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Vulnerability Assessment Of Water Distribution Systems Using Directed And Undirected Graph Theory

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VULNERABILITY ASSESSMENT OF WATER DISTRIBUTION SYSTEMS USING DIRECTED AND UNDIRECTED GRAPH THEORY

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Vulnerability assessment is a promising approach to identifying critical components of complex systems. In such systems, critical components could be crucial links and strategic locations for management and control. Managing those components better could, in turn, improve or reinforce a system's resilience to shock events. However, fully comprehensive vulnerability assessments cannot be guaranteed as enumerating all possible shock events in complex systems is computationally intensive and almost infeasible. Consequently, the important role of some components might be neglected. This paper explores the feasibility of using a dual approach vulnerability assessment for water distribution systems (WDSs). Specifically, complex network analysis (e.g. community detection) and control-theory-based analysis (e.g. controllability analysis) is utilized to identify critical components within WDSs. A real-world benchmark water distribution system is mapped into either undirected or directed graphs by identifying critical pipes and nodes respectively. Here, critical links refer to pipes linking all the subsystems of a WDS together. Critical nodes are defined as actuators based on analogy with control theory. Controlling a suitable set of actuators by different signals can offer full control over a dynamical system. The results from the two different methods are discussed with respect to the criticality of the identified components to water supply during failure, and the correlations among the alternative approaches. It is concluded that, although there is a much larger number of actuators, about 77% of critical links are also identified as actuator pipes (pipes with either of its end points as actuators) and therefore they have the same criticality. For the remaining 23%, the location of critical links is rather close to that of actuators despite differences in criticality. Further study is ongoing to confirm whether any other actuators unexplored in this study could be more critical than the studied actuators and critical links or not. Further, a variety of failures will be compared using the two methods.

INTRODUCTION

Water distribution systems (WDS) are conventionally designed and rehabilitated to be reliable. Here, a reliable system is able to deliver a defined level of service over its design life when subject to a given threat. Nevertheless, the system should also be resilient to unexpected threats. It is further argued that reliability and resilience are foundational in moving towards 'Safe & SuRe' water systems [1]. In order to build resilience, based on reliability, it is necessary to know which parts of the network are vulnerable and some means of identifying critical components is required. This has conventionally been achieved using graph theory [2-10]. This study explores the feasibility of using two graph approaches [11, 12] for identifying critical links and crucial control locations of water distribution systems, respectively. On the basis of analysis results, the correlation between the two different sets of critical components is discussed in respect of their application to WDS management and control, especially for WDS resilience to threats. Note that the critical links refer to interconnections between subsystems of a WDS. Here, the subsystems are clusters, formed along with urban development, serving corresponding communities (e.g. residential zones, industrial sites, business center, etc.) in the urban area [11]. Therefore, in this study, critical links are not constrained to hydraulically critical pipes but are important links providing clues for comprehensive understanding of the WDSs' properties. For instance, the critical links and the corresponding cluster structure could be first detected and then be used to identify all the most hydraulic critical pipes [11]. The crucial control locations are nodes acting as actuators defined in control theory [12]. Controlling a suitable set of actuators by different signals can offer full control over a dynamical system [12].

METHODS

The critical links and actuators are identified by the modularity-based clustering method and controllability analysis, respectively.

Modularity-based clustering

The modularity-based clustering method, originating from complex network research [13], is applied to decompose water distribution systems into clusters formed along with urban development [11]. Each cluster is a subsystem mainly serving a city block. The critical links are pipes connecting the clusters. There are two steps for modularity-based clustering of water distribution systems: (1) Water distribution system mapping: The distribution system is mapped into an undirected graph $G=(V,E)$ in which the vertices V represent the consumers, sources, and tanks - the edges E the connecting pipes, pumps, and valves [5]; (2) The cluster layout of G is identified by using an algorithm proposed by Clauset *et al*. [13]. The method maximizes modularity (Q) to get optimal division, as Q is an indicator to quantify the quality of the graph division into clusters [13]. The modularity is defined to be:

$$Q=\frac{1}{2m}\sum\nolimits_{\upsilon\omega}\left[A_{\upsilon\omega}-\frac{k_{\upsilon}k_{\omega}}{2m}\right]\delta(c_{\upsilon},c_{\omega}) \tag{1}$$

where $A_{\upsilon\omega}$ is an element of the adjacency matrix of the network ($A_{\upsilon\omega}=1$ if vertices υ and ω are connected, and $A_{\upsilon\omega}=0$ otherwise). $m=\frac{1}{2}\sum\nolimits_{\upsilon\omega}A_{\upsilon\omega}$ is the total number of edges; $k_{\upsilon}=\sum\nolimits_{\omega}A_{\upsilon\omega}$ is the degree of vertex υ, defined as the number of edges connected to that vertex; $\delta(c_{\upsilon},c_{\omega})$ is 1 if $c_{\upsilon}=c_{\omega}$, and 0 otherwise. c_{υ} and c_{ω} represents two different clusters; υ and ω represents vertices in c_{υ} and c_{ω} respectively. For more details about the algorithm, refer to the studies by Clauset *et al*. [13] and Diao *et al*. [11].

Controllability analysis

The crucial control locations, termed as actuators or driver nodes, are located through controllability analysis. The controllability of complex water distribution systems is explored based on a differential equation [12]:

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t) \tag{2}$$

where the vector $x(t) = (x_1(t), \ldots, x_N(t))$ represents the state of a system of N nodes at time t. The $N \times N$ matrix A describes the system's wiring diagram and the interaction strength between the components, for instance the flow rate in pipes in water distribution systems. B is the $N \times M$ matrix ($M \le N$) that identifies the actuators controlled by an outside controller. Finally, the time-dependent input vector $u(t) = (u_1(t), \ldots, u_M(t))^T$ denotes the control signals imposed by the controller. Generally, the same $u_1(t)$ can drive multiple nodes. To control the system's dynamics, it is essential to identify the set of actuators that, if fed by signals, can offer full control over the system. The minimum number of actuators is denoted as N_D. If a system described by equation (2) is controllable, it can be driven from any initial state to any desired final state in finite time. This is possible if and only if the $N \times NM$ controllability matrix $C = (B, AB, A^2B, \ldots, A^{N-1}B)$ has a full rank, that is $rank(C) = N$.

Three steps are necessary for identifying actuators in WDSs: (1) Water distribution systems mapping: According to link flow directions from model simulation, the distribution system is mapped into a series of directed graphs $G^d = (G_1^d, \ldots, G_M^d)^T$ with each one referring to the system status at a single simulation time step M. The vertices and links share the same meaning as the undirected graph G introduced above. (2) Controllability analysis: Controllability analysis is made for each graph G_i^d to obtain one set of actuators. 3) Identifying the minimum number of actuators: Finally, the minimum number of actuators is the cardinality of the intersection of the set of actuators for every graph. In other words, the actuators that coincide for different graphs are accounted for just once. More details of the whole process could be found from [14].

CASE STUDY

The case study of both methods focuses on a real-world water distribution system, C-Town [15]. The system is composed of 5 district metered areas (DMAs) with a pump station and tank(s) configured in each DMA. The hydraulic model of the system consists of 444 pipes and 396 nodes (Figure 1). The hydraulic simulation period is 168 hours (i.e. one week).

Identification of critical links and actuators

In terms of critical link identification, 22 critical links are identified by dividing the C-Town network into 20 clusters [Figure 2(A)]. In terms of actuator identification, the system is mapped into 168 directed graphs [$G^d = (G_1^d, \ldots, G_M^d)^T$, $M = 168$] using pipe flow directions at all time steps. After controllability analysis, a minimum number of 163 actuators are located [Figure 2(B)].

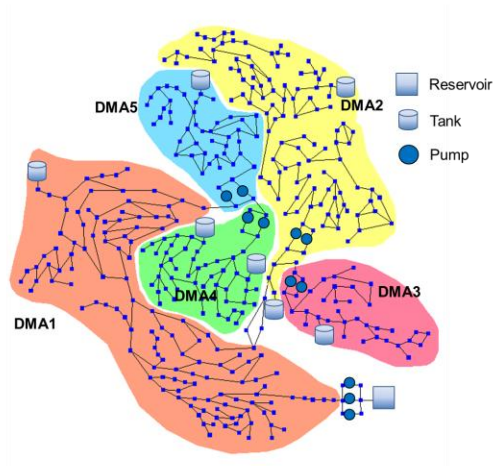

Figure 1. The layout of C-Town water distribution system

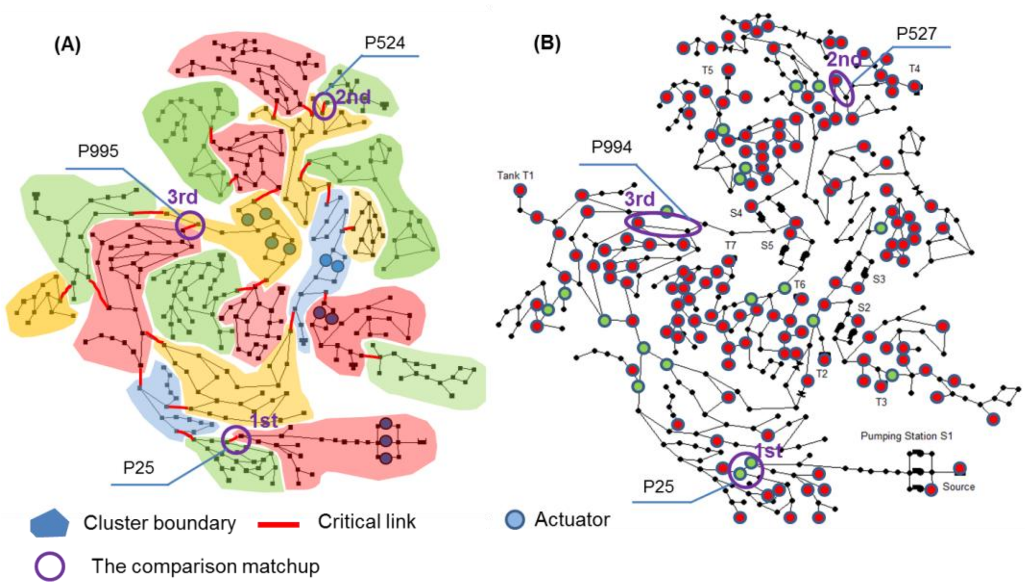

Figure 2. (A) The cluster structure and critical links (B) Layout of actuators

Comparisons on the criticality of critical links and actuators

The criticality of critical links is compared with that of actuator pipes (i.e. pipes with either of its end nodes identified as actuators) in order to deepen insights into their correlations, if any. This study considers the failure shock for criticality comparison. Specifically, the criticality of a component is evaluated by two metrics: 1) the demand shortage resulting from failure of that component (e.g. modeled as a closed pipe during simulation) and 2) time to failure of clusters (TTFC). Failure of cluster means there is no water supply in the cluster. The demand shortage is the actual supplied demand during a failure event minus the total demand required. The actual supplied demand is a sum of actual nodal demands that is estimated based on the following pressure-demand relationship [16]:

$$q_{i,j}(t) = \begin{pmatrix} P_n <= 0 : 0 \\ 0 < P_n < P_{\min} : \; d(t)\left(\dfrac{P_n}{P_{\min}}\right)^{1/2} \\ P_n >= P_{\min} : d(t) \end{pmatrix} \tag{3}$$

where $q_{i,j}(t)$— the estimated actual nodal demand at junction i when component j is closed; $d(t)$— the expected nodal demand at junction i; $P_{\min}$ — the required minimum pressure for delivering demand $d(t)$; P_n— the actual pressure at junction i.

RESULTS AND DISCUSSIONS

As Figure 2 shows, most of critical links (i.e. 17 out of 22) have at least one end node identified as an actuator (labeled with a green color). Hence, the natural boundaries of WDSs, detected by clustering, may not only be the places locating critical links but also important control points. In non-identical cases, critical links and actuator pipes are very close to each other despite differences in criticality. Moreover, analyzing the properties of the WDS could address the origin of the differences in criticality. Comparisons on three typical cases are introduced and discussed. In the first case (1st comparison annotated in Figure 2), the critical link is identical to the actuator pipe as both its ends are chosen as actuators. In the other two cases (2nd and 3rd comparisons annotated in Figure 2), the critical links and actuator pipes are next to each other. The failure is assumed to occur at 12 hours after the simulation starts, since high water demand is reached at that time and more importantly the peak lasts for several hours afterwards.

As for the 1st comparison, since the critical link and the actuator pipe are identical, only the criticality of the pipe is analyzed. The critical link (pipe P25) is the interconnection between the most upstream cluster and the rest part of the system. Since the most upstream cluster is where the only reservoir is located, a break of the critical link would isolate the water source from the system. In this regard, both the graph approaches identified this link as a crucial place for management and control. As further demonstrated by the results (Figure 3), a break of P25 would result in almost total loss of water supply with a 6 hour TTFC. Note that the TTFC is event-dependent. For instance, the TTFC would be shortened to 3 hours if the break of P25 happens at 118 hours after the simulation begins.

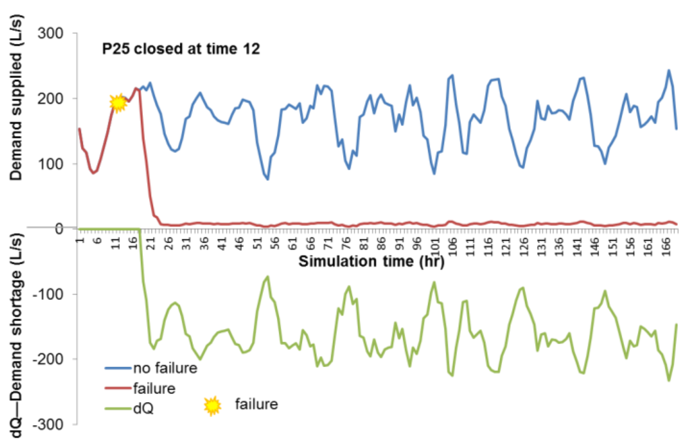

Figure 3. The 1st comparison— identical critical links and actuator pipes

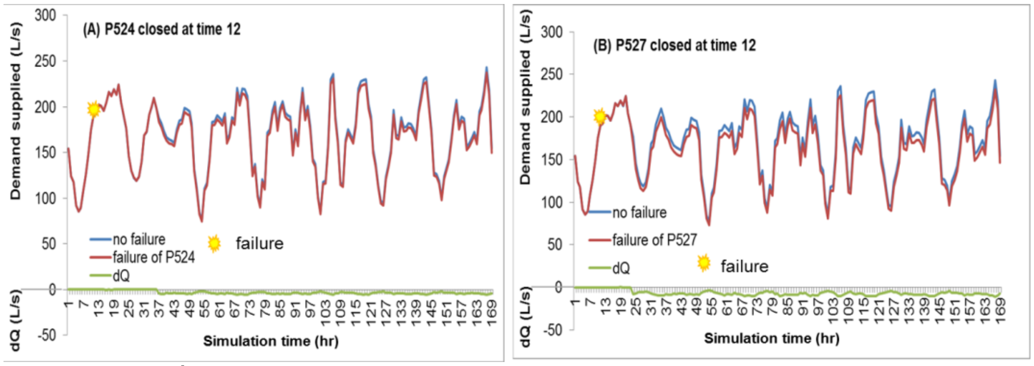

Figure 4. The 2nd comparison— non-identical critical link P524 and actuator pipe P527

In the case 2 both the critical link (P524) and the actuator pipe (P527) are located on the path between the reservoir to the tank in DMA2. Given the comparisons in Figure 4, the failure of the actuator pipe (P527) leads to larger demand shortage and shorter TTFC (11 hrs) than that of P524 (TTFC=24 hrs). The difference in criticality lies in the fact that, the failure of P527 isolates one extra region [the yellow area in Figure 5(B)] with high nodal demands. Subsequently, the total demand in isolated area is nearly doubled than the original case [Figure 5(A)] and the Tank T4 therefore runs out of water much faster. In the 3rd case demand shortage due to failure of the critical link (P995) is slightly higher than the actuator pipe (P994). Note that as both pipes have no significant impacts on water supply, the comparison on demand shortage is not included. There are two reasons for the limited impacts. First, both pipes are located at the secondary path (Figure 6) with comparatively smaller capacity than the primary one. Second, the elevation of the downstream node of the critical link is 22.65 meters higher than that of the upstream node. Subsequently, the average quantity of water delivered by the primary path is more than twice as that of the secondary path. The TTFC is 0 hour (P995) and 6 hours (P994) respectively. This is because failure of P995 will block both the secondary and the third path in Figure 6 instead of the secondary path only as a result of the failure of P994.

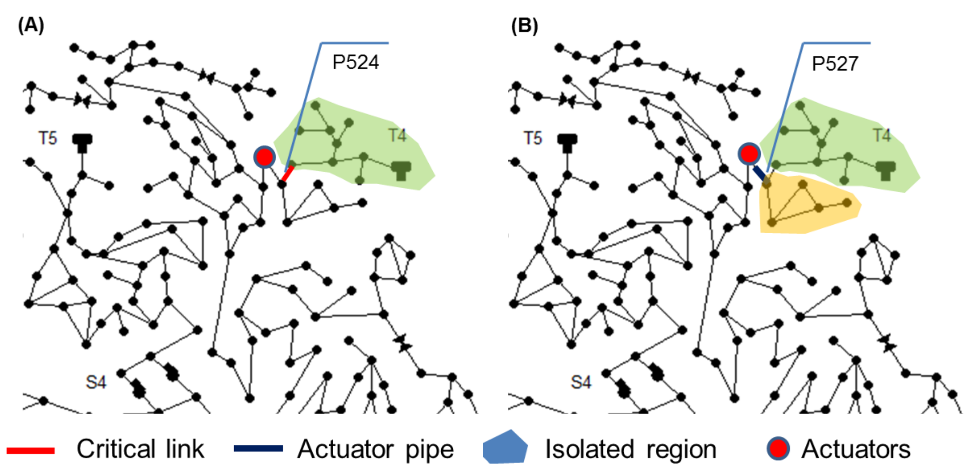

Figure 5. The 2nd comparison: (A) The failure of critical link P524 (B) The failure of actuator pipe P527

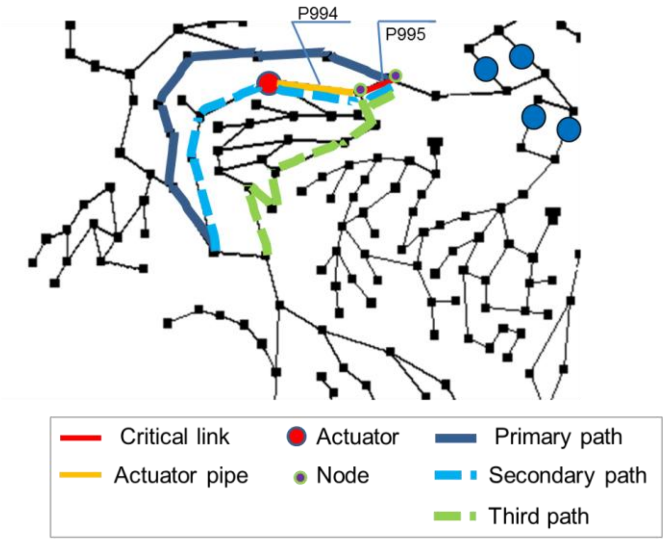

Figure 6. The 3rd comparison: The failure of critical link P995 and the actuator pipe P994

CONCLUSIONS

This study explores the feasibility of using two graph theory approaches to the vulnerability assessment of water distribution systems, with emphasis on identifying critical components to inspire a better understanding of the properties and behaviors of WDSs from the perspective of building resilience for 'Safe & SuRe' water systems. The modularity-based clustering method is applied to identify the critical links in WDSs, while controllability analysis is carried out for locating actuators. Comparisons of the criticality of those identified components are made and reveal that although there are a much larger number of actuators, most of the critical links (77%) have identical criticality to actuators, and hence critical links located at natural boundaries of WDSs may also be crucial control components. In non-identical cases, critical links and actuators are very close to each other, and the origin of their differences in criticality is confirmable. Further study is ongoing to compare the criticality of actuators undetected in this study with the studied ones to confirm if any more critical components have been neglected. In addition, the performance of the two methods will be further tested by considering different failures.

Acknowledgments, appendices, and references

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REFERENCES

[1] Butler, D. "'Safe & SuRe': A sustainable and resilient approach to water management", *Water & Sewerage Journal*, Vol. 1, (2014), pp 43-44.

[2] Deuerlein J., "Decomposition Model of a General Water Supply Network Graph", *Journal of Hydraulic Engineering*, Vol. 134, No. 6, (2008a), pp 822-832.

[3] Deuerlein J., "Efficient Supply Network Management Based on Linear Graph Theory", *Water Distribution Systems Analysis Symposium*, Cincinnati, Ohio, United States, (2006), pp 1-18.

[4] Deuerlein J., Wolters A., Roetsch D. and Simpson A., “Reliability Analysis of Water Distribution Systems Using Graph Decomposition”, *World Environmental and Water Resources Congress*, Kansas City, Missouri, United States, (2009), pp 1-11.

[5] Perelman L. and Ostfeld A., “Topological clustering for water distribution systems analysis”, *Environ. Modell. Softw*., Vol. 26, No. 7, (2011), pp 969-972.

[6] Ostfeld A., “Backups Selection for Water Distribution Systems Using Topological Considerations”, *Conference on Water Resources Planning and Management*, Roanoke, VA, (2002).

[7] Bartolin H., Martinez F. and Cortes J.A., “Topological GIS-based Analysis Of A Water Distribution Network Model. Applications Of The Minimum Spanning Tree”, *Computers and Control in the Water Industry (CCWI 2005)*, Exeter, UK, (2005).

[8] Jacobs P and Goulter I. C., “Optimization of redundancy in water distribution networks using graph theoretic principles”, *Engrg. Optimization*, Vol. 15, No. 1, (1989), pp 71-82.

[9] Jacobs P and Goulter I. C., “Evaluation of methods for decomposition of water distribution networks for reliability analysis”, *Civil Engineering Systems*, Vol. 5, No. 2, (1988), pp 58-64.

[10] Walski T., “Water distribution valve topology for reliability analysis”, Reliability *Engineering and System Safety*, Vol 42, (1993), pp 21-27.

[11] Diao K., Farmani R., Fu G., Astaraie-Imani M, Ward S. and Butler D., “Clustering analysis of water distribution systems: identifying critical components and community impacts”, *Water Science and Technology*, under review, (2014).

[12] Liu Y., Slotine J., Barabási A., “Controllability of Complex Networks”, *Nature*, Vol. 43, (2011), 123-248.

[13] Clauset A., Newman, M. E. J., and Moore, C., “Finding community structure in very large networks”, *Physical Review E*, 70(066111), (2004).

[14] Diao K. and Rauch W., “Controllability analysis as a pre-selection method for sensor placement in water distribution systems”, *Water Research*, 47(16), (2013), pp 6097-6108.

[15] Ostfeld A., Salomons E., Ormsbee L., Uber J G., Bros C M., Kalungi P., Burd R., Zazula-Coetzee B., Belrain T., Kang D., Lansey K., Shen H., Mcbean E., Wu Z Y., Walski T., Alvisi S., Franchini M., Johnson J P., Ghimire S R., Barkdoll B D., Koppel T., Vassiljev A., Kim J H., Chung G., Yoo D G., Diao K., Zhou Y., Li J., Liu Z., Chang K., Gao J., Qu S., Yuan Y., Prasad T D., Laucelli D., Vamvakeridou Lyroudia L S., Kapelan Z., Savic D., Berardi L., Barbaro G., Giustolisi O., Asadzadeh M, Tolson B A., Mckillop R., “The Battle of the Water Calibration Networks (BWCN)”, *J. Water Res. Plan. and Manage.*, Vol. 138, No. 5, (2012), 523-532.

[16] Wagner, J., Shamir, U. and Marks, D. “Water Distribution Reliability: Simulation Methods”. J. Water Res. Plan. and Manage., Vol. 114, No. 3, (1988), pp 276-294.