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ESTIMATION OF OFFTAKE DISCHARGE AND CROSS-DEVICE PARAMETERS USING DATA ASSIMILATION FOR THE AUTOMATIC CONTROL OF AN IRRIGATION CANAL

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This paper presents an application of Kalman Filtering on an open-channel hydraulic system, to estimate unknown discharges at lateral offtakes and head-loss coefficients at cross-regulators using only water level measurements upstream and downstream of each pool. This information is useful for the automatic control of these hydraulic systems, such as irrigation or navigation canals. The convergence properties of such Kalman Filters are also studied and it can be mathematically verified how many measurements are required to allow such identification. The methodology is illustrated on a 15 km-long 5-pool benchmark irrigation canal.

INTRODUCTION

The automatic control of open-channel hydraulic systems such as rivers (with dams and/or hydropower plants), irrigation or navigation canals (with gated cross-devices), and sewage systems almost always use hydraulic models. These models can be used in different manners, either just to test and validate controllers prior to implementation, to tune the controller parameters off-line, or used on-line in real-time as a component of the controller (ex.: in model predictive controllers). The control algorithm then calculates in real-time the control action variables u, using measured variables y obtained from the real system, in order to achieve some objectives for some controlled variables z, in the presence of disturbances w.

These models have always limited precision due to unknown or wrong parameters, input variables and internal states. Among the classical parameters, we find cross-device discharge coefficients C_d and bed friction Manning-Strickler coefficients n. Among input variables, we find the inflow or outflow discharges entering into or leaving from the river Q_p, or taken by users from the canal. Indeed, they are rarely measured, or in the best cases with a limited precision.

This is a problem since the tuning of the control parameters of the feedback loops depends a lot on the dynamics of the system and therefore on the previous listed parameters. In particular, C_d is part of the gate-actuator behavior. Also, a methodology allowing computing in real-time this C_d coefficient could be used to detect a problem at this gate (problem of gate position sensor, object blocking the gate, etc). A feedforward control component, very useful for this class of delayed systems, could benefit from the knowledge of the input variables Q_p at lateral offtakes. Even though this variable will not be known in advance, this can allow improving a model of

the offtake discharges. Knowing the offtake discharge in real-time or even a posteriori is also a usefull information for a canal manager (detecting problems, leaks, stealing of water, etc).
Reconstruction of lateral inflows Q_p has been done in previous work [2] using Kalman Filters. Reconstruction of hydraulic states X and Manning coefficients n has been done in previous work [3] using numerically calculated tangent linear model, which is fastidious, and Kalman Filters. Reconstruction of hydraulic states X and Manning coefficients n has been done in previous work using Monte Carlo particle filter approach [4]. This latter approach does not require a linear model of the system but proved to converge slower that Kalman Filters for still unknown reasons [2]. The estimation of discharge coefficients at cross regulators C_d, and even more the combined estimation of C_d, offtake discharge Q_p and hydraulic states X has never been done to our knowledge. This is the problem that will be solved in this paper and illustrated on a 5-pool irrigation canal.

SYSTEM CONSIDERED

The canal selected for the illustration of the methodology developed in this paper is the canal 1 of the Irstea Benchmark [1]. This is a 5-pool canal with a trapezoidal cross section (bed width = 7 m and bank slope = 1.5) and a longitudinal bed slope of 1e-4 (Figure 1). Each pool is 3 km long and has an offtake at its downstream end. The initial hydraulic state is a steady state with 7 m^3/s head discharge and each offtake taking 0.175 m^3/s at initial time (Q_{pi}, i=1:5). Cross regulators are composed of a gate with a width L=10.18 m, with a C_d contraction coefficient (C_{di}, i=1:5) taken equal to 0.82 at initial time and then changed down to 0.66 latter on. Measurements used by the Kalman Filter will be the water levels upstream and downstream of each pool (y_i, i=1:10). All simulations are carried out using the SIC hydrodynamic model developed by Irstea [1].

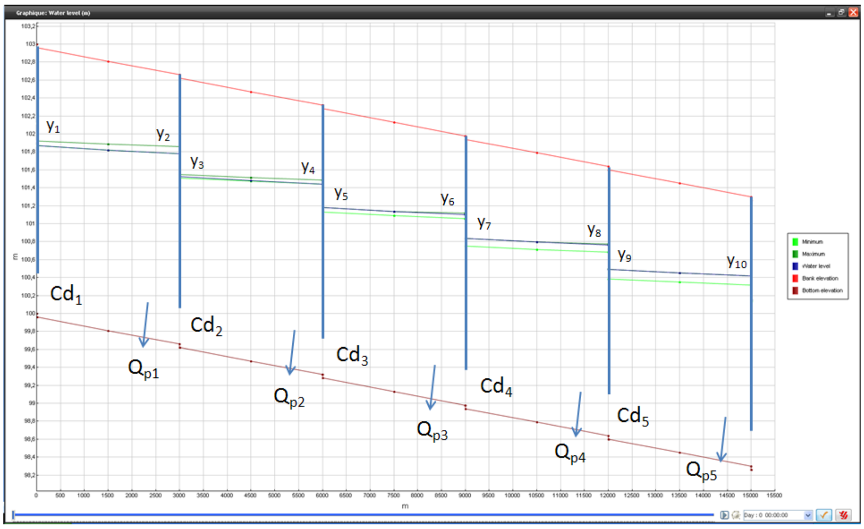

Figure 1. Example of a 5-pool irrigation canal

GENERATION OF THE TANGENT LINEAR MODEL

A Kalman Filter needs a model of the considered system. For a classical linear Kalman Filter, this model needs to be linear. Other variants with non-linear model also exist such as, for example: the Extended, the Ensemble, or the Unscented Kalman Filters. In this paper we will start by testing the linear Kalman Filter. This version will prove to be efficient enough and also provides nice mathematical tool that will allow assessing its convergence properties (convergence of the state estimation error).

The Saint-Venant equations

Since we will use water level measurements and we will reconstruct variables linked to discharges, we need a model manipulating both variables. Simplified models using only discharges are therefore not applicable here. In an irrigation canal, the 1D assumption is usually considered as valid. We will therefore base our model on the 1D Saint-Venant model [1]. The two continuity Eq. (1) and dynamic Eq. (2) equations are completed by external and internal boundary conditions such as upstream water level, downstream rating curve and gate equations at cross regulators Eq. (3).

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{S}\right) + gS\left(\frac{\partial Z}{\partial x}\right) = gS(I - J) \tag{2}$$

With: S: cross section (m^2), Q: discharge (m^3/s), Z: water elevation (m), I: bed slope, J: friction slope, t: time (s), x: longitudinal abscissa (m), g: gravity (9.81 m/s^2).

These Saint-Venant equations are completed by the equations of the intermediate cross regulators. We use equations developed at Irstea [1] allowing the management and the continuous transition between all flow conditions (openflow vs piped-flow, free-flow vs submerged flow). In the case of a piped submerged flow this equations is of the form:

$$Q = C_d \sqrt{2g} L w \sqrt{Z_{up} - Z_{dn}} \tag{3}$$

Where C_d is a discharge coefficient depending on the type of gate and linked to the contraction of the flow under the gate, L is the gate width, w is the gate opening, Z_{up} and Z_{dn} are respectively the water level upstream and downstream of the gate.

The discretization

The Saint-Venant equations cannot be solved analytically. We use the implicit Preissmann's discretization scheme [1] to transform these two partial derivative non-linear hyperbolic equations into a set of ordinary equations:

$$f(x,t) = \frac{\theta}{2}\left(f_{i+1}^{n+1} + f_i^{n+1}\right) + \frac{(1-\theta)}{2}\left(f_{i+1}^{n} + f_i^{n}\right)$$

$$\frac{\partial f}{\partial x} = \theta \frac{\left(f_{i+1}^{n+1} - f_i^{n+1}\right)}{\Delta x} + (1-\theta)\frac{\left(f_{i+1}^{n} - f_i^{n}\right)}{\Delta x}$$

$$\frac{\partial f}{\partial t} = \frac{1}{2}\frac{\left(f_{i+1}^{n+1} - f_i^{n+1}\right)}{\Delta t} + (1-\theta)\frac{\left(f_{i+1}^{n} - f_i^{n}\right)}{\Delta t} \tag{4}$$

With: i the space index, n the time index, θ the weighting coefficient (taken = 0.6). In our example we will select a space step Δx = 600 m and a time step Δt = 5 mn.

The linear system setup

The reaches are divided into calculation cross sections distant by the given space step Δx, and completed by additional sections located at the different hydraulic singularities (eg.: offtakes, cross structures, bed drops, etc). The considered state variables are the discharge Q and water elevation z variables at each cross section i. Since we consider these variables with reference to the linearization hydraulic state, we write these variables δQ_i and δz_i. Between the two sections k and l (see Figure 2), the two Saint-Venant equations are discretized and put into the matrix form:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \end{pmatrix} \cdot \begin{pmatrix} \delta Q_k^+ \\ \delta z_k^+ \\ \delta Q_l^+ \\ \delta z_l^+ \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \end{pmatrix} \cdot \begin{pmatrix} \delta Q_k \\ \delta z_k \\ \delta Q_l \\ \delta z_l \end{pmatrix} \tag{5}$$

The variables with a + superscript are variables at the next time instant $t+\Delta t$. A_{ij} and B_{ij} coefficients are obtained from the Saint-Venant equations and local bathymetry and hydraulic conditions.

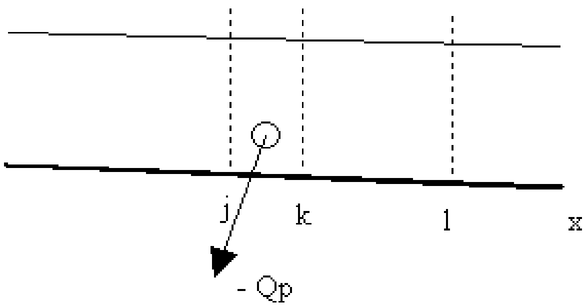

Figure 2. Example of calculation sections around an offtake

For the offtake located between the calculation sections j and k we can write the equations:

$$\begin{cases} \delta Q_k = \delta Q_j + \delta Q_p \\ \delta Q_k^+ = \delta Q_j^+ + \delta Q_p^+ \\ \delta z_k = \delta z_j \\ \delta z_k^+ = \delta z_j^+ \end{cases} \tag{6}$$

With the convention $Qp > 0$ if the discharge is entering the canal, and $Qp < 0$ if the discharge is taken from. After reporting Eq. (6) into Eq. (5) we can eliminate the variables with index k (i.e. δQ_k^+, δz_k^+, δQ_k and δz_k). This allows reducing the dimension of the state vector X since ($\delta z_k = \delta z_j$ and $\delta Q_k = \delta Q_j + \delta Q_p$). We then obtain between sections j and l:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \end{pmatrix} \begin{pmatrix} \delta Q_j^+ \\ \delta z_j^+ \\ \delta Q_l^+ \\ \delta z_l^+ \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \end{pmatrix} \begin{pmatrix} \delta Q_j \\ \delta z_j \\ \delta Q_l \\ \delta z_l \end{pmatrix} + \begin{pmatrix} B_{11} & -A_{11} \\ B_{21} & -A_{21} \end{pmatrix} \begin{pmatrix} \delta Q_p \\ \delta Q_p^+ \end{pmatrix}$$

Through a cross regulator we have the following configuration:

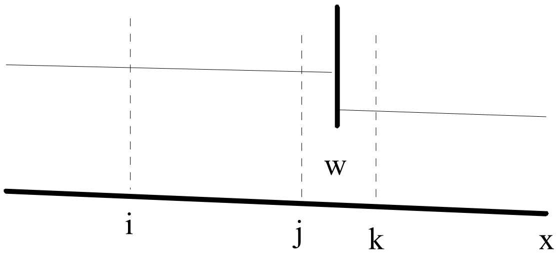

Figure 3. Example of calculation sections around a cross device

Between sections i and j (see Figure 3), the two Saint-Venant equations are discretized and put into the matrix form:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \end{pmatrix} . \begin{pmatrix} \delta Q_i^+ \\ \delta z_i^+ \\ \delta Q_j^+ \\ \delta z_j^+ \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \end{pmatrix} . \begin{pmatrix} \delta Q_i \\ \delta z_i \\ \delta Q_j \\ \delta z_j \end{pmatrix}$$

Between sections j and k we have the gate equation:

$$Q_k(t+\Delta t) = f(z_j(t+\Delta t), z_k(t+\Delta t), w(t+\Delta t), C_d(t+\Delta t))$$

$\Rightarrow \delta Q_k^+ = f(z_j^+, z_k^+, w^+, C_d^+) - f(e)$, where f(e) is the gate discharge at the reference linearization state, w is the gate opening and C_d the gate discharge coefficient.

$$\Rightarrow \delta Q_k^+ = \left(\frac{\partial f}{\partial z_j}\right)_e \delta z_j^+ + \left(\frac{\partial f}{\partial z_k}\right)_e \delta z_k^+ + \left(\frac{\partial f}{\partial w}\right)_e \delta w^+ + \left(\frac{\partial f}{\partial C_d}\right)_e \delta C_d^+$$

And in matrix form:

$$\begin{pmatrix} 0 & -\left(\frac{\partial f}{\partial z_j}\right)_e & 1 & -\left(\frac{\partial f}{\partial z_k}\right)_e \end{pmatrix} . \begin{pmatrix} \delta Q_j^+ \\ \delta z_j^+ \\ \delta Q_k^+ \\ \delta z_k^+ \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix} . \begin{pmatrix} \delta Q_j \\ \delta z_j \\ \delta Q_k \\ \delta z_k \end{pmatrix} + \left(\frac{\partial f}{\partial w}\right)_e . \delta w^+ + \left(\frac{\partial f}{\partial C_d}\right)_e . \delta C_d^+ \quad (7)$$

Combined with the discretisation of the Saint-Venant equation between the section i and j we obtain:

$$\begin{pmatrix} A_{11} & A_{12} & A_{14} & A_{13} & \\ A_{21} & A_{22} & A_{24} & A_{23} & \\ & & -\left(\frac{\partial f}{\partial z_j}\right)_e & 1 & -\left(\frac{\partial f}{\partial z_k}\right)_e \end{pmatrix} \begin{pmatrix} \delta Q_i^+ \\ \delta z_i^+ \\ \delta z_j^+ \\ \delta Q_k^+ \\ \delta z_k^+ \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & B_{14} & B_{13} & \\ B_{21} & B_{22} & B_{24} & B_{23} & \\ & & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta Q_i \\ \delta z_i \\ \delta z_j \\ \delta Q_k \\ \delta z_k \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \left(\frac{\partial f}{\partial w}\right)_e \end{pmatrix} . \delta w^+ + \begin{pmatrix} 0 \\ 0 \\ \left(\frac{\partial f}{\partial C_d}\right)_e \end{pmatrix} . \delta C_d^+$$

After applying the above described procedure to all sections, we obtain an approximate linear model of the true system in state space form [5]:

$$\begin{cases} X_{k+1} = A \cdot X_k + B \cdot U_k + B_p U_p + B_c U_c \\ Y_k = C \cdot X_k \end{cases} \quad (8)$$

Where A: state transition model, B: control-input model, B_p offtake input model, B_c discharge cross-device coefficient model, C: observation model. These matrices are real constant matrices of appropriate dimensions. The X state vector is composed of hydraulic variables (discharges Q and water levels Z) at each calculation cross section, relative to a reference state (δ variables).

THE KALMAN FILTER

The uncertainties on the linear system and the augmented state

In practice, a linear model is not a perfect representation of the reality and the inputs and the measurements are not perfectly precise. There are some uncertainties on the model, on the inputs and on the measurements. This is the reason why noise is added to the process model and to the measurements, in order to better represent a realistic system, when used to generate data in a twin experiment framework. The linear stochastic difference equations become:

$$\begin{cases} X_{k+1} = A \cdot X_k + B \cdot U_k + B_p U_p + B_c U_c + w_k, \\ Y_k = C \cdot X_k + v_k \end{cases} \quad (9)$$

This original state-space representation is modified by augmenting the state with the variables U_p and U_c that will be reconstructed using the Kalman Filter:

$$\begin{cases} \begin{pmatrix} X \\ U_p \\ U_c \end{pmatrix}_{k+1} = \begin{pmatrix} A & B_p & B_c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ U_p \\ U_c \end{pmatrix}_k + \begin{pmatrix} B \\ 0 \\ 0 \end{pmatrix} U_k + w_k \\ Y_k = \begin{pmatrix} C & 0 & 0 \end{pmatrix} \begin{pmatrix} X \\ U_p \\ U_c \end{pmatrix}_k + v_k \end{cases} \quad (10)$$

We can write A_a, B_a and C_a the augmented matrices appearing in the above equation, and X_a the augmented state. The random variables w_k and v_k are assumed to be independent to each other. They are considered like white noises, and with normal probability distributions:

$P(w) \sim N(0,Q)$, (11)

$P(v) \sim N(0,R)$, (12)

The Q and R matrices can be time varying. In our simulations we took these matrices as time invariant and diagonal.

Kalman filter equations

The equations for the Kalman Filter are divided into two groups: time update equations and measurement update equations. The time update equations project forward in time the current state and error covariance estimates to obtain the a-priori estimates for the next time step. The measurement update equations incorporate new measurements into the a-priori estimate to obtain an improved a posteriori estimate.

During the time update equations, we propagate the estimated state vector and the covariance matrix (to simplify the equation we remove the "a" subscript of the augmented state and matrices):

$$\begin{cases} \hat{X}_k^- = A\hat{X}_{k-1} + BU_{k-1} \\ P_k^- = AP_{k-1}A^t + Q \end{cases}$$

During the measurement update equations, we use the measurement y_k to correct the estimated state vector and the covariance matrix, using the Kalman gain K_k:

$$\begin{cases} K_k = P_k^- C^t \left(CP_k^- C^t + R \right)^{-1} \\ \hat{X}_k = \hat{X}_k^- + K_k \left(y_k - C\hat{X}_k^- \right) \\ P_k = \left(I - K_k C \right) P_k^- \end{cases}$$

These equations are simple matrix calculations, requiring little CPU time even for 75×75 matrices of our example (a few seconds for a scenario of a few days). The calculation can even be further reduced by using the asymptotic K matrix (obtained solving a Riccati equation) without loss of performance (this has been tested and verified, but not presented in this paper).

TESTS ON A BENCHMARK CANAL

A Kalman Filter is used to reconstruct unmeasured data on a scenario, representative of real field conditions. In this scenarios twin experiments are conducted. A first simulation is used to generate data. White noises are then added to them to simulate uncertainties on the model and on the measurements (they have been removed on the Figure 4 for a better reading of the graphs and checking of the convergence towards the real values). The initial state X_0 is also taken from random variables (which explains the initial large variations on the Figure 4). The second simulation is then done using the Kalman filter to reconstruct data, without giving, of course, the information to be reconstructed (Q_p and C_d).

Scenario

- Time step for the simulations is 300 seconds.
- The 5 offtakes are operated at time 6 h (-0.100 m^3/s) and again at time 20 h (+0.100 m^3/s) returning to the initial discharge values of -0.175 m^3/s.
- All cross regulators keep the same gate positions, but the discharge coefficient of their gates is changed from 0.82 to 0.66 at time 30 mn.
- The initial state X_0 is taken using random variables.
- For the matrix Q, we choose $\sigma_Q = 0.1$ for the normal hydraulic states and $\sigma_Q = 2$ for the augmented states. For the matrix R, we choose $\sigma_R = 0.02$m.
- There are 10 water level measurements ($y_1, y_2, \ldots, y_{10}$)

We compute the eigenvalues of the A-KCA matrix characterizing the convergence properties of the Kalman Filter (of the error of the state estimation). We observe that the maximum eigenvalue (Max = 0.80014) has a modulus strictly less than one indicating that the estimation error will converge towards 0 (see [5] for the details).

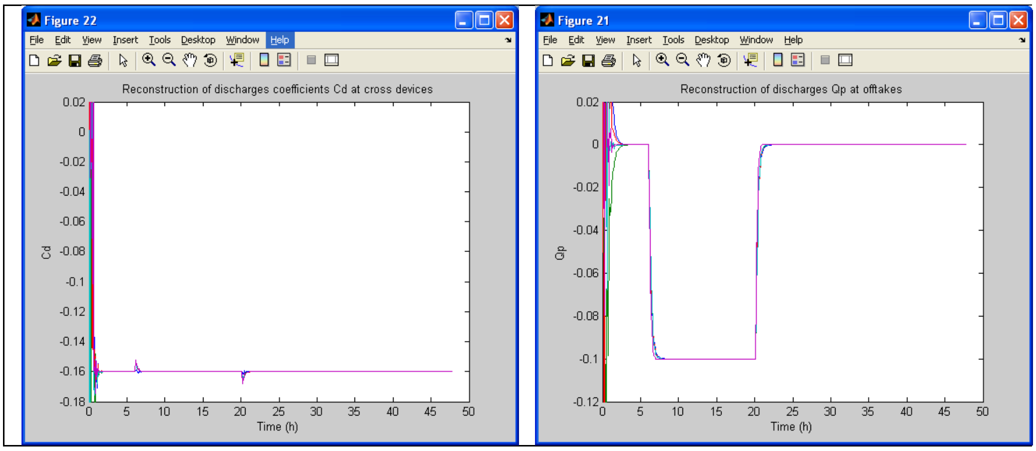

Figure 4. Reconstruction of the discharge coefficients C_p (left) and discharge at offtakes Q_p (right)

We also observed that when only one measurement is removed, then the convergence of the estimation error towards 0 is not obtained any more (the maximum eigenvalue of A-KCA = 1).

CONCLUSIONS

In this paper, we have proposed a way to estimate, in real time, the hydraulic state of a series of pools of an irrigation canal. This state has been augmented with the lateral offtake discharge Q_p and the contraction coefficients C_d of the gates at the cross devices. These variables are correctly calculated by the Kalman filter. This knowledge can then be used to improve the automatic control of the canal using the information of the discharge taken by the users, to improve the gate equation or detect problem at this gate. A mathematic test also allows to verify that this property is satisfied, and to investigate what combination of measurements can still provide this property. In our example the 10 water level measurements are required.

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