1994

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Irvin Sam Schonfeld
CUNY Graduate Center

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Irvin Sam Schonfeld*
School of Education, City College of New York, New York 10031, and Department of Psychiatry, Columbia University, New York, USA
Requests for reprints. Irvin Schonfeld, EDFN, City College, New York, New York 10031, USA.
Received 14 August 1991; revised version received 7 October 1993

Four liquid comparison tasks were designed to assess children's knowledge of functional relations, one-directional compositions of functional relations (greater + greater yield greater) and countervailing compositions (greater + lesser yield ?) in 4- to 7-year-olds. On one task, in which height indexed quantity, children of every age group performed well. Success on the other comparison tasks was related to operative level, as indexed by conservation performance, and age. More advanced pre-operational children evidenced a degree of success on the one-directional composition task. Consistent with Schonfeld (1990), the results suggested that at more advanced operative levels: (1) the understanding of increasingly complex functional relations emerges and (2) thinking becomes increasingly exact. Correlational results revealed consistency across measures and factor-analytical findings suggested that a unitary developmental factor underlies performance differences. An attentional explanation of the findings was ruled out. The findings highlighted the multifaceted nature of children's progress toward integrating information from different dimensions of the comparison tasks.

Piaget and his colleagues developed the psychology of functions (Piaget, Grize, Szeminska & Vinh Bang, 1977) during the latter part of Piaget's career. The psychology of functions was a departure from earlier work that included research related to the classical Genevan tasks (e.g. the conservations, class inclusion, assessments of children's conceptions of space and geometry, etc.). Beilin (1992) labelled the classical theoretical work Piaget completed during this very fertile period of his career as the 'standard theory'. A hallmark of the classical tasks is that they tend to assess the child's knowledge of transformations. By contrast, the psychology of functions is more specifically concerned with the capacity of both pre-operational and concrete operational children to make comparisons among untransformed entities (Piaget, 1977). Moreover, compared with the standard theory, the psychology of functions expresses a greater concern for continuity in cognitive development. It has spurred investigators to develop more detailed representations of the cognitive accomplishments of both pre-operational and concrete operational children (Davidson, 1987, 1988; Dean & Deist, 1980; Schonfeld, 1986, 1990). The psychology of functions remedies the criticism voiced by Miller (1984) and others that Piagetians tend to neglect evidence of 'early but limited competence' (p. 222).

The present study constitutes an extension of other work (Schonfeld, 1986, 1990) on the relevance of the psychology of functions to the child's capacity to compare untransformed quantities. Using discrete arrays, Schonfeld (1990) found that pre- operational children exhibit a rudimentary knowledge of one-way functions involving two-, one- and none-to-one correspondences and one-directional compositions of functional relations. He found that at more advanced operative levels children begin to understand more complicated correspondence relations. Schonfeld (1986) also found that operative level is related to the child's capacity to take advantage of 'solution aids' such as counting and matching in solving problems involving correspondences.

In contrast to studies that examined the child's capacity to compare discrete arrays, the present study examined the capacity of pre-operational and concrete operational children to compare untransformed liquid quantity. Research on the development of the child's capacity to compare liquid quantity has been far less extensive than research on the child's capacity to compare discrete-quantity arrays. A study by Kaplan (1987) linked conservation of continuous quantity (liquid) to the child's capacity to use measurement in comparing liquid quantity. McShane & Morrison (1983, 1985) found that, while 3-year-olds tend to use a relative-fullness strategy in judging liquid quantity, 4-year-olds are more likely to rely on height cues. These studies employed single-item measures of the constructs under investigation; their results were, therefore, subject to some unreliability.

The tasks employed in the study described in this paper assessed the child's capacity to abstract quantitative comparisons *without* the aid of measurement utensils. For example, in one task a child was presented two glasses, of identical diameter, each containing liquid (the same-diameter task depicted in Fig. 1). According to the psychology of functions, judgements of relative quantity can be founded on a one-way mapping of height onto quantity: $y = f(x)$ where x represents height and y, quantity. In other words, quantity is a function of height. The taller column of liquid would be judged the greater.

The Genevans employ the term 'one-way' in two senses. First, one-way functional tasks do not require the 'invertible' operations commonly explored in traditional tests of the concrete operations (e.g. the conservations) in which judgements concerning the restoration of a changed quantity to its original state are implicated. The psychology of functions is concerned with the capacity of the pre-operational child to understand asymmetrical relations such as 'there is more red juice than green juice' or 'Mary is older than Jane'. A limitation is that the pre-operational child will have difficulty simultaneously grasping the logical equivalents of these asymmetrical relations (i.e. 'there is less green juice than red juice' or 'Jane is younger than Mary'; Beilin, 1992). Second, the tasks are 'one-way' in the sense that although y is a function of x, x can be, but is not necessarily, a function of y.

For the Genevans, knowledge of one-way functions emerges, in pre-operational thought, out of action schemes related to points of arrival or the ordering of distances covered by objects that were moved (Piaget, 1968). Everyday activities, accessible to pre-operational children, that involve one entity 'going beyond' another frequently identify the greater quantity or longer distance. Pre-operational children would be expected to perform well on same-diameter (SD) comparisons. Functional relations can also be permuted and made more complex (Piaget *et al.*, 1977). These complexities are seen in the same-height and two-part tasks described below.

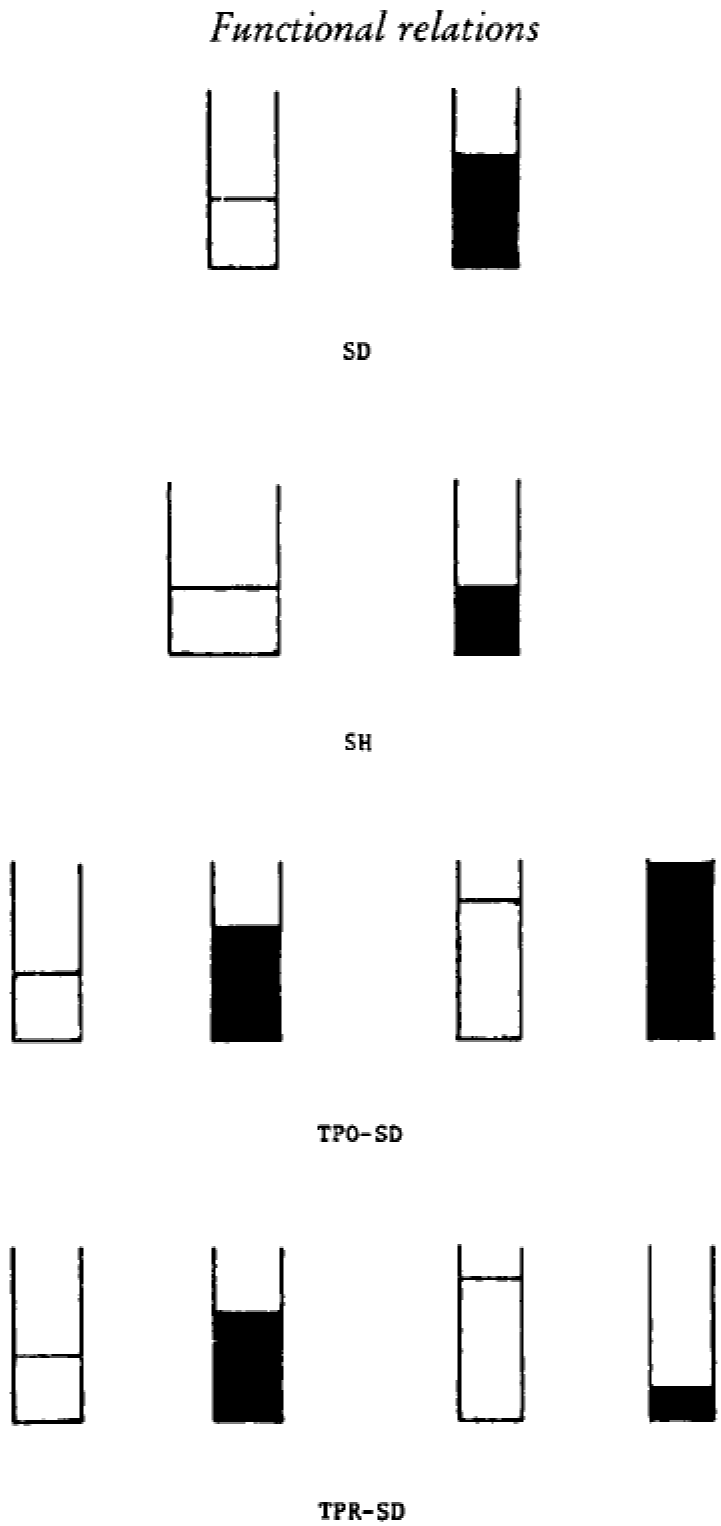

Figure 1. Examples of the comparison tasks.

In the same-height (SH) task, two equally tall glasses having different diameters contained liquids that rose to equivalent heights (see Fig. 1). Judgements concerning the relative quantity of the liquids could be made accurately by indexing quantity to diameter. Height, however, is so strongly fixed as a cue (Miller, 1973), that pre- operational children are unlikely to succeed at comparison tasks in which diameter indexes quantity. In the Genevan view, the 'decentreing' attendant upon, the development of the concrete operations enables the child to use diameter to index quantity. A parallel process occurs with discrete quantity (Schonfeld, 1986, 1990).

Both two-part tasks assessed the child's capacity to compare two amounts of liquid, each amount comprising spatially distinct subquantities. In both tasks, the height of the liquid mapped onto quantity. (The two-part tasks were not really 'height' tasks in the sense that the children were asked to index explicitly quantity by height. The children in the age groups studied [4- to 7-year-olds] related height to quantity virtually without error. The purpose of the tasks was to ascertain the extent to which children compare quantities comprising distinct subquantities given the background that the children map height onto quantity virtually without error when examining containers that have the same diameter.) These tasks, which did not require formal measurement procedures, were particularly apt because children in the age range studied tend to compare fluid quantity by visual inspection (Miller, 1984). In the 4-7 age group numerical cues are potentially interfering (Miller, 1984).

In two of the four trials in the first two-part task, the colour of the greater of the two subquantities on the left was the same as the colour of the greater of the two subquantities on the right (the two-part one-way [TPOJ composition comparison depicted in Fig. 1). In the language of the psychology of functions, a comparison of the total quantities (the same-colour subquantities combined), requires the one-way composition of two same- directional subquantity relations: (Rl > Gl) + (R2 > G2) → (R > G). R represents the red liquid employed in comparison tasks and G the green.

Because each of the two trials requires children to integrate information from four glasses, the trials were expected to be more difficult than the SD trials, which use only two glasses. Schonfeld (1990) found that pre-operational children showed a degree of success on the discrete quantity analogue of this liquid comparison task. It was therefore hypothesized that pre-operational children would similarly demonstrate some success on the TPO liquid trials, although not to the same extent as on the SD trials. If, on the other hand, children judge the total quantity by simply fixing attention on the tallest column of liquid without integrating information from the four glasses, then performance on the above two TPO trials and the SD trials would be very similar.

In another TPO trial, the two tallest quantities of liquid, Rl and G1, were equal in height; of the two remaining (and shorter) quantities R2 and G2, G2 was taller. In the language of the psychology of functions, a comparison of the total quantities requires the following composition: (Gl = Rl) + (G2 > R2) → (G > R). If, however, children base their judgement of the total quantity of G and R by attending to the tallest column of liquid, they would be inclined to err because G1 and Rl are equal. They would mistakenly judge total R to equal total G. Responses to this and other trials were used to provide clues to the bases for the children's judgements.

In the second two-part liquid comparison task, two conditions were satisfied: (1) visually corresponding subquantities were unequal; (2) the direction of the inequality that held between the corresponding subquantities on the left was the reverse of the direction of the inequality that held between the subquantities on the right (see the two- part reverse [TPRJ composition comparison depicted in Fig. 1). In the TPR comparison, the subquantities on the left are G1 and Rl, respectively, and the subquantities on the right are G2 and R2. Although, in total, G is greater than R, G1 is less than R1. This is because the excess of G2 over R2 is greater than the excess of Rl over G1. In order to render an accurate judgement, the child must compare one colour's advantage on the left to the other colour's advantage on the right. If Rand G were equal, the G1-Rl and G2- R2 differences would exactly compensate for each other. Schonfeld (1986, 1990) described discrete quantity analogues of the TPR liquid comparisons.

Accuracy on the discrete quantity analogues requires the coordination or cross- referencing of countervailing subquantity comparisons (Schonfeld, 1990). A feature of concrete operational thought known as compensation involves the capacity to coordinate, or multiply, relations (Piaget, 1965; Silverman & Rose, 1982). Within the framework of the psychology of functions, compensation involves the capacity to coordinate functional relations (Piaget *et al.*, 1977). To compare the total amount of red and green liquid accurately, the child must compare the difference between two differences (R1-G 1 + R2-G2) while taking into account the opposing directions of those differences. Given the complexities of the coordinations, it was expected that children would perform better on a task involving the coordination of one-directional subquantity comparisons (the TPO task) than on a task involving countervailing subquantity comparisons (the TPR task).

Four hypotheses were explored. First, the notion that pre-operational children can succeed on one-way compositional tasks was assessed. Second, the hypothesis that the TPR task is more difficult than the TPO task was examined. Third, because the Genevans advance the view that performance on tasks operationalizing the child's understanding of functional relations is as much an index of operative development as performance on the traditional transformational tests (Schonfeld, 1990), the relation of performance on the one-way function comparison tasks to traditional Piagetian measures was examined. Fourth, factor-analytical evidence was also adduced to assess further the idea that children's knowledge of static liquid comparisons and traditional transformational comparisons reflects continuity in development. The study also evaluated the view that attentional factors shaped performance on the two-part tasks.

METHOD

Subjects

A total of 102 children ranging in age from 4 years, 0 months, to 7 years, 11 months, were administered liquid comparison tasks. Their mean age was 5 years, 11 months. The children attended fee-paying private schools. An informal review of their parents' occupations confirmed that the children came from middle- and upper-middle-class homes. Both sexes were equally represented and about 90 per cent of the children were white.

Materials

All containers used in the comparison tasks were 5 in. (12.70 cm) high, made of clear plastic and cylindrically shaped. The vessels had diameters of 2, 3 and 4 in. (5.08, 7.62 and 10.16 cm) and held water that had been coloured either red or green by food colouring. Figure 1 shows examples of the containers. Sesame Street finger puppets named Bert and Ernie were also used in the tasks. In the test of conservation of continuous quantity, two 4 in. X 3 in. (10.16 cm X

7.62 cm) clear plastic, cylindrically shaped vessels served as standard containers. One 4 in. X 4 in. (10.16 cm X 10.16 cm) vessel, one 9 in. X 2 in. (22.86cm X 5.08cm) vessel and two 4in. X Zin. (10.16cm X 5.08cm) vessels were employed as comparison containers. Black checkers and red checkers were used in the test of conservation of number.

Design

The children were seen in two sessions, randomly ordered, not more than four days apart. Approximately half the children were administered a set of four liquid comparison tasks and two presentations of a test of conservation of continuous quantity (liquid) in the first session. The other children were administered the tasks and the liquid conservation tests in the second session. Schonfeld (1986) described the four numerical comparison tasks, not relevant to this study, and two tests of conservation of number that were administered in the alternate session. Two children present for the liquid-task session were absent for the numerical-task session.

The four liquid comparison tasks included the same-diameter (SD), same-height (SH), two-part one- way (TPO) composition, and two-part reverse (TPR) composition tasks. Within each session were two counterbalanced orders of administration: (1) liquid conservation, SD, SH, TPO, TPR, liquid conservation; (2) liquid conservation, TPO, TPR, SD, SH, liquid conservation.

Procedure

Comparison tasks.

Each comparison task consisted of four trials. An example of one comparison trial from each task is presented in Fig. 1. In each trial the container of green liquid was represented as Bert's juice and the container of red liquid as Ernie's juice. During every trial the Bert puppet stood next to a container of green liquid and the Ernie puppet next to a container of red liquid. In every task two unequal quantities were presented in three of the trials and two equal quantities in one trial. In each task, no puppet received the greater quantity of 'juice' in more than two trials and, over the four tasks, Bert received more as often as Ernie.

In every SD and SH trial the child was presented with two containers, one holding red liquid and the other green liquid. In the SD task the two containers in any trial had the same diameter. The heights of the columns of liquid in the paired containers differed in three of the four trials.

In every trial of the SH task the two liquids, one red and the other green, rose to the same height. In three of the four SH trials the two containers had different diameters. In every SD and SH trial the child was asked, 'Did Bert get more juice? Or did Ernie get more juice? Or did both puppets get the same amount of juice?' The order of the questions was rotated.

In every trial in the two two-part tasks the child was presented with two containers holding red liquid and two holding green liquid, as depicted in the figure. In any one trial, all four containers had the same diameter. In the equality trial in the TPO task the green and red subquantities on the left were equal to each other and the two subquantities on the right were equal to each other. In the equality trial in the TPR task the difference between the pair of subquantities on the left equalled the difference between the pair of subquantities on the right.

The administration of the two-part tasks differed somewhat from the administration of the tasks involving undivided quantities. At the beginning of the first trial in each two-part task, the subquantities on the left were represented as the juice the puppets received in the morning and the subquantities on the right as the juice the puppets received in the afternoon. In each trial, the child initially compared the two subquantities on the left that were represented as the morning juice. Then the child compared the two subquantities on the right that were represented as the afternoon juice. Every child made these comparisons accurately. Then the child compared the total amount of green juice to the total amount of red juice, i.e. the juice that Bert and Ernie got 'for the whole day, morning and afternoon together', using the same three questions described above with orders rotated.

Conservation of liquid test.

Each conservation of liquid test consisted of three trials. In the first trial, the child was shown the two 4 in. X 3 in. standard containers holding equal amounts of red and green 'juice'. The standard containers were half filled. Once the child was satisfied that there was as much red as green liquid, the examiner poured all the red (green) liquid into the 4 in. X 4 in. comparison container. The child was asked if one amount was more or if the two amounts were equal and the reason for his or her judgement. The second trial began when, with the child's agreement, equal amounts of red and green liquid were placed in the two standard containers. During this trial the green (red) liquid was poured into the 9 in. X 2 in. container, and the child was questioned. The order of the first two trials and the colour of the liquid poured were counterbalanced. The third trial parallelled the second except that the red (green) liquid was poured into the two 4 in. X 2 in. containers.

Children who responded incorrectly on all trials of both tests of liquid conservation were classified as non-conservers. Children who, on both tests, responded correctly on at least two of the three trials and supplied adequate justification for their responses (e.g. reversibility, addition-subtraction) were classified as conservers. An 'improver' was operationally defined as a child who failed to respond correctly on all three of the trials on the first liquid conservation test but who responded correctly and supplied adequate justification for his or her correct responses on at least two of the three trials of the second test. Since the performance of only two children conformed to this pattern, the category of improver was excluded from the analyses.

Conservation of number test.

During the alternate session, a conservation of number test was administered to the children twice, before and after the series of numerical comparison tasks not relevant to this study. Each test consisted of three trials. At the beginning of the first trial, the examiner placed before the child a row of eight black (red) checkers and asked the child to take from a bag the same number of red (black) checkers. The examiner recorded whether or not the child aligned the two sets of checkers one-to-one. Once the two rows of checkers were put in one-to-one correspondence (either by the child or the examiner) the child witnessed each of three transformations of one of the rows. The examiner compressed (expanded) a row of checkers and asked the child which row had a greater number or if the two rows had the same number. The examiner also asked the child to give the reasons for his or her response. After restoring the one-to-one correspondence, the examiner expanded (compressed) a row and questioned the child. The order of the first two trials was counterbalanced. Finally, after again restoring the correspondence, the examiner stacked a row of checkers to form a cylinder and questioned the child.

Based on their responses and justifications, the children were post-stratified according to three operative levels: Level I non-conservers (NCs), Level 2 NCs and conservers. The Level I NCs failed to conserve number on both tests and failed to place the checkers in one-to-one correspondence on at least one of the two tests. The Level 2 NCs did not conserve number on either test; however, they placed the checkers in one-to-one correspondence in both tests. The conservers of number responded correctly on at least two of the three trials, on both tests, and adequately justified their responses.

RESULTS

Scale construction

For the purpose of scale construction, each trial in the comparison tasks was treated as an 'item' as in a paper-and-pencil test. Four scales were constructed *a priori* from the items (trials) employed in the SD, SH, TPO and TPR tasks. The KR-20 reliability (r_{11}) for the scale consisting of the four same-diameter items was unsatisfactory (.23) because there was little variance owing to the fact that children of all levels compared the quantities very accurately. The mean score on the SD scale was 3.86 out of a possible 4.00.

The three SH inequality items formed a satisfactory scale (r_{11} = .92). The one excluded SH item involved equal quantities. The item had little variance because the children overwhelmingly identified the quantities as equal. When children erred on the three SH items involving unequal quantities, they judged the two quantities to be equal. The three TPO inequality items (r_{11} = .70) and the three TPR inequality items (r_{11} = .79) formed satisfactory scales.

As a check on the adequacy of the final constructed scales (Rubio-Stipec, Shrout, Bird, Canino & Bravo, 1989), the nine items making up the SH, TPO and TPR scales were submitted to a principal components analysis with a varimax rotation. After rotation the items factored exactly as anticipated in the *a priori* scale construction. The items loaded above .60 on the expected factors (see Table 1).

Operative level

The relation between the children's performance on the liquid and number conservation tests is presented in Table 2. The total number of subjects was reduced from 102 to 98: the two children who completed the session involving liquid quantity but were absent for the session involving discrete quantity and the two 'improvers' were excluded. Consistent with Piaget (1965), Table 2 indicates that *every* child who conserved liquid also conserved number. Most of the children who did not conserve liquid were classified as either Level 1 or Level 2 NCs based on their performance on the number conservation tests. Only 8 per cent of the children who did not conserve liquid were classified as conservers of number.

Based on the distribution of subjects, each child was operationally defined, independently of his or her performance on the comparison tasks, as a member of one of three operative levels (OLs). The children assigned to OL-1 were both non-conservers of liquid and Level **1** NCs. The children assigned to OL-2 were non-conservers of liquid; they, however, could be either Level 2 NCs or conservers of number. The children assigned to OL-3 conserved both liquid and number. The assignment of subjects to developmental levels within this three-level scheme, although somewhat arbitrary, provides for well-known landmarks against which to anchor performance on the comparison tasks.

Table 1. Principal components analysis of the comparison items used m the scale construction: Varimax rotation.

Items	Loadings on the rotated factors		
	Factor 1	Factor 2	Factor 3
SHl	.94	.08	.15
SH 3	.92	.08	.18
SH 4	.84	.28	.16
TPO 1	.21	.02	.82
TP02	.10	.21	.79
TP04	.15	.37	.63
TPR 1	.10	.80	.12
TPR3	.11	.83	.14
TPR4	.18	.78	.23

Note. SH, same-height trials; TPO, two-part one-way trials; TPR, two-part reverse trials.

Table 2. Performance on the liquid and number conservation tests.

	Liquid conservation			
	Non-conservers		Conservers	
Number conservation	*N*	(%)	*N*	(%)
Level 1 NCs	21	(33)	0	(0)
Level 2 NCs	38	(59)	0	(0)
Conservers	5	(8)	34	(100)
Total	64		34	

Performance of pre-operational children

It was expected that pre-operational children would show some success on comparison tasks that reflect elementary functional relations. As described earlier, children of all OLs demonstrated great accuracy on the SD items. It was expected that pre-operational children would demonstrate some success on the TPO items. The expected proportion of correct responses given random responding on each three-item scale was 1.00 since the child had to choose from among three response alternatives. Scale means for the children are presented in Table 3. The mean score of the OL-1 children on the TPO scale was in the expected direction, but not statistically significant ($t(20) = 1.43$, $p < .20$, two-tail). By contrast, the mean score of the OL-2 children on the scale was significantly better than chance ($t(42) = 4.15$, $p < .001$). Five OL-2 children conserved number but not liquid. To be conservative, the second t test was repeated with the five excluded. The group mean still significantly exceeded chance on the TPO scale ($M = 1.66$, $t(37) = 3.40$, $p < .001$). On the TPR scale OL-1 children performed at chance levels *(t(20)* = -1.04, n.s.) and OL-2 children performed at marginally better levels (t(42) = 1.74, p < .10) [with the five conservers of number excluded *(M* = 1.29, *t(37)* = 1.48, *p* < .20).

The OL-1 and OL-2 children responded to the SH items at levels much worse than chance *(t(20)* = -1.91, *p* < .05 and t(42) = -5.92, *p* < .001, respectively), virtually always indicating that the two puppets had the same amount. The result for the OL-2 children was similar if the five conservers of number were excluded *(M* = .34, *t(37)* = -4.98, *p* < .001).

Table 3. The scale means, standard deviations and test statistics.

Measure		OL-1 (*N* = 21)	OL-2 (*N* = 43)	OL-3 (*N* = 34)	*F*	*p*
SH						
	M	0.57	0.30	2.00	26.18	.001
	SD	1.02	.77	1.34		
TPO						
	M	1.33	1.74	2.70	14.40	.001
	SD	1.06	1.17	.68		
TPR						
	M	0.76	1.32	2.23	11.97	.001
	SD	1.04	1.22	1.07		

Note. Scores are presented as number correct; 3 is the highest possible score. SH, same-height scale; TPO, two-part one-way scale; TPR, two-part reverse scale.

Comparisons by operative level

It was expected that children's performance on each of the scales would vary directly with OL as operationalized by their performance on the conservation tests. Three one-way analyses of variance, reported in Table 3, demonstrated an association (p < .001) between the scale means and the OL variable[1]. 1 Pairwise comparisons employing Tukey's Honest Significant Differences (p < .05) indicated that on the SH, TPO and TPR scales, OL-3 children performed significantly better than OL-1 and OL-2 children. Tests for trends revealed that both two-part tasks were linearly (p < .001) related to OL. A quadratic trend (p < .001) was evident for the SH scale, reflecting the observation that performance did not improve from OL-1 to OL-2, but did improve sharply at OL-3.

A test of the relative difficulty of the two two-part scales was conducted by means of a profile analysis

[1] The analyses were repeated with age controlled (each of the tests was one-tailed because the direction of the correlation between O L and each of the scales was thought to be positive). The relation between O L and each scale was reduced when age was controlled: SH scale (p < .007); TPO-SD (p < .08); TPR-SD (p < .16). Age and OL were highly confounded. Only one OL-3 child was found among the 4-year-olds and two were found among the 5-year-olds. No OL-1 children were found among the 6- and 7-year-olds. Age, however, is not considered an exogenous variable that affects knowledge of functional relations (see Wohlwill, 1973, for a discussion of age as an independent variable in developmental research). Problems in comparing the relative predictive power of age and OL are treated in the section on factor analysis.

(Morrison, 1976). In conducting the profile analysis a composite variable was created, a priori, from the difference between the two scales. No interaction between scale differences and OL was detected ($F(2,95) = .10$) indicating that the size of the scale differences was not conditioned on operative level. The analysis revealed significant differences ($F(1,95) = 13.29$, $p < .001$) that were consistent with the view that the TPR scale was more difficult.

Correlations among the scales and factor analyses

The correlations among the scales, the OL variable, and age are presented in Table 4. All correlations were significant ($p < .001$). Table 4 also presents the correlation coefficients corrected for attenuation due to unreliability in the scales, but not in the OL measure or in age. No reliability estimate was obtained for the conservation tests. Age in months was derived from records and assumed to be measured without error. The median correlation among the three scales was .39 and the median corrected correlation was .49.

The median correlation between OL and the three scales was .47 (corrected, .50) and the median correlation between age and the three scales was .54 (corrected, .61). Two factors bear on the finding that, compared with OL, age had slightly higher correlations with the scales: (1) age had considerably more variance than the three-valued OL variable; (2) age was measured with less error.

Using the uncorrected correlations, a principal components analysis (see Table 5) was conducted on the three scales and OL. The purpose of this principal components analysis was different from the item-level analyses that were conducted earlier for the purpose of corroborating the a priori scale construction. The principal components analysis described in Table 5 presumed the completion of scale construction, and was aimed at a broad concern, namely, 'the explication of constructs' (Nunnally, 1978). Since only one factor was extracted a rotation was not warranted. The analysis was repeated with age added. A unitary factor was again extracted. The pattern of loadings on the scales and OL was highly similar to the pattern of loadings obtained in the first analysis.

Table 4. The correlations among the measures: Pearson correlation coefficients below the diagonal and the corrected correlation coefficients above the diagonal.

Measure	1	2	3	4	5
1. SH		.48	.39	.49	.50
2. TPO	.39		.60	.56	.64
3. TPR	.34	.45		.50	.61
4. OL	.47	.47	.44		.70
5. Age	.47	.54	.54	.70	

Note. $p < .001$ for all Pearson correlation coefficients. SH, same-height scale; TPO, two-part one-way scale; TPR, two-part reverse scale; OL, operative level.

Table 5. Principal components analysis of the three scales and OL without and with age.

Measure	Loadings	
	Without age	With age
SH	.72	.68
TPO	.77	.74
TPR	.73	.72
OL	.80	.82
Age		.87

Note. SH, same-height scale; TPO, two-part one-way scale; TPR, two-part reverse scale; OL, operative level.

Response patterns to the TPO and TPR items

An analysis of the pattern of responses provided clues to the processes the children used in judging the quantities. For example, the third TPO item, an 'equals + greater' composition, differed from the other two items, both 'greater+ greater' compositions. In the third item, the two 'morning' liquids were equally tall and taller than the two (unequal) 'afternoon' liquids. If children used a strategy in which they judged the total quantity of liquid on the basis of the height of the tallest column of liquid, one would expect the children to err more frequently on this item (by judging the total quantities of R and G to be the same) more often than on the two 'greater+ greater' TPO items.

Analyses of the responses at the item level indicated that the three TPO items were about equally difficult despite the third item in the scale reflecting an 'equals + greater' composition. Of the subjects who passed the first item, a 'greater+ greater' item, 78 per cent also passed the third item in the scale, the 'equals+ greater' item; 64 per cent of the subjects who did not pass the first item also failed to pass the third item. Similar patterns existed between the second and third items (76 per cent of those who passed the second passed the third and 69 per cent of those who failed the second failed the third) and between the first and second items, two 'greater+ greater' items (88 per cent of those who passed the first passed the second and 58 per cent of those who failed the first failed the second). The three TPR items were also about equally difficult but more difficult than the three TPO items.

When the children erred on the TPO items, they tended to indicate that the two quantities were the same. This pattern did not vary much between the 'equal+ greater' and the 'greater+ greater' items (22 per cent of all responses to the first item; 24 per cent to the second; and 26 per cent to the third). Similarly, when children erred on the three inequality items making up the TPR scale, they also tended to indicate that the puppets received the same amount: 45 per cent of all responses to the first TPR item; 37 per cent to the second; and 36 per cent to the third.

Observational data on the overt spontaneous behaviour incidental to the children's judgements were recorded on a subsample of 20 youngsters, not sufficient for quantitative analysis, but enough to suggest processes that might be implicated in performance. One type of response to the TPO items, a 'logical necessity' type of response, was that the puppet who had more in both the morning and the afternoon had to have had more for the whole day. By contrast, a more 'empirical' type of response to both TPO and TPR items involved the children describing what would happen if the red morning liquid were poured into the glass containing the red afternoon liquid and the green morning liquid into the glass containing the green afternoon liquid. They identified the liquid that would rise highest or overflow as the greater quantity. A different, cross-referencing type of solution to the TPR items involved comparing Rl with G2 and R2 with G1. For example, although Rl > G1 and G2 > R2, the child indicated that Rl > G2 and R2 > Gl, therefore R > G. Another type of response to the TPR items involved a comparison of the size of one puppet's advantage in the morning to the size of the other puppet's advantage in the afternoon in deciding who had more for the whole day. There were two subtypes of this response. Some youngsters would use their fingers to compare the differences; other children would visually inspect the differences.

DISCUSSION

Children of all levels performed well on the SD comparisons, demonstrating virtually error-free performance on a task in which quantity was indexed by height. Partly consistent with the first hypothesis, OL-2, but not OL-1, children performed at better than chance levels on comparisons that involved one-way compositions of functional relations. As anticipated in the second hypothesis, the TPR task was more difficult than the TPO. Consistent with the third hypothesis, performance on the SH, TPO and TPR tasks was highly related to OL as independently indexed by tests of conservation. As anticipated in hypotheses three and four, the correlational and factor-analytical findings reveal consistency across measures and suggest that a unitary, developmental factor underlies performance differences.

Consistent with Schonfeld (1990), the findings suggest that children's performance on comparison tasks improves in two ways: at advanced levels (1) the understanding of more complex functional relations emerges and (2) thinking becomes increasingly exact, facilitating improvement in cognitive performances in which the more advanced pre- operational (i.e. OL-2) children demonstrate some preliminary competence (i.e. TPO items). The findings pertaining to the specific cognitive accomplishments of pre-operational children were consistent with results on cognate discrete-quantity tasks (Schonfeld, 1986, 1990).

The correlations among the liquid scales tended to be lower than the correlations among the discrete-quantity scales (median r = .52, median corrected r = .70 in Schonfeld, 1990; r = .68 for Scales 1 and 2, corrected r = .84 in Schonfeld, 1986). The most reasonable explanation for the difference is that the liquid scales had considerably less variance than the discrete-quantity scales. Each liquid scale comprised only three items and the discrete quantity scales included as many as 14 items (Schonfeld, 1990). The average size of the correlation coefficients among the liquid scales, however, was consistent with findings obtained by de Ribaupierre, Rieben & Lautrey (1985), who employed a different set of Piagetian tasks and a more conservative computational method.

The factor-analytical results were consistent with Lautrey, de Ribaupierre & Rieben (1990), who also found a general developmental factor. Lautrey et al. (1990), however, identified two additional minor factors that they argued reflected intra-individual decalage, or a type of decalage that occurs in opposite directions in different children (Lautrey et al., 1985; Rieben & de Ribaupierre, 1990). Evidence for grossly heterogeneous decalage between tasks (model 1d in Figure 1 in Lautrey et al., 1985; de Ribaupierre et al., 1985), however, was weak.

One explanation of the two-part findings is that the tasks did not tap the children's OL-related understanding of the composition of functions; the children, in comparing the quantities, may have employed alternative procedures that were unrelated to OL. Children's performance could have been governed by the criterion stimuli (cf. Wallach, 1969) they learned to employ in everyday contexts, like determining who around the breakfast table got more milk to drink. The leading criterion stimulus used in comparing liquid quantity is the height of the liquid in a container. Although relative height is an imperfect indicator of quantity, height is often an adequate indicator. By judging quantity purely on the basis of the height of the tallest column of liquid, a child, responding to two of the TPO items, could identify the puppet who obtained more juice to drink 'for the whole day' because the puppet with the one glass having the tallest column of liquid also had more in aggregate.

Such an attentional estimation procedure circumvents the need to integrate information from the morning and afternoon comparisons in comparing the overall quantities; the child needs only to fix his or her attention on the one container having the tallest column of liquid-a variant of Bryant & Trabasso's (1971) 'absolute' response-in order to ascertain the larger quantity. This explanation, however, is inadequate for a number of reasons. First, the tasks were organized to take advantage of the 'affordance' (Miller, 1984) provided by framing them in terms of puppets having quantities of juice to drink. The tasks did not confront the children with unusually abstract notions of quantity; rather, the children were prompted to use 'normal procedures' for interpreting the problems (McGarrigle & Donaldson, 1974). Second, children performed virtually error-free on the SD task. If performance on the TPO task could be explained simply by a strategy in which the child fixed on the tallest liquid, performance on the TPO task (especially for the first two items) should have been equally free of errors. The TPO task turned out to be considerably more difficult than the SD task.

Third, if a fix-on-height strategy governed the children's judgements, their performance on the third TPO (equals+ greater- greater) item would have been worse than their performance on the first two TPO (greater+ greater - greater) items. Analyses of item-level response patterns indicated that the three TPO items were approximately equal in difficulty. Fourth, the item analyses also indicated that, when the children erred, they tended to judge the two-part quantities to be equal. An anonymous reviewer suggested that this finding indicates that the children 'combined two partial comparisons' qualitatively, if not quantitatively.

Finally, the procedure employed to introduce both two-part tasks was designed to draw the child's attention to all four containers trial after trial. The evidence drawn from the spontaneous overt behaviours in the subsample indicates that children who responded correctly considered all four glasses (for example, by imagining what would happen if the two morning juices were poured into the glasses containing the same-colour afternoon juices). Furthermore, the TPR task required no more attentional resources (see Chapman & Lindenberger, 1989) than the TPO task; however, the TPR task, consistent with its greater integrational requirements, was more difficult.

Within the context of the psychology of functions, compensation refers to the ability to coordinate, or integrate, functional relations, including relations involving static, or untransformed, quantity (Piaget et al., 1977), much like the quantities examined here. The spontaneous overt behaviours indicate that children differ in how they integrate dimensions to arrive at the right answer. For example, one type of response to the TPO task found it logically inescapable that greater and greater yield greater. Another type of response to the TPO and TPR tasks involved anticipatory imagery of the results of pouring liquid from one glass into another (the children were not permitted to move the liquid). This procedure probably carried with it some evolving estimation strategies (cf. Klahr & Wallace, 1973) that should be explored in future research. A different type of response to the TPR task involved the mental cross-referencing of Rl with G2 and R2 with G1. Yet another type of response to the TPR task involved a direct comparison of the within-morning and within-afternoon differences, sometimes with the help of crude finger measurement and sometimes by observation of the differences. The findings indicate that children's emergent integrative capacities are multifaceted.

Of course, many of the children erred in approaching the tasks. In distinguishing between invention and learning, Siegler & Yago (1978), in studying children's knowledge of the relative fullness of glasses of liquid, found that 6- and 7-year-olds are generally unlikely to overcome the power of the height cue by developing a proportionality rule. Siegler & Yago (1978) showed, however, that children of this age group are capable of learning to apply a proportionality rule with the help of instruction. In a similar vein, future research that capitalizes on the distinction between invention and learning might explore the conditions under which children who are unlikely to 'invent' the relevant functional schemes may profit from instruction on functional relations. One might, for example, develop teachable solution procedures that help children solve the TPR tasks by physically, or mentally, moving closer glasses that are initially removed from each other (e.g. Rl and G2) or by pouring liquids. Cowan (1987) and Schonfeld (1986) demonstrated that procedural aids can improve the performance of children engaged in discrete-quantity comparison tasks that require knowledge of functional relations.

ACKNOWLEDGEMENTS

This article involves a substantive reanalysis of data collected in one of the studies conducted for my doctoral dissertation. The dissertation was submitted to the City University of New York in partial fulfilment of the requirements for the PhD degree. I thank Harry Beilin, Geoffrey Saxe and three anonymous reviewers for their critical reading of an earlier version of this paper. Special thanks are extended to Bennett Gewirtz, Alan Gross and George Schonfeld.

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