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Assessment of Marine Weather forecasts over the Indian Sector of Southern Ocean

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- Westerly winds
- Sub–seasonal Rainfall Variability
- Sub polar low pressure systems

Abstract
The Southern Ocean (SO) is one of the important regions where significant processes and feedbacks of the Earth's climate take place. Expeditions to the SO provide useful data for improving global weather/climate simulations and understanding many processes. Some of the uncertainties in these weather/climate models arise during the first few days of simulation/forecast and do not grow much further. NCMRWF issued real-time five day weather forecasts of mean sea level pressure, surface winds, winds at 500 hPa & 850 hPa and rainfall, daily to NCAOR to provide guidance for their expedition to Indian sector of SO during the austral summer of 2014–2015. Evaluation of the skill of these forecasts indicates possible error growth in the atmospheric model at shorter time scales. The error growth is assessed using the model analysis/reanalysis, satellite data and observations made during the expedition. The observed variability of sub-seasonal rainfall associated with mid-latitude
systems is seen to exhibit eastward propagations and are well reproduced in the model forecasts. All cyclonic disturbances including the sub-polar lows and tropical cyclones that occurred during this period were well captured in the model forecasts. Overall, this model performs reasonably well over the Indian sector of the SO in medium range time scale.

1.0 Introduction

The atmospheric surface forcing (which includes winds, solar radiation, atmospheric pressure, temperature, humidity and precipitation) of the Southern Ocean (SO) influences the earth’s weather and climate on many scales from minutes and hours through decades and centuries. Accurate estimation and modelling of ocean surface forcing components over the SO are therefore necessary and critical not only for analysis of shorter scale events, but also for modelling and analysis of longer time scales, including climate processes (Cerovecki et al., 2011, Meijers et al., 2014). Many of the biases in the models arise from improper simulations over the southern hemisphere (SH) due to lack of observations.

The strongest westerly winds on the earth are in the SO blowing from 40°S and Antarctic Circle are driven by the strong temperature difference of sea and ice. These winds drive the longest ocean current on the earth, the Antarctic Circumpolar Current (ACC). The weather front between the Polar air masses and the Antarctic trigger storms over the SO. These storms are very intense and influence large scale climate variability. The near-surface temperature, pressure, wind and precipitation over the southern mid- and high-latitudes are sensitive to low-level atmospheric circulation and much on the westerly windstress (Van den Broeke, 2000, Van den Broeke and Van Lipzig, 2004). The SO has a large influence on the Earth's climate through processes like carbon sequestration (Landschützer et al., 2015), oxygen production (Aoki et al., 2005), ocean productivity (Palter et al., 2010), meridional overturning circulation (Orsi et al., 1999) and redistribution of heat & fresh water fluxes. It is suggested that the SO surface fresh water forcing (sea ice melting and freezing, river runoff,
precipitation, and evaporation) not only controls the glacial-interglacial stratification changes in deep ocean but also influences atmospheric carbon dioxide concentration (Sun et al., 2016). The transports of the Aghulas current system in the Indian sector of the SO impacts the weather and climate on various time and space scales (Weijer and Sebille, 2014, Beal et al., 2011, Pilo et al., 2015). More mesoscale cyclones are formed in the Indian Ocean sector and south of Australia than other regions of the SO (Hoskins and Hodges, 2005, Yuan et al., 2009). Despite such a significant role, the SO and is poorly sampled and less understood. Especially the Indian Ocean sector of the SO is less observed. International expeditions are being carried out to study the SO. NCAOR, MoES carried out the 8th International Expedition to the Indian sector of the SO (ISOE-8) onboard ORV-Sagar Nidhi with a main goal to collect high resolution physical, chemical, biological and atmospheric data to understand the various processes in the polar front zone. The cruise track and the study area are indicated in Figure 1. The voyage planned till coastal waters of Antarctica reached upto 57\degree S and could not travel further due to inclement weather. The data collected by NCAOR were largely hydrodynamic and biogeochemical measurements of CTD, ARGO, XCTD, plankton sampler, fluorometer, radiometer etc. The atmospheric data collected was those of radiosonde.

Weather/Climate models are useful tools for studying climate systems. However, models still suffer from unrealistic simulation of many climate features across globe, leading to uncertainty in its use for climate prediction (Palmer et al., 2005). Many of the current generation of weather and climate models struggle to represent key aspects of the SH and polar climate such as near-surface temperature and sea-ice, southern annular mode (SAM) and their variability on all time scales from daily to decadal (Mayewski et al., 2009). Almost all AOGCMs have problems in simulating the SH zonal wind stress maximum in the middle to high latitudes and that this has large detrimental impact on the SO simulation like ACC location, Antarctic intermediate water mass, too diffuse thermocline and warm SST (Randall
et al., 2007). Atmospheric heat transport is tied to the SH westerly winds and shifts in position and intensity of the westerlies is of importance to the climate system (McGlone et al., 2010). Models which include stratosphere have less bias in simulating the surface jet over the SO (Bracegirdle et al., 2013). All GCMs have too strong a radiative contrast between frontal and non-frontal cloud over the SO. This leads to excess shortwave radiation (SWR) reaching the ocean surface which in some models, leads to a warm SST bias (Smith et al., 2015). This warm SST bias is stronger during austral summer (Wang et al., 2014). Due to cloud errors in the SO, most models have spurious ITCZ south of the equator (Hwang et al., 2013). Comparison of five reanalysis precipitation products with GPCP merged data indicated that ERA-Interim offers the most realistic precipitation variability over the SO and that these products can differ by more than 30% in the SO (Bromwich et al., 2011). These uncertainties in fresh water forcing can result in a salinity bias at the surface that erodes the stratification, causing excessive deep convection in the SO (Kjellsson et al., 2015, Behrens et al., 2016).

The seamless weather and climate approach is being advocated by many researchers to tackle such issues (Hurrell et al., 2009, Shukla et al., 2009, Rodwell and Palmer, 2007). The Unified modelling (UM) system is one such system with seamless approach. It has been demonstrated in the UM system that the warm SST bias of the SO can be realised in the numerical weather prediction (NWP) simulations and that the fast atmosphere processes & SWR flux biases are likely to play significant role in this (Williams et al., 2015). The initial tendency in NWP can be used to assess climate models (Rodwell and Palmer, 2007). Over the SO, low and mid-level clouds in the cold-air sector are responsible for most of the surface downward SWR bias (Bodas-Salcedo et al., 2012). Biases in clouds appear very rapidly in climate simulations, often within the first days or weeks of simulation (Bretherton et al., 2012). The doubling time of the small errors in the daily rainfall simulated by CFS is 4-15
days (Rai and Krishnamurthy, 2011). Error patterns observed in the first days of spin-up are very similar to the mature error patterns in climate system but with reduced amplitude (Strachan et al., 2006). Assessing the errors growth during the first few days is therefore very important not only for real-time weather prediction but also contributes to model evaluation and provides for improvement of climate models.

The vast majority of SH systems are found in the two mid-latitude belts. The greatest number of cyclone systems in southern hemisphere are found in the sub-Antarctic belt 50°–70°S, while the second greatest frequency is observed in the belt immediately to its north (30°–50°S) (Simmonds and Keay, 2000). The forecasts of rainfall, mean sea level pressure (MSLP) and winds of the mid-latitude low pressure systems (LPS) in these two bands during summer 2014-15 are validated. Over the 30°-50°S belt, the systems are driven by westerlies and generally move with the zonal flow regime. To explore the systems in this belt, we analysed the rainfall propagations in the 30°-40°S belt and found eastward propagating sub-seasonal variability in the rainfall. Sub-seasonal rainfall variability is observed over the SH by earlier by researchers. Westward propagating wet (dry) spells associated with low level easterly(westerly) winds over 7°-17°S during austral summer were noted by Jury and Nkosi (2000) while subseasonal rainfall patterns over south America are observed by Liebmann et al. (2004). However sub-seasonal rainfall patterns over the SO especially in the Indian Ocean sector of SO have not been studied so far in this manner. The systems in the other 50°-70°S belt known as the sub-polar LPS are studied in a last section which addresses case studies of these systems as well as case studies of tropical cyclones that have occurred in the study region and period.

In this study, an assessment of the error growth of atmospheric model outputs of NCMRWF’s Global Forecast System (NGFS) at shorter time-scales is made covering the expedition period, over the Southern Indian Ocean region using the model analysis fields, reanalysis data
and observations. Standard statistical methods are used to assess these errors. The forecasts are compared with forecasts of other state-of-the-art modelling systems and the results found comparable. In particular, the location of the westerly wind jet, the sub-seasonal rainfall variability and the passage of LPS in sup-polar, mid-latitude and tropical regions are also verified using model analysis, ERA-Interim reanalysis and satellite observations. The ERA-Interim is considered for evaluating the model fields as recent studies have found ERA-Interim to be the most reliable reanalysis over Antarctica (Bromwich et al., 2011, Bracegirdle et al., 2013) and over the SO (Kent et al., 2013, Swart and Fyfe, 2012).

2.0 Data and Methods

The NGFS comprises of an atmosphere model with 574 waves in the zonal direction (T574), a gaussian grid of 1760 X 880 points (approximately 23 km resolution near equator) and 64 hybrid sigma-pressure levels in the vertical and a 3D VAR assimilation system. The details of NGFS can be found in Rajagopal et al. (2007) and Prasad et al. (2011). This NGFS, a version of NCEP’s GFS was used to issue daily medium range weather forecasts for the above mentioned expedition. Forecasts are issued with one to five days lead time for the SO region of the expedition. The study period is selected from 01 December 2014 to 28 February 2015, to cover this expedition period over the Southern Indian Ocean region. This also covers the austral summer 2014-2015. The forecast products from this system used in this study include rainfall, MSLP, winds at various pressure levels, fluxes, air temperature, and humidity over the region 0°E to 100°E and 20°S to 80°S. The model’s daily accumulated rainfall is compared with the satellite observed rainfall. The satellite rainfall for comparison is obtained from Global Rainfall Map in Near-Real-Time (GSMaP_NRT) produced by JAXA Global Rainfall Watch System (Ver. 3.0) Earth Observation Research Center, Japan Aerospace Exploration Agency (Okamoto et al., 2005, Kubota et al., 2007, Ushio et al., 2013). The resolution of the rainfall rate (in mm/hr) data is 0.1 degree in latitude/longitude. This data is
available over 60°N to 60°S every hour, four hours after observation. The system is based on the combined MW-IR algorithm using GPM-Core GMI, TRMM TMI, GCOM-W1 AMSR2, DMSP series SSMIS, NOAA series AMSU, MetOp series AMSU, and Geostationary IR developed by the GSMaP (Global Satellite Mapping of Precipitation) project. The hourly rainfall rate is converted to daily accumulated rainfall in centimetres. Both model and satellite observed rainfall products are smoothed spatially over 0.25 deg.

The model wind and MSLP fields of analysis and forecast are on the same spatial resolution (that of the model) and hence compared directly without re-gridding. The ERA-Interim (Dee et al., 2011) data at 0.25° X 0.25° grid resolution is obtained from ECMWF data server. The surface jet position and strength are calculated based on Swart and Fyfe (2012). According to this, the position is calculated as the maximum in the zonal mean of zonal surface wind stress between 70°S and 20°S. The strength is defined as the stress at this position. The systematic errors viz., bias, variance and root mean square deviation (RMSD) are computed using standard formulae as given under. The bias is taken as the difference of model forecast field and the analysis/observation. The RMSD for the rainfall is computed with respect to the satellite rainfall and in the case of winds computed with respect to the model analysed field. The rainfall is verified against the satellite observations instead of the model analysis fields as the rainfall model analysis is not performed in our modelling system. The THORPEX Interactive Grand Global Ensemble (TIGGE) forecasts from ECMWF, NCEP and UKMO models are used for inter-comparison with NGFS forecasts. The data is obtained from the ECMWF TIGGE data portal. The configurations of these ensemble prediction systems are detailed in Bougeault et al. (2010).

Bias:

Wind bias = model field – analysis field
Rainfall bias = model rainfall - observation rainfall

Monthly bias = model monthly average – analysis/observations monthly mean

RMSD:

\[ RMSD = \sqrt{\frac{\sum_{i=1}^{n} (p_i - q_i)^2}{n}} \]

Where \( p_i \) is the field variable at time ‘i’ and \( q_i \) is observation/analysis variable at the time ‘i’ of the field variable, \( n \) is the number of observation points in time.

Observed tracks of cyclones are obtained from JTWC best tracks data.

3.0 Mean Features and comparison with ERA-Interim data

The climatological features over the Southern Indian Ocean from 20°S to 80°S and 0°E to 100°E are the trade wind-belt, the subtropical highs, the prevailing westerlies, and the subpolar lows. During the austral summer season [December-January-February (DJF)], the mid-latitude flow over the SO is closest to the zonal symmetry for the mean circulation as well as the storm tracks (Arakelian and Cordon, 2012). To verify that the model is reproducing the mean climatological features well, the means of zonal winds at 10 m, zonal winds at 850 hPa and MSLP during austral summer of December 2014 to February 2015 are compared with those of ERA Interim data. These are plotted in Figure 2. The climatological mean features like the easterly trade wind belt, prevailing westerlies and subtropical high are well brought out by both the data and agree well with each other. Although, the general trade-wind belt extends from 5°-30° latitudes, the present study region includes only a part of these. They blow over a 20°S to 30°S as easterlies in the region of interest. During summer, the winds are stormy and tropical cyclones occur in the South West Indian Ocean. These transients pass through along the edges of the wind zones. Figures 2(a) & (b) show that the
magnitude of trade winds is comparable in both model analysis and ERA Interim reanalysis. It is clearly seen from Figures 2(a) & (b), that the DJF mean shows the prevailing westerlies with equal magnitude in both model analysis and ERA Interim data. The correlation with ERA Interim data for the model trade winds is very high and is greater than 0.95. The westerly winds blow poleward out of the subtropical highs and are found between 35°S to 60°S. These are strong winds associated with stormy weather. The tropical systems undergo transition to mid-latitude systems here. The correlation of daily analysed winds at 10 m with ERA interim reanalysis is between 0.90 and 0.95. The 850 hPa zonal winds are also similar to the 10 m winds, but for the narrower width of the westerlies (Figures 2(c) & 2(d)). The correlation of these winds with ERA data over the domain is uniform and is very high (0.95). The subtropical highs are centres of high pressure over the oceans between 20°-40° latitudes in both the hemispheres that are semi-permanent. In the SH the circulation around these highs is counter-clockwise and they move towards the poles during summer. Winds diverging from these highs move towards the equator and form the trade winds. Winds diverging from the highs towards the poles are deflected to the east forming the prevailing westerlies in mid-latitudes. These systems intensify over the ocean during the summer. In the Indian Ocean region the subtropical high is generally centred around 70°E and 35°S during SH summer. The subtropical high is clearly seen in the DJF mean of both model analysed and ERA Interim data (Figures 2(e) & (f)). The correlation of daily model analysed MSLP with that of ERA Interim data for DJF is nearly +1 everywhere over the domain. The sub-polar lows are well formed low pressure centres around Antarctica that occur during summer and winter. These lows form a continuous zone of low pressure in the SH between 50°S and 70°S. This zone is most intense during SH summer. This is clearly seen in Figures 2(e) & 2(f). These features viz., the trade winds, the prevailing westerlies, the subtropical highs and the sub-polar lows are found to be in well captured in the Day 1, Day 3 and Day 5 forecasts. The
correlation of the Day 5 forecast surface zonal winds is found to be above 0.6 with ERA Interim data. The SH surface westerly jet climatological mean position is estimated to be 52°S (Swart and Fyfe, 2012). This is well reproduced in the model analysis and forecasts (Day 1, Day 3 and Day 5). Both the position and magnitude of the westerly jet from model analysis and forecasts compare well with the ERA Interim data (Figures 3a & 3b).

3.1 Inter-comparison of NGFS, NCEP, ECMWF, UKMO Forecasts

The NGFS model forecasts of zonal winds at 850hpa and 500hpa are compared with the TIGGE ensemble prediction system forecasts of NCEP, ECMWF, and UKMO models. Figures 4a and 4b show the taylor diagrams comparing these forecasts. From the figure it is clear that for all the models, the correlations of forecasts with their respective analysis are in the decreasing order Day 1(>0.9), Day 3(>0.8) and Day 5(>0.5) and all models have similar skill for zonal winds at both 850hpa and 500hpa as well as for MSLP. The results fall into three clusters each for 850 hPa and 500 hPa zonal winds, grouped as Day 1 (D1), Day 3(D3) and Day 5(D5) forecasts. The Day 1 and Day 3 forecasts are further observed to be grouped in two sets. The NCEP and the NGFS lie closer to each other considered as the first set while the ECMWF and the UKMO fall closer, considered the second set. The ECMWF and UKMO show slightly higher correlations than the GFS based NGFS and NCEP models for all lead times and pressure levels. Of all the models, the highest correlation is observed for the ECMWF forecasts while the UKMO forecasts shows best agreement with their analysis in representing the variability. This is inferred from the figure showing all the red circles (UKMO) lying exactly on the red curve. The standard deviation of the UKMO and ECMWF analysis fields are very close together and in the plot they overlap and hence a blue dashed line corresponding to ECMWF standard deviation is not seen. The results for MSLP (figure 4b) are also similar to winds and show that they are grouped as Day 1 (D1), Day 3(D3) and Day 5(D5) forecasts. For MSLP, the standard deviations of all the models lie close to one
another as well as to that of the analysis. A comparison of the correlations with respective analysis fields show that the NCEP forecasts are slightly lower than those of the other three models (figure 4b). It can be concluded that the NGFS forecasts are at par with the forecasts of these state-of-the-art models in the study region for the study period. Therefore the NGFS forecasts are reliable and can be used in the SO conveniently.

4.0 Systematic Errors: Bias, RMSD and Variance

The bias and RMSD of predicted zonal winds at 10 m, 850 hPa and 500 hPa for Day 1, Day 3 and Day 5 over the region of interest against the analysis field are presented in Table 1. The bias in Day 1 and Day 3 forecasts of 10 m winds for over most of the region of interest is limited to ±2 m/s. The area average bias is less than 0.05 m/s. Further analysis of individual months showed that the bias of Day 5 forecasts during December 2014 show slightly large along the periphery of Antarctica ranging +4 m/s (figure not shown). However during January 2015 and February 2015 the Day 5 bias is also very less limited to ±2 m/s. The RMSD of the Day 1, Day3 and Day 5 forecasts of daily zonal winds at 850 hPa m with respect to analysed field for the entire summer is shown in Figures 5 (a), (b) & (c) (top row) respectively. Similarly, Figures 5 (a), (b) & (c) (middle row) show the RMSD for zonal winds at 500 hPa and figures 5 (a), (b) & (c) (bottom row) for MSLP. It is seen that the error increases as the lead time of forecast increases. The RMSD of zonal winds for 10m Day 1 is less than 2 m/s north of 50°S and in the range 2 to 4 m/s south of it. For the Day 3 forecasts the RMSD of 10 m zonal wind is in the range 1-6 m/s and for Day 5 forecasts it in the range 1-7 m/s. Overall, at all the lead times the RMSD over 20°S to 50°S is lesser compared to the southern latitudinal band 50°S to 80°S. The RMSD averaged over the domain is 1.7 m/s. The bias of 850 hPa predicted winds for all the lead times, mostly lies in the range of ±2 m/s. Again, similar to 10 m winds, during December 2014, along the periphery of Antarctica, there is an increased positive bias. Another similar feature is that, during January, over the
longitudes 0°E-30°E and along 60°S a region of larger positive bias is observed. It is seen from Figure 5 that the RMSD over the region 20°S to 50°S is less than that over 50°S to 80°S. It is also seen that, the RMSD of 850 hPa winds increases with the increasing lead time. The RMSD of 850 hPa zonal wind forecasts are in the ranges 1-3 m/s, 1-6 m/s and 1-9 m/s for Day 1, Day 3 and Day 5 respectively.

The bias of 500 hPa zonal winds also increases with increasing lead time. Comparing 10 m, 850 hPa and 500 hPa, it is observed that the biases increase with increasing vertical height. However the biases over all the vertical levels (10 m, 850 hPa and 500 hPa) and at all lead times for all the months (December 2014 to February 2015) is not much and is less than ±4 m/s over most of the region. It is also observed from Figure 5 (middle row) that the RMSD of 500 hPa zonal winds is less over the 20°S to 50°S latitudes compared to the latitudinal band 50°S to 80°S. The RMSD is increasing with increasing lead time as in 850 hPa and 10 m. The ranges of RMSD for Day 1, Day 3 and Day 5 forecasts of 500 hPa zonal winds are 1-4 m/s, 1-8 m/s & 1-11 m/s respectively. In general, the rainfall biases are low except for heavy rainfall during tropical cyclones. During December the biases over most of the region are below ±1 cm. During January 2015, the largest biases are along the edges of the tracks of the cyclones BANSI, EUNICE and CHEZDA. During February 2015 also, the largest biases are along the edges of the tracks of the cyclones FUNDI and GLENDA. The bias is not deteriorated much in rainfall from Day 1 to Day 5 forecasts. The RMSD of rainfall forecasts for Day 1, Day 3 and Day 5 are shown in Figures 5(a), (b) & (c) (bottom row) respectively. The larger deviations are associated with heavy rainfall locations.

4.1 Validation with Ship-Board AWS

The model forecasts of surface (2 m) temperature, MSLP and relative humidity are validated with the automatic weather station (AWS) mount on the Ship, ORV Sagar Nidhi, of the SO

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cruise. The correlation coefficient, bias and RMSE of the closest collocated points of the model grid to the ship location are given in Table 2. The 2 m temperature has more than 0.9 correlation coefficient even for the Day3 forecast. The correlation coefficient for MSLP is 0.588 for Day 1 and 0.534 for Day 2 but it falls to 0.377 for Day 3. The correlation coefficient for relative humidity is also more than 0.6 up to Day 3 forecasts.

5.0 Rainfall verification with satellite data

The mean rainfall for the months December 2014, January 2015 and February 2015 over the region of interest from satellite merged rainfall is shown along with Day 1, Day 3 & Day 5 model predicted rainfall in Figure 6. The corresponding mean of MSLP (analysed MSLP on satellite rainfall and forecasted MSLP over forecasted rainfall) is overlaid as contours. Most of the features observed in the satellite observed rainfall are reproduced in Day1 to Day5 forecasts. They are found to match well spatially and quantitatively as can be seen from the figure. Clearly, rainfall is either minimum or absent over the subtropical high during all the three months and this is well represented till Day5 forecasts. During December 2014, rainfall patterns fall into three categories viz., 1) rainfall west of the subtropical high 55°E-70°E and 20°S-40°S, 2) eastward moving rainfall associated with the mid-latitude stormy weather in the westerlies 30°S-40°S and 30°E-60°E and 3) rainfall over Madagascar and south eastern Africa. Figure 6 shows that all these patterns are well captured in all forecasts (Day 1, Day 3 and Day 5). The rainfall associated with the sub-polar lows at 55°-70°S could not be verified as the satellite rainfall extends to 60°S and the rainfall south of 50°S is scanty and of reduced quality. During January 2015, the rainfall amounts are much larger compared to December 2014 and the higher rainfall is located mostly to the north of 35°S which is seen to be well represented in the Day 1, Day 3 and Day 5 forecasts spatially as well as quantitatively. During January 2015, four major tropical cyclones occurred over the region of interest out of which one is a category five cyclone and one is are category four cyclone. These contributed
largely to the rainfall. The rainfall over southern Africa observed during December 2014 is not seen during January 2015 in both observations as well as forecasts. The rainfall during February 2015 is slightly lesser than January 2015 but more than that in December 2014. The last column of Figure 6 clearly shows the rainfall patterns and quantities well captured in Day 1, Day 3 and Day 5 forecasts during February 2015. During February 2015, two cyclonic events took place over the region of interest which contributed to the rainfall.

5.1 Sub-seasonal rainfall associated with the mid-latitude LPS

The sub-seasonal rainfall variability over the 30°S-40°S is found to exhibit eastward propagations during austral summer. This is the frontal zone between the easterly trades and westerly zone over higher latitudes and transients pass through along these edges. These transients are the mid-latitude lows. Figure 7 shows the time longitude plots of rainfall averaged over 30°S to 40°S overlaid with easterly zonal wind at 850hPa contours during 01 December 2014 to 28 February 2015 from (a) satellite observed (shaded) /analysed easterly zonal wind at 850hPa (contours) and NGFS predicted rainfall (shaded)/ easterly zonal wind at 850hPa(contour) with lead times (b) Day 1, (c) Day 3 and (d) Day 5. Eight bands of eastward propagating rainfall can be observed in the satellite merged rainfall viz., the first one starting at 40°E on 8th December, second band starting at 20°E on 20th December, the third one at 20°E on 26th December. The subsequent bands start on 15th January 2015, 26th January, 29th January, 12th February and 20th February. These eastward propagations of the rainfall are well reproduced in the Day 1 forecast, and fairly well in Day 3 and Day 5 forecasts. The Day 3 and Day 5 forecasts also match well with the satellite data in terms of phase speed and time of occurrence but the intensity is slightly reduced in Day 3 and Day 5 forecasts. The time longitude contours of easterly zonal winds at 850 hPa indicate coherent eastward propagations. The phase and speed of the propagations in these zonal winds are also well forecasted till Day 5. All these rainfall propagations are associated with eastward passing
LPS as can be seen from figure 8. These coherent propagations can be observed in MSLP shown in the time longitude plot of MSLP (contours) overlaid on rainfall (shaded) in Figure 8. Again the propagations in MSLP are also well brought out in the model till Day 5 forecast. No lead/lag is noticed in rainfall vs wind or rainfall vs MSLP even till Day 5 forecasts, indicating that the skill of the model is good and reliable.

6.0 Case studies of LPS/cyclones

6.1 Tropical cyclones

The storm tracks in the Indian Ocean sector of SO are linked with large gradients in sea surface temperature (Nakamura and Shimpo, 2004). There were total six tropical cyclones that formed in the region of interest during the study period. Out of six cyclones, four occurred in January 2015 and two occurred in February 2015. The dates, names and category of the cyclones are given in Table 3. Estimates by Météo-France La Reunion (MFR/RSMC La Reunion) suggest that “BANSI (10-18 January 2015)” and “EUNICE” (26th Jan to 1st Feb 2015) were the strongest with very intense tropical cyclone status. They fall under category 4 and category 5 status respectively on Saffir-Simpson scale and EUNICE is one of the strongest recorded cyclones over the South West Indian Ocean basin in terms of 10 minute sustained winds reaching 230 km/h and lowest pressure of 915 hPa. BANSI recorded maximum sustained wind speeds of 220 km/h and lowest pressure of 910 hPa followed by, CHEZDA, FUNDI, GLENDA and DIAMONDRA recording maximum sustained wind speeds of 100 km/h, 95 km/h, 85 km/h and 85 km/h and lowest pressures 975 hPa, 985 hPa, 979 hPa and 986 hPa respectively. CHEZDA made landfall over Madagascar Island and crossed the Island and re-intensified over the ocean. All these cyclones except GLENDA underwent extratropical transition.
The model forecasts of surface winds, MSLP are compared against the analysis and the model forecast rainfall is compared against satellite observations for rainfall during the cyclone event periods (Figures 9 & 10). These figures show the surface winds and MSLP for these cyclones on the respective peak intense day. The rainfall is the aggregate rainfall during the respective cyclone event period. It can be seen from these figures that the forecasts match in general till Day 3. The location and magnitude of the central pressure for all the five cases are observed to match well. Similarly the winds are also well matching with the analysed field for all the cases. The Day 5 forecasts of MSLP and winds are slightly more intense. The location of the central pressure seems to be away from that in the analysis in Day5 forecasts for all cyclones except for BANSI and CHEZDA. The aggregate rainfall is also brought out fairly well.

The mean bias for central minimum pressure for the six cyclones is underestimation by 2.46hPa, 5.06hPa and 7.86hPa for Day1, Day3 and Day5 forecasts with respect to the analysis. The central minimum pressure is mostly underestimated for all cyclones and all lead times. The Day1 and Day3 forecasts have less bias for all the cyclones with a best estimated difference of 0.4hPa and worst estimated difference of -8.9hPa. The mean bias in estimation of maximum winds is overestimation by 0.11m/s, 0.27m/s and 4.2m/s for Day1, Day3 and Day5 forecasts. The maximum winds speed is overestimated for the almost all the cyclones for forecasts of all lead times except for DIAMONDRA and GLENDA. For these two cyclones, they are underestimated for forecasts all lead times. Except for Day5 forecasts of BANSI and CHEZDA and GLENDA, the wind speed bias is within ± 6.8 m/s. The bias curves for central minimum pressure and maximum winds are given in the supplementary figure S1.

Table 4 shows the pattern correlation of the three parameters surface wind speed (10m), MSLP and rainfall. The un-centred pattern coefficients have higher values than the centred
coefficients as they neglect the shift. This indicates that even though there is shift, the general patterns match well. The un-centred pattern coefficients are all 0.9 for MSLP, above 0.84 for wind speed and above 0.43 for rainfall. Of all the cyclones, the best pattern correlation is observed for BANSI cyclone in all the three parameters MSLP, wind speed and rainfall and the weakest is for CHEZDA. DIAMONDRA and EUNICE are not separated far apart falling in the same frame and have occurred on the same dates. Hence their coefficients are represented under the head EUNICE. The un-centred coefficients for MSLP for Day1 and Day3 are above 0.9 indicating reliable forecasts. For Day 5 except for CHEZDA(0.63) and GLENDA(0.53) they are in the range 0.88 to 0.92. For wind speed again, except for CHEZDA(0.32) and GLENDA(0.40), for Day 5 forecasts the coefficients are in the range 0.61 to 0.93. The rainfall coefficients are slight lower than those for MSLP and the wind speed. Except for CHEZDA and GLENDA they are ranging between 0.48 to 0.74 for Day 1, 0.3 to 0.7 for Day 3 and 0.26 to 0.65 for Day 5 forecasts. Overall, the Day1 and Day 3 forecasts are reliable and the Day 5 forecasts need improvement for tropical cyclone forecasting.

6.2 Sub-polar LPS

The sub-polar low pressure belt over the southern hemisphere forms over the periphery of Antarctica and are characterised by occurrence of cyclones. These cyclones usually are very huge spanning between 1000Km to 2000Km. Their base is in the upper troposphere and these are not driven by convection unlike tropical cyclones. Forecasts of three lows formed over the study region during the summer 2014-15 are verified. The three systems fall on the dates 15th Dec 2014, 13th Jan 2015 and 23rd Feb 2015. Analysis of time-longitude plots of MSLP and winds averaged over 60°S to 70°S have indicated that three intense LPS have formed over the study region during summer 2014-15 (supplementary figure S2). These systems are deep and hence we analysed the 500hpa winds along with MSLP. Rainfall over these
latitudes is not available from the satellite and hence could not be verified. However the
hovmoller plots of rainfall (supplementary figure S2-b,c,d) does show occurrence of strong
rainfall bands along these systems. Figures 11(a-d) show plots of MSLP overlaid with
500hPa wind vectors for analysis and Day 1, Day 3 and Day 5 forecasts for these three
systems. From the figure it is observed that the location of the central pressure is well
reproduced in the forecasts of all lead times. The pattern correlations also agree well with the
analysis (supplementary table ST1). They range from 0.88 to 0.99 for the three systems for
Day 1, Day 3 and Day 5 forecasts. For wind speed at 500hpa all the three systems show
correlation of 0.96 for Day1 forecast and range from 0.80 to 0.90 for Day 3 and Day 5
forecasts. Hence the model is able to forecast these systems with good efficiency, five days in
advance.

7.0 Summary and Conclusions

The NGFS model performance is verified and found to be of good skill till Day 5 for rainfall,
winds at surface, 850 hPa & 500 hPa and MSLP in the SO region. The three climatological
bands over the region of interest viz., trade winds , mid latitude highs and the sub-polar lows
are well reproduced till Day 5. The model analysis fields compare well with those ERA
Interim both qualitatively and quantitatively. The correlation coefficient is above 0.9 for
analysis and above 0.8 to 0.6 forecasts, decreasing with increasing lead time. The rainfall
associated with these regions is also well captured. The NGFS performance is found to be at
par with the forecasts of ECMWF, UKMO and NCEP. The sub-seasonal eastward
propagations of the rainfall in the frontal edge between easterlies and westerlies, associated
with mid-latitude cyclones are well brought out in terms of phase and magnitude. It is found
that the model RMSD increases marginally with increasing lead times and increasing height
for the zonal winds. Case studies of tropical and sub-polar LPS show Day1 and Day3
forecasts are reliable and Day5 forecasts need improvement. The results from the study like the sub-seasonal rainfall and their variability can serve as a reference for future expeditions.

Acknowledgements

ERA-Interim and TIGGE data provided courtesy ECMWF. The GSMaP_NRT and JAXA Global Rainfall Watch system was developed based on heritage of the GSMaP Project led by Prof. K. Okamoto (Osaka Prefecture University, Osaka, Japan) under R&D of Hydrological Modeling and Water Resources System in the Core Research for Evolutional Science and Technology program of the Japan Science and Technology Agency (JST). We would like to thank JST and the members of the GSMaP Project.

References


Tables:

**Table 1: Domain averaged Bias and RMSD for Day 1, Day 3 and Day 5 for DJF**

<table>
<thead>
<tr>
<th>Field</th>
<th>Day 1</th>
<th></th>
<th>Day 3</th>
<th></th>
<th>Day 5</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bias</td>
<td>RMSD</td>
<td>Bias</td>
<td>RMSD</td>
<td>Bias</td>
<td>RMSD</td>
</tr>
<tr>
<td>U10m (m/s)</td>
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<td>1.772</td>
<td>0.0322</td>
<td>2.854</td>
<td>-0.0568</td>
<td>3.966</td>
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<tr>
<td>U850hPa (m/s)</td>
<td>0.0401</td>
<td>2.157</td>
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<td>3.581</td>
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<td>U500hPa (m/s)</td>
<td>0.0284</td>
<td>2.843</td>
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<td>--------</td>
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<td>--------</td>
<td>--------</td>
<td>---------</td>
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</table>

622

623
Table 2: Table showing the correlation coefficient, Bias and RMSE of model forecasts with onboard ship AWS measured 2m air temperature, MSLP and Relative humidity.

<table>
<thead>
<tr>
<th></th>
<th>Analysis</th>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
</tr>
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<tr>
<td><strong>2m Temperature (°C)</strong></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Corr. Coef.</td>
<td>0.9969</td>
<td>0.9925</td>
<td>0.9904</td>
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<tr>
<td>Bias</td>
<td>0.82</td>
<td>0.87</td>
<td>0.73</td>
<td>0.73</td>
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<tr>
<td>RMSE</td>
<td>1.09</td>
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<td>1.53</td>
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<td><strong>MSLP (hPa)</strong></td>
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<td>Corr. Coef.</td>
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</tr>
<tr>
<td>Bias</td>
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<td>3.25</td>
<td>3.73</td>
<td>4.42</td>
</tr>
<tr>
<td>RMSE</td>
<td>3.62</td>
<td>5.08</td>
<td>7.26</td>
<td>8.57</td>
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<tr>
<td><strong>Relative Humidity (%)</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr. Coef.</td>
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</tr>
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<td>Bias</td>
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<td>2.32</td>
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<tr>
<td>RMSE</td>
<td>4.20</td>
<td>8.98</td>
<td>8.49</td>
<td>9.24</td>
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Table 3: Details of Cyclones occurred over the region of Interest during the study period

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<th>Name</th>
<th>Period</th>
<th>Category</th>
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<tr>
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<td>11-18 January 2015</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>CHEZDA</td>
<td>16-18 January 2015</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>DIAMONDRA</td>
<td>26-29 January 2015</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>EUNICE</td>
<td>27 January – 02 February 2015</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>FUNDI</td>
<td>06-08 February 2015</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>GLENDA</td>
<td>24-28 February 2015</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4: Pattern correlation of Cyclones occurred over the region of Interest during the study period

<table>
<thead>
<tr>
<th>Cyclone</th>
<th>dy1</th>
<th>dy3</th>
<th>dy5</th>
<th>dy1</th>
<th>dy3</th>
<th>dy5</th>
<th>dy1</th>
<th>dy3</th>
<th>dy5</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSLP</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.984</td>
<td>0.979</td>
<td>0.972</td>
<td>0.835</td>
<td>0.806</td>
<td>0.774</td>
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<tr>
<td>WSP(10m)</td>
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<td>0.999</td>
<td>0.999</td>
<td>0.968</td>
<td>0.943</td>
<td>0.896</td>
<td>0.660</td>
<td>0.475</td>
<td>0.550</td>
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<tr>
<td>RAIN</td>
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<td>0.999</td>
<td>0.999</td>
<td>0.983</td>
<td>0.971</td>
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<td>0.551</td>
<td>0.514</td>
<td>0.442</td>
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<td>0.999</td>
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<td>0.999</td>
<td>0.999</td>
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<td>0.966</td>
<td>0.849</td>
<td>0.685</td>
<td>0.619</td>
<td>0.436</td>
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<tr>
<td>Chezda</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.983</td>
<td>0.971</td>
<td>0.966</td>
<td>0.551</td>
<td>0.514</td>
<td>0.442</td>
</tr>
<tr>
<td>Fundi</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.984</td>
<td>0.966</td>
<td>0.849</td>
<td>0.685</td>
<td>0.619</td>
<td>0.436</td>
</tr>
<tr>
<td>Glenda</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.984</td>
<td>0.966</td>
<td>0.849</td>
<td>0.685</td>
<td>0.619</td>
<td>0.436</td>
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</table>

<table>
<thead>
<tr>
<th>Cyclone</th>
<th>dy1</th>
<th>dy3</th>
<th>dy5</th>
<th>dy1</th>
<th>dy3</th>
<th>dy5</th>
<th>dy1</th>
<th>dy3</th>
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</thead>
<tbody>
<tr>
<td>MSLP</td>
<td>0.996</td>
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<td>0.928</td>
<td>0.875</td>
<td>0.855</td>
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</tr>
<tr>
<td>WSP(10m)</td>
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<td>RAIN</td>
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<td>0.973</td>
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<td>0.679</td>
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<tr>
<td>Eunice</td>
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<td>0.534</td>
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<tr>
<td>Chezda</td>
<td>0.961</td>
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<td>0.795</td>
<td>0.619</td>
<td>0.324</td>
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<tr>
<td>Fundi</td>
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<tr>
<td>Glenda</td>
<td>0.991</td>
<td>0.970</td>
<td>0.534</td>
<td>0.937</td>
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<td>0.400</td>
<td>0.539</td>
<td>0.441</td>
<td>0.164</td>
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</table>
**Figure Captions**

Figure 1: Cruise track and the study area of ISOE-8. The dashed line from Chennai to 56.26°S indicates the cruise track where only surface sampling was carried out at every 2° intervals without stopping the ship, the blue dots indicates the multidisciplinary stations, whereas the red star marks denotes the XCTD launching locations. On the return track, all the measurements were stopped after 30°S.

Figure 2: DJF (2014-2015) mean of zonal winds at 10m from (a) model (b) ERA_Interim; zonal winds at 850 hPa from (c) model (d) ERA_Interim and mslp from (e) model and (f) ERA_Interim.

Figure 3: Surface Jet (a) location and (b) wind stress from ERA Interim, model analysis and Day 1, Day 3 and Day 5 forecasts.

Figure 4: Taylor diagram for (a) 850hPa and 500hPa zonal winds (b) MSLP forecasted for Day 1(D1), Day 3 (D3) and Day 5 (D5) by four models UKMO (red), ECMWF (blue), NGFS(green) and NCEP(black). Dashed curves represent the standard deviation of the analysis of the models with corresponding colours. The diagrams are for the period Dec 2014 to January 2015 over the study region 0°E to 100°E and 20°S to 80°S.

Figure 5: RMSD of predicted Top Row: zonal winds at 850 hPa, Middle Row: zonal winds at 500 hPa against model analysis and Bottom Row: predicted rainfall against satellite observed rainfall during 01 December 2014 to 28 February for (a) Day 1 (b) Day 3 and (c) Day 5.

Figure 6: Total Rainfall (cm) overlaid with MSLP contours (every 5hPa) for the months of December 2014 (a-d), January 2015 (e-h) and February 2015 (i-l) of analysed (row 1) Predicted for Day 1 (row 2), Day 3 (row 3) and Day 5 (row 4).

Figure 7: Time longitude plots of daily rainfall (cm) averaged over 30°S to 40°S overlaid with contours of 850hPa easterly zonal winds for 01 December 2014 to 28 February 2015 from (a) Satellite merged rainfall with analysed easterly zonal wind (contours) and NGFS predicted rainfall(shaded) and zonal wind(contour) for (b) Day 1 (c) Day 3 (d) Day 5. Easterly (positive) zonal wind contours are separated by 2m/s.

Figure 8: Time longitude plots of daily rainfall (cm) averaged over 30°S to 40°S overlaid with contours of MSLP for 01 December 2014 to 28 February 2015 from (a) Satellite merged rainfall with analysed MSLP(contours) and NGFS predicted rainfall(shaded) and MSLP(contour) for (b) Day 1 (c) Day 3 (d) Day 5. MSLP contours are separated by 5hPa.

Figure 9: Surface windspeed (shaded) overlaid with wind vectors and MSLP(contours at 2hPa intervals) (a) analysis predicted for (b) Day 1, (c) Day 3, (d),Day 5 on 15th January 2015 during Bansi cyclone for different cyclones 1st row: Bansi cyclone on15th January 2015 during 2nd row: Chezda cyclone on 16th January 2015 3rd row: cyclone Eunice and Diamondra on 29th January 2015;
4th row: cyclone Fundi and on 6th Feb 2015; 5th row cyclone Glenda on 26th February
Star indicates the observed location of the cyclone on that day

Figure 10: same as figure 9 but rainfall aggregates during different cyclones
1st row: Bansi cyclone, rainfall is aggregated during 11th to 18th January 2015
2nd row: Chezda cyclone, rainfall is aggregated during 16 to 18th January 2015
3rd row: cyclone Eunice and Diamondra cyclones, rainfall is aggregated during 27th January to 02nd February 2015
4th row: cyclone Fundi, rainfall is aggregated during 06 to 08th February 2015
5th row: cyclone Glenda rainfall is aggregated of 24th to 28th February 2015
Stars indicates the observed track of the cyclone

Figure 11: MSLP (shaded and contoured every 2hPa) overlaid with wind vectors at 500hPa.
Top Row: 15th December 2015 Middle Row: 13th January 2015 and Bottom Row: 23rd February 2015 (a) analysis and predicted for (b) Day 1, (c) Day 3, (d) Day 5.

Figures:

Figure 1:
Figure 3:

![Graph showing wind speed vs. latitude](image)

Figure 4:

![Graph showing standard deviation vs. latitude](image)
Figure 5:

- **$u_{850}$ rmsd (m/s)**
  - (a) dy11
  - (b) dy3f
  - (c) dy5f

- **$u_{500}$ rmsd (m/s)**
  - (a) dy11
  - (b) dy3f
  - (c) dy5f

- **Rainfall rmsd (cm)**
  - (a) dy11
  - (b) dy3f
  - (c) dy5f
Figure 6:
**Figure 7:**

![Figure 7](image)

**Figure 8:**

![Figure 8](image)
Figure 9:

Figure 10:
Figure 11: