton.edu/NICHE> or <http://teachqr.org> (accessed January 15, 2015).

systematic misperception in judging fuel efficiency when it is expressed as miles per gallon (MPG). They presented this scenario:

Consider a family that has an SUV that gets 10 MPG and a sedan that gets 25 MPG. Both are driven equal distances in a year. Is the family better off replacing the SUV with a minivan that gets 20 MPG or replacing the sedan with a hybrid that gets 50 MPG?

Seventy-seven Duke undergraduate students were asked to rank-order five pairs of old and new vehicles in the order of reduction of gas consumption. Only one student ordered the pairs correctly. The majority (60%) intuitively thought that the increase in MPG is proportional to the decrease in gas consumption. That assumption is erroneous, as explained below. Larrick and Soll show how the MPG label leads to faulty judgment; they suggest that the United States should express fuel efficiency as a ratio of fuel consumed to distance traveled, for example, gallons per 100 miles (GPM).

The paper attracted a flurry of media attention, but from readers' comments posted on the websites of the *New York Times*, the *Wall Street Journal*, and other news outlets, many people still failed to comprehend the essence of the "MPG illusion." Larick and Soll wrote: "People falsely believe that the amount of gas consumed by an automobile decreases as a linear function of a car's MPG. The actual relationship is curvilinear." Students can relate this statement to the properties of the reciprocal function they have learned in elementary algebra. For a fixed distance, gas consumption by a vehicle is proportional to the **reciprocal** of MPG. Figure 1 shows the diminishing benefit of replacing a relatively high-MPG vehicle with an even higher-MPG vehicle.

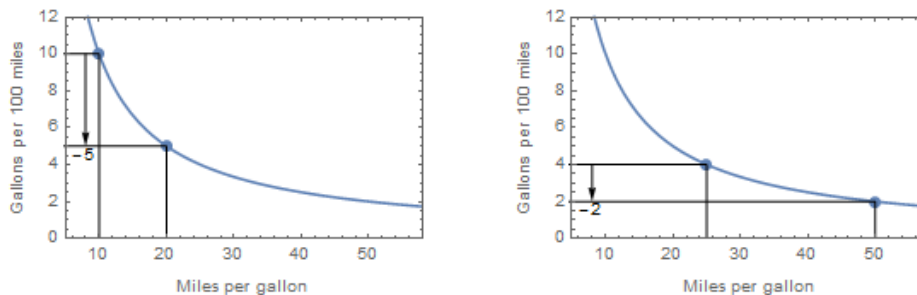


Figure 1. Visualizing how much fuel consumption is reduced for given increase in MPG. For a fixed distance, say 100 miles, replacing a 25-MPG vehicle with a 50-MPG vehicle will reduce fuel consumption by 2 gallons, but replacing a 10-MPG vehicle with a 20-MPG vehicle will reduce consumption by much more: 5 gallons.

Although nothing more than high school math is required to understand the cause of the MPG illusion, it is well known that many college students, including CUNY students, lack such knowledge (Wilder et al. 2013).

This situation prompted one of us (FW) to conduct an experiment in a different setting, using community college students as research subjects. Specifically, Wang and Dedlovskaya (2009a) designed a multiple choice “math quiz” to discover (1) whether the students have the basic understanding of fractions, ratios and rates needed to solve the MPG problem and (2) whether students make the same mistake reported in *Science* when the MPG illusion problem is presented as a math quiz—a situation in which students might be expected to be more alert. The math quiz required students to list fractions ($3/4$, $4/9$, $5/6$) from largest to smallest; to solve distance/time/speed problem;² and to solve the MPG illusion problem. Altogether, 37 of the 44 students in non-remedial courses, or 84%, had the math skills to solve the MPG illusion problem, but most of them made the same mistake as the Duke University students in Larrick and Soll’s (2008) study. Only 16% of the 44 students circled the right answer, but most failed to show their work as instructed. Many students in remedial math courses have difficulty with fraction operations, which precludes them from truly understanding the MPG illusion problem. See Wang and Dedlovskaya (2009a) for details.

Based on these results, we developed a lesson plan to teach elementary math concepts including fraction operations and reciprocal functions through the MPG illusion problem (see Wang and Dedlovskaya 2009b). We solicited several instructors to adapt this lesson in their math courses at various levels and to retest their students by incorporating the MPG illusion problem in one of their examinations. Our initial review of the data indicated that students’ performance improved considerably after instruction. Unfortunately, unfavorable circumstances (e.g., flooding and class cancellations) have prevented us from completing our project and publishing the results. Nevertheless, our analyses of students’ work suggests that the MPG illusion can be mitigated through educational intervention, even for students without a proper grounding in basic skills.

A referee who commented on Wang and Dedlovskaya (2009a) questioned the notion that people think differently in a math test setting than they would in a non-school situation. We formed a hypothesis based on our teaching experience: Students tend to be more cautious during a test, and more suspicious about the obvious answer being the right one. The referee’s comment prompted us to conduct a literature search, and we learned about the Adult Math Project (Lave 1988), which is extensively discussed in Keith Devlin’s *Math Instinct* (2005). By following 25 shoppers in Orange County in southern California, Lave discovered a difference between performance on school-style tests and the use of arithmetic in real life. In the ensuing years, we discovered a wealth of psychological

² Alice drove 320 miles in 6 hours; Bob drove 410 miles in 7 hours. Who has a higher average speed?

research on numerical cognition. The dual-processing theories of reasoning (Sloman 1996; Evans 2008; Kahneman 2011) are especially appealing, since they can explain the MPG illusion as well as many other common mistakes. Because this research area is not widely known to many of our colleagues, we developed a unit in NICHE that introduces the cognitive research literature to prospective QR educators.

Dual-Processing Theory of Reasoning

In 1974, Amos Tversky and Daniel Kahneman published a seminal paper in *Science* entitled “Judgment under Uncertainty: Heuristics and Biases,” in which they documented many of the biases and misconceptions that arise when people make intuitive predictions and judgments: biases in assessments of subjective probability, misconceptions of randomness, and misconceptions of regression, to name but a few. Later, we will give some specific examples. Kahneman and his collaborators developed a dual-process theory to explain biases in probability judgment (Kahneman and Frederick 2002; Evans 2008), and the theory has been popularized in Kahneman’s book *Thinking, Fast and Slow* (2011). Under this framework, there are two distinct but interacting systems of thinking: System 1 relies upon frugal heuristics that yield intuitive responses, while System 2 relies upon deliberative analytic processing. Crudely, we can regard System 1 as intuition and System 2 as deliberation.

Figure 2 demonstrates how easily our eyes can be deceived. The bottom line appears to be longer. If you have already seen this image, you recognize it as the famous Müller-Lyer illusion: the horizontal lines are identical in length. The Müller-Lyer illusion suggests that perception and knowledge derive from distinct systems. Perception provides one answer, yet knowledge (or a ruler) provides a different one (Sloman 1996). Vision is not the only domain of illusions. Illusions of thoughts are called “cognitive illusions.”

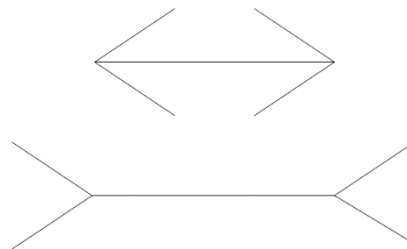


Figure 2. Müller-Lyer illusion

To observe the mind in automatic mode, think about $2+2$. The answer comes to us immediately and intuitively. This is an instance of System 1 thinking. But for the problem 17×24 , probably nothing comes to mind. We have to generate the solution, but first we must decide to do so; we must decide to engage System 2. Kahneman gave examples of System 1 and System 2 activities (see Table 1). Notice that the highly diverse operations of System 2 have one feature in common: they require attention and are disrupted when attention is drawn away.

Table 1.
Comparison of System 1 and System 2 of Kahneman (2011)

System 1	System 2
<ul style="list-style-type: none"> • Detect that one object is more distant than another. • Orient to the source of a sudden sound. • Complete the phrase “bread and ...” • Detect hostility in a voice. • Drive a car on an empty road. • Understand simple sentences. 	<ul style="list-style-type: none"> • Focus attention on the clowns in the circus. • Focus on the voice of a particular person in a crowded and noisy room. • Count the occurrences of the letter <i>a</i> in a page of text. • Compare two washing machines for overall value. • Fill out a tax form. • Check the validity of a complex logical argument.

One particular dual-process model (Kahneman and Frederick 2002) suggests that System 1 quickly proposes intuitive answers to judgment problems as they arise, and that System 2 monitors the quality of these proposals, which it may endorse, correct, or override. Consider this problem.

A bat and a ball cost \$1.10 in total. The bat costs \$1.00 more than the ball. How much does the ball cost?

According to Kahneman and Frederick, almost everyone has an initial tendency to answer *10 cents* because the sum \$1.10 separates naturally into \$1 and 10 cents. Many people yield to this immediate impulse, perhaps because they are not accustomed to thinking hard and are often content to trust a plausible judgment that quickly comes to mind. But anyone who reflects upon it for even a moment would recognize that the difference between \$1.00 and 10 cents is only 90 cents, not \$1.00 as stipulated in the problem. Catching that error is tantamount to solving the problem; According to Frederick (2005), nearly everyone who does not respond “10 cents” gives the correct response: *5 cents*.

We believe that the dual-processing theories of human thinking—the interaction and conflict between intuitive and analytic thinking—are valuable for QR educators. Among other things, they can give the faculty member a handle to respond to students’ doubts about the need for mathematical and quantitative reasoning skills. Nearly every math teacher has been asked the question “When am I ever going to use this [school mathematics]?” by students. Although educators have increasingly been using real-life examples to interest and connect with students, it is still paradoxical for some students to witness that the great majority of people manage to function in our society by using their intuition and common sense, without ever applying the math skills they learned in school. Some successful professionals even take pride in uttering “I’m bad at math” (Bennett 2012; Madison 2012).

While human intuition does meet our needs most of the time (Pinker 1997), psychologists have found that people make systematic, unconscious errors of reasoning that distort their judgment. The MPG illusion is a good example, demonstrating how intuition leads to a suboptimal economic decision. Likewise, many people, including medical professionals, fail to comprehend cancer screening statistics (Casscells et al. 1978; Eddy 1982; Wegwarth et al. 2012). They apparently replace the laws of chance with intuition, which sometimes yields reasonable estimates but quite often do not. These examples should serve as a wakeup call for faculty and students, since they convincingly demonstrate the need to build quantitative reasoning skills.

The question that is most often asked about cognitive illusions is whether they can be overcome (Kahneman 2011). For educators, it is tempting to encourage students to engage in deliberate thinking at all times to avoid mistakes. While it may appear that System 2 is superior to System 1, as the former is often associated with correct mathematical answers and the latter with cognitive biases, it is not possible to engage in deliberative thinking constantly. As a way to live our lives, continuous vigilance is simply not practical. In fact, intense focus on a task can make people effectively blind to events and information outside their narrow range of concentration. This is the main point of *The Invisible Gorilla*, by Christopher Chabris and Daniel Simons (2010). We will discuss the limited capacity of attention in the next section.

In summary, the brain works hard to help us survive and accomplish our goals in a complicated world. We often need to react quickly to emergencies and opportunities, and System 1 is in charge most of the time. Only when System 1 runs into difficulty does it call on the slow-thinking System 2 to support more detailed and systematic processing. The division of labor between System 1 and System 2 is highly efficient, as it minimizes effort and optimizes performance. However, it can lead to predictable mistakes. Under the dual-processing framework, quantitative reasoning is a crucial component of System 2 deliberative thinking. With this understanding of how the mind works, educators can be more persuasive in convincing their students to develop quantitative reasoning skills so that students can be more alert, more intellectually active, less willing to be satisfied with superficially plausible answers, and more skeptical about their intuition.

Content of a NICHE Unit

Each NICHE unit begins with a brief video followed by activities and interactive discussions. Through the course, participants learn to (1) articulate QR learning goals; (2) create QR lessons to help students achieve those goals; and (3) develop instruments to assess student learning. The purpose of Unit 3, “The Brain,

Cognition and QR,” is to familiarize educators with research on cognitive illusions and to help them understand the psychological causes of the human brain’s struggle to store and process numerical information. This knowledge empowers educators to help students identify their quantitative blind spots—to better predict when their intuition is reliable and when it is likely to mislead them.

Faculty participants in NICHE complete a set of readings before starting each of the eight learning units. The main reading assignment for Unit 3 is a *Scientific American Mind* article entitled “Knowing Your Chances” (Gigerenzer et al. 2009),³ which describes the difficulties of correctly interpreting quantitative health-related information. The article also discusses absolute and relative risk, an important concept that is emphasized by Madison et al. (2012) and others. In addition, faculty participants read several *New York Times* articles on the potential harm of mammography and prostate tests (Parker-Pope 2011, 2012). Our experience in professional development seminars suggests that these issues are inherently interesting and relevant. Our intention is to build on that personal interest to deliver a clear message that numeracy can help individuals improve their own lives through better personal choices. One of us (FW) used these methods and the NICHE supporting materials to enhance an elementary statistics course. The second paper in this series describes that initiative.

After a video that reviews the dual-processing theories of human thinking, the NICHE participants perform two experiments to observe how the brain functions. We emulate the approach of E.F. Redish of the University of Maryland Physics Education Research Group (see, for example, Redish 2014).

The first experiment is to watch the YouTube video, the Monkey Business Illusion (Simons 2010). In that video, a group of six students serve as two teams, one with white shirts and one with black. Each team has a basketball. During the short video, they move around quickly, passing their ball among members of their own team. The task is to see how well one can concentrate by counting the number of passes among the members of the white-shirted team.

One needs a partner for the second experiment. The participant asks the partner to read the following strings of numbers and the participant tries to say the numbers back in reverse order. For example, if the partner says “123” the participant responds “321.”

- 4629
- 38271
- 539264
- 9026718
- 43917682

³ In 2014 NICHE we also added a book chapter from Steven Pinker’s (1997) book *How the Mind Works* to this unit to illustrate common errors in judgment.

For the first experiment, many people manage to count the number of passes but somehow fail to see the gorilla that walks through the scene during the clip. This phenomenon, called “inattention blindness,” shows how intense focus can make people effectively blind to stimuli outside their range of attention. In the second experiment, the participant will discover that it gets harder and harder to complete the task, and that above a certain number of digits it is impossible. This illustrates that our brain has a finite processing capacity. This phenomenon has been known for more than 50 years, since George Miller (1956) published his famous paper “The Magic Number Seven, Plus or Minus Two.” Miller’s study is the basis of the important psychological construct of “working memory,” which allows the short-time storage of a few information items in order to compare or manipulate them in useful ways.

These two experiments lead to the concept of “cognitive load”: the demand that is made on a person’s working memory (Reif 2008). A task with a high cognitive load requires the individual to pay attention to many different things at the same time and can reduce his or her effectiveness in carrying out the task. For example, multiplying two seven-digit numbers in one’s head is beyond most people’s ability because it requires too many pieces of information to be remembered at one time. A piece of paper and a pencil make the task doable. For many QR tasks, it is helpful to inform students that having paper or a calculator handy can reduce excessive and unnecessary cognitive load. To design an effective lesson, instructors need to consider the learner’s cognitive load, namely the amount of information processing required to perform a task. If the cognitive load becomes excessive, little or no learning can occur. On the other hand, a very low cognitive load can lead to inattentiveness.

After the experiments, there are three problem sets to further stimulate discussion. We encourage participants to consider whether the dual-processing theory of reasoning provides plausible explanations for their observation of students’ perceptions. Problem set A asks participants to reflect on the following common errors, taken from Stanislas Dehaene’s *Number Sense* (2011).

- 0.25 is greater than 0.5
- $0.2 + 4 = 0.6$
- $\frac{1}{5} + \frac{2}{5} = \frac{3}{10}$
- The temperature is in the 80s today, twice as warm as last night when the temperature was 40°F.
- There is a 50% chance of rain for Saturday, and also a 50% chance of rain for Sunday, so there is a 100% certainty that it will rain over the weekend (heard on the local news by John Allen Paulos 1988).

Problem set B is the Cognitive Reflection Test developed by Shane Frederick (2005). The Test presents the bat and ball question discussed in the preceding section, along with two other problems:

- (2) *If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets?*
- (3) *In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half the lake?*

According to Kahneman (2011), more than 50% of students at Harvard, MIT and Princeton gave the intuitive but incorrect answers: 100 minutes instead of 5 for question 2 and 24 days instead of 47 for question 3. We ask NICHE participants to imagine how well CUNY students will perform, and how we can help them develop the disposition to resist reporting the first response that springs to mind.

Many Harvard Medical School students and staff stumbled when confronted with problem set C, the medical diagnosis problem published in the *New England Journal of Medicine* (Casscells et al. 1978):

If a test to detect a disease whose prevalence is 1/1000 has a false positive rate of 5%, what is the chance that a person found to have positive result actually has the disease, assuming you know nothing about the person's symptoms or signs?

This question can be approached formally using Bayes' rule, which is discussed in the second paper, but Casscells et al. show that common sense alone is sufficient for the correct answer. Despite the existence of a simple solution, Casscells and colleagues found that only 11 of the 60 subjects gave the correct answer (2%), and 27 vastly overestimated the probability by answering 95%. Many other studies show that clinicians routinely miscalculate risk (Eddy 1982; Hoffrage and Gigerenzer 1998; Gigerenzer 2002). Consequently, there is a growing recognition among medical professionals that cancer screening is a double-edged sword. While some individuals may benefit from early detection, others will be diagnosed and treated for cancer unnecessarily (Schwartz et al. 2004). The ability to accurately assess the probability associated with this type of medical diagnosis problem is critical in following or engaging in public discourse, and this is the main reason we chose Gigerenzer et al. (2009) as a reading assignment.

Faculty Responses

Improving the faculty's QR skills is one of the objectives of NICHE. To measure the impact of NICHE, we developed two versions of a QR skills test that we

administered to faculty participants both before and after the program. Each version consists of 20 questions and covers skills including the interpretation of rates, ratios and charts; measures of central tendency; and the comparison of populations (Wilder et al. 2014). The MPG illusion problem and the medical diagnosis problem are included in the test. Among the 25 faculty members who consented to participate in our study and completed both tests in 2013 and 2014, 12 answered the MPG illusion problem correctly in the pre-test; 11 answered the medical diagnosis problem correctly. These rates are much lower than those for the other questions (Table 2), which indicates that some quantitatively sophisticated faculty are not immune to cognitive illusions. In the post-test, 19 got the MPG illusion question right; 16 answered correctly on the medical diagnosis problem. While such an improvement is not statistically significant with the current sample size, we believe that faculty were more cautious and self-critical in the post-test after they were introduced to cognitive illusion examples in Unit 3. The full analysis of the QR skills assessment results will be the topic of a future publication.

Table 2.
NICHE Faculty Participants' Performance on the QR Skills Assessment:
Pretest and Posttest ($n = 25$)

Skill	Number correct	
	Pretest	Posttest
Fuel efficiency and car mileage	12	19
Interpretation of medical testing results	11	16
Interpretation of rate	24	25
Conversion of money	24	24
Independence of events	24	21
% salary increase/calculation of pay raise	25	25
Interpretation. of histogram	24	24
Interpretation of pie chart	6	17
Number sense for very large numbers	21	18
Median	20	20
Mean/Average	25	25
Mode	23	25
Interpretation of table on percent distribution of workers	23	23
Ratio	22	22
Interpret. of percent less than one	24	22
Comparison of % distribution of two populations	22	21
Interpretation of line chart	23	25
Extrapolation of line chart	17	21
Interpretation of multivariate chart	22	23
Interpretation of qualitative chart	18	23

To structure the faculty participants' exchange of teaching ideas, we set up three forums in the Blackboard discussion board. Participants were given three guidelines for discussion:

- (1) After reviewing the math/QR problems in Unit 3, comment on why you think people often make the errors described. How do you think your students would answer these kinds of questions? What strategy would you employ to help them overcome some of the common mistakes?
- (2) Comment on your experience undertaking the two cognitive experiments and how you think managing learners' cognitive load relates to QR instruction.
- (3) Consider the *New York Times* article "Multivitamin Use Linked to Lower Cancer Rates" by Roni Caryn Rabin published on October 17, 2012. How do absolute risk and relative risk differ? How do you explain this to your students?

After reading the participants' postings, we noticed several recurring themes, which are shown in bold type in the following discussion.

It was clear that the faculty participants understood the dual-processing theory of thinking, based on the appearance of the terms **System 1**, **System 2**, and their associated characteristics (intuition/deliberation, fast/slow, and automatic/controlled). For example:

- *I thought the readings and video did a good job of pointing out the differences between our Type 1 and Type 2 thinking systems.... I think that we have made a real culture of doing things quickly. So it takes a real effort to slow down and make sure that you notice where the decimal point is, for example.*
- *Problem sets A and B often times lead to wrong answers because our efficient automated thinking switches on is tricked by what seems an easy problem. Problem set C is actually more difficult.*
- *Students may confuse getting a general impression of a problem (intuitive thinking) with analysis of content (deliberative thinking).*

However, some participants were critical of the theory:

- *I have never accepted idea that our brains (naturally) use different kinds of thinking. Instead, I think of slow v. fast thinking as different in degree."*
- *I think the idea of 'slow' versus 'fast' thinking is a useful distinction for talking about different types of problem solving strategies, but I've been reading a neuroscience textbook this summer and I have to point out that our brains do not actually have different thinking 'systems.'"*

These comments suggest that some participants were led to believe that there are literally two discrete systems of thinking. In the future, we will emphasize that the dual-processing theory is not strictly dichotomous. As one participant wrote

- *human thinking is such a fascinating phenomenon: it's a delicate balance between fast (intuitive) and fast (deliberative) thinking.*

One participant who disagreed with the dual-processing theory wrote,

- *I don't see the difficulties that students might encounter in the unit 3E math/QR problems as simply a problem of lacking deliberative/slow thinking.*

This is indeed a pertinent remark that may reflect the characteristics of CUNY students. Henrich et al. (2010) criticized the overreliance on the so-called WEIRD population (undergraduate students from Western, educated, industrialized, rich and democratic cultures) in psychologists' studies; they pointed out that the analytical reasoning style used by such a group might not be representative of other human populations. Such a criticism resonates with our own experience. We informally tried the bat and ball problem with our students, and many of them were unprepared to answer the question. For instance, some participants were unable to check the validity of a proposed answer.

In addition to math challenges, CUNY students also face language difficulties.⁴ Many discussion threads centered on the **wording** of the problems:

- *I think for our students, particularly the ESL students, they may not be able to answer the questions because they don't understand the sentence construction.*
- *I made those errors myself and I think it is primarily the result of the wording of the problems.*
- *When we teach solving word problems by picking out key words for identifying which operation to use I think that we are leading students astray. The words can be used in different ways and don't always mean the same thing.*

Trick was a term often used to describe classic cognitive-illusion questions:

- *Some people are easily tricked and it doesn't say much about their true capabilities.*
- *Set B has nice 'got you' problems!*
- *I was primed to notice that this was probably a 'trick' question.*
- *These trick questions are designed to trick us by luring us into using a form of reasoning that isn't appropriate for the matter at hand.*

As to how participants may use these questions in their classroom, one wrote,

- *Getting the wrong answer is what the designers of the tests want as an outcome because the wrong answers will help them make a point about how the brain functions."*

One wrote,

⁴ The City University of New York (CUNY) comprises 24 colleges and schools, including 17 senior colleges and 7 community colleges. It has more than 269,000 students enrolled in degree programs and 247,000 adult, continuing and professional education students (City University of New York 2015). Altogether, 57% are female and 56% identify as black, Hispanic or American Indian/Native American. Nearly 30% are 25 or older, 54% have household incomes of less than \$30,000, 42% are first-generation students, 38% are born outside the U.S. mainland, and 42% have a native language other than English (City University of New York Office of Institutional Research 2013, 2014). Among full-time CUNY students, 65% are providing care to other people and approximately 48% are working for pay (City University of New York Office of Institutional Research 2014).

- *It is fun to go over these in class. Students experience firsthand how their intuition tricks them into common biases and become more aware that maybe sometimes they should “think twice.”*

But another one was more cautious:

- *The notion of a trick can be very appealing to people who happen to have curiosity about the material, but very off-putting to people who don't.*

Many people attributed common mistakes to inadequate **attention**:

- *We made mistakes when we are rushing through questions and are not paying attention to details esp. to the wording of the questions.*
- *Paying attention to details can make a difference.*
- *The lake problem seems easier to see once you “slow down” and pay attention to the wording or test the two options.*

While acknowledging the importance of paying attention, several faculty participants noted that

- *many of our students don't know what to focus on when problem solving.*

The term **blindness** also appeared in many postings:

- *I appreciate you touching on the concept of student “blindness”—missing the forest for the trees.*
- *[Students] seem to want to attend to everything while not knowing where to address their focus.*
- *In my statistics class ... the sight of uncomfortable numbers ... and potential for use of one of the several similar looking formulae blind students from transcending to the next stage – the thinking skills. Many students think statistics is another math course and they need to use some formula to provide an answer.*

Responding to this, one wrote

- *I think that giving students an overall idea of where/why those numbers are relevant is important. For my statistics class, I start by putting the topic into real life context before going into the details.*

The concept of **cognitive load** came naturally to many of the participants, “even if we (I) didn't necessarily know that it was called cognitive load” as one said.

- *We use [the concept of cognitive load] all along in our teaching.*

In connection with the discussions on attention, one wrote

- *This makes me think about how we manage cognitive load by sufficiently preparing students to look for the right information. Otherwise, they end up expending their cognitive capacity on extraneous information.*

Several participants plan to introduce the concept to students directly:

- *I will definitely share the concept of cognitive load with students.*

To manage cognitive load, one math professor wrote

- *Calculators or computers can be used to perform computations and then students are more able to focus of the overall procedure and/or more complicated parts of the problem they are trying to solve.*

Nearly everyone brought up or supported the idea of **scaffolding** and breaking problems into smaller parts. From a math professor:

- *I try to scaffold new types of math problems for the students, encouraging them to write down different pieces of information in the problem so they don't have to hold it in their working memory.*

A biology professor wrote,

- *scaffolding assignments is the way we've handled cognitive load – even with non-number related tasks.*
- *Keeping a lab notebook in scientific laboratories is a universally-accepted necessity (among scientists) – it seems like a similar tool could be used for more general problem solving in all sorts of classes.*

From a participant who taught a capstone class on Criminal Law and Policy:

- *Students were generally horrified at the beginning of the semester to hear that they had to write such a long [25-page] paper. However, I broke the paper down into several smaller benchmarks (literature review, outline, etc.), and devoted several classes to students sharing drafts.*

All these comments received positive feedback from peers.

While everyone agreed with the idea of scaffolding, one faculty member mentioned,

- *the question here is how can you remove the scaffolding so that they can effectively fly on their own without the supports.*

Other concerns include students' differences in abilities and attitudes, and the time constraint that instructors face.

- *The variation between students in QR knowledge and ability to learn is always a big challenge.*
- *Tailoring a course to match the cognitive load is always a challenge in mathematics especially if students of low levels of preparation, ability, motivation, etc. are placed with students of higher levels.*
- *[Instructors] teaching those classes have to hurry up to finish all the work in the given class time.*
- *I try to use scaffolding but it often competes with the pace required to cover material.*

Responding to these comments, one participant made a distinction between working memory and long-term memory.

- *Cognitive load refers to working memory, which is your ability to hold digits or different items in your head in a way that you can manipulate them. ... I don't think it really reflects the amount of information that you introduce in a class or the pace of the class.*

Faculty's reaction to the use of a medical theme to stimulate technical discussions was mixed. While some participants were very engaged in deciphering the numerical information presented in a *New York Times* report on the benefit of taking a multivitamin daily (Rabin 2012), others, particularly humanities faculty, considered such a topic too specific and unimportant in their disciplines. An English professor wrote:

- *I'll be honest, I understood the Gigerenzer article [but] I would never be able to replicate this on my own.*

From a history professor:

- *The bigger question for me is how I can teach QR skills (which I value and envy) when I don't have them.*

And from a political science professor:

- *I have almost no interest in actually calculating it for myself.*

A psychology professor reported that

- *I have assigned this Gigerenzer et al. paper as extra reading ... but found that not even my best students were willing to tackle it."*

One anonymous response to a survey question on how to improve Unit 3 was

- *more disciplinary diversity in the reading materials in this unit.*

Our focus on medicine was a difficult decision after an internal debate during the development of the NICHE project. Because many of the NICHE participants are seasoned QR educators, we sensed a need to include a more mathematically sophisticated topic to further strengthen their skills. As predicted, however, the more advanced and technical topic did indeed alienate some participants. To manage this dilemma, we have made several parts optional. Overall, most participants appreciated the exposure to medical issues: One wrote,

- *Surprisingly, even medical professionals can be confused about statistical information presented to them.*

We hope that this revelation will translate to a powerful message when the faculty infuse QR in their courses, whether they use medical examples or not.

Concluding Remarks

The Numeracy Infusion Course for Higher Education (NICHE) was designed to foster the infusion of quantitative reasoning instruction and assessment into

courses in a broad range of disciplines. The use of cognitive illusions illustrates the promise of the project as well as some of the challenges. Through hands-on activities, faculty participants have learned how students and others jump to (faulty) conclusions through an overreliance on System 1 thinking. Their experiences have also helped them to understand the causes of some frequently observed mistakes and led them to think about ways to help our students do better. The dual-processing theory provides faculty with a foundation to convey the idea that quantitative reasoning is an effortful task that requires slow thinking.

While the course materials have generated many excellent ideas for QR instruction, an important issue mentioned by many participants is that a large proportion of CUNY students face additional challenges in language comprehension and elementary math (high school or even lower-level) operations. Additionally, faculty members who implement QR instruction face huge variation among students in their abilities and attitudes, along with a need to cover many topics in a tightly controlled curriculum.

Following enrollment in NICHE, the faculty participants teach QR-infused courses and implement the lessons they have developed during the program. They also use the assessment instruments they have created through NICHE to determine whether students have achieved their learning goals. The next QR assessment results are due at the end of July 2015, and we plan to report the results in upcoming publications. In the next paper, we will present a case study that describes students' learning of the medical diagnosis problem based on the psychological principles introduced in NICHE.

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References

- Bennett, J. 2012. *Math for Life*. Greenwood, CO: Roberts and Company Publishers.
- Casscells, W., A. Schoenberger, and T. B. Graboys. 1978. Interpretation by physicians of clinical laboratory results. *New England Journal of Medicine* 299(18): 999–1001. <http://dx.doi.org/10.1056/NEJM197811022991808>
- Chabris, C., and D. Simons. 2010. *The Invisible Gorilla: How Our Intuitions Deceive Us*. New York: Crown.
- City University of New York. 2015. "About." Retrieved January 31, 2015 (<http://www.cuny.edu/about.html>).
- City University of New York Office of Institutional Research. 2013. *Student Data Book*. Retrieved January 31, 2014

- (http://www.cuny.edu/about/administration/offices/ira/ir/data-book/current/student/ug_student_profile_f13.pdf).
- . 2014. *2014 Student Experience Survey*. Retrieved January 31, 2015 (http://www.cuny.edu/about/administration/offices/ira/ir/surveys/student/SES_2014_Report_Final.pdf).
- Dehaene, S. 2011. *The Number Sense: How the Mind Creates Mathematics*. Revised and Expanded Edition. New York: Oxford University Press.
- Devlin, K. 2005. *The Math Instinct*. New York: Thunder's Mouth Press.
- Eddy, D. M. 1982. Probabilistic reasoning in clinical medicine: Problems and opportunities. In *Judgment under Uncertainty: Heuristic and Biases*, ed. D. Kahneman, P. Slovic, and A. Tversky, pp. 249–267. Cambridge, UK: Cambridge University Press. <http://dx.doi.org/10.1017/CBO9780511809477.019>
- Evans, J. S. 2008. Dual-processing accounts of reasoning, judgment, and social cognition. *Annual Review of Psychology* 59: 255–278. <http://dx.doi.org/10.1146/annurev.psych.59.103006.093629>
- Frederick, S. 2005. Cognitive reflection and decision making. *Journal of Economic Perspectives* 19(4): 25–42. <http://dx.doi.org/10.1257/089533005775196732>
- Gigerenzer, G., W. Gaissmaier, E. Kurz-Milcke, L. M. Schwartz, and S. Woloshin. 2009. Knowing your chances. *Scientific American Mind* 20: 44–51. <http://dx.doi.org/10.1038/scientificamericanmind0409-44>
- Henrich, J., S. J. Heine, and A. Norenzayan. 2010. Most people are not WEIRD. *Nature* 466: 29. <http://dx.doi.org/10.1038/466029a>
- Hoffrage, U., and G. Gigerenzer. 1998. Using natural frequencies to improve diagnostic inferences. *Academic Medicine* 73: 538–540. <http://dx.doi.org/10.1097/00001888-199805000-00024>
- Kahneman, D. 2011. *Thinking, Fast and Slow*. New York: Farrar, Straus and Giroux.
- Kahneman, D., and S. Frederick. 2002. Representativeness revisited: Attribute substitution in intuitive judgment. In *Heuristics and Biases: The Psychology of Intuitive Judgment*, ed. T. Gilovich, D. Griffin, and D. Kahneman, pp. 49–81. Cambridge, UK: Cambridge University Press. <http://dx.doi.org/10.1017/CBO9780511808098.004>
- Larrick, R. P., and J. B. Soll. 2008. The MPG illusion. *Science* 320(5883): 1593–1594. <http://dx.doi.org/10.1126/science.1154983>
- Lave, J. 1988. *Cognition in Practice: Mind, Mathematics and Culture in Everyday Life*. Cambridge, UK: Cambridge University Press. <http://dx.doi.org/10.1017/CBO9780511609268>
- Madison, B. L. 2012. If only math majors could write. *Numeracy* 5(1): Article 6. <http://dx.doi.org/10.5038/1936-4660.5.1.6>
- Madison, B. L., S. Boersma, C. L. Diefenderfer, and S. W. Dingman. 2012. *Case Studies for Quantitative Reasoning: A Casebook of Media Articles*. 3rd edition. Boston, MA: Pearson Learning Solutions.
- Miller, G. A. 1956. The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review* 63(2): 81–97. <http://dx.doi.org/10.1037/h0043158>

- Parker-Pope, T. 2011. Mammogram's role as savior is tested. *New York Times* October 24: p. D1.
- . 2012. Prostate test advice appears unheeded. *New York Times* April 25: p. A17.
- Paulos, J. A. 1988. *Innumeracy*. New York: Hill and Wang.
- Pinker, S. 1997. *How the Mind Works*. New York: Norton.
- Rabin, R. C. 2012. Curbing the enthusiasm on daily multivitamins. *New York Times* October 23: p. D1.
- Redish, E. F. 2014. How should we think about how our students think? *American Journal of Physics* 82(6): 537–551. <http://dx.doi.org/10.1119/1.4874260>
- Reif, F. 2008. *Applying Cognitive Science to Education: Thinking and Learning in Scientific and Other Complex Domains*. Cambridge, MA: MIT Press.
- Schwartz, L. M., S. Woloshin, F. J. Fowler, and H. G. Welch. 2004. Enthusiasm for cancer screening in the United States. *Journal of the American Medical Association* 291(1): 71–78. <http://dx.doi.org/10.1001/jama.291.1.71>
- Simons, D. 2010. The monkey business illusion. YouTube. https://www.youtube.com/watch?v=IGQmdoK_ZfY. (accessed March 25, 2015).
- Sloman, S. A. 1996. The empirical case for two systems of reasoning. *Psychological Bulletin* 119(1): 3–22. <http://dx.doi.org/10.1037/0033-2909.119.1.3>
- Tversky, A., and D. Kahneman. 1974. Judgment under uncertainty: Heuristics and biases. *Science* 185(4157): 1124–1131. <http://dx.doi.org/10.1126/science.185.4157.1124>
- Wang, F., and M. Dedlovskaia. 2009a. Do your students suffer from the MPG illusion? We can help! *New York State Mathematics Teachers' Journal* 59(3): 123–128.
- . 2009b. A lesson for demystifying the MPG illusion. *New York State Mathematics Teachers' Journal* 59(3): 129–135.
- Wegwarth, O., L. M. Schwartz, S. Woloshin, W. Gaissmaier, and G. Gigerenzer. 2012. Do physicians understand cancer screening statistics? A national survey of primary care physicians in the United States. *Annals of Internal Medicine* 156(5): 340–349. <http://dx.doi.org/10.7326/0003-4819-156-5-201203060-00005>
- Wilder, E. I. 2012. Teaching quantitative literacy/quantitative reasoning (QL/QR skills): A numeracy infusion course for higher education. *Proceedings of the 2012 Joint Statistical Meetings*, Alexandria, VA: American Statistical Association. Pp. 2676–2690.
- Wilder, E. I., E. Waring, F. Wang, and D. Hurley. 2013. The key components of a numeracy infusion course for higher education (NICHE): Using best practices to train faculty. *Proceedings of the 2013 Joint Statistical Meetings*, Alexandria, VA: American Statistical Association. Pp. 1850–1864.
- . 2014. A numeracy infusion course for higher education (NICHE): Strategies for effective quantitative reasoning (QR) instruction. *Proceedings of the 2014 Meeting of the International Conference on Teaching Statistics (ICOTS)*.