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## **BAYESIAN MODEL AVERAGING APPROACH FOR REDUCING URBAN FLOODING DAMAGE ESTIMATION UNCERTAINTY**

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Uncertainty analysis is useful to define the level of reliability of a modelling application, but operational methods are needed to identify the best modelling structure for a specific problem based on uncertainty reduction criteria. One interesting example is given by flood damage estimation problem where different possible modelling solution and flood damage estimation can be used depending on the case study. Past literature showed that several modelling structures may be equally reliable in terms of calibration ability but they may produce different uncertainty levels. The use of the Bayesian approach for uncertainty analysis has been stimulated by its rigorous theoretical framework and by the possibility of evaluating the impact of new knowledge on the modelling predictions. Model structure uncertainty is associated with the assumptions reflected in model conceptualization and mathematical structure. An unfortunate truth in model development is that no matter how many resources are invested in developing a particular model, there remain conditions and situations in which the model is unsuitable to give an accurate forecast. Reliance on a single model typically overestimates the confidence and increases the statistical bias of the forecast. Bayesian model-averaging (BMA) techniques look to overcome the limitations of a single model by linearly combining a number of competing models into a single new model forecast. This method showed that a pooled forecast of competing models outperformed any single model forecast. In the present paper, BMA was applied to several flood damage estimation models in order to identify the best combination of models to analyse urban flooding distribution in Palermo city centre (Italy).

### **INTRODUCTION**

As result of the ongoing climate change and imperviousness of urban environment, frequency and impacts of urban flooding have increased in the last decades rising the interest of researchers and practitioners on this topic. A sustainable management of flooding in urban areas plays an important role in protecting people safety and their socio-economic activities.

According to a proactive management of natural disasters, the hydraulic analysis of urban flooding phenomena and the evaluation of the expected damages offer essential information both for stakeholders and for involved population. A quick estimation of flood damage may support the first ones in allocating resources for recovery and reconstruction after a flooding event or in planning adequate flood control measures in long term and in carrying out reliable

cost-benefit analysis of these measures [1]. At the same time the knowledge about the expected consequences of a flooding may facilitate the birth of a flood resilient society, that is the preparedness of involved people about flood risks and damages and how to act in the event of a flood [2].

The international literature includes several procedures for flood damage estimation in urban areas which often differ about methods adopted, aims pursued and availability of source data required. A rough classification can be done between ex-post or ex-ante analysis. In ex-post analysis, a damage appraisal at local scale is obtained by accounting in detail the object-specific damages after a flooding event. Ex-ante analysis provides the expected damage for a potential flooding event in the investigated area. The expected damage results from an a-priori appraisal obtained by interpolating real damage data related to historical flooding events [3,], or by accounting the effects of flood in terms of the depreciation of assets (based on historical values or replacement values) or a percentage of the market value of the flooded properties [4].

In this kind of analysis, the expected flood damage is usually evaluated by means of damage functions [5]. Damage functions describe the relationship between the level of damage and the hydraulic characteristics of flood (e.g. the flooding depth, the combination of water depth and velocity, the duration [1], or the load of sediments) with respect to different land uses, characteristics and types of harmed goods (buildings, household furnishings, vehicles, etc.), and socio-economic conditions of the affected area [4]. The analysis is usually focused only on direct tangible damages of public and private properties (e.g., buildings, cars, roads) as a function of inundation depth. Direct tangible damage is easily turned into monetary costs and related to flooding hydraulic features [3, 7]. Depth-damage functions are normally defined by interpolating flooding depth and damage data usually obtained by systematic survey procedures that analyze historical flood events, insurance claims data, or synthetic damage data. Several regression laws with different level of simplification can be used as depth-damage functions thus influencing the damage appraisal. Moreover, flooding data are often piecemeal, affected by measuring errors and spatially aggregated [7, 8]. In consequence, the flood damage assessment is usually affected by a degree of intrinsic uncertainty that cannot be realistically eliminated [9].

Despite the tremendous amount of resources invested in developing a model, no one can convincingly claim that any particular model in existence today is superior to other models for all type of applications and under all conditions. Different models have strengths in capturing different aspects of physical processes. Relying on a single model often leads to overestimates the confidence and increases the statistical bias of the forecast. This has motivated a number of researchers to advocate multi-model methods.

Bayesian model averaging (BMA) is a statistical procedure that looks to overcome the limitations of a single model by combining a number of competing models into a single new model forecast [10]. BMA predictions are weighted averages of the individual predictions from the competing models. The BMA weights, all positive and summing up to 1, reflect relative model performance because they are the probabilistic likelihood measures of a model being correct given the observations. This method showed that a pooled forecast of competing models could outperform any single model forecast. BMA also provides a more realistic description of the predictive uncertainty that accounts for both between-model variances and in-model variances [10, 11]. Recently, BMA has been used in weather forecasting [12], in groundwater simulation, and to estimate the uncertainty of hydrological model structures [10, 13].

This paper explores the use of BMA for flood damage predictions from different flood damage estimation models (depth damage curves).

We are interested in how BMA scheme can be used to improve both the accuracy and reliability of the damage analysis predictions in urban area. To this aim the uncertainty linked to the choice of the depth-damage function adopted in the damage analysis was investigated by analyzing and comparing the predictions of four different depth damage functions (individual models) and of a BMA multi-model ensemble with the real damage data observed in a case study watershed. Particularly, BMA was applied to identify more reliable damage predictions for urban flooding occurring in Palermo city centre (Italy).

### THE BAYESIAN MODEL AVERAGING (BMA)

The Bayesian Model Averaging (BMA) is a statistical methodology that aims to combine inferences and predictions of several different models and to jointly assess their predictive uncertainty [14]. To describe Bayesian Model Averaging methodology, consider a quantity,  $y$ , to be forecasted, such as the magnitude of the flooding damage for a given flooding depth.

Assume we have a set of  $K$  models denoted by  $M_k$  with  $k=1, 2, \dots, K$ , giving us independent model forecasts and let  $D$  the observed values of  $y$ . According to the given observed data  $D$ , the model ensemble posterior density function (PDF) of  $y$  is given by the BMA method as:

$$p(y_{\text{BMA}} | D) = \sum_{k=1}^K p(y | M_k, D) \cdot p(M_k | D) \quad (1)$$

where  $p(y | M_k, D)$  is the posterior distribution of  $y$  on the condition of the given sample  $D$  and model  $M_k$  and  $p(M_k | D)$  is the posterior model probability (PMP) $_k$  of  $M_k$  or the probability that the model  $M_k$  is the optimal model on the condition of the given data  $D$ . (PMP) $_k$  represents the likelihood of model  $M_k$  being the correct model or the weights  $w_k = p(M_k | D)$  of model  $M_k$ . According to the Bayes' law the weight,  $w_k$ , related to  $M_k$  can be expressed as follow:

$$w_k = p(M_k | D) = \frac{p(D | M_k) \cdot p(M_k)}{\sum_{n=1}^K p(D | M_n) \cdot p(M_n)} \quad (2)$$

where  $p(M_k)$  is the prior probability of the model  $M_k$  and  $p(D | M_k)$  is the marginal likelihood of the model  $M_k$ . In the present study we worked with the log-likelihood function of Eq.2 because more easily to compute than the likelihood function itself. According to Eq. 1, the posterior mean and the variance of the BMA prediction  $y_{\text{BMA}}$  can be expressed as:

$$E[y_{\text{BMA}} | D] = \sum_{k=1}^K p(M_k | D) \cdot \int_{-\infty}^{+\infty} y \cdot p(y | M_k, D) dy = \sum_{k=1}^K w_k \eta_k \quad (3)$$

$$\text{Var}[y_{\text{BMA}} | D] = \sum_{k=1}^K w_k \cdot \left( \eta_k - \sum_{k=1}^K w_k \eta_k \right)^2 + \sum_{k=1}^K w_k \sigma_k^2 \quad (4)$$

where  $\eta_k$  and  $\sigma_k^2$  are the expectation and the variance of  $y$ , respectively, on the condition of the given sample  $D$  and model  $M_k$ . The posterior mean of the BMA prediction, is usually used as quantitative forecasting, and is obtained by weighting the individual model predictions  $\eta_k$  by the likelihood  $w_k$  that the individual model  $M_k$  is the optimal model on the condition of the

given data  $D$  [12, 13]. The variance of the BMA prediction, (Eq.4) is essentially obtained as sum of two terms: the first one, denotes the variance between models, while the second one, expresses the weighted average of the within model variance. It represents an important uncertainty measure that better describes the predictive uncertainty than in a non-BMA scheme where uncertainty is estimated based only on the variance between models and consequently results in under-dispersive predictions [10].

In summary, the application of BMA scheme requires to evaluate the posterior distribution of  $y$ ,  $p(y | M_k, D)$ , and the weight,  $w_k$ , for each model being considered in the ensemble.

## METHODOLOGY APPLICATION

In the present study the Bayesian Model Averaging methodology was applied for account for the uncertainty linked to the structure of the damage curve adopted in the flood damage appraisal. To this aim, for each historical flooding event analyzed, the simulated damage obtained by 4 different formulations of damage curve functions such as linear (POLY1), polynomial-2ord (POLY2), exponential (EXP) and power with upper limit (POWER) and by the prediction of BMA methodology were compared to measured damage data and their inherent uncertainty was analyzed. The analysis was applied to a real case study, the Centro Storico catchment of Palermo (Italy), the oldest part of the city, strongly urbanised and with a very old drainage system, where local surface flooding due to the system insufficiency often occurs even for high-frequency rainfalls. During 1993-2008, several parts of the watershed were affected by about 30 flooding events due to the system's surcharge [8, 9]. A detailed database on flooded area, water depth and volume, duration and damaged properties have been collected for these events by querying fire brigades and insurance companies [8]. Figure 1 shows together with the damage curve obtained by the application of BMA methodology the four formulations of damage curves for vehicles adopted in the present study. Those functions are the median damage curves of a set of 464 families of curves obtained by adopting the least squares minimization approach to interpolate insurance claims dataset related to the historical flooding events affecting the investigated watershed by excluding information drawn from one flooding location or one flooding event. For more details see [7].

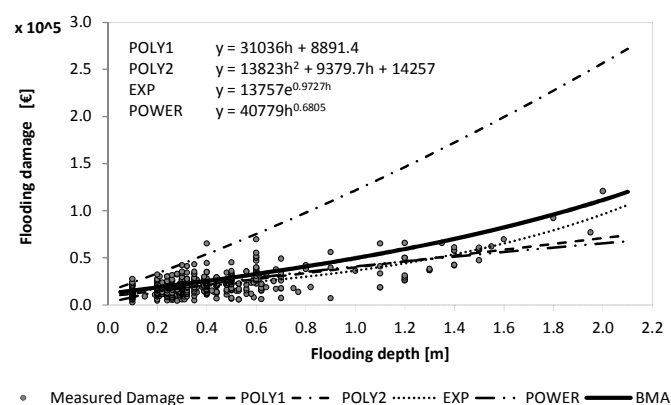


Figure 1. Depth-damage curves and measured damages for vehicles related to the case study

In literature several hypothesis are usually make on the posterior distribution,  $p(y | M_k, D)$ , and on the prior probability  $p(M_k)$  of the model  $M_k$  for an easily BMA implementation [10,14]. In the present study some of those assumptions were made to carry out the analysis.

According to the original BMA method [12], for each individual model (damage curve)  $M_k$  and for each historical flooding event analyzed, the posterior distribution of  $y$ ,  $p(y | M_k, D)$ , was assumed Gaussian with mean  $\mu_k$  and variance  $\sigma_k^2$  equal to mean and variance of the individual model prediction  $y^k$ . In the present study the Gaussian assumption was made for computational convenience. BMA scheme in fact could be applied by assuming other probability distributions thanks to the adoption of statistical techniques such as Markov Chain Monte Carlo (MCMC) method capable of simulating any complex probability distribution [10]. However using different statistical distribution to describe  $p(y | M_k, D)$  resulted in very similar conclusions as the normal conditional distribution presented in the study [10].

Moreover, to compute model weights was made the hypothesis regarding the Gaussian distribution of the residuals between the model and the observations assuming the null average and variance  $\sigma_{k,e}^2$  [21]. According to such hypothesis, the term  $p(D | M_k)$  presents in the Eq. 2 can be written in the multiplicative form as follow:

$$p(D | M_k) = \prod_{i=1}^m \frac{1}{\sqrt{2\pi\sigma_{e,k}^2}} \exp\left(-\frac{(D_i - y_i^k)^2}{2\sigma_{e,k}^2}\right) \quad (5)$$

where, for the considered historical event,  $y_i^k$  are the modelling responses corresponding to the  $m$  available measurements  $D_i$  of flooding damage in the analyzed watershed, and  $\sigma_{k,e}^2$  is the variance of the  $k^{\text{th}}$  model residual. The application of Eq. 5 is based on the hypothesis that residuals are homoscedastic, independent and identically distributed in time. That hypothesis should be verified considering that the probability distribution of damage and of damage residuals is usually non-Gaussian. Therefore, for each historical flood event, both modeled and observed damage data were pre-processed using the Box–Cox transformation prior to the BMA procedure, so that the transformed variables were close to the Gaussian distribution [10, 16].

According to the previous hypothesis, the individual model weights  $w_k$  were obtained solving Eq.2 by a Bayesian updating approach based on a recursive definition of Bayes law.

According to this approach the model weights,  $w_k$ , resulted as a weighted average of its current forecast performance weighted by the conditional probabilities of the previous step. Namely, at first step, the weights  $w_k$  were obtained considering as observed data  $D$  the damages related to the first historical flooding event occurring in the analysed watershed, and assuming equal to  $1/K$  the prior probability  $p(M_k)$  of all individual model  $M_k$ . At second step, individual model weights  $w_k$  were obtained by Eq.2 considering the damages related to the second historical flooding event as observed data  $D$ , and assuming as prior probability  $p(M_k)$  of the model  $M_k$ , the related weight obtained in the previous step and so on. At the end of the updating approach were obtained the weights of the individual models taking into account the information linked to all flooding event monitored in the watershed. Figure 2 illustrates an example of how the BMA methodology produces a multi-model forecast PDF. Figure 2b shows the individual model weights obtained at the end of the Bayesian updating approach: the exponential damage curve (EXP) obtained the higher likelihood to be the best model with weight equal to 0.5661 while the polynomial -2ord (POLY2) presented the lower likelihood equal to 0.0623.

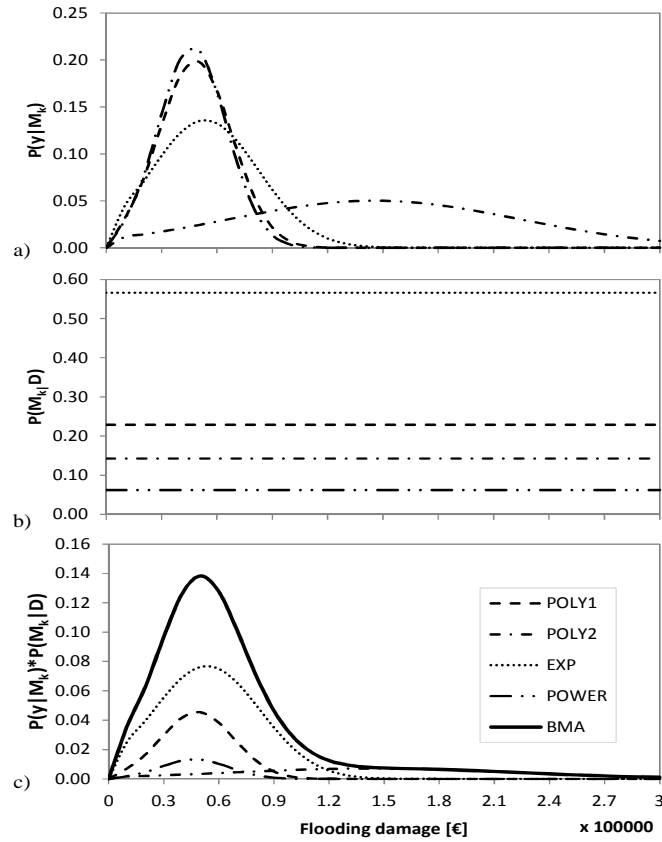


Figure 2. Implementation of Bayesian model averaging on four models (depth-damage curves): a) posterior distribution of damage forecast for each model, b) individual model weights obtained at the end of the Bayesian updating, c) model forecasts weighted by normalized likelihood, and weighted forecast summed to form BMA density (continuous line).

To evaluate the performance of model predictions two measures associated with accuracy and forecast skill were computed (Eq.6) on flooding damage values in original space (not the Box-Cox transformed space): the percentage bias (PBIAS) and the percentage root mean square error (PRMSE) computed as the percentage improvements of RMSE over the reference values  $RMSE^*$  related to the best individual model prediction.

$$PBIAS = 100 * \left( \frac{\sum_{i=1}^m (D_i - y_i^k)}{\sum_{i=1}^m D_i} \right); \quad PRMSE = 100 * \left( 1 - \frac{\sum_{i=1}^m \frac{(D_i - y_i^k)^2}{m-1}}{RMSE^*} \right) \quad (6)$$

PBIAS is an accuracy measure taking into account the error between a prediction and the corresponding observation. It measures the average tendency of simulated values to be larger or smaller than their observed ones. The optimal value of PBIAS is zero, with low magnitude values indicating accurate model simulations. Positive values indicate overestimation bias, whereas negative values indicate model underestimation bias. PRMSE is a forecast skill, being

closely related to the variance, represents for BMA an important uncertainty measure that better describes the predictive uncertainty than in a non-BMA scheme.

For all 28 historical flooding events analysed, Figure 3 shows the PRMSE (Fig. 3a) and PBIAS statistics of the expected BMA predictions, together with that related to the simple model average predictions (SMA) (Fig. 3b). Fig. 3a show that the PBRMSE statistics of the expected BMA predictions are better than that of the best individual predictions for 15 events, and are clearly better than that the SMA predictions for all 28 events. Similar consideration can be done with regard to the PBIAS statistics showed in Fig 3b where for each event are also showed the PBIAS statistics of the best individual model. Even if BMA predictions overestimate damage bias, the related accuracy are better than SMA predictions and for some events better than the individual model. This indicates that simply averaging the original ensemble predictions would not necessarily lead to improved accuracy of the predictions.

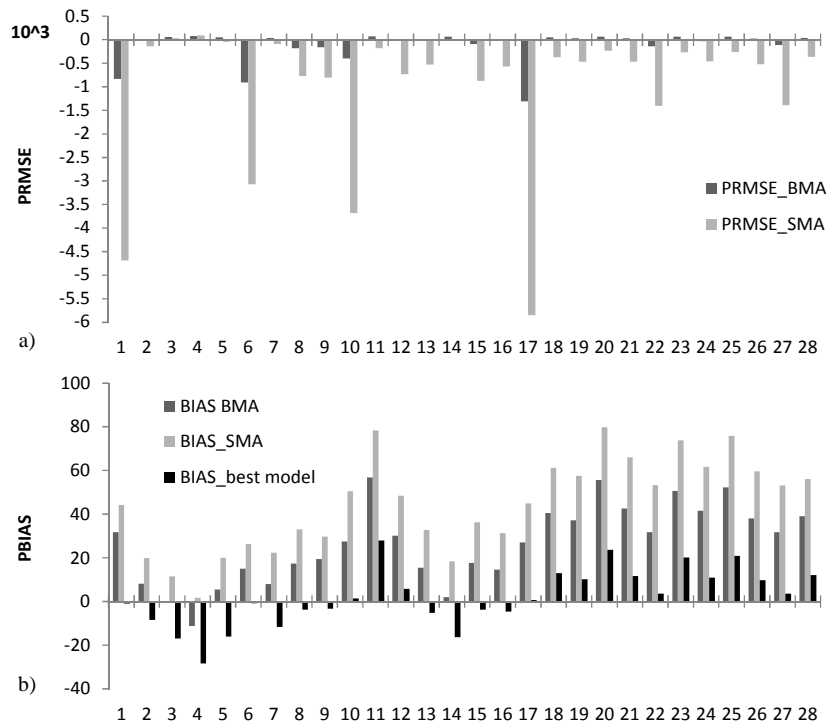


Figure 3. PBIAS and PRMSE statistics of BMA and SMA predictions

## CONCLUSIONS

An unfortunate truth in model development is that no matter how many resources are invested in developing a particular model, there remain conditions and situations in which the model is unsuitable to give an accurate forecast. Reliance on a single model typically overestimates the confidence and increases the statistical bias of the forecast. Bayesian model-averaging (BMA) techniques look to overcome the limitations of a single model by linearly combining a number of competing models into a single new model forecast. In the present study the application of BMA scheme to flooding damage analysis has shown to be an useful statistical scheme that generates probabilistic predictions from different competing predictions. The expected BMA predictions has shown performances better or comparable to the best individual model predictions in terms of PBRMSE and PBIAS statistics. Moreover, for all analysed events the BMA prediction performance was clearly better than that the SMA predictions thus confirming that simply averaging the original ensemble predictions would not necessarily lead to improved accuracy of the predictions.



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