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Sequential Nonlinear Model Predictive Control Of A Drainage Canal Using Implicit Diffusive Wave Model

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ABSTRACT

Model predictive control (MPC) is an efficient control method to regulate water systems, for both water quantity and quality. It can generate optimal control solutions based on model predictions over a finite horizon. There are many ways of categorizing MPC. In this research, the predictive control uses a nonlinear internal model and solves optimization in a sequential manner, thus called Sequential Nonlinear Model Predictive Control (SeNMPC). An implicit scheme is applied on the internal diffusive wave model to avoid time step limitation due to model stability. This is important in many real-time control applications where large control time step and fine model grids are used. In order to speed up the computation of optimization, an adjoint method is applied to analytically calculate derivatives of the objective function with respect to control variables. The time reduction is significant. SeNMPC is successfully tested on a drainage canal to regulate water levels.

Key words: adjoint method, diffusive wave model, implicit scheme, sequential model predictive control

INTRODUCTION

In the past and even nowadays, most water systems are operated manually. This traditional approach lacks accuracy, in particular when water systems become more complex, such as canal networks, and control criteria become strict. More efficient and objective management is required and, therefore, automated real-time control and decision-making techniques are introduced.

Real-time control uses system information to calculate control actions, in order to maintain a desired state of a water system by properly operating hydraulic structures, such as pumps, weirs and orifices. A widely used and advanced technique is Model Predictive Control (MPC). Because of the advantages of considering forecasting information and constraints explicitly within an optimization, MPC can achieve better control accuracy and it is attracting more and more attentions [1].

In this research, a Sequential MPC approach is applied by integrating a nonlinear internal model into the optimization problem of the MPC. It uses a fully implicit scheme to keep the solution stable in the case of large control time step. Analytical gradients of the objective function with respect to control variables are calculated through an adjoint method [2] and provided to the optimizer to speed up the calculation. Ipopt [3], an open source optimizer, is used as the optimization engine in this research.

METHOD

The predictive control optimization problem can be formulated through an objective function subject to certain constraints. The common goal of controlling open channel flow is to keep the water level at setpoint by adjusting hydraulic structures as smoothly as possible. The mathematical formulation of the objective function is:

$$\min_{u} \sum_{k=1}^{n} J(x^k, u^k) \tag{1}$$

$$subject\ to: \begin{cases} x^k = f(x^{k-1}, x^k, u^k) \\ u_{min} \leq u^k \leq u_{max} \end{cases}$$

where x is the state, u is the control input, $f(\,)$ is the hydraulic model, u_{min} and u_{max} minimal and maximal structure settings.

The diffusive wave model is used to predict future system states. It neglects the inertia and advection terms of the Saint-Venant equations. The spatial discretization uses one dimensional staggered grids, where water level nodes are connected by flow branches. The momentum equation can be discretized as:

$$Q_{i+1/2}^k = -sign(h_{i+1}^k - h_i^k) C \tilde{A}_{i+1/2}^k \sqrt{\frac{|h_{i+1}^k - h_i^k|}{\Delta x} \tilde{R}_{i+1/2}^k} \tag{2}$$

where Q is the mean discharge, h is the water level, C is the Chezy friction coefficient, $\tilde{A}$ and $\tilde{R}$ are cross section area and hydraulic radius, respectively, which are unknown at branches. They are calculated by taking the average of the up- and downstream nodes for a central spatial schematization or the upstream node in the case of an upwind schematization. Δx is spatial grid size, i, k are the indices of the spatial and temporal discretization, respectively.

The discharge can be substituted into the continuity equation shown in equation (3) to calculate the state of water level.

$$F(h) = h_i^k - h_i^{k-1} + \frac{\Delta t}{\Delta x}\left(Q_{i+1/2}^k - Q_{i-1/2}^k\right) = 0 \tag{3}$$

The discharges are computed based on the water levels of the new time step. Therefore, equation (3) gets a system of nonlinear equation that can be solved through Newton-Raphson method. It is an iterative method to solve a nonlinear equation, where the first order Taylor expansion is applied and model Jacobian needs to be calculated [4].

$$\Delta h = -\frac{F}{\partial F / \partial h} \tag{4}$$

The new state becomes $h_{iter} = h_{iter-1} + \Delta h$ and the iteration terminates when Δh is under a certain tolerance.

The analytical first-order derivatives are calculated through a Lagrangian form, which does not trace back the iterations of the nonlinear equation solver. This is different from the classical algorithmic differentiation. The Lagrangian formulation over a prediction horizon is as follows:

$$L = \sum_{k=1}^{n} \left[J^k + \lambda^k F^k \right] \tag{5}$$

where F is the model constraints (equality constraints), λ is the Lagrange multiplier.

Taking the differential of the Lagrangian function with respect to each variable, it becomes:

$$dL = \sum_{k=1}^{n} \partial \lambda^k F^k + \sum_{k=1}^{n} \partial x^k \left(\frac{\partial J^k}{\partial x^k} + \lambda^k \frac{\partial F^k}{\partial x^k} \right) + \sum_{k=1}^{n} \partial u^k \left(\frac{\partial J^k}{\partial u^k} + \lambda^k \frac{\partial F^k}{\partial u^k} \right) \tag{6}$$

The first term on the right hand side of equation **Error! Reference source not found.** is the simulation model. The second term is the adjoint model used to calculate the Lagrange multipliers backwards. The adjoint model includes a series of linear equations, which can be generalized as:

$$\sum_{k=1}^{n} M^k \Lambda^k = \sum_{k=1}^{n} b^k \tag{7}$$

The differential of a certain water level is connected to all the neighbouring branches. This implies that the matrix M contains the topology of the water networks.

The last term of equation **Error! Reference source not found.** is the derivative of the objective function with respect to control inputs. For a certain prediction step k, with the help of Lagrange multipliers calculated backwards, the objective function gradient can be written as:

$$\frac{dJ^k}{du^k} = \frac{\partial J^k}{\partial u^k} + \frac{\partial F^k}{\partial u_j^k} \lambda^k \tag{8}$$

CASE STUDY

A virtual case is generated for testing the aforementioned method. It is a simple canal network with two branches merging together and flow down to a third reach. Water level at each reach is regulated by a sluice gate at the end of the reach. Two up reaches both have a constant inflow of $0.1 m^3/s$, downstream water level boundary of the third reach is set to 2m. Figure 1 shows the topology of the network. Parameters of the canal and control settings are listed in Table 1.

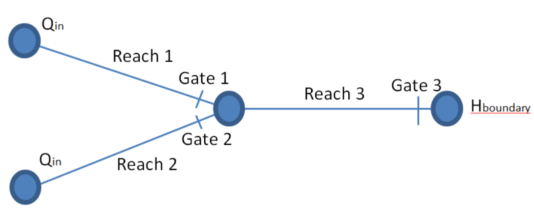
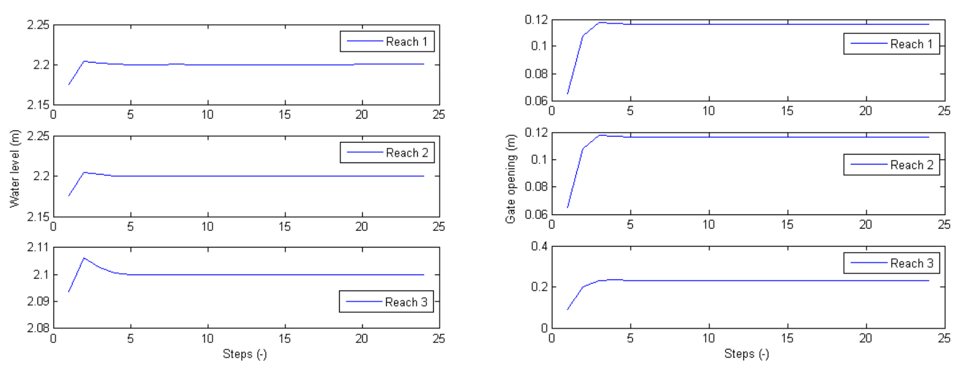


Figure 1. Topology of the canal network

Table 1. Parameters of the test canal network and control settings

Length / Width / Bed level (rectangular)	1000 / 1000 / 1000; 1 / 1 / 1; 0 / 0 / 0 (m)
Gate crest level / Width	0 / 0 / 0; 1 / 1 / 1 (m)
Setpoint	2.2 / 2.2 / 2.1 (m)
Weighing factor (state / control input)	10 / 10 / 10; 1 / 1 / 1
dt / dx	3600 (s) / 200 (m)

Figures 2(a) and 2(b) show the control results of open loop MPC with a prediction horizon of 24 hours. Water level in each reach is well controlled to the setpoint over the horizon. The optimizer of Ipopt takes 46.15s to find the optimal solutions with 33 iterations.



(a) (b)

Figure 2. (a) Controlled water level; (b) Optimal gate opening

CONCLUSIONS

The presented canal network can be controlled well with sequential model predictive control using implicit diffusive wave model. The novel approach enables us to use a sequential MPC setup, which limits the dimensions of the optimization problem to the control instants, in combination with an implicit time stepping scheme without any stability-related time step restrictions. The Newton-Raphson method iteratively solves the system of nonlinear equations in the simulation mode for predicting future system states. The additional adjoint mode is an efficient way of calculating the first-order derivatives analytically to speed up the optimization.

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