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## **A COMPARATIVE STUDY OF ACCURACY AND PERFORMANCE BETWEEN A FULLY 2D GPU BASED AND A 1D-2D COUPLED NUMERICAL MODEL IN A REAL RIVER**

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Hydraulic simulation is becoming a very widely used tool to provide information about the duration, the extension and the magnitude of a flooding episode. In the last decade, the advances in the numerical schemes, connected to the evolution experimented in the power of computers have allowed to aim for maximum accuracy in the results instead of working with simplified models. In particular, two main implementations have been developed in recent years: 1D-2D coupled numerical models and fully 2D models under GPU technologies. These strategies allow simulating a wide range of applications over large domains and time scale problems, computing them at an affordable cost. Both the 2D model and the 1D-2D coupled model are implemented using a finite volume framework using a conservative upwind cell-centered formulation based on Roe's Riemann solver across the edges. In particular, the 1D-2D model is constructed by imposing mass or mass/momentum conservation at each coupling zone depending on the flow conditions and the 2D model uses the mentioned numerical scheme, performing the computation by means of the last GPU technology. The topography is represented by means of triangular unstructured meshes for the 2D model and with cross sections for the 1D domain inside the coupled model. Both models were previously validated with experimental measurements, analytical solutions in academic and realistic configurations. A real river configuration with different scenarios and field data is presented. Not only the error made with respect the measurements is analyzed, but also the CPU time is evaluated in order to decide whether or not to use the 1D-2D coupled or the GPU model.

### **INTRODUCTION**

Hydraulic simulations are a useful tool to provide information about the extension, the duration and the magnitude of a flooding event. In particular, 1D and 2D shallow water models are widely popular in the context of rivers. The main characteristic of the 1D model is the speed of the computations, although they are unable to represent correctly the behavior of the floodplain phenomena. On the other hand, 2D models capture well all kind of flood scenarios in rivers and flood areas nevertheless there exists a notable increase in the required computational time [5]. In order to be able to improve the existent models detailed above and to achieve correct results in a reasonable time, two possible strategies are compared in this article: a 1D-2D coupled model and the implementation of a 2D GPU model. The former approach combines the best of both 1D and 2D models: while the river bed is represented by the 1D model, the 2D model covers the floodplain adjacent areas. With this strategy, a large number of 2D computational cells (usually the cells that govern the time step size) are eliminated of the computation, decreasing the computational time. Adopting the conservative method explained in [4], it is feasible to link both models in a simple way, imposing mass or mass and momentum

conservation depending on the flow conditions. The second technique analyzed in this work consists of implementing a fully 2D GPU model able to deal with triangular unstructured grids. As detailed in [2], this approach is able to obtain a very good performance with respect to the simple 2D model, achieving promising speed-ups. Moreover, the accuracy is guaranteed. The work is concerned with the aim of comparing both strategies (1D-2D vs. 2D GPU) in a real river in Zaragoza (Spain) with different flooding scenarios. The accuracy is measured in terms of flooded area and extension and the computational time is also evaluated for each simulation.

## GOVERNING EQUATIONS AND NUMERICAL SCHEME

### 1D model

The 1D shallow water equations can be expressed in a conservative form as follows:

$$\frac{\partial \mathbf{U}(x,t)}{\partial t} + \frac{d\mathbf{F}(x, \mathbf{U})}{dx} = \mathbf{H}(x, \mathbf{U})$$

$$\mathbf{U} = \begin{pmatrix} A \\ Q \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} Q \\ \frac{Q^2}{A} + gI_1 \end{pmatrix}, \quad \mathbf{H} = \begin{pmatrix} 0 \\ gI_2 + A(S_0 - S_f) \end{pmatrix} \quad (1)$$

$A$  is the wetted cross sectional area,  $Q$  is the discharge and  $I_1$  and  $I_2$  represent hydrostatic pressure force integrals.  $I_2$  accounts for the pressure forces in a volume of constant depth  $h$  due to longitudinal width variations.  $S_0$  represents the bed slope source term and  $S_f$  stands for the friction losses expressed in terms of the Manning's roughness coefficient  $n$ .

### 2D model

The 2D water flow under shallow conditions can be formulated by means of the depth averaged set of equations expressing water volume conservation and water momentum conservation. That system of partial differential equations will be formulated here in a conservative form as follows:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} + \frac{\partial \mathbf{G}(\mathbf{U})}{\partial y} = \mathbf{S}(\mathbf{U}) \quad (2)$$

where

$$\mathbf{U} = (h, q_x, q_y)^T$$

$$\mathbf{F} = \left( q_x, \frac{q_x^2}{h} + \frac{1}{2}gh^2, \frac{q_x q_y}{h} \right)^T, \quad \mathbf{G} = \left( q_y, \frac{q_x q_y}{h}, \frac{q_y^2}{h} + \frac{1}{2}gh^2 \right)^T \quad (3)$$

$$\mathbf{S} = \left( 0, gh(S_{ox} - S_{fx}), gh(S_{oy} - S_{fy}) \right)^T$$

where the bed slopes of the bottom level  $z$  are

$$S_{ox} = -\frac{\partial z}{\partial x}, \quad S_{oy} = -\frac{\partial z}{\partial y} \quad (4)$$

and the friction losses are written in terms of the Manning's roughness coefficient  $n$ :

$$S_{fx} = \frac{n^2 u \sqrt{u^2 + v^2}}{h^{4/3}}, \quad S_{fy} = \frac{n^2 v \sqrt{u^2 + v^2}}{h^{4/3}} \quad (5)$$

### Numerical scheme

Both 1D and 2D shallow water models are based on an upwind first order finite volume schemes. The system can be written compactly:

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \mathbf{E} = \mathbf{S} \quad (6)$$

When formulating the upwind cell-centered finite volume method, the normal flux  $\mathbf{E} \mathbf{n}$  and its Jacobian  $\mathbf{J}_n$  will be a question of interest. Moreover  $\mathbf{J}_n$  can be diagonalized

$$\mathbf{J}_n = \mathbf{P} \mathbf{\Lambda}_n \mathbf{P}^{-1}, \quad \mathbf{\Lambda}_n = \mathbf{P}^{-1} \mathbf{J}_n \mathbf{P} \quad (7)$$

where the diagonal matrix  $\mathbf{\Lambda}_n$  is formed by the eigenvalues of  $\mathbf{J}_n$  and  $\mathbf{P}$  is constructed with its eigenvectors. Applying Roe's linearization [6] it is possible to decouple the original hyperbolic system (4) and to define locally an approximate matrix  $\tilde{\mathbf{J}}_n$  at each wall  $k$  whose eigenvalues  $\tilde{\lambda}^m$  and eigenvectors  $\tilde{\mathbf{e}}^m$  can be used to express the differences in vector  $\mathbf{U}$  and  $\mathbf{S}$  across the grid edge  $k$  as a sum of waves [5]:

$$\delta \mathbf{U}_k = \mathbf{U}_i - \mathbf{U}_j = \sum_m (\tilde{\alpha} \tilde{\mathbf{e}})_k^m \quad \mathbf{S}_k = \sum_m (\tilde{\beta} \tilde{\mathbf{e}})_k^m \quad (8)$$

In the 1D model,  $m = 2, k = 2$  and the explicit upwind numerical scheme can be written as follows [1],[3]:

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\Delta t_{1D}}{\delta x} \left[ \left( \sum_m \tilde{\lambda}^+ \tilde{\gamma} \tilde{\mathbf{e}} \right)_{i-1/2}^m + \left( \sum_m \tilde{\lambda}^- \tilde{\gamma} \tilde{\mathbf{e}} \right)_{i+1/2}^m \right]^n \quad (9)$$

where  $i + 1/2$  represents the computational edge between cells  $i$  and  $i + 1$  (analogous with  $i - 1/2$  and cells  $i - 1$  and  $\tilde{\gamma}_{i+1/2}^m = \left( \tilde{\alpha} - \frac{\tilde{\beta}}{\tilde{\lambda}} \right)_{i+1/2}^m$  and  $\tilde{\lambda}_{i+1/2}^{\pm m} = \frac{1}{2} (\tilde{\lambda} \pm |\tilde{\lambda}|)_{i+1/2}^m$ . The time step  $\Delta t$  is dynamically chosen following:

$$\Delta t_{1D} = CFL \frac{\delta x}{\max_{m,i} |\tilde{\lambda}^m|_i} \quad CFL \leq 1 \quad (10)$$

Analogously, the 2D numerical upwind explicit scheme can be formulated using the finite volume approach for the updating of a single cell whose area is  $S_i$  dealing with the contributions that arrive to the cell from the neighboring walls:

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\Delta t_{2D}}{S_i} \sum_{k=1}^{NE} \sum_m \left[ (\tilde{\lambda}^- \tilde{\gamma} \tilde{\mathbf{e}})_k^m l_k \right]^n \quad (11)$$

In this expression,  $m = 3$ ,  $NE$  indicates the number of involved neighbouring walls in the computation and  $l_k$  is the length of each edge. When considering unstructured meshes in the 2D scheme, the time step is defined according to  $\chi_i$  [5]:

$$\Delta t_{2D} = CFL \frac{\min(\chi_i, \chi_j)}{\max_m |\tilde{\lambda}^m|} \quad \chi_i = \frac{S_i}{\max_{k=1,NE} l_k} \quad (12)$$

### 1D-2D COUPLED MODEL 2D GPU MODEL

The numerical coupling of both 1D and 2D models is detailed in [4]. As a summary, the models are linked geometrically by means of a new element of discretization called coupling zone, composed by one 1D cell and  $N$  triangular 2D cells. The procedure is as follows for each time step: each model computes its own conserved variables from (7) and (9), which will be called star variables. After that, an exact balance of water volume is done at each coupling zone:

$$V_{CZ} = A_{1D}^* \delta x + \sum_i^{N_C} h_i^* S_i + Q_{1D}^n n_{1D} \Delta t + \sum_i^{N_C} (\mathbf{F}_{1i}^n \cdot \mathbf{n}_i l_i) \Delta t \quad (13)$$

where  $\mathbf{F}_{1i}^n = (q_x, q_y)$ ,  $\mathbf{n}_i$  the outward normal direction. Note that  $n_{1D} = \pm 1$  in the frontal coupling and  $n_{1D} = 0$  in the lateral coupling configuration. Once the total volume of the coupling zone is computed, it is shared out between the involved computational 1D and 2D cells, enforcing a common water level surface. The same procedure can be extended to achieve the total momentum conservation, which will impose common average velocities in x and y directions at the coupling zone. More information about the momentum conservation as well as about the choice of the adequate strategy (mass or mass-momentum) can be found in [4].

On the other hand, the 2D GPU model is implemented on CUDA technology, developed by NVIDIA. This technology provides an efficient and friendly way to take advantage of the architecture of graphic cards to perform the corresponding computations. As expected, this 2D GPU model is based on the same numerical method presented for the 2D shallow water equations and is applied over triangular unstructured grids. More information can be found in [2].

Apart from the technology used, the main difference between the 2D GPU model and the 1D-2D model is that the former must do the computations over all the computational cells in each time step (due to the architecture) while the latter is implemented to run only over the wet cells. Therefore, the wet cells during the computations will be an important factor to be taken into account [2].

### THE EBRO RIVER: TEST CASES

A case study based on a reach of the Ebro river near an urban area has been selected to evaluate the proposed techniques. A Digital Terrain Model (DTM) is used in this work to provide great accuracy about the topography. The uncertainty on the particular shape of the river bed under the water surface has been eliminated by reconstructing the river able to convey the water discharge that was flowing in the moment of the measurements and so that it reproduces the

water surface extension and slope as measured. The DTM plus the river bed reconstruction were used as full bed topography to provide information to both 2D and coupled models. In particular the 2D GPU model is discretized into 201208 triangular unstructured cells while the 1D-2D model involves 121 cross sections for the 1D subdomain and 152729 cells covering the 2D subdomain of the 1D-2D coupled model. It is worth noting that, in order to achieve a good comparison between the models in terms of accuracy and computational time, the resolution of the cell size in the 2D subdomain of the coupled model is the same as in the 2D GPU model.

Three flooding scenarios (from now on test case 1, 2 and 3) are chosen to compare the numerical results obtained by the 1D-2D coupled model and by the 2D GPU model. They are based on a 8-day real flooding event occurred in Zaragoza in January 2013. In fact, test case 3 corresponds to the exact event, while test case 1 and test case 2 represent the same event but scaled in the discharge with a factor of 0.5 and 0.75 respectively. Figure 1 shows the hydrographs introduced as inlet boundary condition. The outlet boundary condition is set to free flow and the Manning roughness coefficient is set to  $0.035 \text{ s} / \text{m}^{1/3}$  in the river bed and  $0.05 \text{ s} / \text{m}^{1/3}$  in the floodplain areas. However, the 1D scheme 'inside' the coupled model needs a greater coefficient in order to diminish the differences with a 2D model [4]. For this purpose,  $0.037 \text{ s} / \text{m}^{1/3}$  has been chosen in the 1D sub-domain of the coupled model.

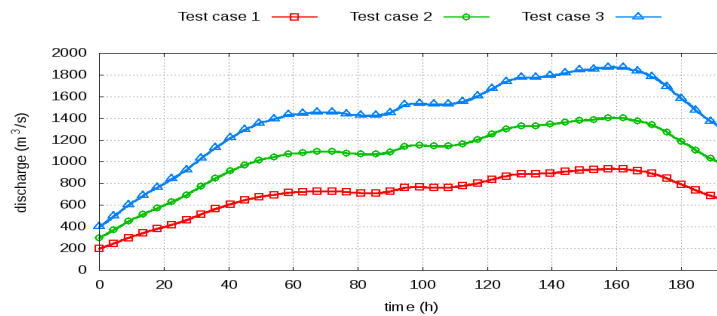


Figure 1. Inlet hydrographs for test case 1, 2 and 3

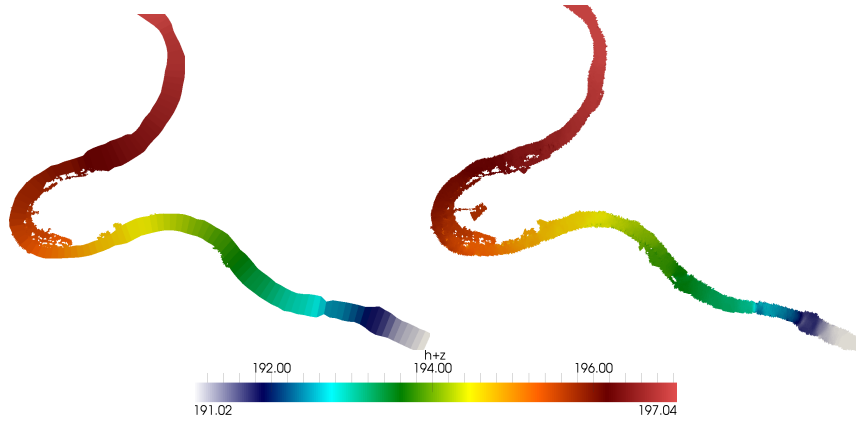
## NUMERICAL RESULTS

The numerical results in terms of water surface elevation achieved by the 1D-2D model (left) and by the fully 2D GPU (right) are displayed in Figure 2 for test case 1, 2 and 3 (from upper to lower respectively) It shows a qualitative approach of the flooding extension at  $t=166\text{h}$ , that is, when the flooding discharge is decreasing.

### Test case 1

1D-2D

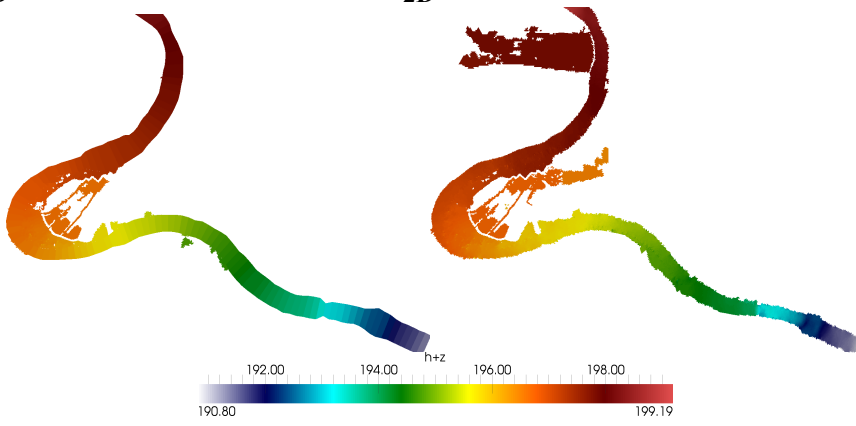
2D



**Test case 2**

**1D-2D**

**2D**



**Test case 3**

**1D-2D**

**2D**

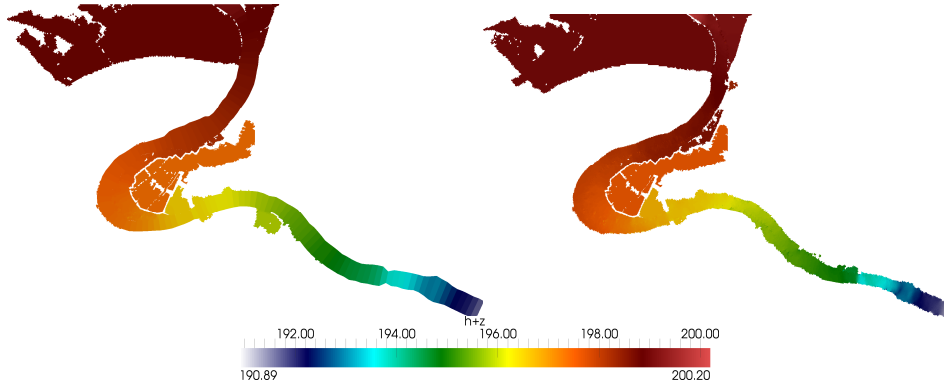


Figure 2. Water surface elevation and flooding extension at  $t=166h$  achieved by the 1D-2D model (left) and by the 2D GPU model (right) for test cases 1, 2 and 3 (from upper to lower respectively)

As can be seen, results are very similar in test case 1 and test case 3. However, test case 2 shows an area close to the beginning of the river reach considered which is partially flooded. It may be due to a bad representation of the levees in the fully 2D model. In order to have a quantitative measure of the flooding extension, the evolution of the wetted area in  $m^2$  for test cases 1, 2 and 3 is displayed in Figure 3. The inundation area is very similar until certain discharge from which the results in test cases 2 and 3 differ.

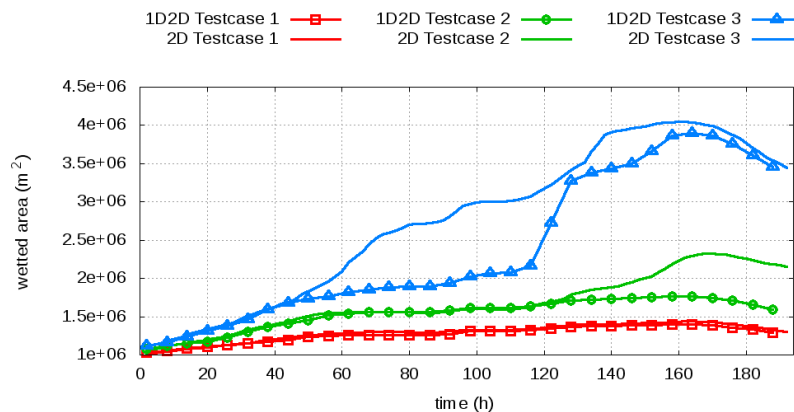


Figure 3. Evolution in time of the flooded area achieved by the 1D-2D coupled model and the 2D GPU model for test cases 1, 2 and 3.

The number of maximum wet cells as well as the CPU time consumed by each model is shown in Table 1. When the number of wet cells in the 2D subdomain of the coupled model is small, the 1D-2D model is able to achieve better results in terms of speed of computation. However, when the number of wet cells increases, the 2D GPU model is able to obtain the numerical solution by consuming less CPU time.



| Test case | Wet cells 1D-2D | Wet cells 2D GPU | Time 1D-2D (s) | Time 2D GPU (s) |
|-----------|-----------------|------------------|----------------|-----------------|
| 1         | 1292            | 21604            | 316            | 4503            |
| 2         | 7504            | 34856            | 2692           | 5236            |
| 3         | 29836           | 57584            | 12075          | 7222            |

Table 1. Maximum wet cells and CPU time used by the 1D-2D model and by the 2D GPU model

## CONCLUSIONS

Two possible techniques to accelerate the computational time when performing the simulation of flooding events are compared: a 1D-2D coupled model and a 2D GPU shallow water model. The comparison is done by means of different flooding scenarios in a real river and the performance of each scheme is analysed in terms of flooding extension and CPU time.

## REFERENCES

- [1] Burguete, J. and García-Navarro, P. "Efficient construction of high resolution TVD conservative schemes for equations with source terms: application to shallow water flows" *Int. Journal for Num. Methods in Fluids* 37 (2001) pp 209-248
- [2] Lacasta, A, Morales-Hernández, M., Murillo, J., García-Navarro, P. , García, R.High performance GPU-Speed up strategies for the computation of 2D inundation models., *HydroInformatics*, 2014
- [3] Morales-Hernández, M., García-Navarro, P., Murillo, J., A large time step 1D upwind explicit scheme (CFL> 1): Application to shallow water equations, *Journal of Computational Physics*,231, 19, pp. 6532-6557,2012
- [4] Morales-Hernández, M., García-Navarro, P., Burguete, J., Brufau, P., A conservative strategy to couple 1D and 2D models for shallow water flow simulation, *Computers & Fluids*, 81, pp.26-44, 2013
- [5] Murillo J., García-Navarro P. "Weak solutions for partial differential equations with source terms: Application to the shallow water equations", *J. Comput. Phys.* 229. (2010) pp 4327-4368.
- [6] Roe P.L. "A basis for Upwind Differencing of the Two-Dimensional Unsteady Euler Equations" *Numerical Methods in Fluid Dynamics*, Vol II. (1986) Oxford University Press, Oxford