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Bioelectrical Circuits: Lecture 5

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BME 205 L05

Operational Amplifiers

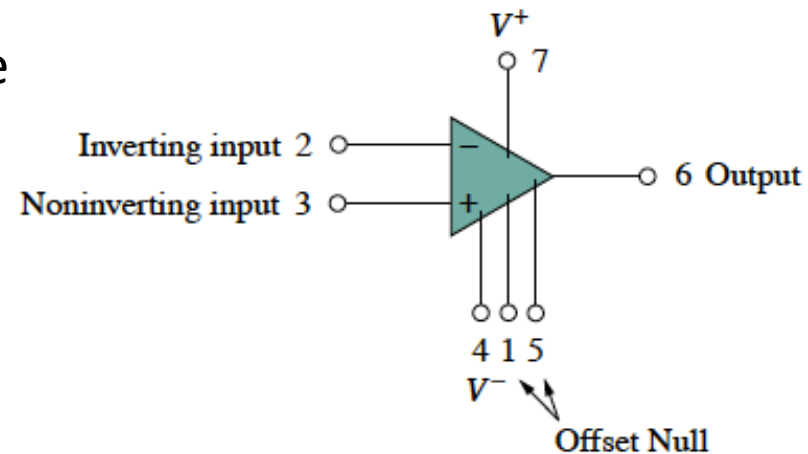
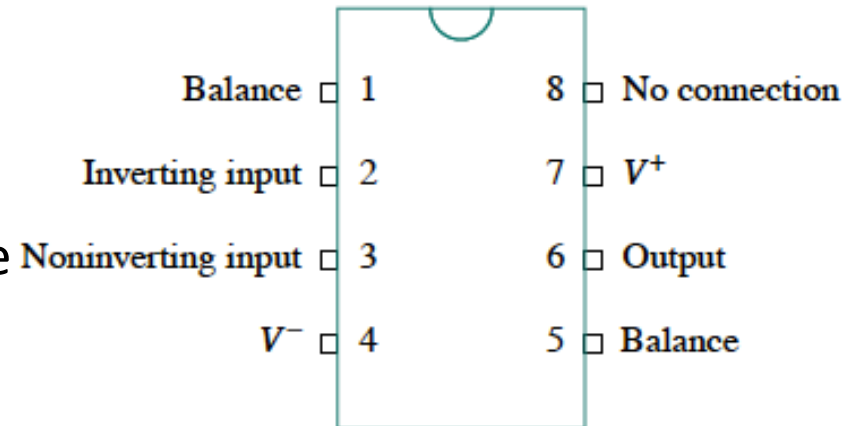
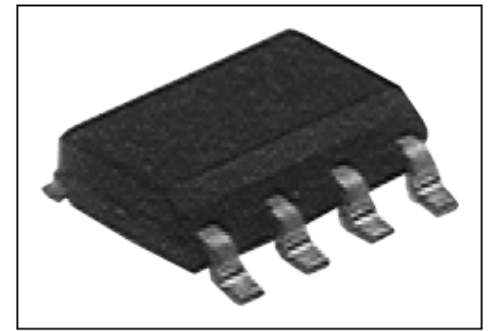
Introduction

- The operational amplifier (“op amp”) is a versatile active circuit element that behaves like a voltage-controlled voltage source.
- It can also be used to create Voltage- or current-controlled current sources.
- An op amp can sum signals, amplify a signal, integrate it, or differentiate it (hence the name “operational”).
- Op amps are popular in practical circuit designs because they are versatile, inexpensive & easy to use.
- We will briefly look at the nonideal op amp, then use the ideal op amp to build circuits such as the inverter, voltage follower, summer, and difference amplifier.

Operational Amplifiers

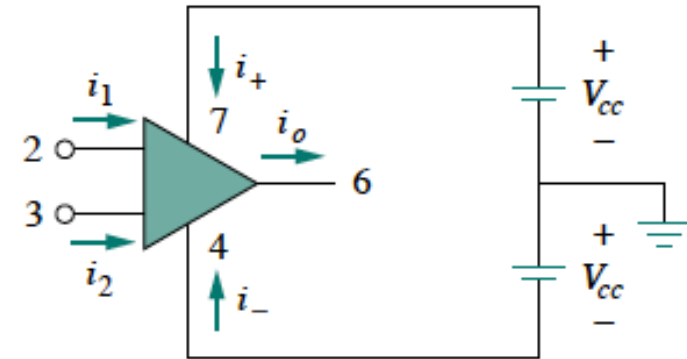
- An operational amplifier is designed so that it performs some mathematical operations when external components, such as resistors and capacitors, are connected to its terminals. Thus a good definition is:
- **An op amp is an active circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division, differentiation, and integration.**
- The op amp is an electronic device consisting of a complex arrangement of resistors, transistors, capacitors, and diodes. We won't discuss what's inside – we will treat the op amp as a building block and study what happens at its terminals.
- Op amps are commercially available in integrated circuit packages in several forms. A typical op amp package is the dual in-line package (DIP).

- Pin or terminal 8 is unused, and terminals 1 and 5 are of little concern to us. The five important terminals are:
- 1. The inverting input, pin 2.
- 2. The noninverting input, pin 3.
- 3. The output, pin 6.
- 4. The positive power supply V^+ , pin 7.
- 5. The negative power supply V^- , pin 4.
- The circuit symbol for the op amp is the triangle shown below, with two inputs and one output.
- An input applied to the *noninverting* terminal (marked +) will appear with the same polarity at the output, while an input applied to the *inverting* terminal (marked -) will appear inverted at the output.

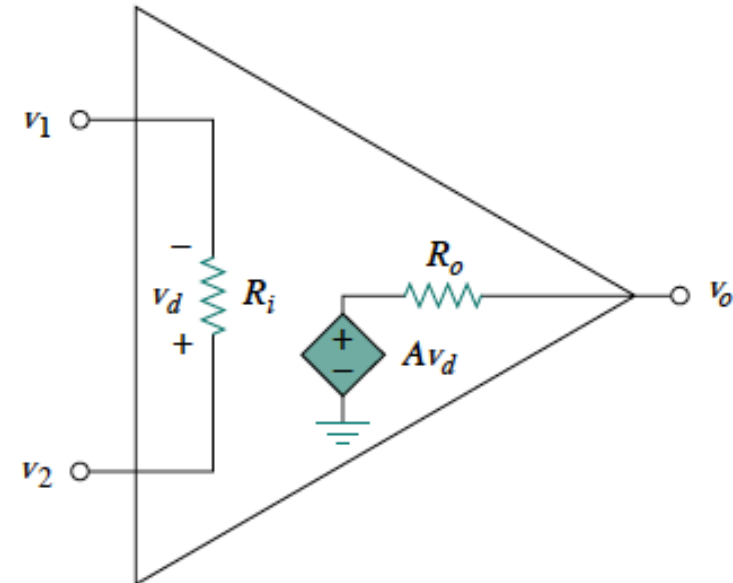


- As an active element, the op amp must be powered by a voltage supply as shown.
- Although the power supplies are often ignored in op amp circuit diagrams for the sake of simplicity, the power supply currents must not be overlooked. By KCL,

$$i_o = i_1 + i_2 + i_+ + i_-$$



- The equivalent circuit model of an op amp is shown in lower panel.
- The output section consists of voltage-controlled source in series with the output resistance R_o .
- It is evident that the input resistance R_i is the Thevenin equivalent resistance seen at the input terminals, while the output resistance R_o is the Thevenin equivalent resistance seen at the output.



- The differential input voltage v_d is given by

$$v_d = v_2 - v_1$$

where v_1 is the voltage between the inverting terminal and ground and v_2 is the voltage between the noninverting terminal and ground.

- The op amp senses the difference between the two inputs, multiplies it by the gain A , and causes the resulting voltage to appear at the output. Thus, the output v_o is given by

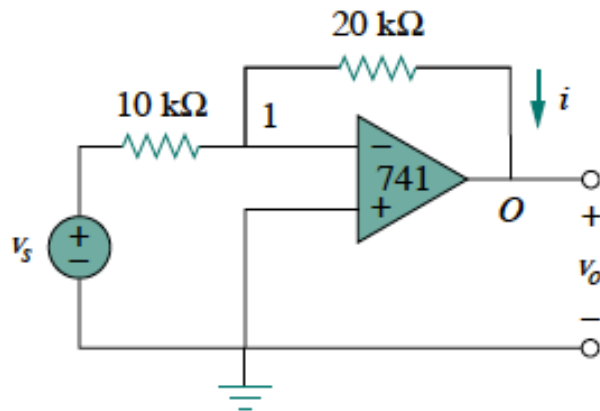
$$v_o = Av_d = A(v_2 - v_1)$$

- A is called the *open-loop voltage gain* because it is the gain of the op amp without any external feedback from output to input.
- The table shows typical values op amp parameters:

Parameter	Typical range	Ideal values
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_i	10^6 to $10^{13} \Omega$	$\infty \Omega$
Output resistance, R_o	10 to 100 Ω	0 Ω
Supply voltage, V_{cc}	5 to 24 V	

Feedback

- Op amps are almost always used in circuits with “feedback paths.”
- A negative feedback is achieved when the output is fed back to the inverting terminal of the op amp.
- As the following example shows, when there is a feedback path from output to input, the ratio of the output voltage to the input voltage is called the *closed-loop gain*.
- As a result of the negative feedback, it can be shown that the closed-loop gain is almost insensitive to the open-loop gain A of the op amp.



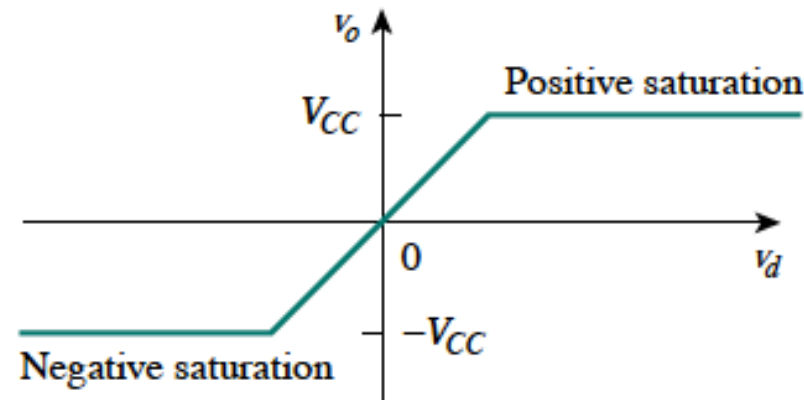
(upcoming example)

Saturation

- A practical limitation of the op amp is that the magnitude of its output voltage cannot exceed $|V_{CC}|$. In other words, the output voltage is dependent on and is limited by the power supply voltage.

- The op amp can operate in three modes depending on the differential input voltage

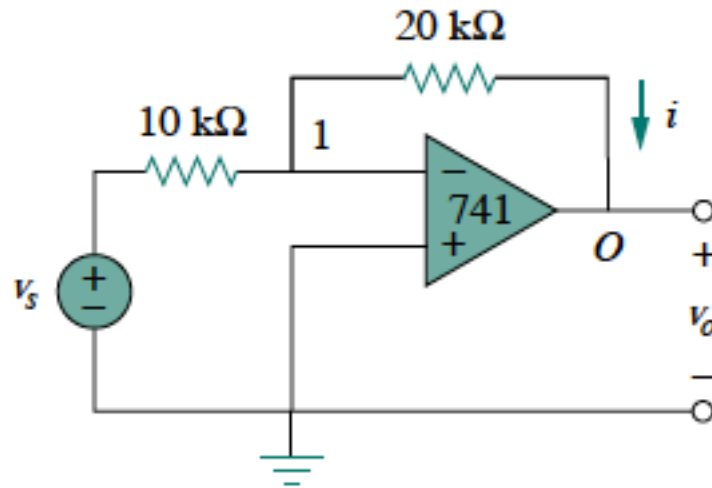
1. Positive saturation, $v_o = V_{CC}$.
2. Linear region, $-V_{CC} \leq v_o = Av_d \leq V_{CC}$.
3. Negative saturation, $v_o = -V_{CC}$.



- If we attempt to increase v_d beyond the linear range, the op amp becomes saturated and yields $v_o = V_{CC}$ or $v_o = -V_{CC}$. Throughout this course, we will assume that our op amps operate in the linear mode.
- Although we'll always operate the op amp in the linear region, the possibility of saturation must be kept in mind when designing circuits with op amps that will work in the laboratory.

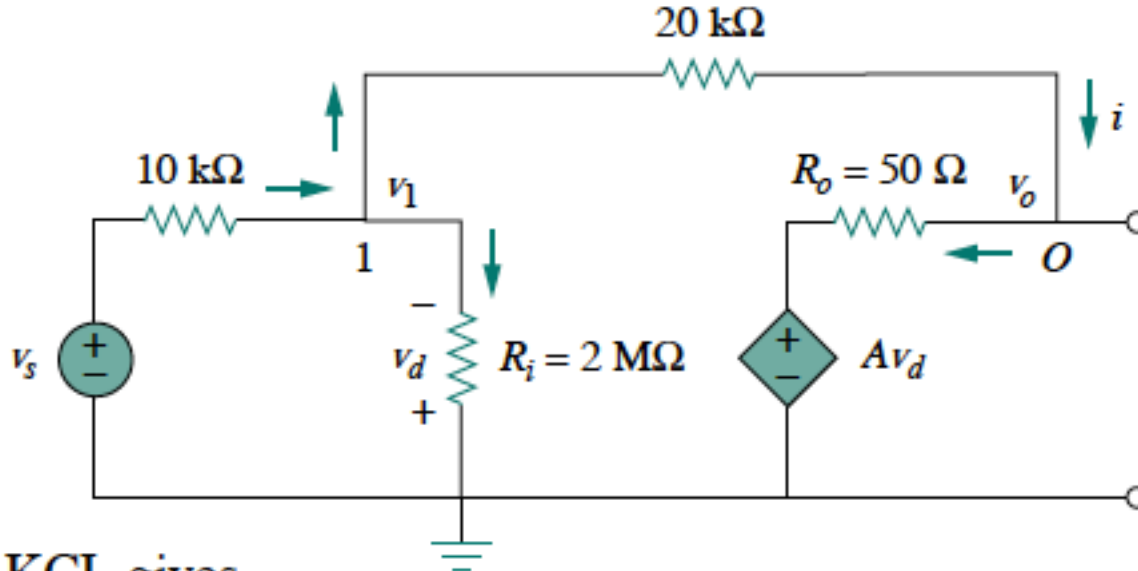
Example

- A 741 op amp has an open-loop voltage gain of 2×10^5 , input resistance of $2 \text{ M}\Omega$, and output resistance of 50Ω . It's used in the circuit shown.
- Find the closed-loop gain v_o/v_s and determine current i when $v_s = 2 \text{ V}$



Solution:

- We use the equivalent circuit and solve using nodal analysis...



At node 1, KCL gives

$$\frac{v_s - v_1}{10 \times 10^3} = \frac{v_1}{2000 \times 10^3} + \frac{v_1 - v_o}{20 \times 10^3}$$

Multiplying through by 2000×10^3 , we obtain

$$200v_s = 301v_1 - 100v_o$$

$$2v_s \simeq 3v_1 - v_o \quad \Rightarrow \quad v_1 = \frac{2v_s + v_o}{3}$$

At node O ,

$$\frac{v_1 - v_o}{20 \times 10^3} = \frac{v_o - Av_d}{50}$$

But $v_d = -v_1$ and $A = 200,000$. Then

$$v_1 - v_o = 400(v_o + 200,000v_1)$$

Substituting:

$$\frac{v_o}{v_s} = -1.9999699$$

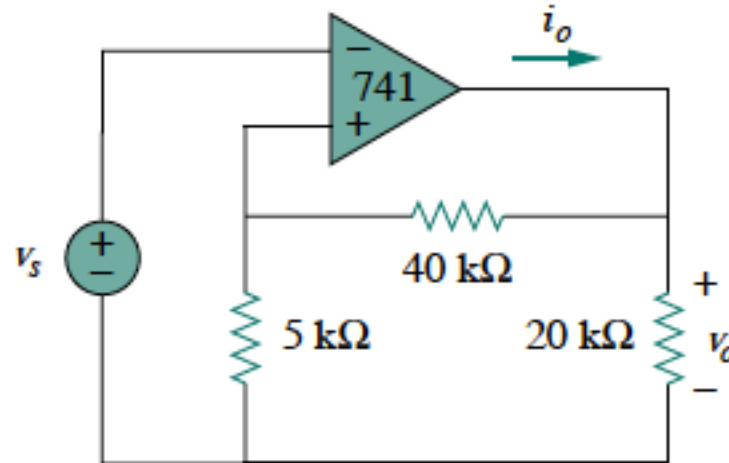
(actually exactly -2 if we hadn't approximated).

This is the closed loop gain.

When $v_s = 2V$, $v_o = -4V$, so $v_1 = 0$. Hence $i = 0.2 \text{ mA}$.

Practice

- Same 741 op amp as before; calculate closed loop gain v_o/v_s and find i_o when $v_s = 1V$.



Ideal OpAmp

It's much easier to understand op amp circuits if we assume ideal op amps. An op amp is ideal if it has the following characteristics:

1. Infinite open-loop gain, $A = \infty$.
 2. Infinite input resistance, $R_i = \infty$.
 3. Zero output resistance, $R_o = 0$.
- Although assuming an ideal op amp provides only an approximate analysis, most modern amplifiers have such large gains and input impedances that the approximate analysis is a good one. Unless stated otherwise, we will assume from now on that every op amp is ideal.
 - For circuit analysis, the ideal op amp is derived from the nonideal model.

- Two important characteristics of the ideal op amp are:

- The currents into both input terminals are zero:

$$i_1 = 0, \quad i_2 = 0$$

This is due to infinite input resistance. An infinite resistance between the input terminals implies that an open circuit exists there and current cannot enter the op amp. But the output current is not necessarily zero, according to our KCL equation $i_o = i_1 + i_2 + i_+ + i_-$

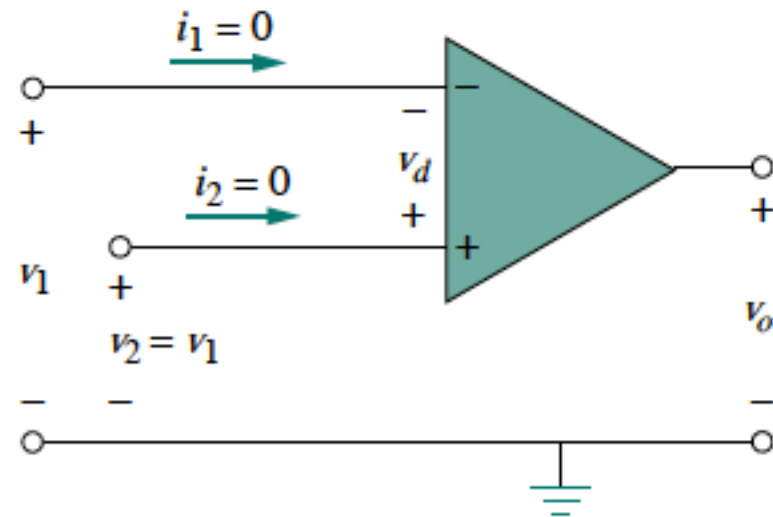
- The voltage across the input terminals is negligibly small; i.e.,

$$v_d = v_2 - v_1 \simeq 0$$

or

$$v_1 = v_2$$

Thus, an ideal op amp has zero current into its two input terminals and negligibly small voltage between the two input terminals. These equations are extremely important and should be regarded as the key handles to analyzing op amp circuits.



The two characteristics can be exploited by noting that for voltage calculations the input port behaves as a short circuit, while for current calculations the input port behaves as an open circuit.

Example

Rework the last Practice Problem using the ideal op amp model.

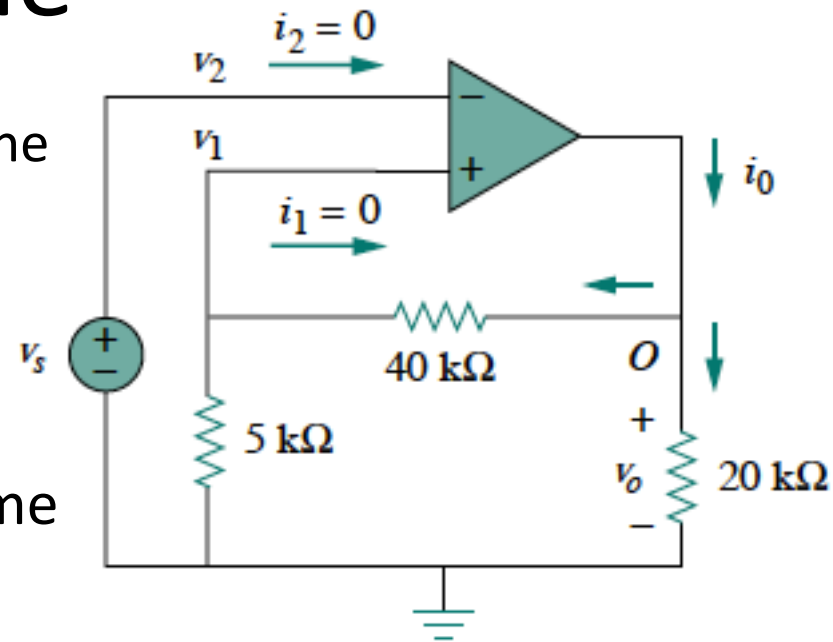
Solution:

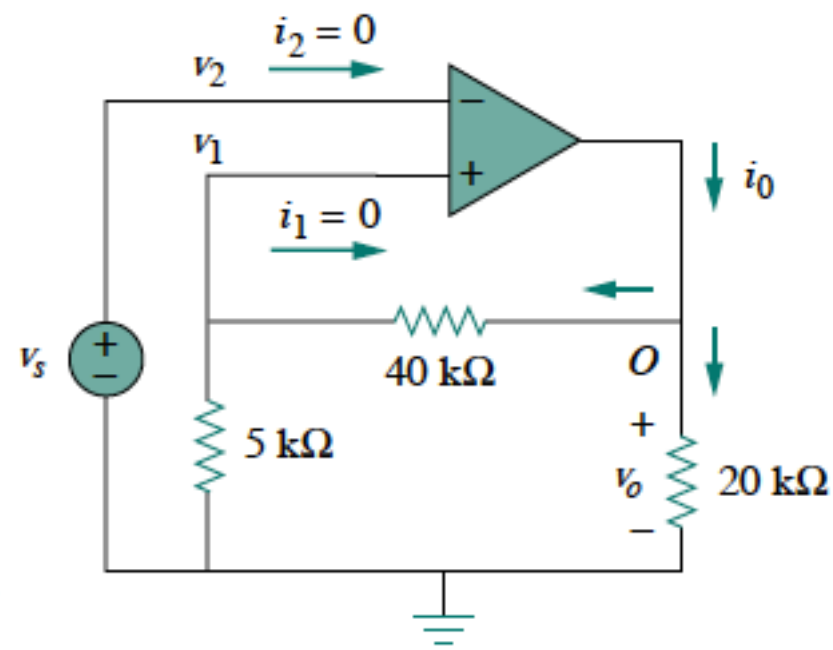
Note that $v_2 = v_s$

Since $i_1 = 0$, the 40-k Ω and 5-k Ω resistors are in series because the same current flows through them. v_1 is the voltage across the 5-k resistor.

Hence, using the voltage division principle,

$$v_1 = \frac{5}{5 + 40} v_o = \frac{v_o}{9}$$





- Because the op amp is ideal, $v_2 = v_1$
- Substituting yields the closed loop gain:

$$v_s = \frac{v_o}{9} \quad \Longrightarrow \quad \frac{v_o}{v_s} = 9$$

→ very close to the answer from the practice problem.

At node O ,

$$i_o = \frac{v_o}{40 + 5} + \frac{v_o}{20} \text{ mA}$$

when $v_s = 1\text{ V}$, $v_o = 9\text{ V}$. Substituting for $v_o = 9\text{ V}$

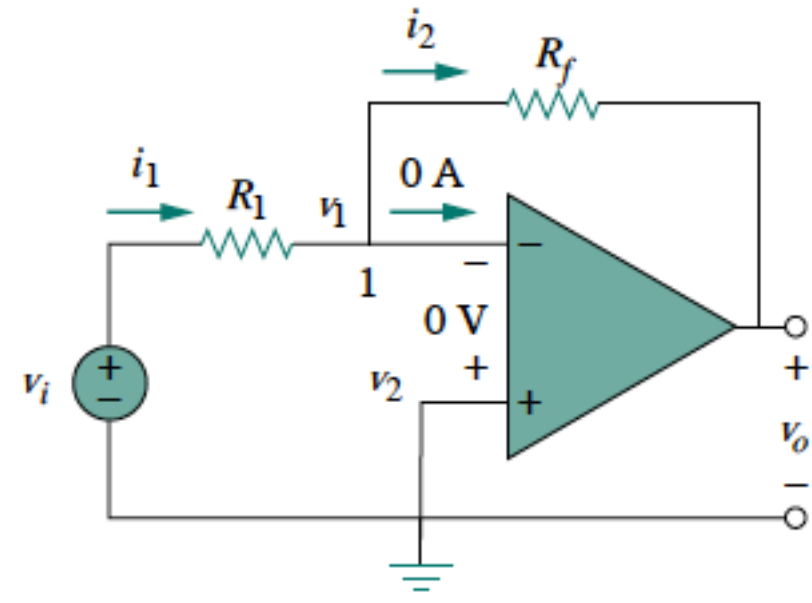
$$i_o = 0.2 + 0.45 = 0.65 \text{ mA}$$

Inverting Amplifier

- In this circuit, the noninverting input is grounded, v_i is connected to the inverting input through R_1 , and the feedback resistor R_f is connected between the inverting input and output.
- Our goal is to obtain the relationship between the input voltage v_i and the output voltage v_o .
- Applying KCL at node 1,

$$i_1 = i_2 \quad \Rightarrow \quad \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

- But $v_1 = v_2 = 0$ for an ideal op amp, since the noninverting terminal is grounded. Hence \rightarrow



$$\frac{v_i}{R_1} = -\frac{v_o}{R_f} \quad \text{or} \quad v_o = -\frac{R_f}{R_1} v_i$$

- The voltage gain is $A_v = v_o/v_i = -R_f/R_1$. The designation of the circuit as an *inverter* arises from the negative sign. Thus,

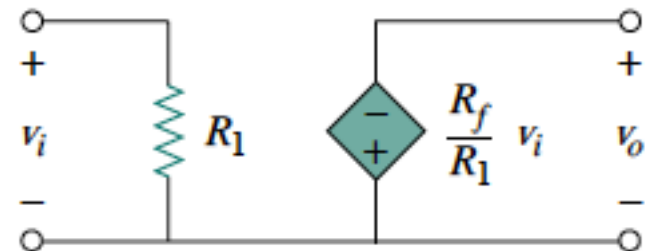
An inverting amplifier reverses the polarity of the input signal while amplifying it.

Notice that the gain is the feedback resistance divided by the input resistance which means that the gain depends only on the external elements connected to the op amp.

In view of the equation

$$v_o = -\frac{R_f}{R_1} v_i$$

an equivalent circuit for the inverting amplifier is as shown. The inverting amplifier is used, for example, in a current-to-voltage converter.



Example

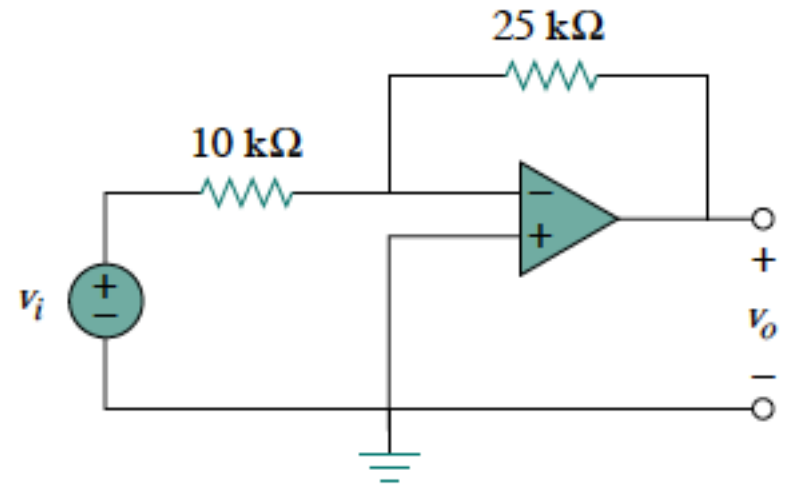
- If $v_i = 0.5$ V, calculate: (a) the output voltage v_o , and (b) the current in the 10 k resistor.

$$(a) \quad \frac{v_o}{v_i} = -\frac{R_f}{R_1} = -\frac{25}{10} = -2.5$$

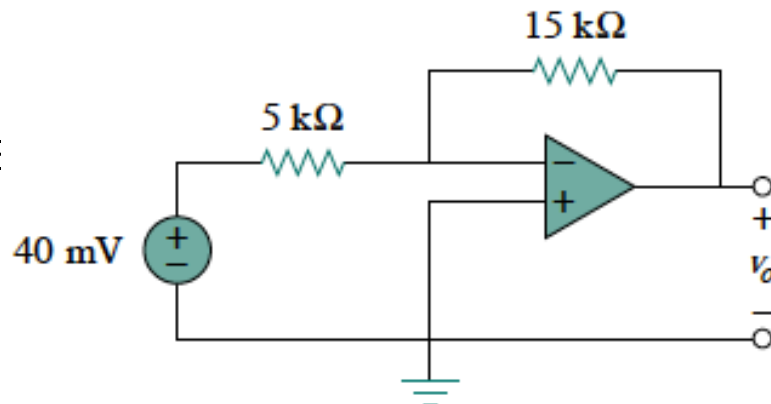
$$v_o = -2.5v_i = -2.5(0.5) = -1.25 \text{ V}$$

(b) current through 10k resistor:

$$i = \frac{v_i - 0}{R_1} = \frac{0.5 - 0}{10 \times 10^3} = 50 \mu\text{A}$$



PRACTICE



Example

- Determine v_o in the op amp circuit shown.
- Solution:

Applying KCL at node a ,

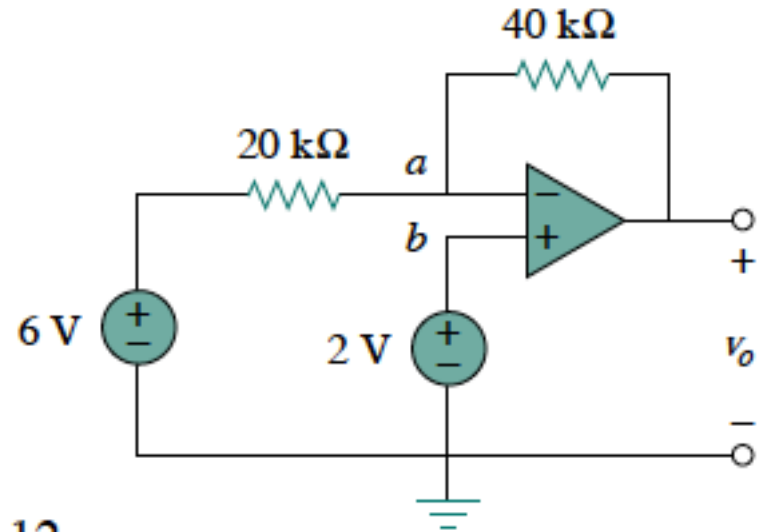
$$\frac{v_a - v_o}{40} = \frac{6 - v_a}{20}$$

$$v_a - v_o = 12 - 2v_a \quad \implies \quad v_o = 3v_a - 12$$

But $v_a = v_b = 2$ V for an ideal op amp, because of the zero voltage drop across the input terminals of the op amp. Hence,

$$v_o = 6 - 12 = -6 \text{ V}$$

Notice that if $v_b = 0 = v_a$, then $v_o = -12$, as expected



Noninverting Amplifier

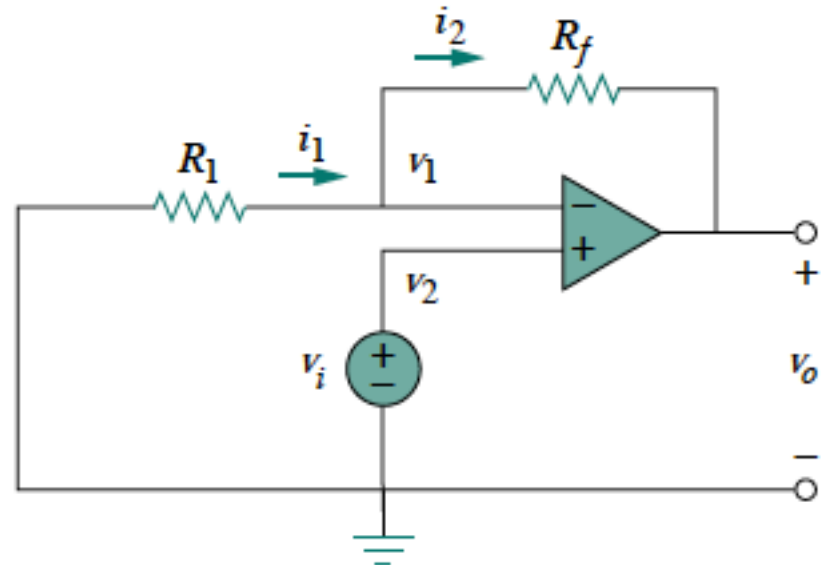
- In this case, the input voltage v_i is applied directly at the noninverting input terminal, and resistor R_1 is connected between the ground and the inverting terminal. We are interested in the output voltage and the voltage gain. Application of KCL at the inverting terminal gives

$$i_1 = i_2 \implies \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

But $v_1 = v_2 = v_i$.

$$\implies \frac{-v_i}{R_1} = \frac{v_i - v_o}{R_f}$$

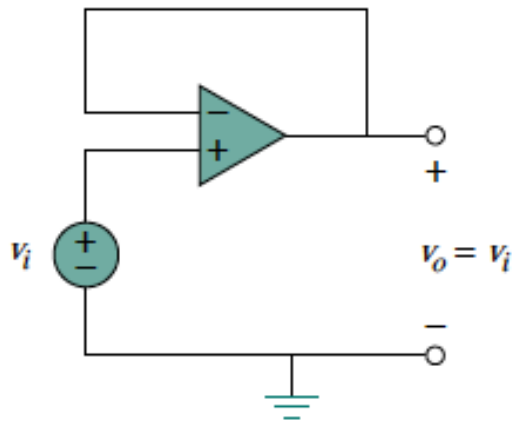
or
$$v_o = \left(1 + \frac{R_f}{R_1}\right) v_i$$



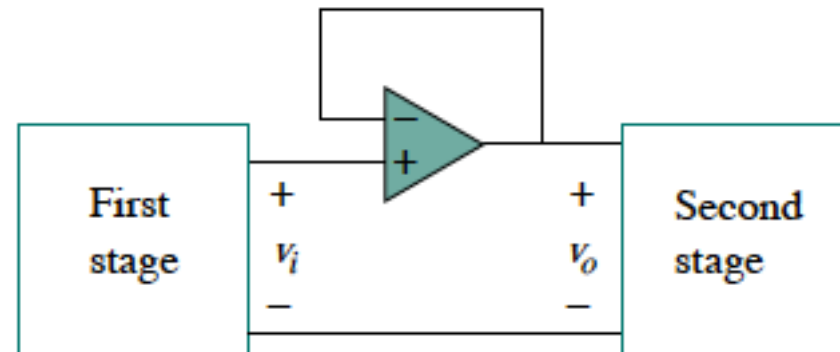
- The voltage gain is $\left(1 + \frac{R_f}{R_1}\right)$ which does not have a negative sign.
- Thus, the output has the same polarity as the input.

A noninverting amplifier is an op amp circuit designed to provide a positive voltage gain.

- Again we notice that the gain depends only on the external resistors.
- Notice that if feedback resistor $R_f = 0$ (short circuit) or $R_1 = \infty$ (open circuit) or both, the gain becomes 1. Under these conditions, the circuit becomes that shown below, which is called a voltage follower (or unity gain amplifier) because the output follows the input. Thus, for a voltage follower $v_o = v_i$



useful as an intermediate-stage (or buffer) amplifier to isolate one circuit from another →

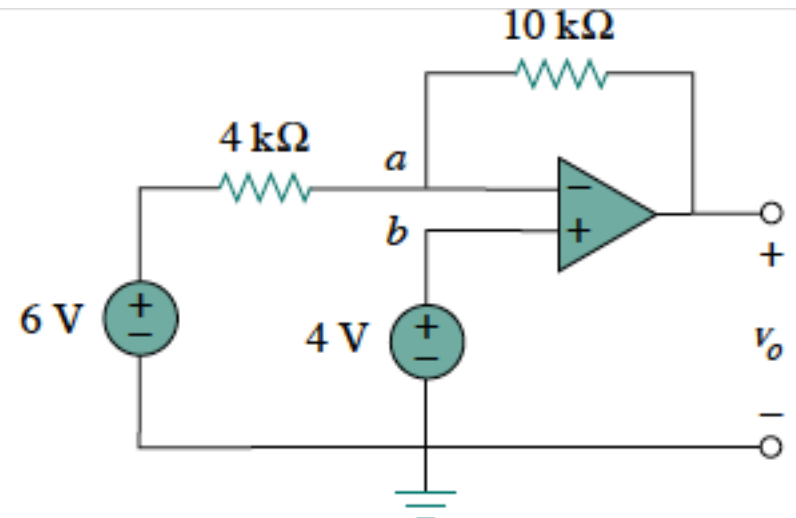


Example

- For this op amp circuit, calculate the output voltage v_o .

Solution:

- One way to solve it is using superposition: let $v_o = v_{o1} + v_{o2}$



where v_{o1} is due to the 6-V voltage source, and v_{o2} is due to the 4-V input.

To get v_{o1} , we set the 4-V source equal to zero. Under this condition, the circuit becomes an inverter, so

$$v_{o1} = -\frac{10}{4}(6) = -15 \text{ V}$$

To get v_{o2} , we set the 6-V source equal to zero. The circuit becomes a noninverting amplifier, so

$$v_{o2} = \left(1 + \frac{10}{4}\right) 4 = 14 \text{ V}$$

Thus, $v_o = v_{o1} + v_{o2} = -15 + 14 = -1 \text{ V}$

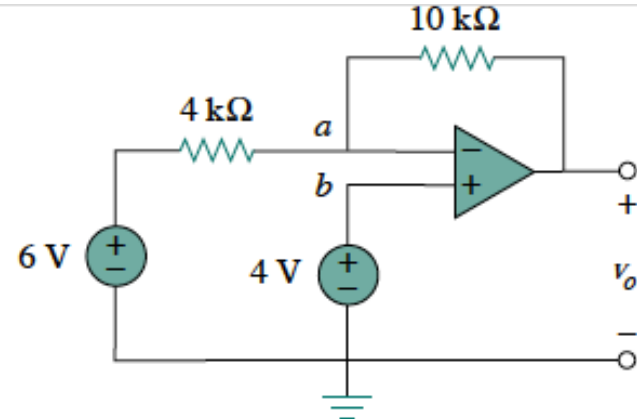
METHOD 2 Applying KCL at node a ,

$$\frac{6 - v_a}{4} = \frac{v_a - v_o}{10}$$

But $v_a = v_b = 4$, and so

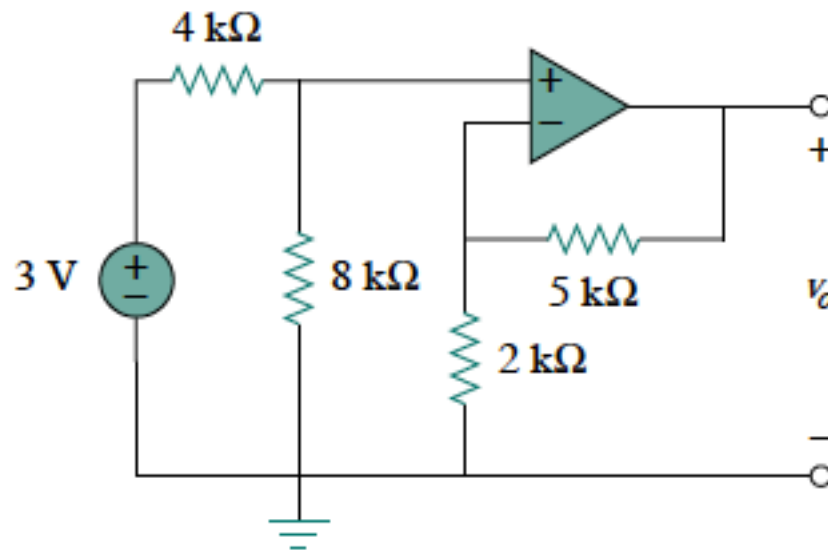
$$\frac{6 - 4}{4} = \frac{4 - v_o}{10} \quad \Rightarrow \quad 5 = 4 - v_o$$

or $v_o = -1$ V, as before.



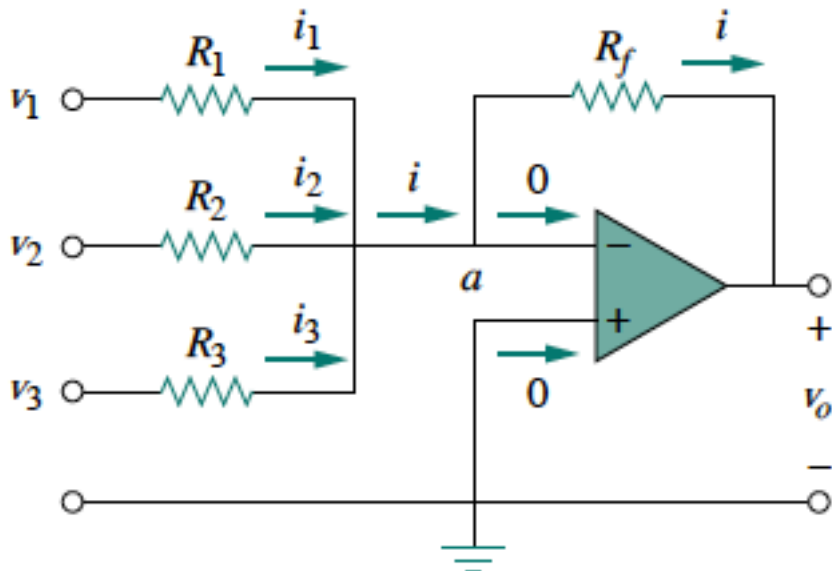
Practice Problem:

Calculate v_o in the circuit



Summing amplifier

- Besides amplification, the op amp can perform addition and subtraction.
- A summing amplifier is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.
- The summing amplifier, shown below, is a variation of the inverting amplifier. It takes advantage of the fact that the inverting configuration can handle many inputs at the same time.



Applying KCL at node a

$$i = i_1 + i_2 + i_3$$

$$i_1 = \frac{v_1 - v_a}{R_1}, \quad i_2 = \frac{v_2 - v_a}{R_2}$$

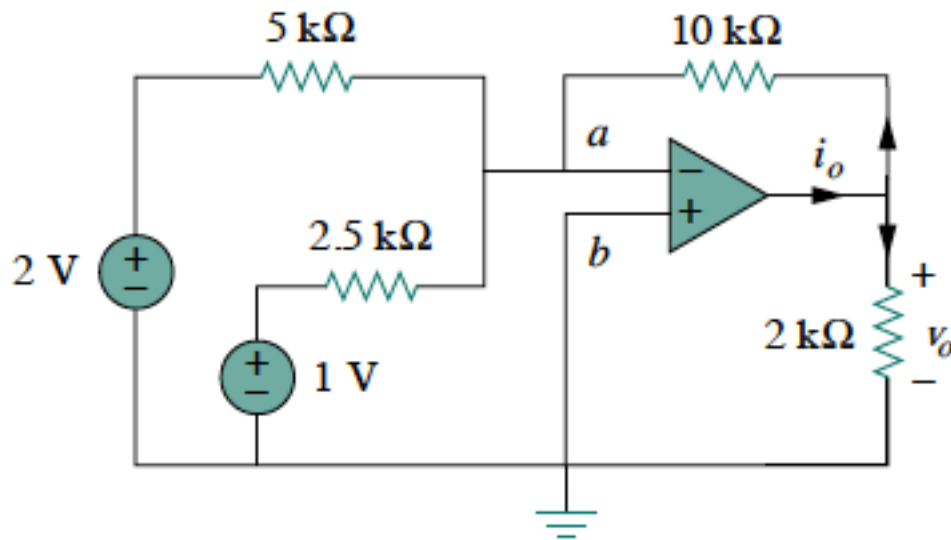
$$i_3 = \frac{v_3 - v_a}{R_3}, \quad i = \frac{v_a - v_o}{R_f}$$

$$v_o = - \left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 \right)$$

- Hence the output voltage is a weighted sum of the inputs. For this reason, the circuit is called a *summer*. Needless to say, the summer can have more than three inputs.

Example

Calculate v_o and i_o in the op amp circuit



This is a summer with two inputs

$$v_o = - \left[\frac{10}{5} (2) + \frac{10}{2.5} (1) \right]$$

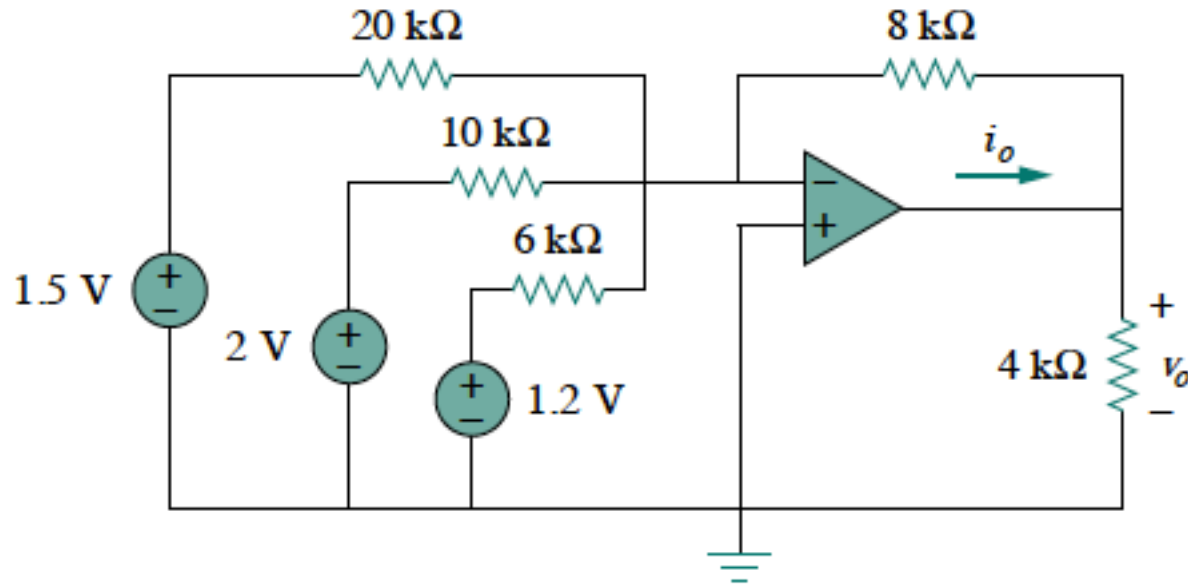
$$= -(4 + 4) = -8 \text{ V}$$

The current i_o is the sum of the currents through the 10-k and 2-k resistors. Both of these resistors have voltage $v_o = -8 \text{ V}$ across them, since $v_a = v_b = 0$. Hence,

$$i_o = \frac{v_o - 0}{10} + \frac{v_o - 0}{2} \text{ mA} = -0.8 - 0.4 = -1.2 \text{ mA}$$

Practice Problem

- Find v_o and i_o in the op amp circuit shown



Difference Amplifier

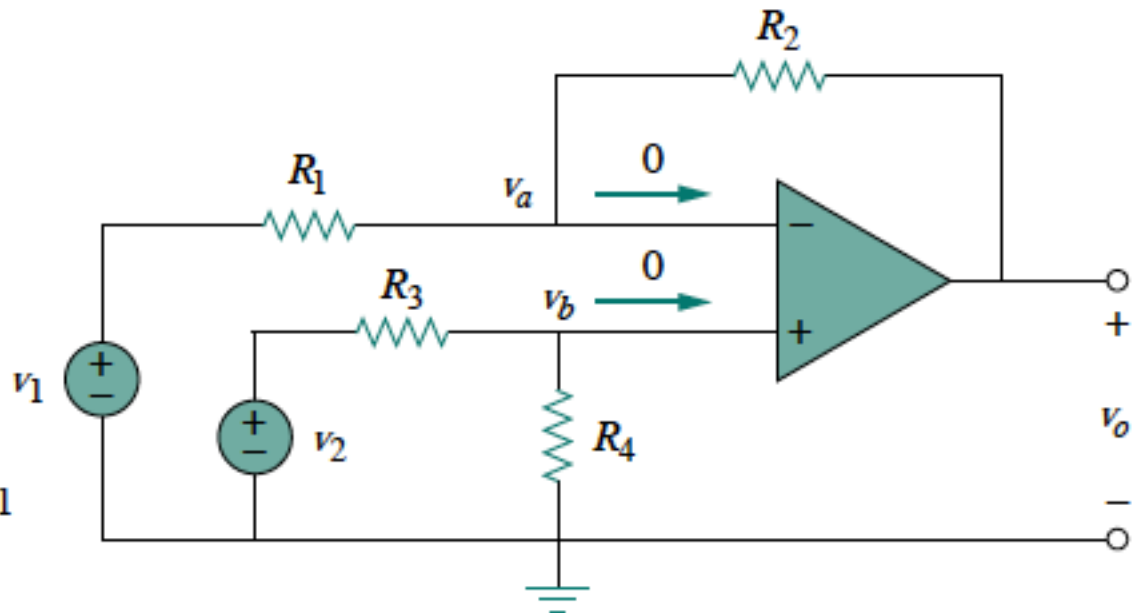
- Difference (or differential) amplifiers are used in various applications and are first cousins of the *instrumentation amplifier*, the most useful and popular amplifier.
- A difference amplifier is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.
- Consider the op amp circuit shown

Applying KCL to node a ,

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2}$$

or

$$v_o = \left(\frac{R_2}{R_1} + 1 \right) v_a - \frac{R_2}{R_1} v_1$$



Applying KCL to node b ,
$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4}$$

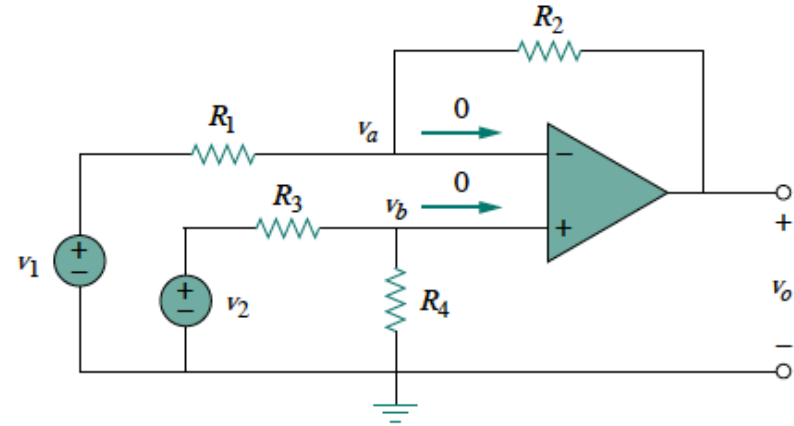
or
$$v_b = \frac{R_4}{R_3 + R_4} v_2$$

But $v_a = v_b$.

substituting:
$$v_o = \left(\frac{R_2}{R_1} + 1 \right) \frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} v_1$$

or

$$v_o = \frac{R_2 (1 + R_1/R_2)}{R_1 (1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$$



Since a difference amplifier must reject a signal common to the two inputs, the amplifier must have the property that $v_o = 0$ when $v_1 = v_2$. This

property exists when
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Thus, when the op amp circuit is a difference amplifier:
$$v_o = \frac{R_2}{R_1} (v_2 - v_1)$$

If $R_2 = R_1$ and $R_3 = R_4$, the difference amplifier becomes a *subtractor*, with the output

$$v_o = v_2 - v_1$$

Example

Design an op amp circuit with inputs v_1 and v_2 such that $v_o = 3v_2 - 5v_1$

Solution: This circuit can be realized in two ways.

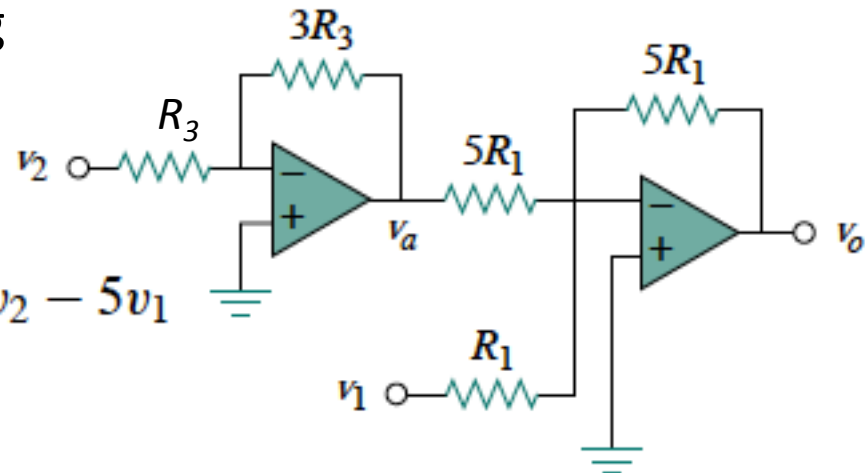
DESIGN 1: If we desire to use only one op amp, we can use the difference amplifier circuit we just saw. From its input-output equation

we just set $\frac{R_2}{R_1} = 5$ and $5 \frac{(1 + R_1/R_2)}{(1 + R_3/R_4)} = 3 \implies R_3 = R_4$

If we choose $R_1 = 10 \text{ k}\Omega$ and $R_3 = 20 \text{ k}\Omega$, then $R_2 = 50 \text{ k}\Omega$ and $R_4 = 20 \text{ k}\Omega$

DESIGN 2: we can cascade an inverting amplifier and a two-input inverting summer as shown.

For the summer, $v_o = -v_a - 5v_1$
 and for the inverter, $v_a = -3v_2 \implies v_o = 3v_2 - 5v_1$



Instrumentation Amplifier

- An *instrumentation amplifier* is an amplifier of low level signals used in process control or measurement applications and commercially available in single-package units.

A3 is a difference amplifier, so

$$v_o = \frac{R_2}{R_1}(v_{o2} - v_{o1})$$

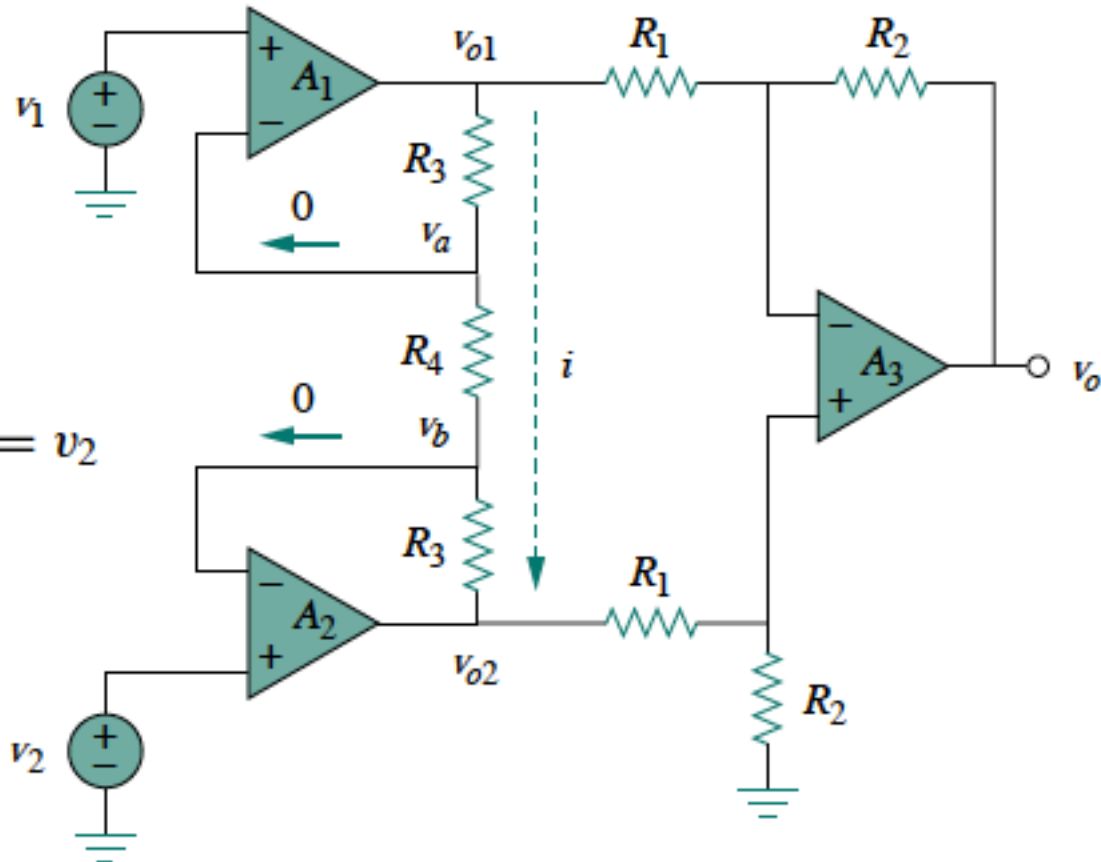
A1 and A2 draw no current, so

$$\begin{aligned} v_{o1} - v_{o2} &= i(R_3 + R_4 + R_3) \\ &= i(2R_3 + R_4) \end{aligned}$$

But $i = \frac{v_a - v_b}{R_4}$ $v_a = v_1, \quad v_b = v_2$

So $i = \frac{v_1 - v_2}{R_4}$ and

$$v_o = \frac{R_2}{R_1} \left(1 + \frac{2R_3}{R_4} \right) (v_2 - v_1)$$



Practice Problem

Obtain i_o in the instrumentation amplifier circuit of Fig. 5.27.

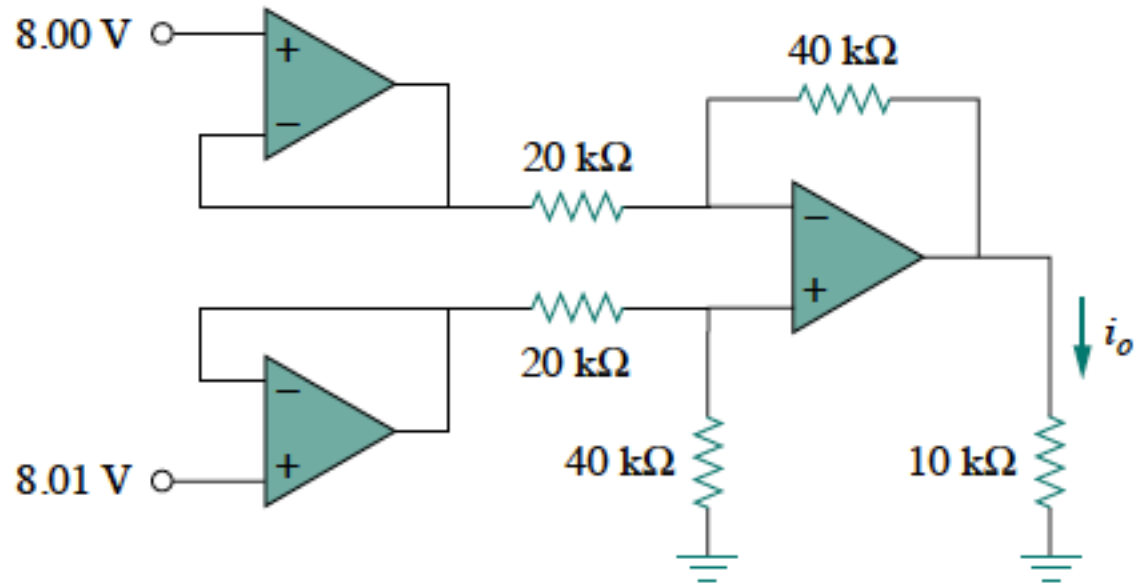


Figure 5.27 Instrumentation amplifier; for Practice Prob. 5.8.

Cascaded Op Amp circuits

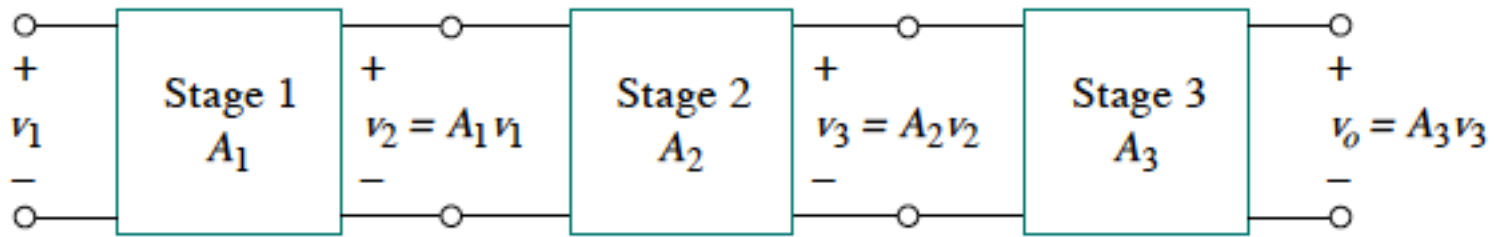
- As we know, op amp circuits are modules or building blocks for designing complex circuits. It is often necessary in practical applications to connect op amp circuits in cascade (i.e., head to tail) to achieve a large overall gain.
- In general, two circuits are cascaded when they are connected in tandem, one behind another in a single file.
- When op amp circuits are cascaded, each circuit in the string is called a *stage*; the original input signal is increased by the gain of the individual stage. Op amp circuits have the advantage that they can be cascaded without changing their input-output relationships. This is due to the fact that each (ideal) op amp circuit has infinite input resistance and zero output resistance.

Cascaded Op Amp circuits

- Here is a block diagram representation of three op amp circuits in cascade. Since the output of one stage is the input to the next stage, the overall gain of the cascade connection is the product of the gains of the individual op amp circuits, or

$$A = A_1 A_2 A_3$$

- Although the cascade connection does not affect the op amp input-output relationships, care must be exercised in the design of an actual op amp circuit to ensure that the load due to the next stage in the cascade does not saturate the op amp.



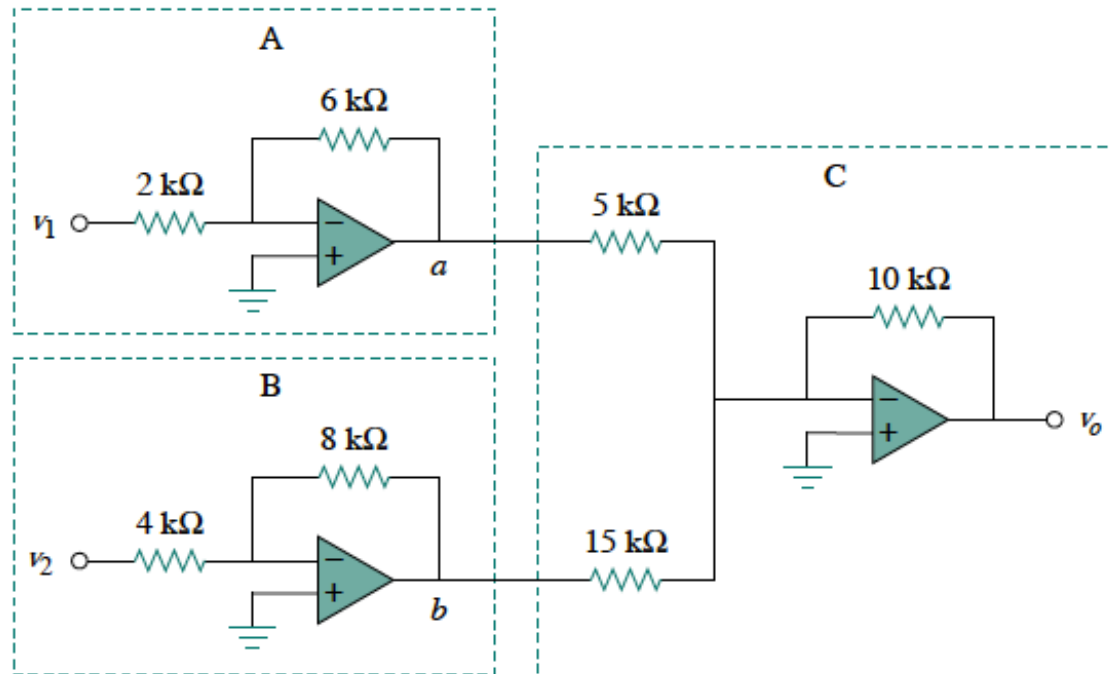
Example

If $v_1 = 1$ V and $v_2 = 2$ V,
find v_o in this op amp
circuit.

First find outputs of
inverters A & B:

$$v_a = -\frac{6}{2}(v_1) = -3(1) = -3 \text{ V},$$

$$v_b = -\frac{8}{4}(v_2) = -2(2) = -4 \text{ V}$$



These become the inputs to the summer so that the output is obtained as

$$v_o = -\left(\frac{10}{5}v_a + \frac{10}{15}v_b\right) = -\left[2(-3) + \frac{2}{3}(-4)\right] = 8.333 \text{ V}$$

Practice Problem

If $v_1 = 2\text{ V}$ and $v_2 = 1.5\text{ V}$, find v_o in the op amp circuit of Fig. 5.32.

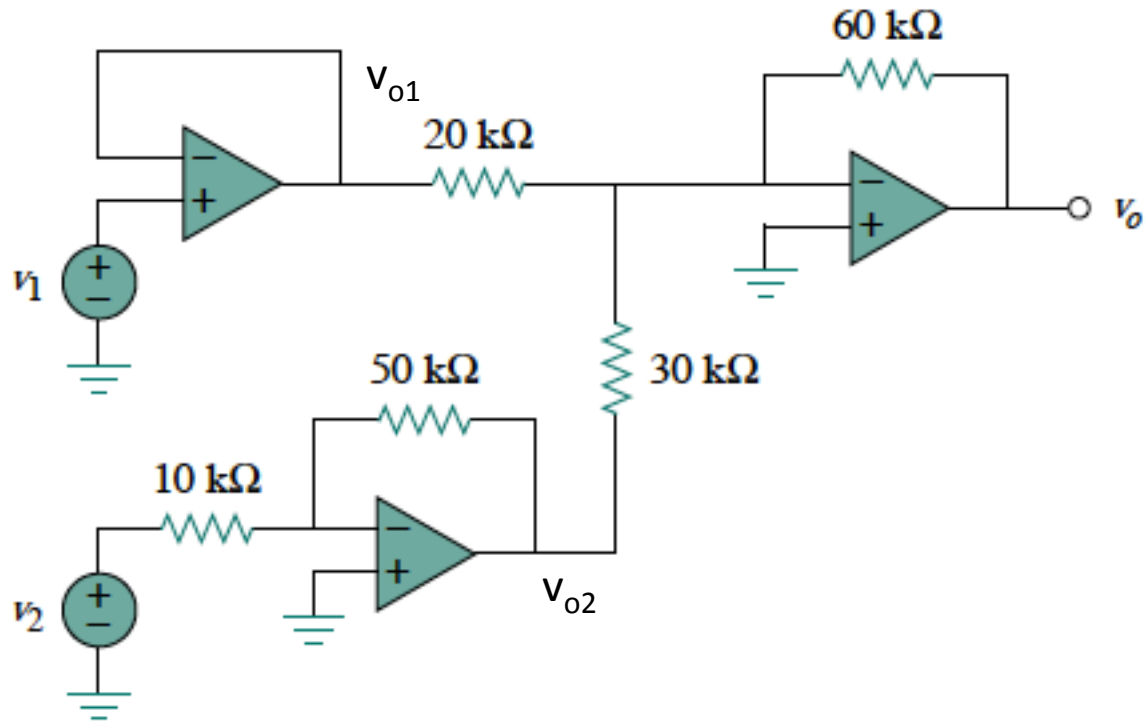
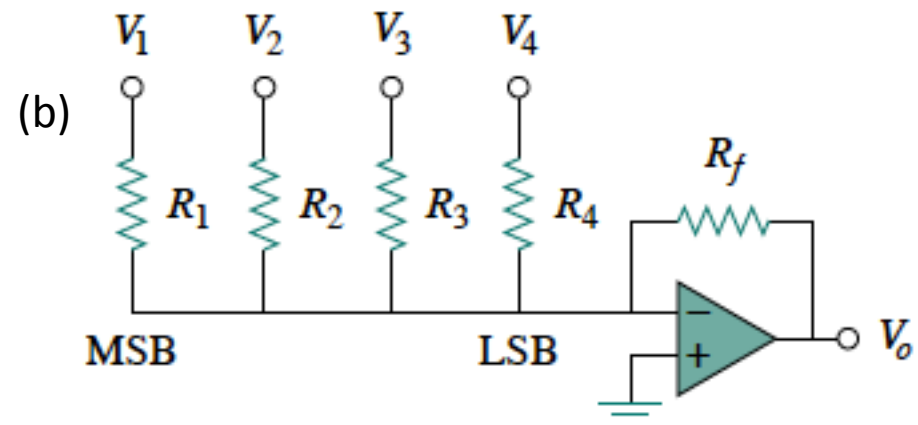
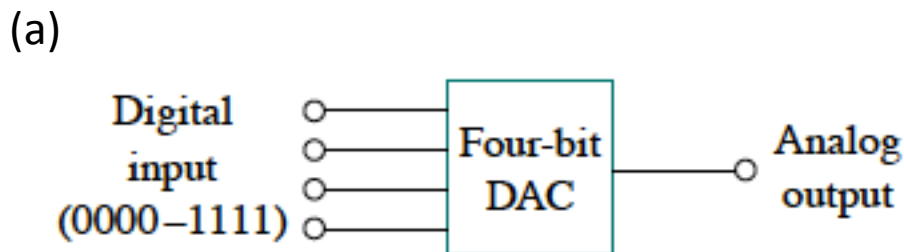


Figure 5.32 Practice Prob. 5.10.

Application: Digital-to-Analog Converter

- The digital-to-analog converter (DAC) transforms digital signals into analog form. A typical example of a four-bit DAC is illustrated in (a).
- The four-bit DAC can be realized in many ways. A simple realization is the *binary weighted ladder*, shown in (b). The bits are weights according to the magnitude of their place value, by descending value of R_f/R_n so that each lesser bit has half the weight of the next higher. This is obviously an inverting summing amplifier. The output is related to the inputs as

$$-V_o = \frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3 + \frac{R_f}{R_4} V_4$$



- Input V_1 is called the *most significant bit* (MSB), while input V_4 is the *least significant bit* (LSB). Each of the four binary inputs V_1, \dots, V_4 can assume only two voltage levels: 0 or 1 V. By using the proper input and feedback resistor values, the DAC provides a single output that is proportional to the inputs.
- Example: In this op amp circuit, let $R_f = 10 \text{ k}\Omega$, $R_1 = 10 \text{ k}\Omega$, $R_2 = 20 \text{ k}\Omega$, $R_3 = 40 \text{ k}\Omega$, and $R_4 = 80 \text{ k}\Omega$. Obtain the analog output for binary inputs [0000], [0001], [0010], \dots , [1111].

Solution:

- Substituting into previous Equation:

$$\begin{aligned}
 -V_o &= \frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3 + \frac{R_f}{R_4} V_4 \\
 &= V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4
 \end{aligned}$$

- So a digital input $[V_1 V_2 V_3 V_4] = [0000]$ produces an analog output of $-V_o = 0 \text{ V}$; [0001] gives $-V_o = 0.125 \text{ V}$, [0010] gives $-V_o = 0.25 \text{ V}$ and so on.

Instrumentation Amp revisited

We can make the resistors all equal except for one that we pull out to allow us to vary the gain. We saw before that $v_o = A_v(v_2 - v_1)$

where

$$A_v = 1 + \frac{2R}{R_G}$$

Amplifies small differential signal voltages.
Common-mode voltages cancel.

