Bioelectrical Circuits: Lecture 10

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BME 205 L10

SINUSOIDAL STEADY STATE ANALYSIS
Introduction

• We just learned that the forced or steady-state response of circuits to sinusoidal inputs can be obtained by using phasors. We also know that Ohm’s and Kirchhoff’s laws are applicable to ac circuits.

• Now we want to see how nodal analysis, mesh analysis, Thevenin’s theorem, Norton’s theorem, superposition, and source transformations are applied in analyzing ac circuits. Since these techniques were already introduced for dc circuits, we’ll mostly illustrate with examples.

• Analyzing ac circuits usually requires three steps.
  – 1. Transform the circuit to the phasor or frequency domain.
  – 2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
  – 3. Transform the resulting phasor to the time domain

• Step 1 is not necessary if the problem is specified in the frequency domain. In step 2, the analysis is performed in the same manner as dc circuit analysis except that complex numbers are involved.
Nodal Analysis

- The basis of nodal analysis is Kirchhoff’s current law. Since KCL is valid for phasors, we can analyze ac circuits by nodal analysis. The following examples illustrate this.

**EXAMPLE**

Find \(i_x\) by nodal analysis.

Solution:
We first convert the circuit to the frequency domain:

\[
\begin{align*}
20 \cos 4t &\quad \Rightarrow \quad 20 / 0^\circ, \quad \omega = 4 \text{ rad/s} \\
1 \text{ H} &\quad \Rightarrow \quad j \omega L = j4 \\
0.5 \text{ H} &\quad \Rightarrow \quad j \omega L = j2 \\
0.1 \text{ F} &\quad \Rightarrow \quad \frac{1}{j \omega C} = -j2.5
\end{align*}
\]
Thus, the frequency-domain equivalent circuit is as shown

![Circuit Diagram](image)

Applying KCL at node 1,

\[
\frac{20 - V_1}{10} = \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4} \Rightarrow (1 + j1.5)V_1 + j2.5V_2 = 20
\]

At node 2,

\[2I_x + \frac{V_1 - V_2}{j4} = \frac{V_2}{j2}\]

But \(I_x = V_1 / -j2.5\). Substituting this gives

\[
\frac{2V_1}{-j2.5} + \frac{V_1 - V_2}{j4} = \frac{V_2}{j2}
\]

By simplifying, we get

\[11V_1 + 15V_2 = 0\]
We can put these equations in matrix form:

\[
\begin{bmatrix}
1 + j1.5 & j2.5 \\
11 & 15
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
= \begin{bmatrix}
20 \\
0
\end{bmatrix}
\]

We obtain the determinants as

\[
\Delta = \begin{vmatrix}
1 + j1.5 & j2.5 \\
11 & 15
\end{vmatrix} = 15 - j5
\]

\[
\Delta_1 = \begin{vmatrix}
20 & j2.5 \\
0 & 15
\end{vmatrix} = 300, \quad \Delta_2 = \begin{vmatrix}
1 + j1.5 & 20 \\
11 & 0
\end{vmatrix} = -220
\]

\[
V_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97 \angle 18.43^\circ \text{ V}
\]

\[
V_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91 \angle 198.3^\circ \text{ V}
\]

The current \( I_x \) is given by

\[
I_x = \frac{V_1}{-j2.5} = \frac{18.97 \angle 18.43^\circ}{2.5 \angle -90^\circ} = 7.59 \angle 108.4^\circ \text{ A}
\]

Transforming this to the time domain,

\[
i_x = 7.59 \cos(4t + 108.4^\circ) \text{ A}
\]
Practice problem

Using nodal analysis, find $v_1$ and $v_2$

Answer: 

$$v_1(t) = 11.33 \sin(2t + 60.02^\circ)$$

$$v_2(t) = 33.02 \sin(2t + 57.13^\circ)$$
Example

Compute $V_1$ and $V_2$

Nodes 1 and 2 form a supernode as shown. Applying KCL at the supernode gives

$$3 = \frac{V_1}{-j3} + \frac{V_2}{j6} + \frac{V_2}{12}$$

or

$$36 = j4V_1 + (1 - j2)V_2$$
But a voltage source is connected between nodes 1 and 2, so that

\[ V_1 = V_2 + 10\angle 45^\circ \]

Substituting into the last equation gives

\[ 36 - 40\angle 135^\circ = (1 + j2)V_2 \quad \implies \quad V_2 = 31.41\angle -87.18^\circ \text{ V} \]

Then

\[ V_1 = V_2 + 10\angle 45^\circ = 25.78\angle -70.48^\circ \text{ V} \]
Calculate $V_1$ and $V_2$

$15/0^\circ \text{ V}$

$20/60^\circ \text{ V}$

$4 \Omega$

$j4 \Omega$

$-j1 \Omega$

$2 \Omega$

Answer: $V_1 = 19.36/69.67^\circ \text{ V}$,
$V_2 = 3.376/165.7^\circ \text{ V}$. 
Mesh Analysis

Recall: Mesh analysis is all about applying KVL. Now we have complex numbers.

**EXAMPLE**

Determine current $I_o$

Solution:

Applying KVL to mesh 1, we obtain

$$(8 + j10 - j2)I_1 - (-j2)I_2 - j10I_3 = 0$$

For mesh 2,

$$(4 - j2 - j2)I_2 - (-j2)I_1 - (-j2)I_3 + 20/90^\circ = 0$$

For mesh 3, $I_3 = 5$. Therefore:

$$(8 + j8)I_1 + j2I_2 = j50$$

$$j2I_1 + (4 - j4)I_2 = -j20 - j10$$
In matrix form:

\[
\begin{bmatrix}
8 + j8 & j2 \\
 j2 & 4 - j4 \\
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\end{bmatrix}
= 
\begin{bmatrix}
j50 \\
-j30 \\
\end{bmatrix}
\]  

from which we obtain the determinants

\[
\Delta = \begin{vmatrix}
8 + j8 & j2 \\
 j2 & 4 - j4 \\
\end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68
\]

\[
\Delta_2 = \begin{vmatrix}
8 + j8 & j50 \\
 j2 & -j30 \\
\end{vmatrix} = 340 - j240 = 416.17 \angle -35.22^\circ
\]

\[
I_2 = \frac{\Delta_2}{\Delta} = \frac{416.17 \angle -35.22^\circ}{68} = 6.12 \angle -35.22^\circ \text{ A}
\]

The desired current is 

\[
I_o = -I_2 = 6.12 \angle 144.78^\circ \text{ A}
\]
Practice Problem

Find $I_o$ using mesh analysis.

Answer: $1.194/65.45^\circ$ A.
Example

Find $V_o$ using mesh analysis
Solution:
As shown below, meshes 3 and 4 form a supermesh due to the current source between the meshes. For mesh 1, KVL gives

$$-10 + (8 - j2)I_1 - (-j2)I_2 - 8I_3 = 0$$

or

$$(8 - j2)I_1 + j2I_2 - 8I_3 = 10$$

For mesh 2

$$I_2 = -3$$

For the supermesh

$$(8 - j4)I_3 - 8I_1 + (6 + j5)I_4 - j5I_2 = 0$$

Due to the current source between meshes 3 and 4, at node A,

$$I_4 = I_3 + 4$$

With substitutions we can get two equations in $I_1$ and $I_3$:

$$(8 - j2)I_1 - 8I_3 = 10 + j6$$

$$-8I_1 + (14 + j)I_3 = -24 - j.$$
Matrix equation is:
\[
\begin{bmatrix}
8 - j2 & -8 \\
-8 & 14 + j
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_3
\end{bmatrix} =
\begin{bmatrix}
10 + j6 \\
-24 - j35
\end{bmatrix}
\]

We obtain the following determinants:

\[
\Delta = \begin{vmatrix}
8 - j2 & -8 \\
-8 & 14 + j
\end{vmatrix} = 112 + j8 - j28 + 2 - 64 = 50 - j20
\]

\[
\Delta_1 = \begin{vmatrix}
10 + j6 & -8 \\
-24 - j35 & 14 + j
\end{vmatrix} = 140 + j10 + j84 - 6 - 192 - j280 = -58 - j186
\]

Current \( I_1 \) is obtained as

\[
I_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618/274.5^\circ \text{ A}
\]

The required voltage \( V_o \) is

\[
V_o = -j2(I_1 - I_2) = -j2(3.618/274.5^\circ + 3)
\]

\[
= -7.2134 - j6.568 = 9.756/222.32^\circ \text{ V}
\]
Practice problem

Calculate current $I_o$ in the circuit

Answer: $5.075 \angle 5.943^\circ \text{ A.}$
Superposition theorem

- Since ac circuits are linear, the superposition theorem applies to ac circuits the same way it applies to dc circuits.
- The theorem becomes important if the circuit has sources operating at different frequencies. In this case, since the impedances depend on frequency, we must have a different frequency-domain circuit for each frequency. The total response must be obtained by adding the individual responses in the time domain.
- It is incorrect to try to add the responses in the phasor or frequency domain. Why? Because the exponential factor $e^{j\omega t}$ is implicit in sinusoidal analysis, and that factor would change for every angular frequency $\omega$. It would therefore not make sense to add responses at different frequencies in the phasor domain. Thus, when a circuit has sources operating at different frequencies, one must add the responses due to the individual frequencies in the time domain.
Example

Use superposition to re-solve this circuit, to find $I_o$. 
Let \( I_o = I'_o + I''_o \)

where \( I'_o \) and \( I''_o \) are due to the voltage and current sources, respectively.

To find \( I'_o \), consider the circuit in (a). If we let \( Z \) be the parallel combination of \(-j2\) and \( 8 + j10 \), then

\[
Z = \frac{-j2(8 + j10)}{-2j + 8 + j10} = 0.25 - j2.25
\]

and so the current is

\[
I'_o = \frac{j20}{4 - j2 + Z} = \frac{j20}{4.25 - j4.25}
\]

or \( I'_o = -2.353 + j2.353 \)

To get \( I''_o \), consider circuit in (b)

mesh 1: \((8 + j8)I_1 - j10I_3 + j2I_2 = 0 \)

mesh 2: \((4 - j4)I_2 + j2I_1 + j2I_3 = 0 \)

mesh 3: \( I_3 = 5 \)
Substituting for $I_3$

$$(4 - j4)I_2 + j2I_1 + j10 = 0$$

Expressing $I_1$ in terms of $I_2$ gives

$$I_1 = (2 + j2)I_2 - 5$$

Then the mesh 1 equation becomes

$$(8 + j8)(2 + j2)I_2 - 5 - j50 + j2I_2 = 0$$

$$\Rightarrow I_2 = \frac{90 - j40}{34} = 2.647 - j1.176$$

Current $I''_o$ is obtained as

$$I''_o = -I_2 = -2.647 + j1.176$$

Putting it all together:

$$I_o = I'_o + I''_o = -5 + j3.529 = 6.12 \angle 144.78^\circ \text{ A}$$

Note that this took much longer than when we used mesh analysis. But as we’ll see in the next example superposition is the best way to deal with more than one frequency in a circuit.
Practice Problem

Find $I_o$

using superposition

Answer: $1.194/65.45^\circ$ A.
Find $v_o$ using the superposition theorem.

The circuit contains sources at 3 different frequencies (including the DC source, for which $\omega = 0$). To solve using superposition, we break the problem into three single-frequency problems.

We let

$$v_o = v_1 + v_2 + v_3$$

where $v_1$ is due to the 5-V dc voltage source, $v_2$ is due to the $10 \cos 2t$ V voltage source, and $v_3$ is due to the $2 \sin 5t$ A current source.
To find $v_1$, we set to zero all sources except the 5-V dc source. We recall that at steady state, a capacitor is an open circuit to dc while an inductor is a short circuit to dc. There is an alternative way of looking at this.

Since $\omega = 0$, $j\omega L = 0$, $1/j\omega C = \infty$. Either way, the equivalent circuit is as shown in (a). By voltage division,

$$-v_1 = \frac{1}{1+4} (5) = 1 \text{ V}$$

To find $v_2$, source and the $2 \sin 5t$ current source and transform the circuit to the frequency domain.

\[
\begin{align*}
10 \cos 2t & \Rightarrow 10/0^\circ, \quad \omega = 2 \text{ rad/s} \\
2 \text{ H} & \Rightarrow j\omega L = j4 \Omega \\
0.1 \text{ F} & \Rightarrow \frac{1}{j\omega C} = -j5 \Omega
\end{align*}
\]
Let \( Z = -j5 \parallel 4 = \frac{-j5 \times 4}{4 - j5} = 2.439 - j1.951 \)

By voltage division,

\[
V_2 = \frac{1}{1 + j4 + Z} (10/0^\circ) = \frac{10}{3.439 + j2.049} = 2.498/ -30.79^\circ
\]

In the time domain,

\[v_2 = 2.498 \cos(2t - 30.79^\circ)\]

To obtain \( v_3 \), we set the voltage sources to zero and transform what is left to the frequency domain. Equivalent circuit in (c).

\[2 \sin 5t \implies 2/ -90^\circ, \quad \omega = 5 \text{ rad/s}\]

\[2 \text{ H} \implies j\omega L = j10 \Omega\]

\[0.1 \text{ F} \implies \frac{1}{j\omega C} = -j2 \Omega\]
Let \( Z_1 = -j2 \parallel 4 = \frac{-j2 \times 4}{4 - j2} = 0.8 - j1.6 \, \Omega \)

By current division,

\[
I_1 = \frac{j10}{j10 + 1 + Z_1} \left(2 \angle -90^\circ\right) \, A
\]

\[
V_3 = I_1 \times 1 = \frac{j10}{1.8 + j8.4} (-j2) = 2.328 \angle -77.91^\circ \, V
\]

In the time domain,

\[
v_3 = 2.33 \cos(5t - 80^\circ) = 2.33 \sin(5t + 10^\circ) \, V
\]

Adding the three components to finish:

\[
v_o(t) = -1 + 2.498 \cos(2t - 30.79^\circ) + 2.33 \sin(5t + 10^\circ) \, V
\]
Practice problem

Find $v_o$ using superposition

Answer: $4.631 \sin(5t - 81.12^\circ) + 1.051 \cos(10t - 86.24^\circ)$ V.
Source transformation

As shown below, source transformation in the frequency domain involves transforming a voltage source in series with an impedance to a current source in parallel with an impedance, or vice versa. As we go from one source type to another, we must keep the following relationship in mind:

\[ V_s = Z_s I_s \quad \iff \quad I_s = \frac{V_s}{Z_s} \]
Calculate $V_x$ in the circuit of Fig. 10.17 using the method of source transformation.
Solution:
We transform the voltage source to a current source and obtain the circuit in Fig. 10.18(a), where

\[ I_s = \frac{20}{5} \frac{-90}{-90} = 4 \frac{-90}{-90} = -j4 \text{ A} \]

The parallel combination of 5-Ω resistance and (3 + j4) impedance gives

\[ Z_1 = \frac{5(3 + j4)}{8 + j4} = 2.5 + j1.25 \text{ Ω} \]

Converting the current source to a voltage source yields the circuit in Fig. 10.18(b), where

\[ V_s = I_s Z_1 = -j4(2.5 + j1.25) = 5 - j10 \text{ V} \]
By voltage division,

\[ V_x = \frac{10}{10 + 2.5 + j1.25 + 4 - j13} (5 - j10) = 5.519 \angle -28^\circ \text{ V} \]
Practice problem

Find $I_o$ using source transformation

Answer: $3.288 \angle 99.46^\circ$ A.
Thevenin & Norton Equivalent circuits

- Thevenin’s and Norton’s theorems are applied to ac circuits in the same way as they are to dc circuits (but in ac complex numbers come in).
- The frequency-domain version of a Thevenin equivalent circuit is depicted below left, where a linear circuit is replaced by a voltage source in series with an impedance. The Norton equivalent circuit is illustrated below right, where a linear circuit is replaced by a current source in parallel with an impedance. Keep in mind that the two equivalent circuits are related as

\[ V_{Th} = Z_N I_N, \quad Z_{Th} = Z_N \]

\( V_{Th} \) is the open-circuit voltage while \( I_N \) is the short-circuit current.

If the circuit has sources operating at different frequencies, the Thevenin or Norton equivalent circuit must be determined at each frequency. This leads to entirely different equivalent circuits, one for each frequency, not one equivalent circuit with equivalent sources and equivalent impedances.
Example

Obtain the Thevenin equivalent at terminals $a-b$ of the circuit in Fig. 10.22.

Solution:
We find $Z_{Th}$ by setting the voltage source to zero. As shown in Fig. 10.23(a), the $8$-Ω resistance is now in parallel with the $-j6$ reactance, so that their combination gives

$$Z_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3.84 \Omega$$
Similarly, the 4-Ω resistance is in parallel with the \( j12 \) reactance, and their combination gives

\[
Z_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2 \, \Omega
\]

![Figure 10.23](image)

The Thevenin impedance is the series combination of \( Z_1 \) and \( Z_2 \); that is,

\[
Z_{Th} = Z_1 + Z_2 = 6.48 - j2.64 \, \Omega
\]

To find \( V_{Th} \), consider the circuit in Fig. 10.23(b). Currents \( I_1 \) and \( I_2 \) are obtained as

\[
I_1 = \frac{120/75}{8 - j6} \, \text{A}, \quad I_2 = \frac{120/75}{4 + j12} \, \text{A}
\]
Applying KVL around loop $bcdeab$ in Fig. 10.23(b) gives

$$V_{Th} - 4I_2 + (-j6)I_1 = 0$$

or

$$V_{Th} = 4I_2 + j6I_1 = \frac{480/75^\circ}{4+j12} + \frac{720/75^\circ + 90^\circ}{8-j6}$$

$$= 37.95/3.43^\circ + 72/201.87^\circ$$

$$= -28.936 - j24.55 = 37.95/220.31^\circ \text{ V}$$
Practice problem

Find the Thevenin equivalent at terminals $a-b$ of the circuit in Fig. 10.24.

Figure 10.24

Answer: $Z_{Th} = 12.4 - j3.2 \, \Omega$, $V_{Th} = 18.97 \, \sqrt{51.57^\circ} \, V$. 
Example

Find the Thevenin equivalent of the circuit in Fig. 10.25 as seen from terminals a-b.

Solution:
To find $V_{Th}$, we apply KCL at node 1 in Fig. 10.26(a).

$$15 = I_o + 0.5I_o \quad \implies \quad I_o = 10 \text{ A}$$

Applying KVL to the loop on the right-hand side in Fig. 10.26(a), we obtain

$$-I_o(2 - j4) + 0.5I_o(4 + j3) + V_{Th} = 0$$

or

$$V_{Th} = 10(2 - j4) - 5(4 + j3) = -j55$$

Thus, the Thevenin voltage is $V_{Th} = 55\sqrt{-90^\circ} \text{ V}$
To obtain $Z_{Th}$, we remove the independent source. Due to the presence of the dependent current source, we connect a 3-A current source (3 is an arbitrary value chosen for convenience here, a number divisible by the sum of currents leaving the node) to terminals $a-b$ as shown in Fig. 10.26(b). At the node, KCL gives

$$3 = I_o + 0.5I_o \quad \implies \quad I_o = 2 \text{ A}$$

Applying KVL to the outer loop in Fig. 10.26(b) gives

$$V_s = I_o(4 + j3 + 2 - j4) = 2(6 - j)$$

The Thevenin impedance is

$$Z_{Th} = \frac{V_s}{I_s} = \frac{2(6 - j)}{3} = 4 - j0.6667 \ \Omega$$
Determine the Thevenin equivalent of the circuit in Fig. 10.27 as seen from the terminals $a-b$.

Answer: $Z_{Th} = 4.47 < -7.64^\circ$

$V_{Th} = 30.3 < -3.07^\circ$  (I do not agree with this)
Example

Obtain current $I_o$ in Fig. 10.28 using Norton’s theorem.

**Figure 10.28** For Example 10.10.

Solution:

Our first objective is to find the Norton equivalent at terminals $a-b$. $Z_N$ is found in the same way as $Z_{Th}$. We set the sources to zero as shown in Fig. 10.29(a). As evident from the figure, the $(8 - j2)$ and $(10 + j4)$ impedances are short-circuited, so that

$$Z_N = 5 \, \Omega$$
To get $I_N$, we short-circuit terminals $a-b$ as in Fig. 10.29(b) and apply mesh analysis. Notice that meshes 2 and 3 form a supermesh because of the current source linking them. For mesh 1,

$$-j40 + (18 + j2)I_1 - (8 - j2)I_2 - (10 + j4)I_3 = 0 \quad (10.10.1)$$

For the supermesh,

$$(13 - j2)I_2 + (10 + j4)I_3 - (18 + j2)I_1 = 0 \quad (10.10.2)$$

![Diagram](image)

**Figure 10.29** Solution of the circuit in Fig. 10.28: (a) finding $Z_N$, (b) finding $V_N$, (c) calculating $I_o$. 
At node $a$, due to the current source between meshes 2 and 3,

$$I_3 = I_2 + 3 \quad \text{(10.10.3)}$$

Adding Eqs. (10.10.1) and (10.10.2) gives

$$-j40 + 5I_2 = 0 \quad \implies \quad I_2 = j8$$

From Eq. (10.10.3),

$$I_3 = I_2 + 3 = 3 + j8$$

The Norton current is

$$I_N = I_3 = (3 + j8) \text{ A}$$

Figure 10.29(c) shows the Norton equivalent circuit along with the impedance at terminals $a-b$. By current division,

$$I_o = \frac{5}{5 + 20 + j15} I_N = \frac{3 + j8}{5 + j3} = 1.465 / 38.48^\circ \text{ A}$$
Practice problem

Determine the Norton equivalent of the circuit in Fig. 10.30 as seen from terminals $a-b$. Use the equivalent to find $I_o$.

![Circuit Diagram]

**Figure 10.30** For Practice Prob. 10.10.

**Answer:** $Z_N = 3.176 + j0.706 \ \Omega$, $I_N = 8.396 \angle -32.68^\circ \ \text{A}$, $I_o = 1.971 \angle -2.101^\circ \ \text{A}$. 
Op Amp AC circuits

• The three steps for solving AC circuits stated at the start of this lecture also apply to op amp circuits, as long as the op amp is operating in the linear region.
• As usual, we will assume ideal op amps. As always, the key to analyzing op amp circuits is to keep two important properties of an ideal op amp in mind:
  • 1. No current enters either of its input terminals.
  • 2. The voltage across its input terminals is zero.
• The following examples will illustrate these ideas.
Example

Determine $v_0(t)$ for the op amp circuit in (a) if $v_s = 3 \cos 1000t$ V.

Solution:
We first transform the circuit to the frequency domain, as shown in (b), where

$V_s = 3\angle 0^\circ$, $\omega = 1000$ rad/s.
Applying KCL at node 1, we obtain

\[
\frac{3/0^\circ - V_1}{10} = \frac{V_1}{-j5} + \frac{V_1 - 0}{10} + \frac{V_1 - V_o}{20}
\]

or

\[6 = (5 + j4)V_1 - V_o\]

At node 2, KCL gives

\[
\frac{V_1 - 0}{10} = \frac{0 - V_o}{-j10}
\]

which leads to

\[V_1 = -jV_o\]

Substituting gives

\[6 = -j(5 + j4)V_o - V_o = (3 - j5)V_o\]

\[V_o = \frac{6}{3 - j5} = 1.029/59.04^\circ\]

Hence,

\[v_o(t) = 1.029 \cos(1000t + 59.04^\circ) \text{ V}\]
Practice problem

Find $v_o$ and $i_o$ in the op amp circuit of Fig. 10.32. Let $v_s = 2 \cos 5000t$ V.

![Circuit Diagram](image)

Answer: $0.667 \sin 5000t$ V, $66.67 \sin 5000t$ $\mu$A.
Example

Compute the closed-loop gain and phase shift for the circuit shown. Assume that $R_1 = R_2 = 10 \text{k}$, $C_1 = 2 \mu\text{F}$, $C_2 = 1 \mu\text{F}$, and $\omega = 200 \text{ rad/s}$.

Solution:
The feedback and input impedances are calculated as

$$Z_f = R_2 \left| \frac{1}{j\omega C_2} \right| = \frac{R_2}{1 + j\omega R_2 C_2}$$

$$Z_i = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$

Since the circuit is an inverting amplifier, the closed-loop gain is given by

$$G = \frac{V_o}{V_s} = -\frac{Z_f}{Z_i} = \frac{j\omega C_1 R_2}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)}$$
Substituting the given values of \( R_1, R_2, C_1, C_2, \) and \( \omega \), we obtain

\[
G = \frac{j^4}{(1 + j^4)(1 + j^2)} = 0.434 / -49.4^\circ
\]

Thus the closed-loop gain is 0.434 and the phase shift is \(-49.4^\circ\).
Obtain the closed-loop gain and phase shift for the circuit. Let $R = 10 \, \text{k}\Omega$, $C = 1 \, \mu\text{F}$, and $\omega = 1000 \, \text{rad/s}$.

**Answer:** $1.015, -5.599^\circ$. 
Summary

1. We apply nodal and mesh analysis to ac circuits by applying KCL and KVL to the phasor form of the circuits.

2. In solving for the steady-state response of a circuit that has independent sources with different frequencies, each independent source must be considered separately. The most natural approach to analyzing such circuits is to apply the superposition theorem. A separate phasor circuit for each frequency must be solved independently, and the corresponding response should be obtained in the time domain. The overall response is the sum of the time-domain responses of all the individual phasor circuits.

3. The concept of source transformation is also applicable in the frequency domain.

4. The Thevenin equivalent of an ac circuit consists of a voltage source $V_{Th}$ in series with the Thevenin impedance $Z_{Th}$.

5. The Norton equivalent of an ac circuit consists of a current source $I_N$ in parallel with the Norton impedance $Z_N (= Z_{Th})$. 