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Reservoir Management For Flood And Drought Protection Using Infinite Horizon Model Predictive Control

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RESERVOIR MANAGEMENT FOR FLOOD AND DROUGHT PROTECTION USING INFINITE HORIZON MODEL PREDICTIVE CONTROL

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Model Predictive Control (MPC) is a control method that can be employed for optimal operation of adjustable hydraulic structures. MPC selects the control to apply on the system by solving in real time an optimal control problem over a finite horizon. The finiteness of the horizon is both the reason of MPC's success and its main limitation. MPC has been in fact successfully employed for short-term reservoir management. Short-term reservoir management deals effectively with fast processes, such as floods, but it is not capable of looking sufficiently ahead to handle long-term issues, such as drought.

We propose an Infinite Horizon MPC solution tailored for reservoir management. The proposed solution structuring the input signal by use of triangular basis functions. Basis functions reduce the optimization argument to a small number of variables, making the control problem solvable in a reasonable time. Constraints on the input are easy to implement.

. We tested this solution on Manantali Reservoir, on the Senegal River. The long-term horizon offered by IH-MPC is necessary to deal with the strongly seasonal climate of the region, reducing both floods and droughts.

INTRODUCTION

Reservoirs are valuable assets to ensure water security. They balance the variability of water availability, storing water when it is abundant and releasing it when it is more necessary.
Design of optimal reservoir operation is a complex control problem that has been typically solved with Stochastic Dynamic Programming (SDP) [1]. SDP is an off-line optimal control method, i.e. its solution provides the optimal control rules. Optimal control rules suggest the optimal control in function of the system state. However, SDP is a functional optimization particularly complex to solve numerically. The so-called "curse of dimensionality" limits its application to simple systems, made of few variables. During the years, different approximations have been proposed to reduce the computational complexity [6]. Another drawback of SDP is the requirement to express all state transitions explicitly. This is called the

"curse of modeling". SDP is often employed at a monthly time step, and it is used for strategic management only, not for operational management.
Model Predictive Control (MPC) is a real time optimal control technique [3]. MPC offers different advantages on other control methods, suffering neither the curse of dimensionality, nor the curse of modeling. In reservoir management, for example, the hydrological input trajectory can be used straightforwardly in the optimal control problem.
MPC finds a control that is optimal for a finite horizon. However, reservoirs memory is generally longer than the optimization horizon. In this case MPC can be employed for short-term optimal control method, but it does not ensure long-term optimality, as effects after the optimization horizon are not included.
There are different options to integrate the long-term effects within the MPC optimal control problem. They refer to infinite horizon MPC [3]. Among these, a suitable approach is input structuring by use of basis function [7]. We propose the use of triangular basis functions, by which constraints on the inputs can be easily included, preserving the linearity of the optimization problem. We show an application on a test case.

METHOD

In MPC, at each control instant t_0, the control actions are obtained by solving on-line, i.e. at each control time step, the following optimal control problem.

$$\min_{\{u\}_{t=0}^{h-1}} \sum_{t=1}^{h-1} g_t\left(x_t, u_t, d_t\right) + g_h\left(x_h\right) \tag{1.1}$$

$$x_{t+1} = Ax_t + Bu_t + Cd_t \tag{1.2}$$

$$0 \leq u_t \leq u_{\max} \tag{1.3}$$

$$x_{\min} \leq x_t \leq x_{\max} \tag{1.4}$$

$$\mathrm{c}_t\left(x_t, u_t\right) \leq 0 \tag{1.5}$$

$$x_0, \{d\}_{t=1}^{h} \text{ given} \tag{1.6}$$

In Equations (1), t is the time index, going from 1 to the final time step of the control horizon, h; g_t is the time-step cost function, g_h the final penalty that sums up all the future costs beyond the control horizon; $x_t \in R^{Nx}$ is the state, $u_t \in R^{Nu}$ the control, and $d_t \in R^{Nd}$, the disturbances, all vectors at time t. Matrixes A, B, and C, in Equation (2), define the system model, and $c(\cdot)$ are other inequality constraints that can be present. Control and disturbances are respectively controllable and uncontrollable system inputs. The initial conditions, x_0, and the series of deterministic disturbances, $dt\ \forall t$, are given.
If MPC is to be employed for reservoir management, the state is the volume, release and spillage are controls, and the hydrological inputs are the disturbances. If downstream routing is also included, the state must include river state and the disturbances the affluent to the controlled river.
MPC uses the system model Equation (1.2) to predict the system behavior in response to the control actions over a finite future horizon, called prediction horizon. The model takes the current state of the system as initial state, and the deterministic forecasts of the disturbances. The cost function defines the operational goal. Once system model, cost function, initial state and forecasted disturbance are given, MPC selects the control trajectory for the adjustable

structures that offers the optimal behavior for the future prediction horizon. At each time step, only the first value is applied to the real system, then the horizon is shifted ahead and the procedure is repeated at the next controlling instant using the latest up-to-date information.

The cost-to-go function g_h should theoretically sum up all the costs from the instant h to infinite for having left the system in x_h at the end of the control horizon. In practice, however, this function is difficult to obtain. If g_t is a Liapunov function, and the control horizon is sufficiently long, MPC ensures stability [3], even without g_h. An example of Liapunov function widely used in MPC for trajectory following problems is a quadratic penalty on the state deviance from the optimal trajectory. This property is extensively used in MPC applications, where the objective is trajectory tracking. In reservoir operation, however, step-costs are rarely Liapunov functions.

An alternative way to guarantee stability is adding a constraint on the final state. However, this solution requires the identification of a desired final state, which can be unknown. This is often the case in reservoir operation. Moreover, if the horizon is too short, this MPC configuration run the risk of having an undetermined problem.

Generally speaking, infinite horizon MPC are different solutions to deal with the finiteness of the optimization horizon [3]. Input structuring [7] is a type of infinite Horizon MPC that is particularly suited for reservoir operation. In input structuring, the control are not optimized directly, but they are arranged according to a convenient form. Among the different forms of input structuring, we selected basis functions, for they can follow the yearly periodicity of natural systems. Equation (2) shows input structuring using basis function.

$$u_t = \sum_{i}^{N} \lambda_i \cdot f_i(t) \tag{2}$$

where $f_i(t)$ are fixed time-variant functions and λ_i are N parameters to optimize. Thanks to basis function, we can extend the control horizon without having an explosive growth of control variables.

Basis functions have been already used for system identification [8]. However, in MPC, constraints on u_t imply constraint on λ_i. For reservoir management, u_t are generally limited between zero and a maximum value, i.e. the control variables are constrained within a hypercube. Using triangular basis functions, hypercube constraints become linear constraints on λ_i. In this case, the infinite horizon MPC problem can be written as in Equations (3).

$$\min_{\lambda_k} \sum_{t=1}^{h-1} e^{-rt} \cdot g_t\left(x_t, u_t, d_t\right) \tag{3.1}$$

$$x_{t+1} = Ax_t + Bu_t + Cd_t \tag{3.2}$$

$$u_t^T = M_t \cdot \left[\lambda_1, \ldots, \lambda_K\right] \tag{3.3}$$

$$\lambda_k \geq 0 \tag{3.4}$$

$$C_k \lambda_k \leq u_{k,\max} \tag{3.5}$$

$$x_0, \{d\}_{t=1}^{h} \text{ given} \tag{3.6}$$

In Equation (3), r is the discount rate, $\lambda_k \in R^N$ are the vectors of optimization parameters, for $k=\{1,\ldots,N_u\}$. M_t is a $1 \times N$ vector defined by the triangles, as in Equation (4). Each triangle is defined by its peak instant, T_i, its left base L_i, and its right base R_i. Figure 1 shows a graphical visualization of the triangles and their parameters.

$$M_t(i) = \begin{cases} 1 - \dfrac{T_i + t}{L_t} & \text{for } T_i + L_i < t \le T_i \\ 1 + \dfrac{T_i + t}{R_i} & \text{for } T_i < t \le T_i + R_i \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

C_k are $N \times N$ matrixes of constraints on λ_k. If triangles are symmetric, i.e. $L_i = R_i$, then each cell $C_k(i,j)$ is $max(1-(T_i-T_j)/L_i, 0)$. The discount rate is required for convergence, but it can also be interpreted as discount factor. Constraints on the state x can be integrated as soft constraints in the objective function. Using this formulation, the number of optimization variables increase with the number of triangles, N, and not with the horizon length, h. Then, h can be extended to a much larger number than in classic MPC.

Extending the long term far beyond the horizon where forecast are reliable requires the inclusion of climatic information and add large uncertainty. Uncertainty that jeopardizes MPC's robustness, can be dealt with method for syntetic robust methods [2], such as Tree-Based MPC [5] or other methods [2].

Triangles selection

Triangles selection is an analyst's choice that depends on the system characteristics. We give here some general indication, highlighting the advantages of some specific shapes.

We suggest selecting progressive triangles, i.e. $L_i < R_i$, $L_{i+1} > L_i$, in the early stage of the horizon. In MPC in fact, only the first control value will be applied to the systems. The first control is more sensitive to controls that are closer in time; therefore it is better to have a higher degree of freedom in the initial part of the horizon. The first triangle should have its peak T at the initial time step.

Sufficiently far from present condition, periodicity becomes dominating. For $t>P$, where P is the system periodicity, triangles having L_i and R_i equal to $P/2$ are able to follow the periodic trend. In this part of the horizon, T should be equal to $P \times j$ and multiple of $P \times (j+1/2)$, where j is an integer going from zero to the number of years contained in the control horizon.

Selection of independent triangles, such that $L_{i+1} = R_i$, and $T_{i+1} = T_i + L_i$, makes constraints independent. In this case the constraints can be written as $0 < \lambda_k < u_{k,max}$, $\forall i,k$.

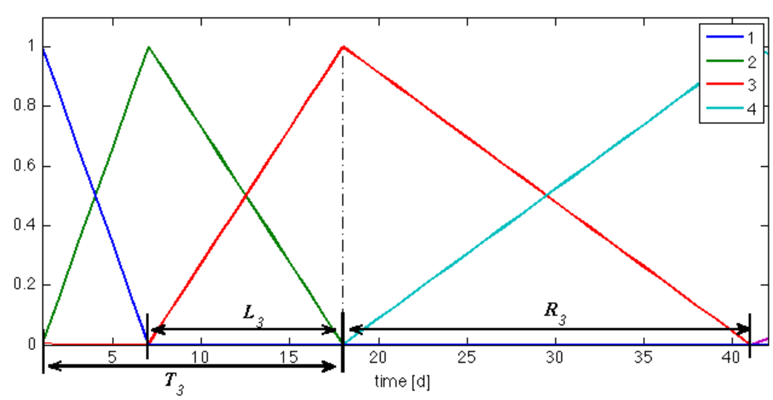

Figure 1. Example of basis triangular functions. For triangles 3, the peak time T, the left base L and the right base R are highlighted.

TEST-CASE

The method is tested on Manantali reservoir. Manantali is located in Mali, on the Senegal River, and it is present use is mainly for electricity only. Plans for agro-business on the Senegal River valley could change the management in the short future [4]. In this case, the objective of energy production must be balanced with flood and drought protection. The hydrology on the Senegal River is strongly seasonal, influenced by the tropical rainy season in the upper basin.

The reservoir is modeled by the continuity equation.

$$v_{t+i} = \mathrm{v}_t + \Delta t \cdot \left(d_t - r_t - s_t\right) \tag{5}$$

Where v is the volume, which is the system state. The system controls are the release through the turbines, r, and the spillage, s. Controls are constrained between zero and maximum release, $r_{\max}$, and maximum spillage, $s_{\max}$. The operatiove volume is constrained between v_{max} and v_{min}. In this experiment, the operative volume is reduced to increase the difficulty of the reservoir operation. The inflow to the reservoir d is the system disturbance. State constraint will be included in the problem as soft constraint. Evaporation from the reservoir and other losses are neglected.

The hydrological input d_t uses both real time forecast and climatic information, gliding from the real time information into the climatic one going ahead on time. Therefore d_t is the Bayesian Model Averaging of the forecasted inflow, d^{fr}, and the climatic one, d^{cl}, weighted by their reliability.

$$d_t = B_t \cdot d_t^{fr} + \left(1 - B_t\right) \cdot d_t^{cl} \tag{6}$$

Where B_t is the product of the inflow autocorrelation ϕ_t, from 0 to t. $\Pi_{\tau=1,\dots,t}\ \phi_t$. Use of an average climatic year as climatic disturbance, d^{cl} , would filter out the extremes. Instead of an average value, the controller will use a different observed inflow at each control time step, selected from the observed inflow data. When the reservoir is big enough, its slow dynamic will serve as low pass filter, which will average out the effects of different inflow years used at each time step. This is expected to have little effects on each single control decision. Nonetheless, the question on how to deal with the presence of a relevant uncertainty affecting the hydrological input beyond the forecast predictability in infinite horizon MPC is still open.
The reservoir management, in this experiment, is designed for three objectives, flood and drought protection, and energy production. The first two have higher priority on the latter, and are weighted more than the electricity. Flood and drought protection are represented by the cost function, g_t^{tg} , in Equation (7.2). Keeping the total discharge as close as possible to the target flow, q_{tg}, set to 200 m3/s, attains both flood and drought protection. The electricity production objective, Equation (7.1), is the linearized energy production function. The negative sign means that its value must be maximized.

$$g_t^e = -(r_t + k_0 \cdot v_t) \tag{7.1}$$

$$g_t^{tg} = \left(r_t + s_t - q_{tg}\right)^2 \tag{7.2}$$

Table 1. Peak time (T), Left base (L), and right side (R) defining the 10 triangles.

i	1	2	3	4	5	6	7	8	9	10
T	1	7	18	41	87	178	269	360	543	726
L	0	6	11	23	46	91	91	91	182	182
R	6	11	23	46	91	91	91	182	182	182

Where w_{tg} is 0.8 and w_e is 0.2. The decaying factor r is set to *0.973*, chosen to loose *97%* of its memory at the end of the 3 years horizon. We use 10 independent triangles, defined by T_i, S_i, and L_i as in Table 1.

The aggregated objective function is therefore the weighted sum of these components.

$$g_t^{tot} = w_e \cdot g_t^e + w_{tg} \cdot g_t^{tg} \quad (8)$$

RESULTS

To evaluate the proposed method, we separately analyze the role of input structuring and that of uncertainty, isolating their effects in departing from the optimal solution. We analyze three solutions: i) Infinite Horizon MPC using triangular input structuring and realistic forecast, ii) Infinite Horizon MPC using triangular input structuring and perfect forecast, iii) Infinite Horizon MPC with no input structuring and perfect forecast. Comparing first and second case shows the loss due to uncertainty; comparing second and third shows the loss due to input structuring. In the third case, solving the optimal control problem requires a large computation time, and it is not applicable in reality. However, it serves as upper boundary of system performance. Three indicators are employed to evaluate performance: "Yearly energy production" for electricity production, "percentage of days when flow is lower than 100 m^3/s" for drought protection, percentage of days when flow is larger than 800 m^3/s, for flood protection. The first indicator is to be maximized, the others to be reduced. We run a four-year simulation, from the 1st January 2005 to the 31st December 2008.

Table 2 summarizes the results for the three cases under evaluation for the three indicators. Data shows that the perfect forecast contributes more to flood protection than for energy production, and even less flood reduction. In fact, the low flow recession curve is much more predictable, whereas the rising part, in the rainy season, is much more affected by uncertainty. Results between the second and third case, i.e. MPC with input structuring and MPC without structuring, show that the higher freedom of the latter can largely improve the drought protection. This is counterintuitive and it should be further analyzed.

Figure 2 shows the results of an optimization run at a given control instant, for the first 365

Table 2. Results for the three analyzed configuration

From 1/1/2005 To 31/12/2008 Case\Indicator	Electricity production [MWh/year]	Flow below 100 m^3/s [%]	Flow above 800 m3/s [%]
IH-MPC, basis functions	9.1 E5	18%	2%
IH-MPC, basis funct.-perf.forecast	9.5 E5	21%	0%
IH-MPC, perfect forecast	9.6 E5	9%	0%

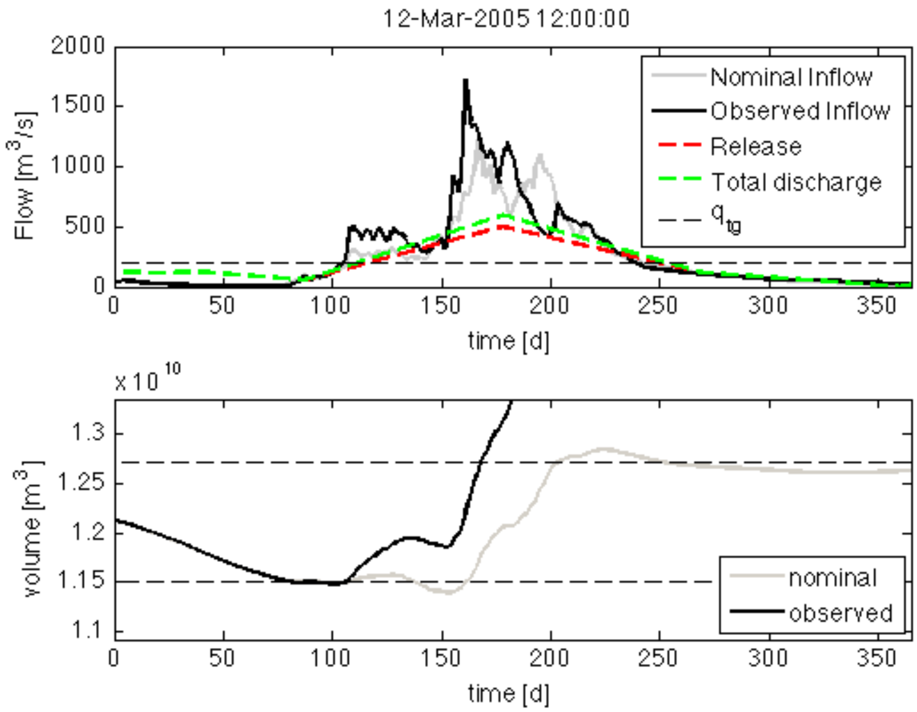

Figure 2. Optimization results at a 12 March 2005. First year. Plot above: Inflow and outflow on time. Nominal Inflow (gray line) is the inflow used in the optimization. Observed Inflow is the actual inflow (black line). Release (red line) is the water released through the turbines. Total discharge is the sum of release and spillage (green line). Plot below, water volume in case of nominal (gray line) and observed inflow (black line) on time. The operational volume is the space between v_{max} and v_{min} (dashed lines).

days. The plot above shows the hydrologic inflow and the controlled outflow from the reservoir on time. The controlled outflow tries to balance the hydrological variability. In the dry season outflow is higher than the inflow. The balance is negative and the volume in the reservoir decreases, as it is possible to see in the plot below of Figure 2, showing the water volume on time. The control slowly empties the reservoir in the dry season, keeping a low water volume until initial part of the high-flow, in preparation of the peak. The reservoir is eventually filled, and spillages are minimized. The plot below shows small state constraints violation, at around $t=150$ and $t=220$. These constraints violations are small and happen sufficiently ahead on time, therefore they do not constitute a problem.

CONCLUSIONS

This paper presented an Infinite Horizon Model Predictive Control method specifically designed for reservoir operations. Input structuring can be employed thanks to the slow dynamic of reservoirs. Basis functions, often employed in system identification, were used here for control. We selected triangular basis function for their ability to handle hypercube

constraints on inputs, and we gave some indication on how to select these triangles. We suggested the selection of progressive independent triangles in the early stage, and periodic ones ahead on time. In water systems, in fact, both water demand and hydrological processes are periodic. The proposed method largely reduces the number of variables to be optimized, reducing the problem complexity.
We tested the proposed method for the operational management of Manantali reservoir, on the Senegal River, with the objective of flood and drought protection, and energy production. Analysis shows that both uncertainty and input structuring reduce performance. This requires further investigation. An open question in Infinite Horizon MPC is how to deal with the presence of a relevant uncertainty affecting the hydrological forecast, which enters in the optimal control problem as deterministic disturbance. Nonetheless, The proposed method can potentially handle much larger systems, made of multiple reservoirs or routing downstream of the reservoir.

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