8-1-2014

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Boran Ekin Aydin

Peter-Jules van Overloop

Xin Tian

11th International Conference on Hydroinformatics
HIC 2014, New York City, USA

OFFSET FREE MODEL PREDICTIVE CONTROL OF AN OPEN WATER REACH

BORAN EKIN AYDIN (1), PETER JULES VAN OVERLOOP (1), XIN TIAN (1)

(1): Delft University of Technology, Section of Water Resources Management, Stevinweg 1, 2628 CN, Delft, The Netherlands

Model predictive control (MPC) is a powerful tool which is used more and more to managing water systems such as reservoirs over a short-term prediction horizon. However, due to unknown disturbances present in the water system and other uncertainties, there is always a mismatch between the model and the actual system. To overcome this mismatch and achieve offset free control of the water system, the internal model of the MPC is updated by adding the disturbance dynamics of the actual system by means of a disturbance model. In this paper, the conditions to achieve offset free control for an open water reach are provided. A disturbance model is designed and used to achieve offset free control in a test canal assessed from simulation results.

INTRODUCTION

Model Predictive Control (MPC) is an optimization based control strategy which makes use of a process model to predict the future process outputs within a specified prediction horizon [1]. At each sample time the system state is estimated and a new open-loop optimization is carried out [3]. The model accuracy directly affects the performance of the MPC. Due to the modelling error, unknown disturbances and other uncertainties in the system, there is always a mismatch between the model and the real system. To overcome this mismatch and achieve offset free control there are two main ways: augmenting an integral action to the MPC controller [2] or modelling the disturbances by a disturbance model which augments the system states with integrating disturbances [5]. The latter is the focus in this study.

In this work, offset free MPC method described by Pannocchia et al. [5], is used to control the first pool of the laboratory canal UPC-PAC (Technical University of Catalonia - Control Algorithms Test Canal) located in Barcelona, at the Northern Campus of the University. Canal Automation Model (CAM), an unsteady flow simulation program for irrigation canal with automatic gates developed by the Irrigation Training and Research Center is used for the simulations.

Offset free control is obtained by augmenting the internal model with an integrating disturbance as an additional state. A Kalman filter is designed for the augmented model to adjust the integrating disturbance and the states using the measurements. This paper will first introduce the test canal and the internal model used which will be followed by design guides for the disturbance model and the estimator. The results of the simulations will be followed by conclusions and future work.

TEST CANAL AND INTERNAL MODEL

The test canal modelled and controlled in this study is the first pool of the UPC-PAC. The canal length is 220 m, depth is 1 m, width is 0.44 m, and has a zero bottom slope in order to achieve the largest possible time delay. The maximum discharge is 0.150 m^3/s. In this article, the first pool of this canal is modelled and controlled; its length is 87m. An undershot gate at the upstream end of the canal is used to separate the pool from a constant level reservoir. At the downstream end there is an undershot gate.

The model used is the integrator resonance model (IR) taken from Overloop et al. [4].

$$h_2 = \frac{\omega_0^{\;2}}{A_s s^3 + \dfrac{s^2}{M} + A_s \cdot \omega_0^{\;2} \cdot s} \cdot Q_1 - \frac{2 \cdot s^2 + \dfrac{2}{A_s \cdot M} \cdot s + \omega_0^{\;2}}{A_s s^3 + \dfrac{s^2}{M} + A_s \cdot \omega_0^{\;2} \cdot s} \cdot Q_2 \tag{1}$$

where A_s = 38.28 m^2, ω_0 = 0.101 rad/s and M = 1.2092 which is valid for a flow of 0.010 m^3/s.

In state-space form the model is given by:

$$\begin{aligned} X_{k+1} &= AX_k + Bu_k + B_d d_k \\ y_k &= CX_k \end{aligned} \tag{2}$$

The objective of the controller is to keep the downstream water level at set point (0.8 m). The downstream flow Q_2 acts as a known disturbance. Sampling time is 10 seconds and the prediction horizon is 20 steps.

The objective function is:

$$J = \sum_{i=1}^{20} \left\{ W_e \cdot e(k+i)^2 + W_{\Delta Q} \cdot \Delta Q_1 (k-1+i)^2 \right\} \tag{3}$$

where

$$e(k) = h_2(k) - h_{2,ref} \tag{4}$$

$$\Delta Q_1(k+1) = Q_1(k+1) - Q_1(k) \tag{5}$$

is minimized over the prediction horizon with the change of ΔQ_1. The penalties of deviation of water level from set point (W_e) and the upstream flow change ($W_{\Delta Q}$) are used as 100 and 10000, respectively.

States of the internal model are the errors $e(k)$, $e(k\text{-}1)$, e(k-2) given in Eq. (4) and the inflow discharges $Q_1(k)$, $Q_1(k\text{-}1)$, $Q_1(k\text{-}2)$ (number of states = n = 6). The controlled variable is the error $e(k+1)$ given in Eq. (4) (number of controlled variable = nc = 1). The manipulated variable is the change of inflow discharge $\Delta Q_1(k+1)$ given in Eq. (5) (number of manipulated variable = m = 1) and the measured variable is the error $e(k)$ given in Eq. (4) (number of measured variable = p = 1).

According to Pannocchia et al. [5] one can control a system whose number of measured variables (p) is smaller than or equal to the number of manipulated variables (m). In this study, since both p and m are 1 this condition holds, so we can apply the method described by Pannocchia et al. [5] to control our model without offset. Further restrictions and details of the method can be found in Pannocchia et al. [5] which will not be described in this paper, however are checked for this application.

DISTURBANCE MODEL

A disturbance model is required to achieve offset free control of the controlled variables by removing the unmeasured nonzero disturbances [5]. One way of disturbance modelling is to augment the original internal model by adding integrating disturbances (d_{aug}) to each controlled variable.

The state space form of this augmented system is given by;

$$
\begin{aligned}
X_{k+1} &= AX_k + Bu_k + B_d d_k + B_{d,aug} d_{aug,k} \\
d_{aug,k+1} &= d_{aug,k} \\
y_k &= CX_k + C_{d,aug} d_{aug,k}
\end{aligned}
\tag{6}
$$

Where $d_{aug,k}$ is the integrating disturbance vector and since there is only one controlled variable in the internal model, it is a scalar in this case. The selection of $B_{d,aug}$ and $C_{d,aug}$ matrices directly affects the disturbance model. In this study, $B_{d,aug}$ is used similar as B_d and $C_{d,aug}$ as identity matrix. The integrating disturbance vector in the augmented model cancels the effect of unmeasured nonzero disturbances in the controlled variables.

ESTIMATOR

In order to estimate the states, X_k, and the integrating disturbance, d_k, a steady state kalman filter is used which uses the measurements, y_k, of the system. The Kalman filter is designed for the following augmented system.

$$
\begin{aligned}
\begin{bmatrix} X_{k+1} \\ d_{aug,k+1} \end{bmatrix} &= \begin{bmatrix} A & B_{d,aug} \\ 0 & I \end{bmatrix} \begin{bmatrix} X_k \\ d_{aug,k} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} B_d \\ 0 \end{bmatrix} d_k \\
yk &= \begin{bmatrix} C & C_{d,aug} \end{bmatrix} \begin{bmatrix} X_k \\ d_{aug,k} \end{bmatrix}
\end{aligned}
\tag{7}
$$

The Kalman predictor for this model is given as:

$$
\begin{bmatrix} X_{k+1|k} \\ d_{aug,k+1|k} \end{bmatrix} = \begin{bmatrix} A & B_{d,aug} \\ 0 & I \end{bmatrix} \begin{bmatrix} X_{k|k-1} \\ d_{aug,k|k-1} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} B_d \\ 0 \end{bmatrix} d_k + \begin{bmatrix} K_{k,1} \\ K_{k,2} \end{bmatrix} (y_k - CX_{k|k-1} - C_d d_{aug,k|k-1}) \tag{8}
$$

In this equation $K_{k,1}$ and $K_{k,2}$ are the Kalman gain matrices for the state and the disturbance respectively.

For computational easiness the following notation is used in the application of the Kalman Filter design

$$
A = \begin{bmatrix} A & B_{daug} \\ 0 & I \end{bmatrix}, C = \begin{bmatrix} C & C_{daug} \end{bmatrix}, K_k = \begin{bmatrix} K_{k,1} \\ K_{k,2} \end{bmatrix}
$$

The measurement error covariance matrix (R) and process noise matrix (Q) are defined according to the Kalman Filter used by Overloop et al. [4].

In the operation of the Kalman filter, kalman gain (K_k) and the error covariance (P_k) are updated at every step using the filter update equations [6] given below. The filter update equations have two parts: measurement and time update equations. The time update equations are responsible for estimating (predicting) the *a priori* estimates of the current state and error covariance for the following step. The measurement update equations are used to improve (correct) *a priori* estimate to obtain *a posteriori* estimate [6]. During the simulation, previous *a posteriori* estimates are used to predict the new *a priori* estimates.

Measurement Update Equations ("Correct")

1) Compute the Kalman Gain, K_k

$$K_k = P_k^- C_k^T (C_k P_k^- C_k^T + R)^{-1} \tag{9}$$

2) Update estimate, $\hat{X}_k$, with measurement, y_k

$$\hat{X}_k = \hat{X}_k^- + K_k (y_k - \tilde{C}_k \hat{X}_k^-) \tag{10}$$

3) Update the error covariance, P_k

$$P_k = (I - K_k \tilde{C}_k) P_k^- \tag{11}$$

Time Update Equations ("Predict")

1) Project the State ahead, X_{k+1}^-

$$X_{k+1}^- = \tilde{A}_k \hat{X}_k + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} B_d \\ 0 \end{bmatrix} d_k \tag{12}$$

2) Project the error covariance ahead, P_{k+1}^-

$$P_{k+1}^- = \tilde{A}_k P_k \tilde{A}_k^T + Q \tag{13}$$

The filter needs an initial estimate of the state ($\hat{X}_k^-$) and the error covariance (P_k^-). The initial state is used as the steady state values while the initial error covariance is obtained from the Kalman operator.

OFFSET FREE CONTROL AND SIMULATION RESULTS

At 30 minute, the outflow discharge (0.010 m^3/s up to 30 min) is increased by 0.010 m^3/s as a known disturbance. To add an offset to the model, the downstream flow is increased by 10 % over the entire simulation. This extra discharge is unknown to the controller. The downstream flow during the simulation can be seen in figure 1.

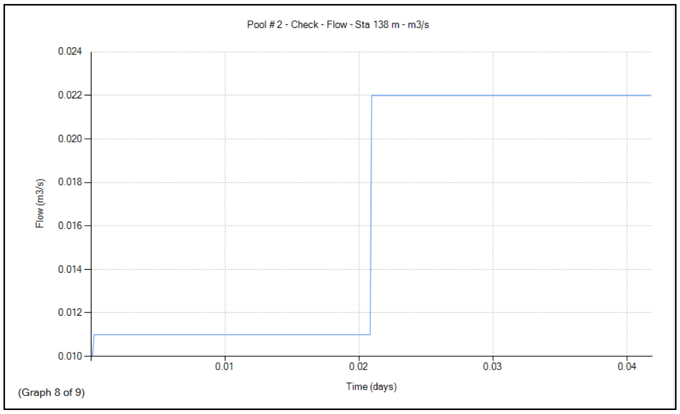

Figure 1. Downstream flow, Q_2 (m3/s), throughout the simulation

The reaction of the controller without an offset controller is shown in figure 2.

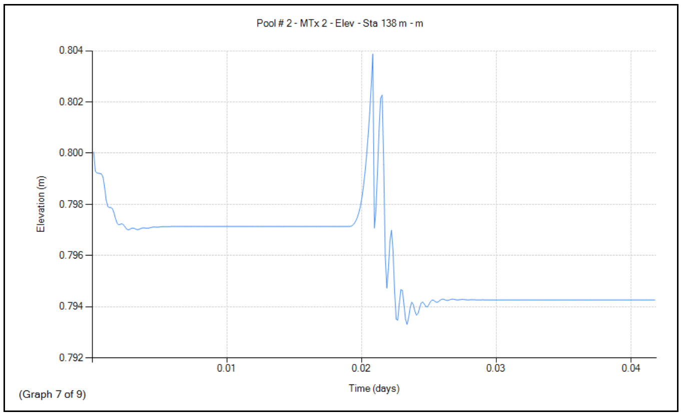

Figure 2. Offset in downstream water level, h_2 (m)

As it is seen in figure 2, the controller cannot reach to set point (0.8 m) due to the fact that the internal model is lacking the information about the unknown 10% disturbance in the outflow discharge.

To overcome this problem, the internal model is augmented with an integrating disturbance and an estimator is used to update the states and the disturbance using the measurements of the system. The results of the controller are provided in figure 3 and figure 4.

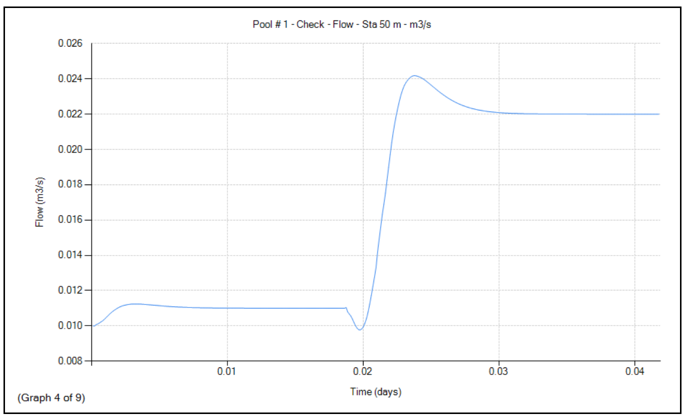

Figure 3. Upstream flow, Q_1 (m3/s), throughout the simulation

As can be seen there is a slight decrease in the flow just before the step occurs. Remember that the prediction horizon is 20 steps so the controller reacts on the step about 3.3 minutes before it occurs. The controller quickly responds to the step and the upstream flow is stabilized in a short time.

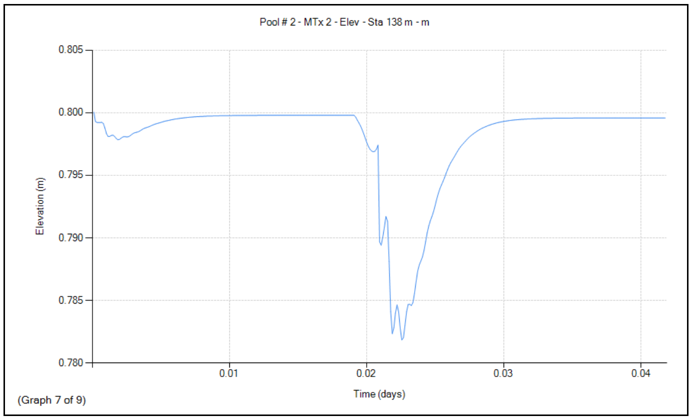

Figure 4. Downstream water level, h_2 (m) obtained by offset free MPC

As can be seen from figure 4, the offset free MPC quickly removes the offset at the start of the simulation and the water level stabilizes at the set point. Then at 30 minutes after the step in the downstream flow occurs, the controller again removes the offset very quickly and smoothly.

CONCLUSION AND FUTURE STUDY

This study shows that offset free control of an irrigation canal is possible by augmenting integrating disturbances to the controlled variables of the system. Comparing the simulation results one can clearly see that the offset in the water level in an irrigation canal can be removed by augmenting the internal model with integrating disturbances.

Moreover, this paper can be used as a guideline of applying offset free control on irrigation canals by providing the required knowledge about the method. General conditions and restrictions of applying this method can be found on Pannocchia et al. [5].

As future work, the writers are focusing on comparing this method to other methods that can obtain offset free control of irrigation canals.

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