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# A Particle Dispersion Model For Analysis Of Two-Dimensional Mixing In Open Channels

Il Won Seo

In Hwan Park

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# A PARTICLE DISPERSION MODEL FOR ANALYSIS OF TWO-DIMENSIONAL MIXING IN OPEN CHANNELS

IL WON SEO (1), INHWAN PARK (1)
*(1): Department of Civil and Environmental Engineering, Seoul National University, Korea*

Non-Fickian dispersion model, 2D PDM (2D Particle Dispersion Model) was developed to simulate the contaminant transport in both the initial and the Taylor period. The 2D PDM was based on the shear flow dispersion theory and consisted of two stages: the shear advection and the vertical mixing. The 2D PDM used the particle to visualize the soluble pollutant transport and analyzed the particle mixing without determination of the dispersion coefficient. The 2D PDM was applied to the straight channel and the meandering channel for analysis of the conservative pollutant mixing. In the straight channel, concentration curves from the 2D PDM showed skewed distribution in the initial period and then turned into the Gaussian distribution in the Taylor period. The concentration distributions in the meandering channel showed good agreement with the tracer test results.

## 1. **INTRODUCTION**

Contaminant transport in open channel flow was analyzed using the Fickian dispersion model which follows the Taylor's assumption. The Taylor's assumption includes that the Fickian dispersion model is available in the Taylor period which a balance between the advective transport and the turbulent diffusion is reached (Taylor, [1]; Fischer *et al*., [2]). During the Taylor period, concentration distribution shows the Gaussian distribution which has symmetric shape in the uniform flow. However, from the several field measurements, concentration distribution showed the skewed distribution. From the tracer test results in natural streams, Day [3] observed the non-Fickian behavior of dispersion process and Nordin and Troutman [4] calculated the skewness coefficient which showed the distorted shape of the concentration curves due to channel irregularities. Atkinson and Davis [5] also measured skewed concentration curves even though the tracer tests were conducted in the statistically uniform channel bed to test the mathematical theory of dispersion.

Alternative models for the non-Fickian dispersion process due to channel irregularities were suggested to compensate the Fickian dispersion model. Bencala and Walters [6] developed the transient storage model to simulate solute transport in the pool-and-riffle stream and Czernuszenko *et al.* [7] compared the Fickian dispersion model and the dead-zone model which showed the distorted concentration curves associated with the tails. Seo and Cheong [8] used the moment matching method to calculate the parameters of the storage zone model and demonstrated distorted concentration curves which were quite similar with the experimental results. Deng *et al.* [9] developed the FRADE (FRactional Advection-Dispersion Equation) with revising the Fick's law and simulated the long-tailed dispersion processes in natural rivers. However, these models are based on the Fickian dispersion model and inappropriate to apply in the initial period which is still not in equilibrium state between shear advection and vertical mixing. For demonstrating the asymmetric concentration distribution in the initial period, the one-dimensional analytic solutions and the non-Fickian dispersion models were developed. Chatwin [10] derived the one-dimensional analytic solution of the pollutant mixing using the edgeworth series for the skewed distribution in initial period and Schmid [11] used the Pearson type III distributions. Based on physical interpretation of pollutant mixing, Seo and Son [12] developed the SMM (Sequential Mixing Model) which is the conceptual model for analysis of the contaminant transport. And, Jung and Seo [13] expanded the SMM to the 2D numerical model, TMM (Time-split Mixing Model) which can be applied in both the initial and the Taylor period. However, these numerical models were developed to apply only in the straight channel. Therefore, the two-dimensional numerical model which can be applied in curved channels is necessary for the versatile applications of the non-Fickian dispersion model.

In this research, the two-dimensional particle dispersion model (2D PDM) was developed to analyze the pollutant mixing in both the initial and the Taylor period without determination of the dispersion coefficient. In straight channel, the 2D PDM was applied to investigate the asymmetric concentration curves during the initial period. For the general application, the 2D PDM was used in the meandering channel.

## 2. DEVELOPMENT OF THE 2D PDM

### 2.1 Fickian dispersion model

Soluble pollutant mixing in natural rivers is analyzed by using the advection-diffusion equation as Eq. (1)

$$\frac{\partial c}{\partial t}+\frac{\partial(u_i c)}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\varepsilon_i\frac{\partial c}{\partial x_i}\right) \qquad (i=1,2,3) \qquad (1)$$

where $c$ is the time-averaged concentration; $u_i$ is the velocity component; $\varepsilon_i$ is the turbulent diffusion coefficient. In natural rivers, the two-dimensional depth-averaged advection-dispersion equation (2D ADE) in Eq. (2) is used for the analysis of the contaminant transport due to the rapid completion of the vertical mixing.

$$\frac{\partial C}{\partial t}+\frac{\partial(\overline{u}_i C)}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\varepsilon_i\frac{\partial C}{\partial x_i}\right)-\frac{\partial}{\partial x_j}\left(\frac{1}{h}\int_0^h u_j'c'\,dx_3\right) \qquad (i,\ j=1,2) \qquad (2)$$

where $C$ is the depth-averaged concentration; $\overline{u}_i$ is the depth-averaged velocity; $h$ is the water depth; $x_3$ is the vertical direction. The additional transport term on the right hand side of Eq. (2) is assumed to be proportional to the concentration gradient by Taylor ([1]) as written in Eq. (3)

$$\frac{1}{h}\int_0^h u_i' c' \, dx_3 = -D_{ij} \frac{\partial C}{\partial x_j} \qquad (i, j = 1, 2) \tag{3}$$

where $D_{ij}$ is the dispersion tensor. However, $D_{ij}$ incorporating the hydraulic properties has difficulties to determine due to the complex channel geometries and flow conditions.

The 2D ADE model using Eq. (3) is available only in the Taylor period which defined in Eq. (4) (Jung and Seo, [13]).

$$0.4\frac{h^2}{\varepsilon_z} < t_T < 0.4\frac{W^2}{\varepsilon_y} \tag{4}$$

where $t_T$ is the Taylor period; $W$ is the channel width. However, most open channel flow has long initial period which makes the skewed concentration distribution due to the unbalance between the shear flow advection and the vertical mixing (Chatwin, [10]). Therefore, the non-Fickian dispersion model is necessary to compensate the limitations of the 2D ADE model.

### 2.2 Description of the 2D PDM

In this study, the non-Fickian dispersion model, 2D PDM was developed using the physical interpretation of the pollutant mixing by shear flow. In the 2D PDM, pollutant particles were introduced to visualize physical mixing process according to the complicate flow variation in open channels. The particle distribution in each time step was converted to the concentration field for various analysis. The 2D PDM is based on the shear flow dispersion theory and adopted the operator split method which divides the shear advection stage and the turbulent diffusion stage as depicted in Figure 1. In the shear advection stage, particles were separated by the vertical velocity deviations in the longitudinal and transverse directions. A particle which was introduced at $x_i(t)$ was transported by the velocity as written in Eq. (5).

$$x_i^k(t+\Delta t) = x_i^k(t) + u_i^k \Delta t \tag{5}$$

where $x_i^k(t)$ is the particle position on the *k*-th layer; $u_i^k$ is the velocity on the *k*-th layer; $\Delta t$ is the time step. The separated particles according to the shear flow were mixed across the vertical in the turbulent diffusion stage like Figure 1 (b). For $\Delta t$, particles were evenly distributed in vertical layers and the vertical mixing was completed. After completion of the vertical mixing, the number of particle on each grid was converted to the concentration field using Eq. (6)

$$C(x_i, t) = \frac{mn(x_i, t)}{h\Delta x \Delta y} \tag{6a}$$

$$n(x_i, t) = \sum_{k=1}^{L} n^k(x_i, t) \tag{6b}$$

where $m$ is the mass for a single particle; $h$ is the water depth; $\Delta x$ , $\Delta y$ is the computational grid size; $n$ is the number of particle in the computational grid; $n^k$ is the number of particle on the *k*-th layer; $L$ is the number of layer.

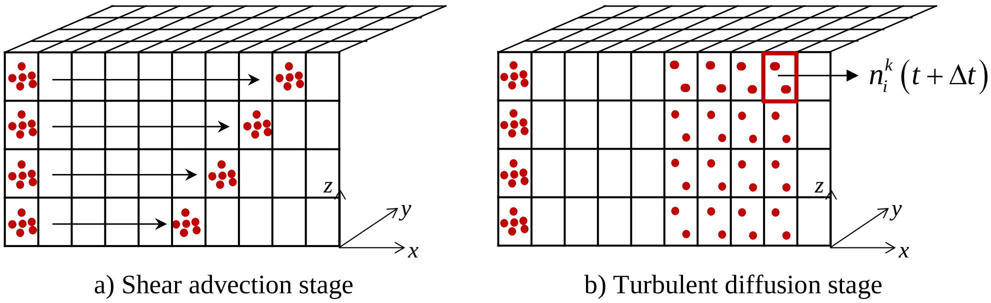



Figure 1. Conceptual description of the 2D PDM

For the shear advection, the longitudinal particle transport was determined using Eq. (7a) (Rozovskii, [14]) which has the logarithmic distribution, and Eq. (7b) which has the linear distribution (Odgaard, [15]) was employed for the particle displacement in transverse direction.

$$u_s(z) = \bar{u}_s\left[1+\frac{\sqrt{g}}{\kappa C_h}\left(1+\ln\frac{z}{h}\right)\right] \tag{7a}$$

$$u_n(z) = \bar{u}_n + 2v_s\left(\frac{z}{h}-\frac{1}{2}\right)\quad \left(v_s = \bar{u}_s\frac{2m+1}{2\kappa^2 m}\frac{h}{r_c},\; m=\frac{\kappa\bar{u}_s}{u^*}=\frac{\kappa C_h}{\sqrt{g}}\right) \tag{7b}$$

where $u_s$, $u_n$ is the stream-wise, span-wise velocity, respectively; $\kappa$ is the von Karman constant; $C_h$ is the Chezy coefficient; $r_c$ is the radius of curvature; $u^*$ is the shear velocity; $g$ is the gravity acceleration. Transported particles which were out of the boundary were absorbed at the wall and excluded for the next computations.

## 3. MODEL APPLICATIONS

With the 2D PDM, the instantaneously injected pollutant mixing in the artificial channels was simulated to investigate the non-Fickian dispersion during the initial period. Simulation conditions were listed in Table 1 and a number of particles were introduced at a point.

Table 1. Simulation conditions for the 2D PDM

| Channels | $Q$ (m$^3$/s) | $h$ (m) | $W$ (m) | No. of layer | No. of particles |
|---|---|---|---|---|---|
| ST | 0.06 | 0.3 | 4.0 | 100 | 40,000 |
| M2 | 0.06 | 0.3 | 1.0 | 40 | 2,000 |

### 3.1 Skewed concentration distribution in initial period

2D PDM was applied to the ST channel with the simulation condition in Table 1. Simulation results of the 2D PDM was compared with the analytic solution which has the Gaussian distribution. In the uniform flow, the analytic solution of the 2D ADE is written in Eq. (8).

$$C(x, y, t) = \frac{M}{4\pi t\sqrt{D_L D_T}} \exp\left[-\frac{(x - Ut)^2}{4D_L t} - \frac{y^2}{4D_T t}\right] \quad (8)$$

where $C(x, y, t)$ is the concentration; $D_L$, $D_T$ is the dispersion coefficient in each direction; $U$ is the uniform velocity. *C*-*x* curves using Eq. (8) were plotted with the simulation results in Figure 2. The initial period ($t_I$) in the ST channel is determined with Eq. (9) (Jung and Seo, [13]) and the initial period sustained for 18 sec in this case.

$$t_I > 0.4\frac{h}{0.067u^*} \quad (9)$$

In the initial period, concentration curves from the simulation results show that asymmetric distributions and have large discrepancies with the analytic solution. After 20 sec, concentration curves gradually change to the symmetric distribution. Shape of the concentration curves was estimated with the skewness coefficient in Figure 3. Immediately after the particle injection, the skewness coefficient soared to 1.2 and rapidly decreased to about 0.3 during the initial period. In the Taylor period, the shear advection and the vertical mixing were achieving balance and the concentration curves approached to the symmetric distribution which follows the Fick's law. From the results, the 2D PDM was available both the initial period and the Taylor period in natural rivers.

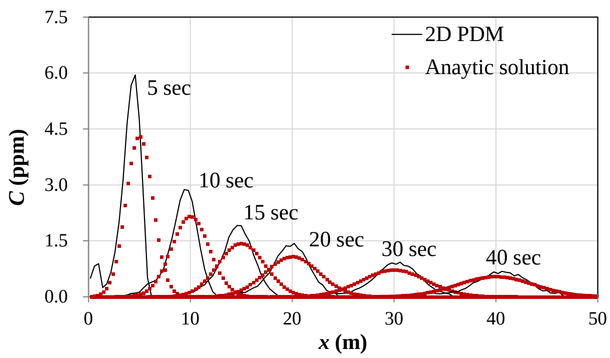



Figure 2. Comparison of C-x curves between the simulation results and the analytic solution

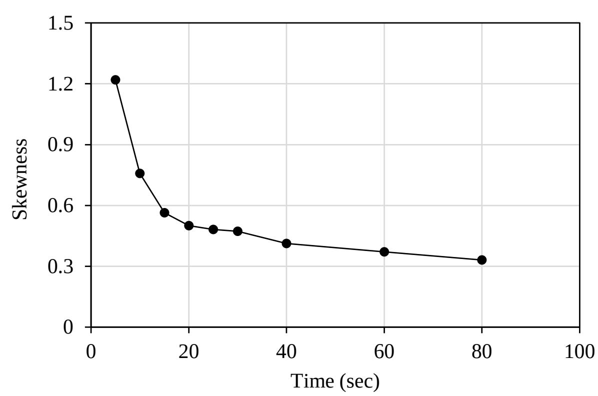
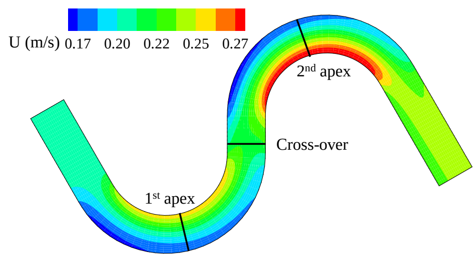


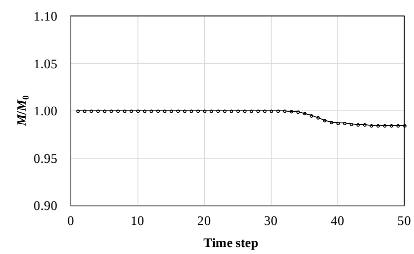

Figure 3. Time evolution of the skewness coefficient from the simulation results

### 3.2 Meandering channels

The 2D PDM was applied to the pollutant mixing in the M2 channel which has rectangular cross section and various flow direction. From the simulation condition in Table 1, flow was simulated using the hydrodynamic model developed by Song *et al.* [16] and Figure 4(a) shows the flow analysis results. With changing flow direction, injected particles have possibilities to run into the channel boundaries. From the simulation results of the 2D PDM in the M2 channel, mass conservation was checked in Figure 4 (b). Total mass was maintained when the particle cloud passed the cross-over part, but some particles were absorbed to the boundary and total mass decreased to 98.4 % which was insignificant loss.




a) Velocity distribution




b) Change of total mass

Figure 4. Mass conservation with the absorbing boundary condition in the meandering channel

Figure 5 shows the particle dispersion and concentration conversion results in the M2 channel. Injected particles were stretched in longitudinal direction and concentration cloud made a tail after passing the 1st apex. The concentration conversion results were compared with the two-dimensional tracer test results which were conducted by Seo and Park [17] in Figure 6. From the comparison results, transverse averaged concentration curves show good agreement with the simulation results on the 1st apex, but the simulation results on the 2nd apex underestimated the experimental results.

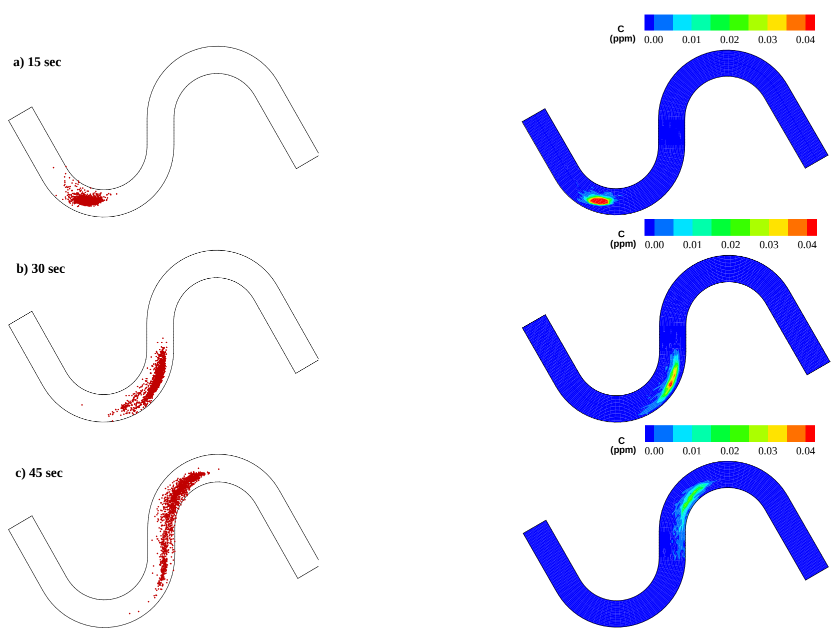



Figure 5. Simulation results of 2D PDM in the M2 channel

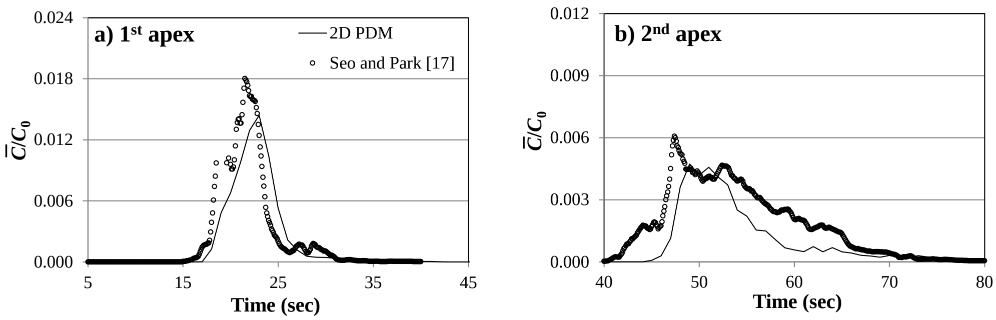



Figure 6. Comparison results of C-t curves

## 4. CONCLUSIONS

The 2D PDM which was available in both the Taylor and the initial period was developed to analyze contaminant transport in the open channel flow. With the shear flow dispersion theory, the 2D PDM was consisted of the shear advection stage and the turbulent diffusion stage. Introduced particles were transported using the vertical velocity profiles in the shear advection stage and the particles were evenly distributed to the vertical layers to complete the vertical mixing. At each time step, particle distributions were converted to the concentration field for the quantitative analysis.

In the ST channel, simulation results of the 2D PDM show the skewed distribution in the initial period and has the large differences with the analytic solution. The skewness coefficient of the simulation results decreased to about 0.3 in the Taylor period and the concentration curves were approaching to the symmetric shape. For the changing flow direction, the absorbing boundary was adopted at the channel wall and the simulation results of the 2D PDM were compared with the tracer test results in the M2 channel. From the comparison results, the concentration curves show good agreement with the experimental results on the 1$^{st}$ apex, but, on the 2$^{nd}$ apex, simulation results underestimated the experimental results.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