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# HYBRID CONTROL-ORIENTED MODELING OF COMBINED SEWER NETWORKS: BARCELONA CASE STUDY

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A hybrid linear model for real-time optimization-based control of a combined sewer network has been developed to be used for the minimization of pollution during storm events. The model takes into account delays and attenuation in sewers together with piecewise linear approximations for flow over weirs, overflows in junctions and flow re-entering the network after overflows. Using the proposed model, an *Optimal Control Problem* (OCP) is formulated, which can be efficiently solved by means of a mixed integer linear or quadratic programming problem. The performance of a *Model Predictive Control* (MPC) strategy solving consecutive OCPs is assessed by means of closed-loop simulations using a physically-based complex model as virtual reality.

## INTRODUCTION

Combined sewer systems carry both storm-water and wastewater together. These infrastructures are designed to convey all the water to *wastewater treatment plants* (WWTP), where it is treated before being released to the natural environment. However, heavy rain episodes may cause the network capacity to become overloaded, forcing untreated water discharges to the natural environment *(combined sewer overflows*, CSO) and flooding urban areas.

In the municipalities where such heavy rain events are common, additional infrastructure in the form of flow-regulation and storage elements such as gates, weirs, pumps and reservoirs is usually available. The efficient regulation of these elements is, therefore, critical to fully take advantage of the network capacity and prevent untreated water discharges.

*Real-time control* (RTC) provides a solution to the problem of the infrastructure regulation that can benefit at each time instant form the last network measurements as well as from future rain forecasts [1]. Moreover, optimization-based control can also take into account the operational constraints of the network elements to provide optimal regulation action with respect to given management objectives. *Model predictive control* (MPC) is a RTC optimization-based technique that uses a mathematical model of the sewer network to formulate and solve successive *optimal control problems* (OCPs) based on the last system measurements and rain forecasts. Only the first regulation action computed by each OCP is applied and the procedure is repeated again.

The physically-based model for water motion in sewer networks is based on the 1D Saint-Venant equations with constant channel cross-sectional area and constant channel bed slope [2]. These equations are hyperbolic nonlinear partial differential equations (PDE) relating both the flow and water level in an open channel/sewer. The Saint-Venant equations lack an explicit solution for arbitrary sewer geometries and need to be solved by means of numerical methods. For mid- to large-scale networks, the required time to solve the coupled system consisting of the whole network elements is too long to be included in a RTC scheme. Therefore, simplified mathematical control-oriented models are developed that, while still describing the flows with reasonable accuracy, allow to formulate OCPs with suitable properties that can be solved within short times [3, 4, 5, 6].

In this work, a control-oriented sewer network model is described. Using this model, an OCP can be formulated to be used in a MPC strategy. This controller acts as an upper level controller that computes optimal gate flows used as set-points for local PID controllers. Simulation results of the control strategy using a physically-based model simulator as virtual reality are provided.

## SEWER NETWORK MODEL

The following model aims to be general enough to be used in a wide range of network instances. Therefore, each network element is modeled individually, so that specific network topologies may be modeled using the interconnection between elements. This approach also facilitates the calibration process, which can be carried out using real data or data generated by a physically-based model simulator.

The model consists of discrete-time equations, which are used directly in the OCP formulation. In the following, $t \in \mathbb{Z}^+$ is the discrete-time variable and $\Delta t$ [s] the time step. Thus, the discrete-time instant t corresponds to $t \cdot \Delta t$ seconds after the modeled event starts.

### Flow Model

The main network equations are the flow model equations, describing the flow delay and attenuation along sewers and the mass balance in the network junctions. For each sewer, two flow variables are used: $q_{in}$ as the inflow to the sewer and $q_{out}$ as its outflow.

The inflow $q_{in}$ to a sewer is computed by adding the contributions of all the upstream elements entering the node where the sewer is attached. In case there are several outgoing sewers at the same junction each one takes a fixed proportional part of the total inflow.

The outflow in a sewer is computed by means of a convex combination of is inflow at two consecutive time steps with a coefficient $a_i \in (0,1]$:

$$q_{out}(t) = a\, q_{in}(t-\tau) + (1-a)\, q^{in}(t-\tau-1),$$

thus, taking into account the transportation delay $\tau$.

### Reservoir Model

The reservoir equation used in the model is simply a discretization of the usual mass balance reservoir equation using a forward Euler method

$$v(t) = v(t-1) + \Delta t \big( g_{in}(t-1) - g_{out} t(t-1) \big),$$

where $v(t)$ is the volume contained in the reservoir and $g_{in}(t)$ and $g_{out}(t)$ the (controlled) inflow and outflow.

**Weir Model**

Weirs are flow regulation elements that divert part of the flow in a sewer to a secondary one, called a spillway, when a certain water level is achieved. Weirs can have fixed or movable position. In the latter case the flows would become decision variables, the OCP would compute their optimal values to be used as set-points for local PID controllers and no particular modeling would be required. Therefore, the following equation applies only to the fixed weir case. The following formula is an approximation in terms of flow of the weir behavior, which actually depends on water levels.

The spillway flow for a weir, $w(t)$ is computed as

$$w(t) = \max\{0, a_w\left(z_w(t) - q_w^{max}\right)\},$$

where $z_w(t)$ is the total inflow to the junction and $q_w^{max}$ is an estimated value of the maximum inflow to the sewer before the flow starts flowing through the spillway. Parameter $a_w \in (0,1]$ allows the flow through the main sewer reach values higher than $q_w^{max}$, as observed in data generated by a physically-based model.

**Overflow and flood runoff model**

Overflows are defined at some network junctions, where flooding or overflows are known or expected to occur. As in weir flow equation, the overflow $f(t)$ is defined as the excess flow above an estimated maximum inflow $q_f^{max}$

$$f(t) = \max\{0, a_f(z_f(t) - q_f^{max})\}.$$

Again, $z_f(t)$ is the total inflow to the junction and a parameter $a_f \in (0,1]$ is introduced to allow the flow through the sewer to reach values higher than $q_f^{max}$, as observed in real cases or in a physically-based model.

Unlike weir flows, overflows are not redirected to another network junction. The overflow volume is kept in a fictitious reservoir until the overflow event finishes and then, is returned to the same junction. The fictitious reservoir holds a mass-balance equation like the reservoir one

$$v_t(t) = v_t(t-1) + \Delta t\left(f(t-1) - q_t(t-1)\right),$$

with an outflow computed as follows:

$$q_t(t) = \min\left\{\max\left\{0, b_f\left(q_f^{max} - z_f(t)\right)\right\}, \frac{v_t(t)}{\Delta t}\right\}.$$

U sing this formula the overflow volume re-entering the network (flood runoff flow) is equal to zero while overflow is occurring (i.e., $f(t) > 0$). When the overflow ends, the flood runoff takes a value proportional to the difference between the maximum flow $q_f^{max}$ and the junction inflow $z_f(t)$, without exceeding the maximum flow available due to the volume stored in the reservoir ($v_t(t)/\Delta t$). The proportionality parameter $b_f$ is introduced for calibration reasons to best fit real or physically-based model generated data.

**Collector Model**

Collectors are big sewers with an in-line retention capacity of the same order as a reservoir. In order to take into account the volume contained in a collector to take full advantage of its capacity, three different models have been developed and compared in this study. In all cases a manipulated gate $g_c(t)$ is assumed to be placed at the downstream end of the collector and an overflow variable $f_c(t)$ is added to model flooding.

*Single Reservoir*: The first model consists simply of modeling the collector as a reservoir

$$v(t) = v(t-1) + \Delta t\left(q_{in}(t-1) - g(t-1) - f_c(t-1)\right),$$

with an overflow defined as

$$f_c(t) = \max\left\{0, v(t-1) + \Delta t\left(q_{in}(t-1) - g_c(t-1)\right) - v_c^{max}\right\},$$

where $v_c^{max}$ is the total collector volume.

*Single Reservoir Plus Delay*: The second model consists of adding a delay to the inflow to the reservoir, thus making the volume available to be released through the downstream gate only some time steps after it has entered the collector. An easy way to implement this is to represent the collector as a series of $N$ reservoirs, each one adding a one time step delay to the flow, with only the last one acting as a storage element. The reservoir equations are the same as in the reservoir model with correspondingly modified in- and outflows:

$$\begin{aligned} v_1(t) &= v_1(t-1) + \Delta t\left(q_{in}(t-1) - q_1(t-1) - f_c(t-1)\right), \\ v_i(t) &= v_i(t-1) + \Delta t\left(q_{i-1}(t-1) - q_i(t-1)\right),\ i = 2, \ldots, N-1, \\ v_N(t) &= v_N(t-1) + \Delta t\left(q_{N-1}(t-1) - g_c(t-1)\right). \end{aligned}$$

The communicating flows $q_i(t)$ between the reservoirs are defined as

$$q_i(t) = \frac{v_i(t)}{\Delta t},\ i = 1, \ldots, N-1.$$

This means that each reservoir completely empties towards the next one every time step except the last one, which is controlled by a gate flow $g_c(t)$. In this case, the overflow variable is defined as

$$f_c(t) = \max\left\{0, \sum_{i=1}^{N} v_i(t-1) + \Delta t\left(q_{in}(t-1) - g_c(t-1)\right) - v_c^{max}\right\},$$

where $v_c^{max}$ is the total collector volume.

*N Reservoirs*: The last model consists also of a series of $N$ reservoirs. In this case each of the reservoirs has the same maximum capacity $v_N^{max} = v_c^{max}/N$, where $v_c^{max}$ is the total collector volume. The last reservoir is again controlled by a gate, acting as a decision variable. This can cause the last reservoir to become full. If this happens the second downstream reservoir starts filling. The same procedure applies to the other reservoirs on until the first one. If the first reservoir becomes full, any additional inflow is regarded as overflow.

The equations for the reservoirs and the overflow variable are the same as for the previous model. However, in this case the flows communicating the reservoirs are defined as follows:

$$q_i(t) = \min\left\{\frac{v_i(t)}{\Delta t}, \frac{v_N^{max} - v_{i+1}(t)}{\Delta t} + q_{i+1}(t)\right\},\ i = 1,\ldots,N-2,$$

with the modified expression for the last reservoir, accounting for the controlled outflow

$$q_{N-1}(t) = \min\left\{\frac{v_{N-1}(t)}{\Delta t}, \frac{v_N^{max} - v_N(t)}{\Delta t} + g_c(t)\right\}.$$

Notice that, even when several reservoirs are full, there is still flow through the collector: all the full reservoirs provide the next one a flow equal to the downstream gate flow $g_c(t)$.

In this case, the overflow variable takes only into account the first reservoir

$$f_c(t) = \max\left\{0, v_1(t-1) + \Delta t\big(q_{in}(t-1) - g_c(t-1)\big) - v_N^{max}\right\}.$$

## OPTIMAL CONTROL PROBLEM

### Hybrid Linear Model

The maximum and minimum functions involved in the model equations are reformulated by means of the *Mixed Linear Dynamic* (MLD, [7]) systems approach into a set of linear equalities and inequalities. This procedure involves the definition of binary variables that describe which of the maximum and minimum function branches is chosen at each time step, thus turning the system into a *hybrid system*.

Once all dynamic equations and MLD inequalities are put together, the system can be written in the following form:

$$\begin{aligned} \sum_{i=0}^{T} M_i X(t-i) &= m(t), \\ \sum_{i=0}^{T} N_i X(t-i) &\leq n(t), \end{aligned} \tag{1}$$

where $T$ is the maximum system delay, $X(t-i)$, $i=1,\ldots,T$, contain all the system variables at time step $t-i$ and $M_i$ , $N_i$ , $i=1,\ldots,T$, contain the coefficients of the equations and inequalities corresponding to $X(t-i)$.

### Optimal Control Problem

The optimal control problem is built by imposing the system of equations and MLD inequalities (1) together with some additional constraints (described below) at $H$ consecutive future time steps (where $H$ is called the *prediction horizon*). The whole resulting set of constraints can be arranged by means of block-structured matrices $\mathcal{M}_1$ and $\mathcal{N}_1$ related to the future unknown variables to be solved for, $\mathcal{M}_2$ and $\mathcal{N}_2$ related to the initial conditions and $\mathcal{M}_3$ and $\mathcal{N}_3$ related tot the rain inflows. The resulting optimization problem has the form

$$\begin{aligned} \min_{X(t)}\ & J(X(t)) = c^\top X(t), \\ \text{s.t.}\quad & \mathcal{M}_1 X(t) = \mathcal{M}_2 X_0(t) + \mathcal{M}_3(t), \\ & \mathcal{N}_1 X(t) \leq \mathcal{N}_2 X_0(t) + \mathcal{N}_3(t), \\ & A_{eq} X(t) = b_{eq}(t), \\ & A_{ineq} X(t) \leq b_{ineq}(t), \end{aligned}$$

where the vector of unknowns is defined as $\mathcal{X}(t) = ( X(t + H)^T, \ldots ,X(t + 1)^T )^T$, and the vector of initial conditions as $\mathcal{X}_0(t) = ( X(t)^T, \ldots ,X(t - T + 1)^T )^T$. Additional constraints $A_{eq}\, \mathcal{X}(t) = b_{eq}$ and $A_{ineq}\, \mathcal{X}(t) \leq b_{ineq}(t)$ take into account bounds on variables, mass balances at junctions with outflowing gates and bounds on the variation rates of the controlled variables to ensure smooth actions. Moreover, since the MPC controller provides set-points to local PID controllers, the gate flows are constrained to remain constant for intervals of 5 time steps in order to take into account the fact that the PID controllers need some time to reach the desired values.

The objective function $J(\mathcal{X}(t))$ contains a weighted sum of variables to be minimized. The specific expression for $J(\mathcal{X}(t))$ will depend on the management objectives of the network. The model is flexible enough to accommodate many different objectives. The management objectives used in this work consist of: (i) minimizing overflows (urban flooding), (ii) minimizing CSO, and (iii) maximizing WWTP usage. Maximization of a variable can be achieved by means of a negative weight in the objective function. Quadratic objective functions could also be used to describe other objectives such as tracking of a desired volume in a reservoir.

## CASE STUDY

The Riera Blanca sewer network is a part of the Barcelona sewer network comprising an area of about 20 $km^2$ converging at its downstream end to a big collector with a total in-line capacity of $6 \cdot 10^4$ $m^3$. Ten gates can be used regulate the network flow and eventually redirect part of the volume to two reservoirs with capacities of $5 \cdot 10^4$ $m^3$ and $10^4$ $m^3$ respectively. Figure 1 shows a simplified diagram of the network. The modeling approach presented above has been applied to this network together with calibration and validation procedures, which are out of the scope of this paper. The resulting control model consists of one collector, 145 sewers, 3 weirs and 10 overflow points in addition to the 10 gates and 2 reservoirs mentioned above. A time step of $\Delta t$ = 1 minute and a maximum delay of $T$ = 6 minutes have been used.

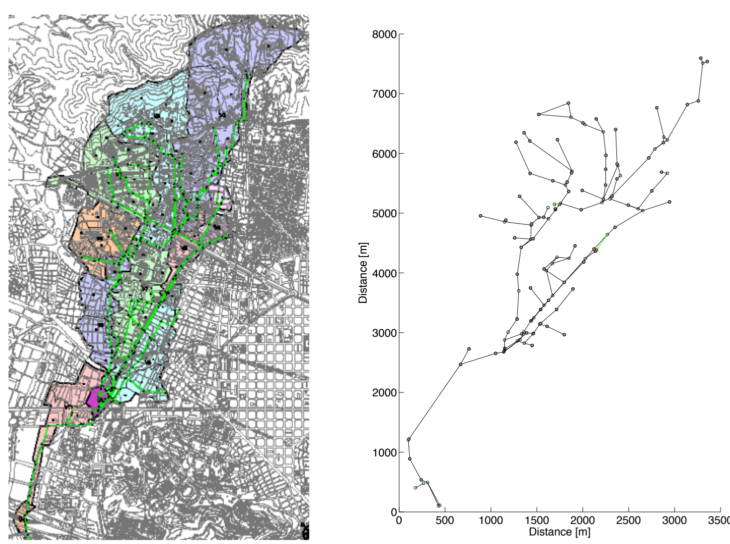


Figure 1: Map and diagram of the simplified interconnection scheme for the Riera Blanca sewer network.

Full information of the network has been made available by the company responsible of its management, CLABSA (CLavegueram de Barcelona S.A.), by means of an implementation in the physically-based model simulator MOUSE [8]. MOUSE solves the complete Saint-Venant equations and is also able to simulate local PID controllers located at the network gates to regulate its outflow by means of their opening. The results presented in the next section are based on using this simulator as a virtual reality.

MOUSE has also provided the rainfall-runoff model used in this study, that is, the model computing net flow entering the network from pluviometer data. The model is based on defining rain catchments which are modeled as reservoirs with an outflow computed by means of the Manning equation [9]. To model the Riera Blanca sewer network 68 rain catchments were defined by CLABSA.

**CLOSED-LOOP SIMULATION ALGORITHM AND RESULTS**

Model predictive control is a control strategy which involves solving consecutive OCPs (based on a model of the system) and only applying to the system the control action corresponding to the first time step. After the system is left to evolve for one time step, measures of the system state are taken and are used to update the initial conditions of a new OCP, whose solution will be used for the next time step. In the case of external disturbances, such as the rain inflows to a sewer network, forecasts of their values must also be included in each OCP.

To simulate this procedure, instead of letting the system evolve, simulations of the network using the physically-based simulator MOUSE are performed. Hence, once the OCP is solved the (constant) gate flow values corresponding to the first 5 minutes of the solution are used as PID set-points for a 5 minute simulation of the network. Then the flow values corresponding to the last $T$ minutes of the simulation are used to update the OCP initial conditions $\mathcal{X}_0$ and the OCP is solved again to compute the PID set-points for the next 5 time minutes. All the OCPs have been solved using a prediction horizon of $H$ = 30 time steps. Figure 2 shows a diagram of this closed-loop simulation algorithm.

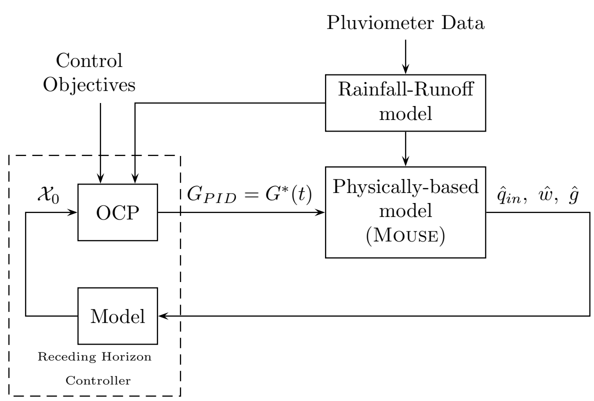


Figure 2: Diagram of the closed-loop simulation algorithm.

Results of these simulations are shown in Table 1 for the three different collector models for a real rain event, with rain data provided by CLABSA. It can be noticed that no relevant differences appear between the three models. This is due to the fact that although the multiple reservoir models describe the dynamics of the collector with better accuracy, the hard constraint forcing not to exceed the collector maximum volume does not allow for better performance in any of these cases.

In all cases, overflows downstream of the controlled gates are completely avoided and maximum volumes at the reservoirs and collector are respected. This, together with the maximum computation times needed to solve the OCPs, show that the proposed control procedure is suitable for RTC.

Table 1: Closed-loop simulation results.

| Model | Overflow [ $\times 10^3$m$^3$] | CSO [ $\times 10^3$m$^3$] | WWTP [ $\times 10^3$m$^3$] | Maximum OCP Time [s] |
|---|---|---|---|---|
| Single Reservoir | 250.09 | 5186.59 | 53054.41 | 1.09 |
| SR, delay = 5 | 250.09 | 5026.79 | 53132.86 | 1.84 |
| SR, delay = 10 | 250.09 | 4582.24 | 53104.61 | 2.20 |
| 5 Reservoirs | 250.09 | 5106.18 | 53157.49 | 2.25 |
| 10 Reservoirs | 250.09 | 3905.40 | 53526.97 | 3.82 |

## CONCLUSIONS

A control-oriented model for sewer networks has been developed together with a model-based formulation of an *Optimal Control Problem* (OCP) to minimize pollution in presence of heavy rain events. The modeling procedure and OCP formulation has been applied to a real network. By solving consecutive OCPs a *Model Predictive Control* (MPC) strategy has been assessed using a physically-based model simulator as virtual reality. The overall modeling and control approach has proven to meet real-time control requirements, fulfill the physical and operational constraints and provide good performance results.

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