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## **SHORT-TERM HYDROPOWER OPTIMIZATION USING A TIME- DECOMPOSITION ALGORITHM**

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A time-decomposition algorithm is proposed which uses long-, mid-, and short-term planning models to efficiently optimize sub-daily operation of a reservoir system with an annual or seasonal planning horizon. Sampling Stochastic Dynamic programming (SSDP) is used to address long-term and mid-term planning for a hydropower system. Re-optimization is used to select the actual releases. The upper-level models supply the terminal value function to the lower-level models rather than specifying rigid release targets. The structure of the algorithm allows for the comparison of the value of different representations of uncertainty and evaluation of the value of forecasts and forecast precision to operations optimization. A case study of a small reservoir system in Maine, USA is presented. It is found that short-term forecasts are most important to summer operation.

### **1. INTRODUCTION**

There has been a resurgence of interest in hydropower as a renewable energy source to meet society's ever increasing demand for clean energy. Since new reservoir construction has nearly halted in developed countries, there is an increasing focus on improving the efficiency of existing facilities. While upgrades to generation and transmission infrastructure can improve efficiency, implementation of effective operation policies can also provide real value. Labadie [7] speculates that such improvements are often neglected once a project is constructed. Moreover Labadie [7] states that greater integration of the operation of interconnected water resources could significantly increase their benefit to society.

Operation of a hydropower reservoir involves successive decisions concerning when and how much water to release. These decisions are difficult because the value of future inflows is not known with certainty. Moreover, in competitive electrical markets, the future value of energy is uncertain. Thus the optimization of hydropower reservoir operations is a stochastic problem in terms of both supply and demand.

This study demonstrates a multi-tiered time-decomposition stochastic optimization algorithm applied to the sub-daily optimization of a hydropower reservoir. Sampling Stochastic Dynamic Programming (SSDP) is used to solve the nested optimization problems in the time-decomposition algorithm.

### **2. RESERVOIR OPTIMIZATION USING TIME-DECOMPOSITION**

Time decomposition of the reservoir optimization problem into overlapping sub-problems with different time-steps and optimization horizons is well suited to the decision structure of real-time hydropower operations [21]. The challenge is that we must optimize short-term (often hourly) operations in light of seasonal or even inter-annual objectives. With such a long planning horizon, modeling hourly operations is often computationally infeasible. Furthermore, the long-term uncertainties and forecasts might be of little relevance to real-time operations within a week or even a day. For example, the forecasted price of crude oil in 5 years may influence the annual operation of a reservoir, but is likely of little concern on an hourly basis. Yeh [22] used a time decomposition approach to optimize hourly operation of a hydrothermal power system with a yearly planning horizon. That algorithm consists of a monthly model with a time horizon of one year; a daily model with a time horizon of one month; and an hourly model with a time horizon of one week. The models are updated with periodicity corresponding to the time step of the particular model. The lower-level models represent the system in more detail than the higher-level models.

An important consideration of the time decomposition approach is how the different models are linked. One approach is for the upper-level models to pass explicit release targets (constraints) to the lower-level models. For example a monthly model might select an optimal release for each month, while a weekly model distributes that release within the month. Such an approach ensures that the resulting optimal policy is consistent across the models and stable through the planning horizon. Yeh [22] took this approach, as have others [1,2,12,17,18,19,20]. A potential problem with this approach is that the upper-level policy may not be optimal or even feasible in the short-term because it uses a coarser representation of the system and uncertainty.

A second approach is for the upper-level models to pass the terminal value of storage to the lower-level models. This approach frees the lower-level model from meeting release constraints imposed by an upper-level model, potentially resulting in improved policies. This approach is closely related to stochastic dual dynamic programming [11,4,16]. A potential problem arises if the terminal value function provided to lower-level model is poor, the lower-level model might engage in myopic behavior. For example, if a monthly model consistently underestimates the terminal value of storage at the end of each week, a nested weekly model will attempt to draw the reservoir down in each week. To avoid this it is critical that the value function of the upper-level model is sufficiently accurate. This approach is essential

The two approaches can be understood by considering similar decompositions in linear programming. The Dantzig-Wolfe decomposition is a method for solving large linear programming problems with a special structure. The method decomposes the original problem into a master program and independent subprograms. The master program sets parameters for the subprograms, which in turn pass their solution back to the master program [8, pp. 144]. This is somewhat analogous to the first approach to time decomposition, in which the upper level models supply a total release volume to the lower-level model. Benders' partitioning algorithm divides linear (or nonlinear) programming problems into two stages [8, pp 370]. The stage-two model can be thought of as providing a terminal value for the stage-one model. This is similar to the second approach to time decomposition, in which the upper-level models pass the terminal value of storage to the lower-level models.

This project takes the second approach to time decomposition, and seeks to exploit the clear advantages of SSDP in representing uncertainty when deriving an optimal strategy.

The proposed algorithm has three steps.

1. Long-term: SSDP model with a weekly time step and a seasonal or annual horizon determines the value of storage for end of each week from the current time period to the end of the planning horizon. This step is to be run once or twice per season, or as important new information becomes available.
2. Mid-term: SSDP model with a six-hour time step and a weekly horizon determines the value of storage in each six hour period in the coming week. This algorithm is to be run each week, or as new information becomes available.
3. Short-Term: A one step re-optimization is run at the start of each six-hour period to determine the optimal release for that period.

This algorithm has several advantages. First it allows for sub-daily optimization over a seasonal or annual planning horizon without incurring major computational effort. Second, by using SSDP rather than traditional SDP models, the algorithm uses a better representation of uncertainty, and a more realistic model of the persistence of flows (i.e. potential inflows are represented by realistic sequences rather than Markov processes). Also, the decomposition allows the analyst to use different forecast products which are available for different time scales. For example, a major concern in seasonal reservoir planning in Maine is the spring snowmelt, and snowmelt forecasts on a weekly or monthly basis are of great value to long-term planning. On the other hand, sub-daily variability is largely driven by rainfall runoff, so daily and hourly precipitation forecasts are of great value to short-term planning.

By varying the model parameterization in each of the three algorithm steps, a wide variety of representations of uncertainty can be constructed. This allows examination of the utility of different uncertainty structures for hydropower optimization in variable hydrology.

### 3. SSDP AND RE-OPTIMIZATION

SSDP was proposed by Kelman et al. [6] as an alternative to the more commonly applied SDP model. SSDP differs from SDP in two respects: First rather than representing future inflows as a Markov process as is common in SDP, SSDP represents future inflows by a range of time-series scenarios which might occur starting in the next time period and running through the planning horizon. The second difference is that SDP uses the same model to select an optimal release and to assess its benefits, whereas SSDP selects an optimal release with a Decision model (equation 1) and determines the value of that release with a Simulation model (equation 3). The SSDP algorithm is given by

$$\max_{R_t} \left\{ B_t(S_t, Q_t(i), R_t) + \alpha E_{j|i} [f_{t+1}(S_{t+1}, j)] \right\}, \quad \forall S_t \text{ and } t \in \{1, \dots, T\} \quad (1)$$

$$S_{t+1} = S_t + Q_t - R_t - e_t(S_t, S_{t+1}) \quad (2)$$

$$f_t(S_t, i) = B_t(S_t, Q_t(i), R_{t,i}^*(S_t)) + \alpha f_{t+1}(S_{t+1}, i), \quad \forall S_t \text{ and } t \in \{1, \dots, T\} \quad (3)$$

$$S_{t+1} = S_t + Q_t - R_{t,i}^*(S_t) - e_t(S_t, S_{t+1}) \quad (4)$$

where  $Q_t(i)$  is the reservoir inflow in stage  $t$  in scenario  $i$ ,  $S_t$  is the reservoir storage in stage  $t$ ,  $R_t$  is the release in stage  $t$ ,  $\alpha$  is a discount factor,  $f_t$  is the value function at stage  $t$ , and  $e_t(S_t, S_{t+1})$  is an evaporation loss for stage  $t$ . For each stage,  $t$ , storage state,  $S_t$ , and streamflow scenario,  $i$ , an optimal release,  $R_{t,i}^*(S_t)$ , is selected which maximizes the sum of the present benefits,  $B_t$ , and the expected future benefits,  $E_{j|i}[f_{t+1}(S_{t+1}, j)]$ , from the resulting storage state  $S_{t+1}$ . The expectation in equation 1 employs the conditional probability of transitioning from trace  $i$  in time  $t$  to trace  $j$  in time  $t + 1$ . An implicit assumption in equation 1

is that  $Q_t$  is known when selecting  $R_{t,i}^*(S_t)$ , which allows  $B_t$  to remain out of the expectation. To solve for  $f_T$  requires a terminal value function, which in the time-decomposition framework is provided to the lower-level models by the higher-level models.

Numerical evaluation of SSDP models typically involves recursively solving equations (1) and (3) at discrete storage values (points in the state-space), at which  $R_{t,i}^*(S_t)$  are determined. However in actual operation, the real storage is unlikely to coincide with one of the discrete storage levels. One solution might be to interpolate inside a policy table between storage levels. Instead Tejada-Guibert et al. [15] recommend a one-stage forward moving SDP re-optimization process. This step selects the optimal release given the current storage and reservoir inflow, and uses the SSDP value of  $f_{t+1}$  as the terminal value of storage. This step is repeated at each decision stage. If one considers the current stage's flow  $Q_t$  to be known, and the latest flow forecast for the next stage is denoted  $H$ , the re-optimization step can be expressed

$$\max_{R_t} \left\{ B_t(S_t, Q_t, R_t) + \alpha \mathbb{E}_{i|Q_t, H} [f_{t+1}(S_{t+1}, i)] \right\}, \quad \forall t \in \{1, \dots, T\} \quad (5)$$

In this case, the expectation uses the conditional probability of transitioning to scenario  $i$  given current reservoir inflow  $Q_t$  and the current flow forecast for the next stage inflow  $H$ .

#### 4. TRANSITION PROBABILITY CASES AND FORECAST PRECISION

The SSDP transition matrix describes the probability of transitioning from trace  $i$  in stage  $t$  to trace  $j$  in stage  $t + 1$ . The choice of transition matrix in the decision step determines the representation of uncertainty in the optimization. If the transition matrix is the identity matrix, then transitions between traces are not considered and the optimization is deterministic. This will be referred to as the "I" case.

Alternatively, if every element of the transition matrix is  $1/m$ , where  $m$  is the number of traces, then it is equally likely that the system will transition into any trace in the next decision step. This will be referred to as the "M" case.

If the hydrologic state of the system is described by some variable, then it can be desirable to condition the transition probability between traces on that variable. Common hydrologic state variables are the current or previous decision stage inflow [9,10]. Stedinger et al. [13] use an inflow forecast as a hydrologic state variable for an SDP case study on the High Aswan Dam in Egypt. Kelman et al. [6] and Faber and Stedinger [3] use forecasts as hydrologic state variables for single reservoir SSDP applications in California and Colorado respectively. In this study, inflow forecasts for the next time period are used as hydrologic state variables. Thus, the probability of transitioning from trace  $i$  to trace  $j$  in time  $t + 1$  is conditioned on the inflow forecast in time  $t$ . This will be referred to as the "F" case.

Kelman et al. [6] and Faber and Stedinger [3] use a Bayesian method to estimate the transition probability between traces. The same procedure was adopted for this work. Bayes theorem takes into account both the forecasted volume and its precision. Thus more precise forecasts should result in a narrower posterior distribution for future flows, and consequently more certain performance. This point is explored through the use of synthetic forecasts.

Given the three steps of the algorithm described in the previous section, it is possible to achieve a variety of representations of uncertainty by selecting the transition case for each step. For example, one might choose the "I" case for the two backwards optimization steps, then the "M" case for the forwards re-optimization step. The resulting algorithm would be referred to as the I/I/M configuration.

By comparing various model configurations we can examine the utility of different transition cases for the long-, mid-, and short-term planning for this reservoir system in

Northern New England. For example, we can examine whether using medium-term forecasts, say on a weekly scale, are of any value to the overall operation of the hydro system by comparing the performance of the I/M/F and I/F/F configurations.

This research does not use an existing forecast product, but instead uses synthetic forecasts created using the generalized maintenance of variance extension (GMOVE) proposed by Grygier et al. [5] and the model of forecast errors proposed by Stedinger and Kim [14]. This procedure was utilized to generate synthetic streamflow forecasts with the desired correlation to the actual streamflow and the desired variance. Assuming that the forecast is the product of a linear regression model, the  $R^2$  is the square of the correlation of the forecasted flow and the actual flow. This allows us to consider the benefit of forecast precision to operations optimization. For example, how much do reservoir operations improve if forecasts with  $R^2 = 0.95$  rather than  $R^2 = 0.65$ ? We can examine this by comparing the I/I/F95 and I/I/F65 configurations, where F65 is the “F” case with  $R^2 = 0.65$ .

Another question is at what planning scale does forecast precision help the most. For example, is higher precision for sub-day planning more valuable than higher precision on a weekly scale. A related question is how the value of forecast precision at different time scales changes seasonally. In Maine, a major concern for long- and mid-term planning during the spring is the snowmelt runoff, whereas in the summer and fall a major concern is heavy rainfall from localized thunderstorms. Perhaps long-term forecast precision is more important during the spring and short-term precision in the summer? By taking advantage of the structure of the time decomposition algorithm these questions are explored.

## 5. CASE STUDY

The three-step time decomposition algorithm described in Section 2 is applied here to summer operation of a single hypothetical reservoir based on Harris Station on the Upper Kennebec River in Maine, USA, with no upstream regulation. The total drainage area is 1365 sq. mi., with a storage capacity of 2.0 BCF, and a generation capacity of 89 MW. A variety of model configurations were applied. These configurations fall into three categories: configurations with no forecasts, configurations with forecasts for the re-optimization step only, and configurations with forecasts for all steps.

To measure the performance of each configuration, the ‘perfect foresight’ case was used as a standard. This corresponds to the I/I/I configuration. In this case, a deterministic backwards SSDP is used for the first two optimization steps, then re-optimization is performed with perfect foreknowledge of future flows. Table 1 reports the average summer benefit and the average efficiency and inefficiency obtained from 20 years of simulated operation using each of the model configurations. Efficiency is the ratio of the summer benefit achieved and the best possible summer benefit with perfect foresight. Inefficiency is one minus the efficiency.

Remarkably, even configurations with no forecasts perform very well: M/M/M and I/I/M are on the order of 97% efficient. This is in part because the average summer flows are small compared to the available storage, so variation in future inflows do not usually affect planning decisions. The I/I/M case returns a better result than the M/M/M case because it better represents the persistence of flows in the longer-term optimization steps.

Forecast precision for the re-optimization (short-term model) appears to be important to overall model performance. The improvement of the M/M/F50 model over the M/M/M model is not statistically significant at the 10% level (using a one-sided paired t-test). However, the M/M/F95 model performs significantly better than the M/M/M model, suggesting that model

performance is sensitive to short-term forecast precision, which is not surprising in summer operation.

Table 1: Average summer benefit, average efficiency, and average inefficiency of various time-decomposition models over 20 years of simulated operation

Model	Average summer benefits (\$M)	Average summer efficiency	Average summer inefficiency
M/M/M	8.235	0.966	0.034
I/I/M	8.254	0.968	0.032
M/M/F50	8.248	0.967	0.033
M/M/F95	8.267	0.969	0.031
I/I/F95	8.333	0.977	0.023
F50/F50/F50	8.304	0.974	0.026
F50/F75/F95	8.335	0.977	0.023
F95/F95/F95	8.333	0.977	0.023
Perfect	8.529	1	0

Interestingly, the M/M/F95 case does not outperform the I/I/M case. This is because assuming any trace is equally likely in the first two stages of the optimization resulted in a poor representation of the future value of storage, resulting in poor decisions, even when good short-term forecasts are available. This conclusion is supported by the excellent performance of the I/I/F95 model, which achieved much better results with the same short-term forecasts. Also, the F50/F50/F50 model outperforms the M/M/F95 model despite having poor forecasts for the re-optimization step. These results seem to indicate that good estimation of the future value of storage is also very important to overall model performance.

The F50/F50/F50, I/I/F95, F50/F75/F95, and F95/F95/F95 models outperformed the other models. These models better represent the persistence of flow when estimating the future value of storage than the other models tested. This results in better estimates of the future value of storage, and thus better optimization results. There was no statistical difference between the I/I/F95, F50/F75/F95, and the F95/F95/F95 models, suggesting in this case that good forecasts in the final re-optimization stage are highly advantageous. This seems consistent with the findings of Faber and Stedinger [3], who found that a two-step I/F model performed as well as the more sophisticated F/F model in a variety of cases.

## 6. FUTURE WORK

The results in the previous section compare relatively few configurations of the time decomposition model and are limited to only summer operation. Future work will consider an extended suite of model configurations, including more granularity in the  $R^2$  of the forecasts considered. While short-term forecast precision is most important in summer operations, it is speculated that medium- and long-term forecast precision will be of more importance in different seasons. In particular, during the spring-time, we anticipate that longer-term forecasts of when the spring freshet will arrive will be of more importance to operational planning than sub-daily flows.

## 7. DISCUSSION

As described before, one difference between the proposed time-decomposition algorithm and many previous models is that this model uses the upper-level models to estimate the terminal value of storage for the lower-level models rather than to impose constraints. This difference stems from a very different view of what DP algorithms provide. If you take the view that SDP provides an optimal policy across the state space and time, then using the upper-level models to impose constraints on the lower-level models makes sense, as does using policy tables for operations. However, if you take the view that SDP provides the value function across the state space and time, then passing the value function to lower-level models makes sense, as does re-optimization for operations rather than policy table interpolation. Tejada-Guibert et al. [15] demonstrate the benefits of re-optimization over policy table interpolation, but taking the latter view of DP for time-decomposition makes sense for another reason. The upper-level models represent processes more coarsely and compute the expected benefits with different uncertainty structures than the lower-level models. Why then should the optimal releases of the upper-level models be allowed to constrain the lower-level models, which potentially use more precise, updated forecasts?

Another important question is the value of forecasts, and forecast precision to reservoir operations optimization. The proposed time-decomposition algorithm provides a framework in which different operational forecasting products can be compared, or the value of hypothetical forecasts can be quantified. For instance, the results presented in Section 3 suggest that long- and mid-term forecasts are of little value for summer operation of the study reservoir, and that good short-term forecasts are vital to improved operational efficiency. The use of SSDP easily facilitates the incorporation of ensemble streamflow predictions (ESP), which provide a rich description of potential hydrology. This allows a system operator to assess the potential benefit of utilizing ESP products, a topic which Faber and Stedinger [13] explored for a two-stage model for a Colorado hydropower system.

## 8. CONCLUSION

This work introduces a time-decomposition algorithm which optimizes sub-daily (6-hour) operations of a hydropower reservoir system, while efficiently considering seasonal uncertainty and objectives. The algorithm consists of several linked SSDP models with varying time-steps and optimization horizons. This approach varies from many previous time-decomposition algorithms in that the upper-level models supply the future value of storage to the lower-level models rather than constraints as is more common. Through parameterization of the various nested models, a wide variety of representations of uncertainty can be constructed, providing a laboratory in which various model structures can be compared. By using synthetic forecasts of varying precision, the value of forecast precision is examined.

The proposed model is applied to a hypothetical single reservoir system based on Harris Station on the Kennebec River in Maine, USA. In simulation of summer operations it is observed that short-term forecasts are most important to operational efficiency and that long- and medium-term forecasts are of little value to hydropower operations. The precision of short-term forecasts seems most important, while long-term forecast precision seems to have little effect on the optimization of summer operation. Ongoing work seeks to determine how this conclusion might change for different seasons, and for multi-reservoir systems.

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