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# Bioelectrical Circuits: Lecture 1

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# **BME 205 L01**

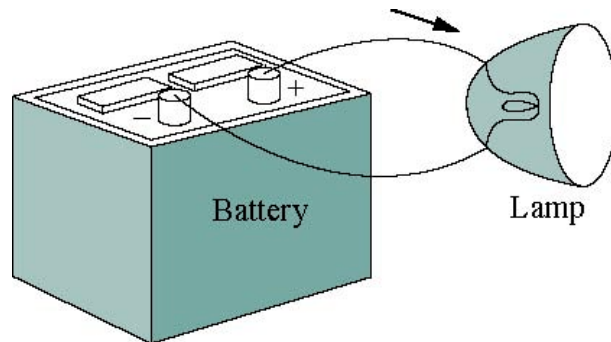
DC Circuits – Basic Concepts

# Circuit theory

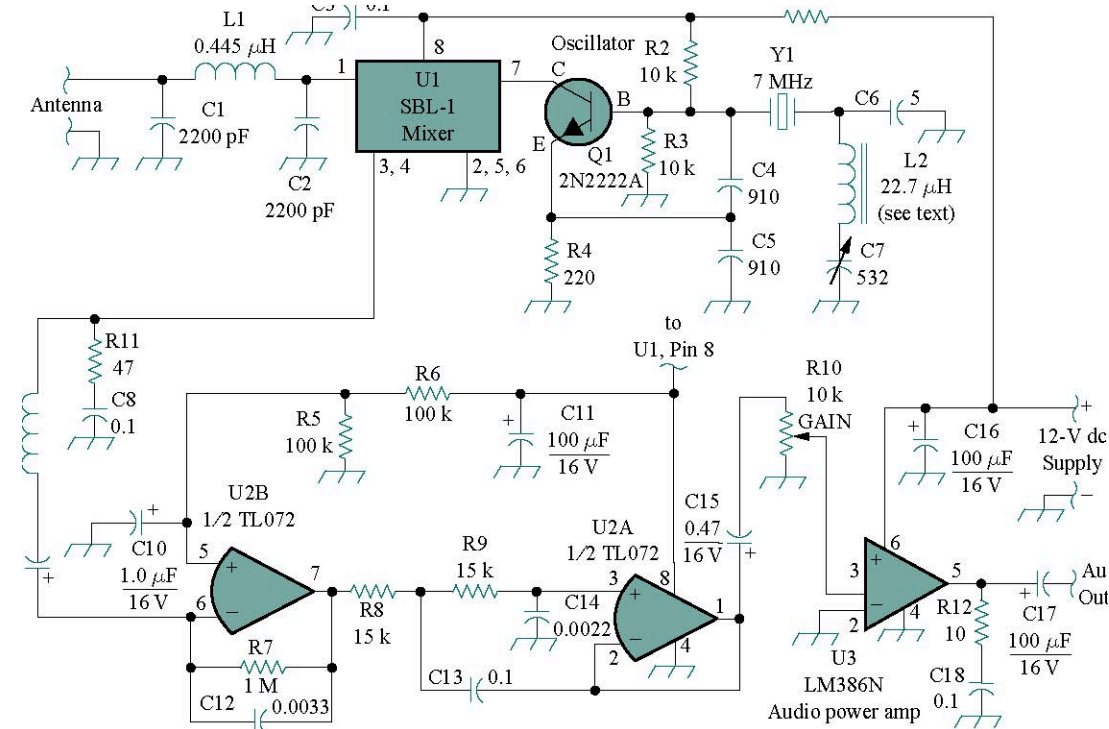
- The study of idealized or theoretical “elements” connected together – what’s the point?
- Because those elements are governed by simple laws that are easy to understand, combine and scale up to build bigger things.
- Because REAL circuits and systems can be very well approximated by the right combinations of those elements
- **For the biomedical engineer:**
- For bioinstrumentation – medically relevant signals created by the body can be captured by transducers or sensors and converted into electrical signals.
- Electric devices monitor patient conditions, acquire information for diagnostic and research purposes (e.g. ECG, EEG), and even control physiological functions (e.g. pacemakers, ventilators).
- Many physiological processes, whether they are electrical in nature (e.g. neural activity) or not (e.g. cardiovascular system) can be modeled using circuit theory.

# What is a Circuit?

- One definition: An electric circuit is an interconnection of electrical elements.
- Here's a simple electric circuit, consisting of three basic components: a battery, a lamp, and connecting wires:



- Better definition: A circuit is a **closed path** through which electrical **charge** can **flow**.



- A complicated real circuit, representing the schematic diagram for a radio receiver.
- Although it seems complicated, this circuit can be analyzed using the techniques we cover in this course.
- Our goal is **to learn various analytical techniques** (and a little computer software) for **describing the behavior of a circuit** like this.

# Circuit Analysis

The study of the behavior of a circuit:

- How does it respond to a given input?
- How do the interconnected elements and devices in the circuit interact?

Today we'll cover basic concepts such as charge, current, voltage, circuit elements, power and energy.

We must first establish a system of units that we will use throughout the course...

# Systems of Units

- The International System of Units (SI) enables engineers to communicate their measurements.

**TABLE 1.1** The six basic SI units.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd

- There are six principal units from which the units of all other quantities can be derived.

- We'll deal with current (ampere), voltage (volt), resistance (ohm), capacitance (farad), inductance (Henry)

# SI Prefixes

$$10^6 = 1\,000\,000$$

$$10^5 = 100\,000$$

$$10^4 = 10\,000$$

$$10^3 = 1000$$

$$10^2 = 100$$

$$10^1 = 10$$

$$10^0 = 1$$

$$10^{-6} = 0.000001$$

$$10^{-5} = 0.00001$$

$$10^{-4} = 0.0001$$

$$10^{-3} = 0.001$$

$$10^{-2} = 0.01$$

$$10^{-1} = 0.1$$

**TABLE 1.2** The SI prefixes.

Multiplier	Prefix	Symbol
$10^{18}$	exa	E
$10^{15}$	peta	P
$10^{12}$	tera	T
$10^9$	giga	G
$10^6$	mega	M
$10^3$	kilo	k
$10^2$	hecto	h
10	deka	da
$10^{-1}$	deci	d
$10^{-2}$	centi	c
$10^{-3}$	milli	m
$10^{-6}$	micro	$\mu$
$10^{-9}$	nano	n
$10^{-12}$	pico	p
$10^{-15}$	femto	f
$10^{-18}$	atto	a

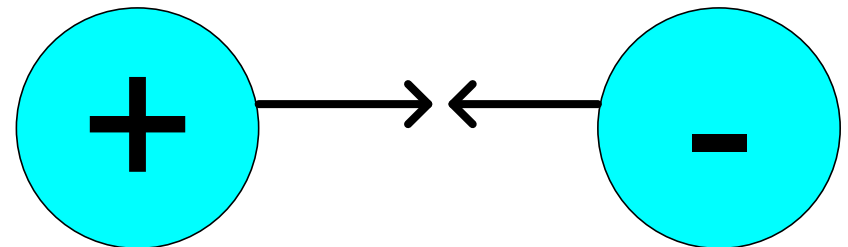
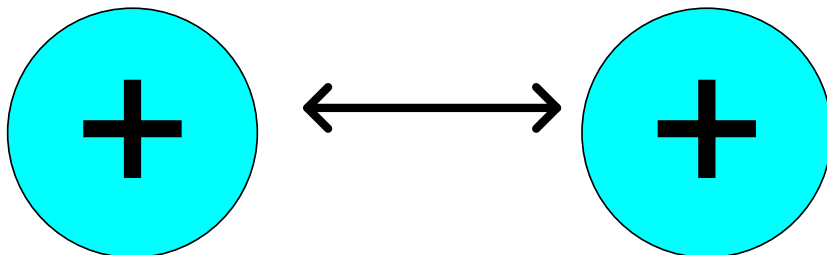


# Charge and Current

The most basic quantity in an electric circuit is an *electric charge*. We all experience the effect of electric charge when we try to remove our wool sweater and have it stick to our body or walk across a carpet and receive a shock.

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

Like charges repel



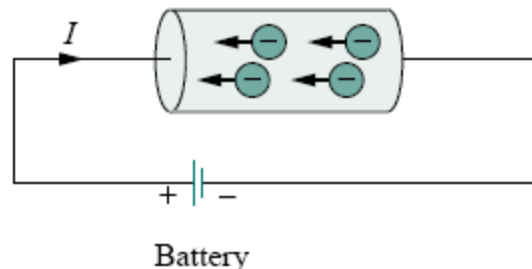
Unlike charges attract

# Electric Charge facts

- The coulomb is a large unit for charges. In 1C of charge, there are  $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$  electrons. Thus realistic or laboratory values of charges are on the order of pC, nC, or  $\mu\text{C}$ .
- According to experimental observations, the only charges that occur in nature are integral multiples of the electronic charge  $= -1.602 \times 10^{-19} \text{ C}$ .
- The *law of conservation of charge* states that charge can neither be created nor destroyed, only transferred. Thus the algebraic sum of the electric charges in a system does not change.

# Flow of electric charges

- A unique feature of electric charge or electricity is the fact that it is mobile; that is, it can be transferred from one place to another, where it can be converted to another form of energy.
- When a conducting wire (consisting of several atoms) is connected to a battery (a source of electromotive force), the charges are compelled to move; positive charges move in one direction while negative charges move in the opposite direction. This motion of charges creates **electric current**.
- **CONVENTION: we take the current flow as the movement of positive charges, that is, opposite to the flow of negative charges.**
- We now know that it's mostly electrons that flow in conductors, but the above convention is still adhered to universally.



# Electric current

Electric current is the time rate of flow of charge, i.e., the charge per second passing any cross section of the circuit.

Mathematically, the relationship between current  $i$ , charge  $q$ , and time  $t$  is

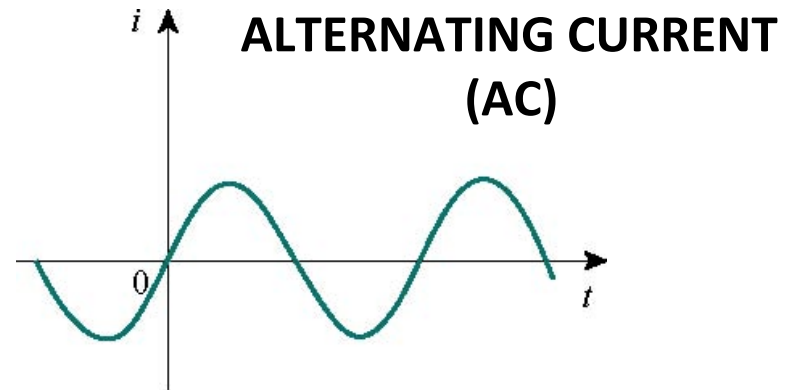
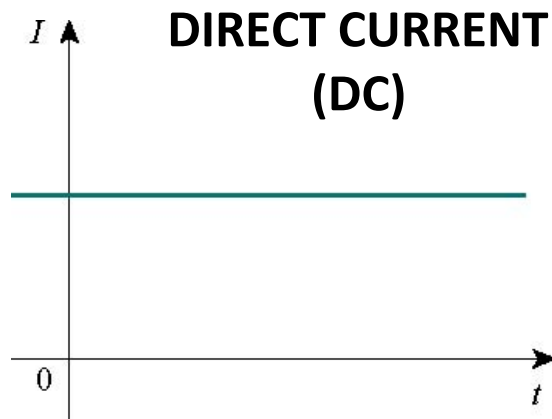
$$i = \frac{dq}{dt}$$

where current is measured in amperes (A), and 1 ampere = 1 coulomb/second.

The charge transferred between time  $t_0$  and  $t$  is obtained by integrating both sides. We obtain

$$q = \int_{t_0}^t i dt$$

- Two types of current:



A **direct current** (dc) is a current that remains constant with time.

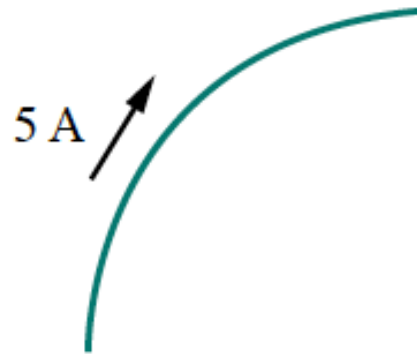
- By convention a capital **I** is used to represent a DC current.

An **alternating current** (ac) is a current that varies sinusoidally with time.

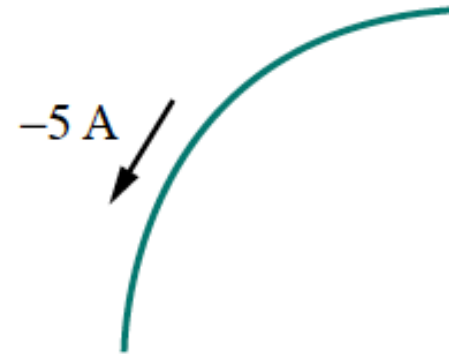
- A time-varying current is represented by the small letter  $i$ .
- A common form of time-varying current is the sinusoidal current or *alternating current* (AC).

# Current flow convention

- By convention, the direction of current flow is taken as the direction of positive charge movement.
- In any wire or element in a circuit, we indicate a reference direction for the positive flow of positive charge,
- This is just like indicating a direction for positive displacement using axes in a mechanical problem ( “let’s say up is positive” )
- *Remember that the reference direction (an arrow) does not tell you which way charge is actually flowing – its VALUE with respect to that reference direction does!*
- E.g. (a) and (b) are the same!



(a)



(b)

# Example

- The total charge entering a terminal is given by  $q = 5t \sin 4\pi t$  mC.
- Calculate the current at  $t = 0.5$ s.
  
- Ex 2: what's the total charge entering terminal between  $t=1$ s and 2s if current is  $i = (3t^2 - t)$  A ?

# Voltage

- Moving an electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by a battery. This emf creates a *voltage* or *potential difference*.
- The voltage  $v_{ab}$  between two points a and b in an electric circuit is defined as the energy (or work) needed to move a unit charge from a to *b*; mathematically,

$$v_{ab} = \frac{dw}{dq}$$

...where *w* is energy in joules (J) and *q* is charge in coulombs (C).

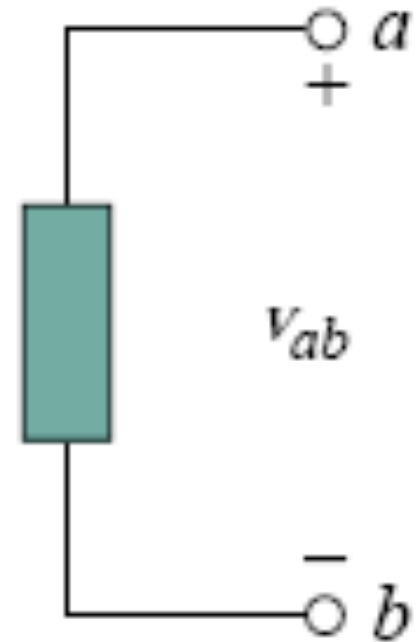
- Voltage is measured in **volts (V)**.
- From this, it is evident that  
1 volt = 1 joule/coulomb = 1 newton meter/coulomb



# Polarity convention

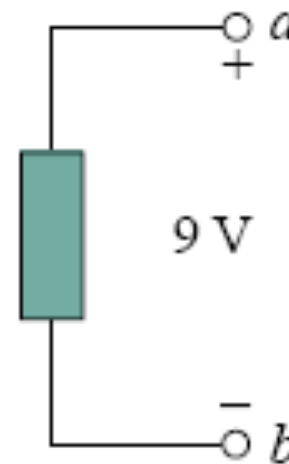
- We're typically concerned with the voltage *across an element*.
- Consider an element (represented by rectangular block) connected to points  $a$  and  $b$ .
- The plus (+) and minus (-) signs are used to define reference direction or voltage polarity, in the same way that an arrow defines the reference direction of a current.
- The  $v_{ab}$  can be interpreted in two ways:
  - (1) point  $a$  is at a potential of  $v_{ab}$  volts higher than point  $b$ , or
  - (2) the potential at point  $a$  with respect to point  $b$  is  $v_{ab}$ . It follows logically that in general

$$v_{ab} = -v_{ba}$$

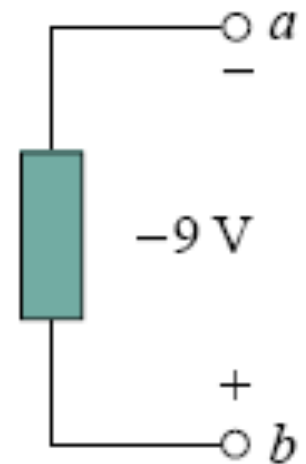


# Polarity convention

- Here we have two representations of the same voltage. In (a), point  $a$  is  $+9\text{V}$  above point  $b$ . In (b), point  $b$  is  $-9\text{V}$  above point  $a$  – *same thing!*
- We may say that in (a), there is a  $9\text{-V}$  **voltage drop** from  $a$  to  $b$  or equivalently a  $9\text{-V}$  **voltage rise** from  $b$  to  $a$ .
- In other words, a voltage drop from  $a$  to  $b$  is equivalent to a voltage rise from  $b$  to  $a$ .



(a)



(b)

## Current and voltage are the two basic variables in electric circuits.

- The common term *signal* is used for an electric quantity such as a current or a voltage (or even electromagnetic wave) when it is used for conveying information.
- Engineers prefer to call such variables signals rather than mathematical functions of time because of their importance in communications and other disciplines.
- Like electric current, a constant voltage is called a dc voltage and is represented by  $V$ , whereas a sinusoidally time-varying voltage is called an ac voltage and is represented by  $v$ .
- A dc voltage is commonly produced by a battery; ac voltage is produced by an electric generator.

# Power and energy

- Current and voltage aren't everything.
- For practical purposes, we need to know how much *power* an electric device can handle or deliver.
- E.g. a 100-W bulb gives more light than a 60-W one.
- *POWER*,  $p$ , is the time rate of expending or absorbing energy (w, joules), measured in watts (W)

$$p = \frac{dw}{dt} \quad \text{(time in seconds)}$$

- It follows that

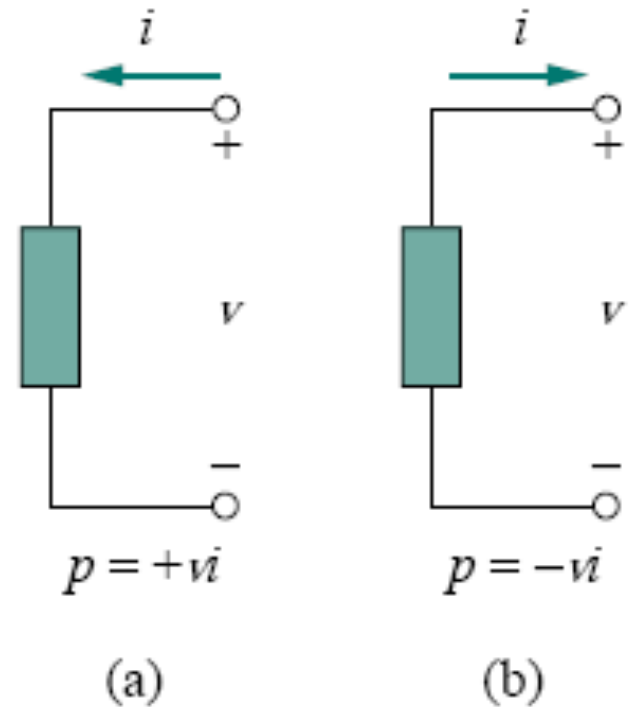
$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi$$

- Power,  $p=vi$ , is a time-varying quantity and is called the *instantaneous power*.
- The power absorbed or supplied by an element is the product of the voltage across the element and the current through it.
- **Convention:** If the power has a + sign (i.e., is positive), power is being delivered to or absorbed by the element. If, on the other hand, the power has a – sign (negative), power is being supplied by the element.
- But how do we know when the power has a negative or a positive sign?

# Current direction and voltage polarity

- **Passive sign convention:**

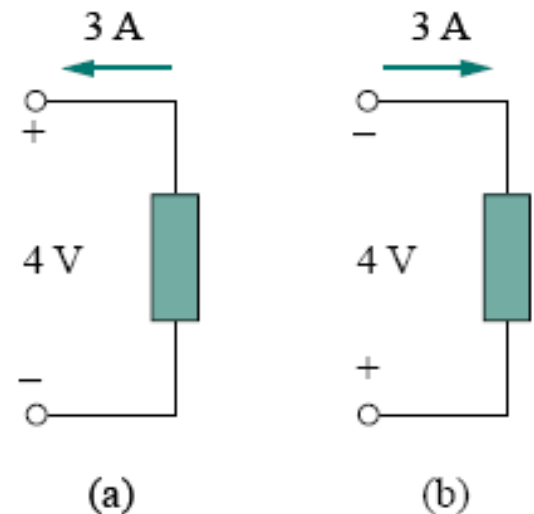
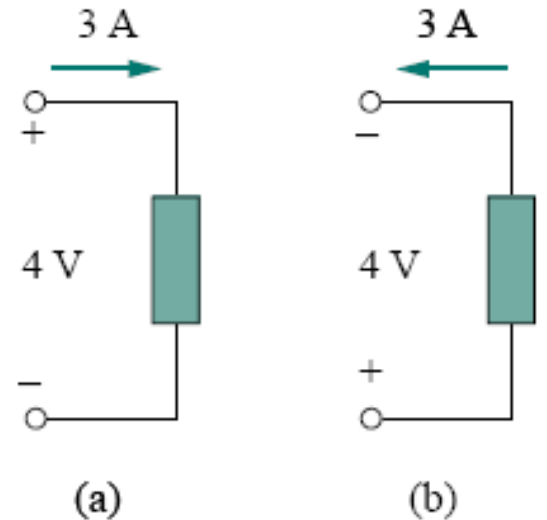
The relation  $p=vi$  assumes that the reference direction for current  $i$  and the reference polarity for voltage  $v$  is as in (a). In this configuration, if  $v$  and  $i$  are both positive, then  $p > 0$  and the element is **absorbing power**. Conversely, if  $v$  and  $i$  have different signs then  $p < 0$  and the element is **releasing or supplying power**.



- If the reference direction/polarities are opposite, as in (b), that is not a problem at all – you just have to use the relation  $p = -vi$  in that case.

# passive sign convention

- For example, the element in both upper circuits has an absorbing power of +12 W because a positive current enters the positive terminal of the block element in both cases.
- In both lower circuits, however, the element is supplying power because a positive current enters the negative terminal of the block element .
- In general
  - + Power absorbed
  - = - Power supplied

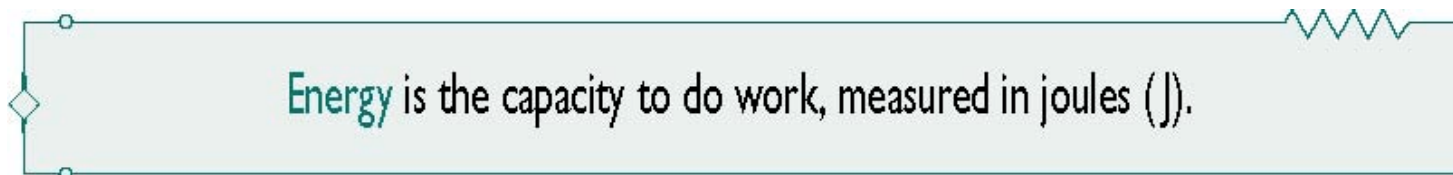


- The *law of conservation of energy* must be obeyed in any electric circuit. For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero:

$$\boxed{\sum p = 0}$$

- This again confirms the fact that the total power supplied to the circuit must balance the total power absorbed.
- The energy absorbed or supplied by an element from time  $t_0$  to time  $t$  is

$$w = \int_{t_0}^t p \, dt = \int_{t_0}^t vi \, dt$$





# Example

- An energy source forces a constant current of 2A to flow through a lightbulb for 10s .
- If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

# Practice Problem

- To move charge from point a to point b requires  $-30$  J. Find the voltage drop  $v_{ab}$  *if*:  
*(a)  $q = 2$ C, (b)  $q = -6$ C.*

- Find the power delivered to an element at  $t=3$  ms if the current entering its positive terminal is  $i = 5 \cos 60\pi t$  A and the voltage is: (a)  $v=3i$ , (b)  $v=3 di/dt$ .

# Practice Problem

- Find the power delivered to the element in the last example at  $t = 5\text{ms}$  if the current remains the same but the voltage is:

(a)  $v = 2i \text{ V},$

(b)  $v = (10 + 5 \int_0^t i(\tau) d\tau) \text{ V}.$

# Example

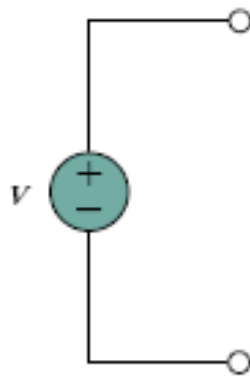
- How much energy does a 100-W electric bulb consume in two hours?

# Circuit Elements

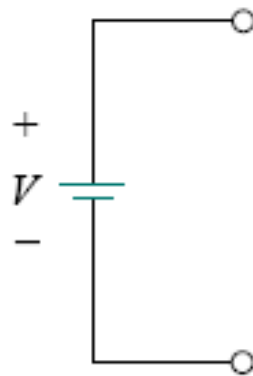
- The circuit element is the basic building block of a circuit. An electric circuit is simply an interconnection of the elements. Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.
- There are two types of elements found in electric circuits: *passive elements* and *active elements*. An active element is capable of generating energy while a passive element is not. Examples of passive elements are resistors, capacitors, and inductors. Typical active elements include generators, batteries, and operational amplifiers.

- We'll just look at active elements for now.
- The most important are voltage or current sources that generally deliver power to the circuit connected to them. There are two kinds of sources: independent and dependent sources.

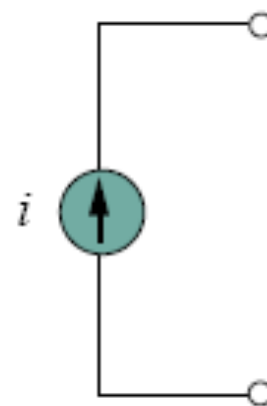
An **ideal independent source** is an active element that provides a specified voltage or current that is completely independent of other circuit variables.



(a)

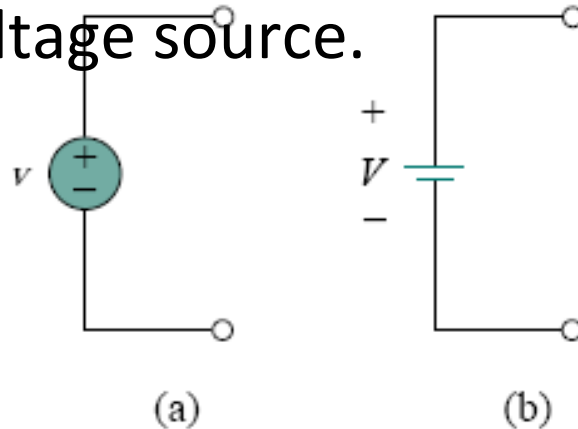


(b)



# Ideal independent voltage source

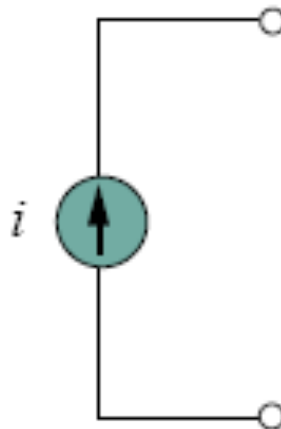
- An ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage.
- Physical sources such as batteries and generators may be regarded as approximations to ideal voltage sources. Shown below are the symbols for independent voltage sources. Notice that both symbols in (a) and (b) can be used to represent a dc voltage source, but only the symbol in (a) can be used for a time-varying voltage source.





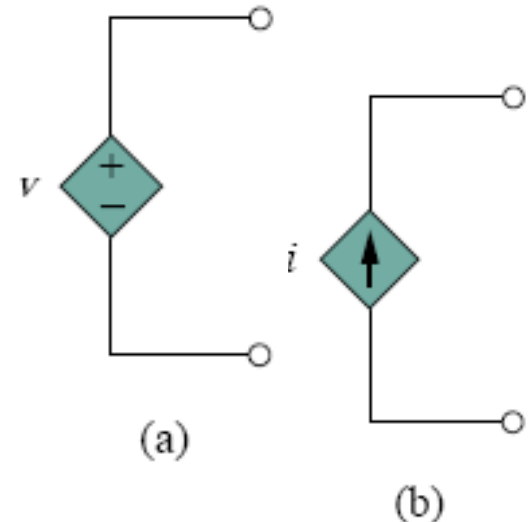
# Ideal independent current source

- Similarly, an ideal independent **current** source is an active element that provides a specified current completely independent of the voltage across the source. That is, the current source delivers to the circuit whatever voltage is necessary to maintain the designated current. The symbol for an independent current source is displayed below, where the arrow indicates the direction of current  $i$ .

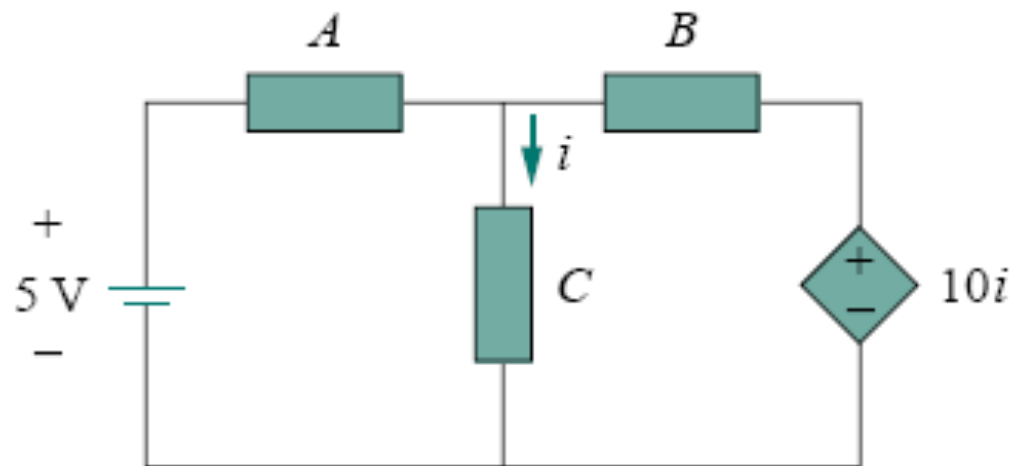


# Dependent sources

- Dependent sources are usually designated by diamond-shaped symbols, as shown. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:
  - A voltage-controlled voltage source (VCVS).
  - A current-controlled voltage source (CCVS).
  - A voltage-controlled current source (VCCS).
  - A current-controlled current source (CCCS).



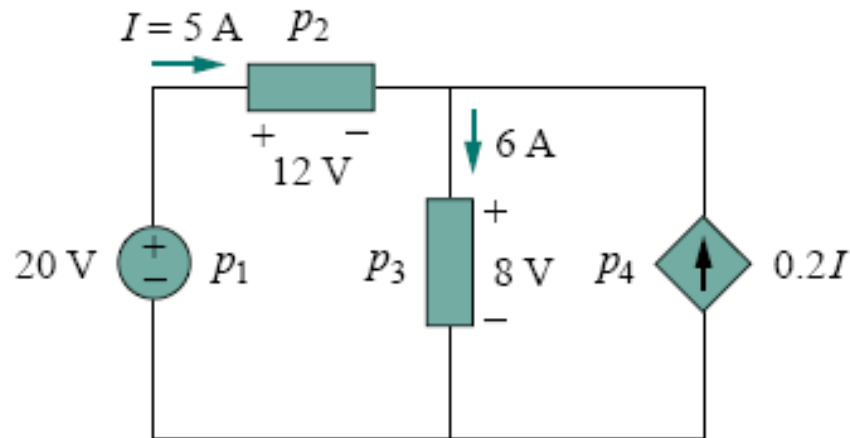
- Dependent sources are useful in modeling elements such as transistors, operational amplifiers and integrated circuits.
- An example of a current-controlled voltage source is shown on the right-hand side of the circuit below, where the voltage  $10i$  of the voltage source depends on the current  $i$  through element  $C$ . Students might be surprised that the value of the dependent voltage source is  $10i$  V (and not  $10i$  A) because it is a voltage source. The key idea to keep in mind is that a voltage source comes with polarities (+-) in its symbol, while a current source comes with an arrow, irrespective of what it depends on.



- It should be noted that an ideal voltage source (dependent or independent) will produce any current required to ensure that the terminal voltage is as stated, whereas an ideal current source will produce the necessary voltage to ensure the stated current flow.
- Thus an ideal source could in theory supply an infinite amount of energy.
- It should also be noted that not only do sources supply power to a circuit, they can absorb power from a circuit too. For a voltage source, we know the voltage but not the current supplied or drawn by it. By the same token, we know the current supplied by a current source but not the voltage across it.

# Example

- Calculate the power supplied or absorbed by each element in the circuit below.



# Practice Problem

Compute the power absorbed or supplied by each component of the circuit in Fig. 1.16.

