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A MULTI-OBJECTIVE OPTIMIZATION MODEL FOR OPERATIONS PLANNING OF MULTI-RESERVOIR SYSTEMS

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This paper presents the development and evaluation of a multi-objective optimization tool for the operations planning platform (OPP) at BC Hydro. The optimization model incorporates two main objectives: (1) maximize revenue from power generation; and (2) to minimize penalties resulting from deviations of reservoir elevations and spill releases from a preferred operating regime. We analyze the use of penalty functions in the objective function and propose an alternative formulation using Chance Constraints and Linear Decision Rules. We present results of a case study to illustrate the capabilities of the tool to provide decision makers with timely information on trade-off between different objectives and the impacts of using chance constraints in lieu of penalty functions.

GENERAL OVERVIEW OF THE OPERATIONS PLANNING TOOL (OPT)

The Operations Planning Tool (OPT) is an in-house application developed by BC Hydro to aid the Operation Planning Engineers (OPEs) to make decisions regarding the operation of a multi-reservoir system. It consists of three main components: the graphical user interface (GUI), the optimization model and the solver software. The GUI allows the user to configure the optimization study, change model configurations, run the optimization and retrieve and display the output data. The optimization model is formulated in AMPL, while the CPLEX solver is used to solve the optimization problem. The GUI is used to configure the study and prepare input data in the user's workstation; the problem is sent and solved at the server workstation where the AMPL and CPLEX solver resides and then the solution is sent back to the client workstation.. This process is illustrated in Figure 1. This paper will focus on the optimization model formulation.

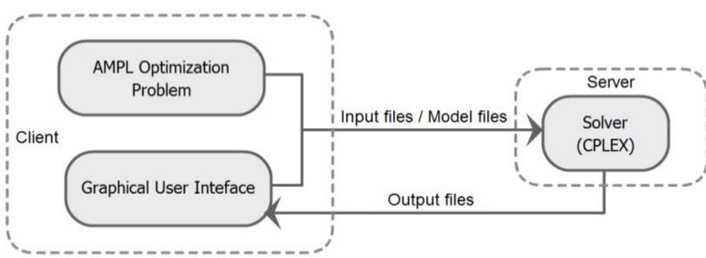

Figure 1: Process diagram of the Operations Planning Tool

The Optimization Model

The OPT is a deterministic Linear Programming optimization model and it can be divided into three basic components: (1) the decision variables, (2) the objective function and (3) the model constraints. A brief description of each of these components is presented in the following sections.

Decision Variables

The major decision variables of the model are the following:

$V_{p,t}$	Volume (storage) of reservoir *p* at the time step *t*
$spilQ_{p,n,t}$	Spillway release from reservoir *p*, through the release structure *n*, at time step *t*
$turbQ_{p,t}$	Turbine release from reservoir *p* at time step *t*
$turbQZone_{p,t,z}$	Turbine release from reservoir *p*, at time step *t* , during sub-time step *z*
$P_{p,t,z}$	Power generated at reservoir *p*, at time step *t* , during sub-time step *z*

It can be observed that there are two variables related to turbine releases. The first variable ($turbQ_{p,t}$) is the average turbine flow during the time step t, while $turbQZone_{p,t,z}$ is the average turbine flow during a shorter time step *z* (sub-time step) defined by the user. These sub-time steps depend on the variability of the electricity prices within a time step. For example, a typical day the user can define two price zones: Heavy Load Hours (hlh) for the heavy load hours, and Light Load Hours (llh) for the rest of the day. The electricity prices in these two zones are different and therefore it is of interest to know the average turbine release in each of them.

The Objective Function

The OPT multi-objective problem is solved making use of an optimization method known as the *Weighting Method of Multi Objective Optimization*, where a grand objective is established adding all the individual objectives, each one multiplied by a weighting coefficient as follows:

$$Minimize\ \{W_V * \sum_{p,t} PFV\left[V_{p,t}\right] + W_S * \sum_{p,n,t} PFS\left[spilQ_{p,n,t}\right] \\ -W_R * \sum_{p,t,z} P_{p,t,z} * \Delta t * priceZoneFraction_{t,z} * price_{t,z}\} \qquad (1)$$

The first two terms of the objective function refer to the minimization of the volume and spillway deviations from some preferred operating regimes, respectively. The minimization is accomplished through the use of “penalty functions” denominated PFV for storage and PFS for spill. These functions, which must be defined by the user, are piece-wise linear curves where a penalty number is assigned to all the possible values that the elevation and the spillway releases can have. When the values are within the preferred range a penalty value of zero is produced, while those outside of the target range must result in a non-zero penalty. The farther a value is from the target, the greatest its penalty will be. Figure 2 presents an example of a penalty function for spillway releases.

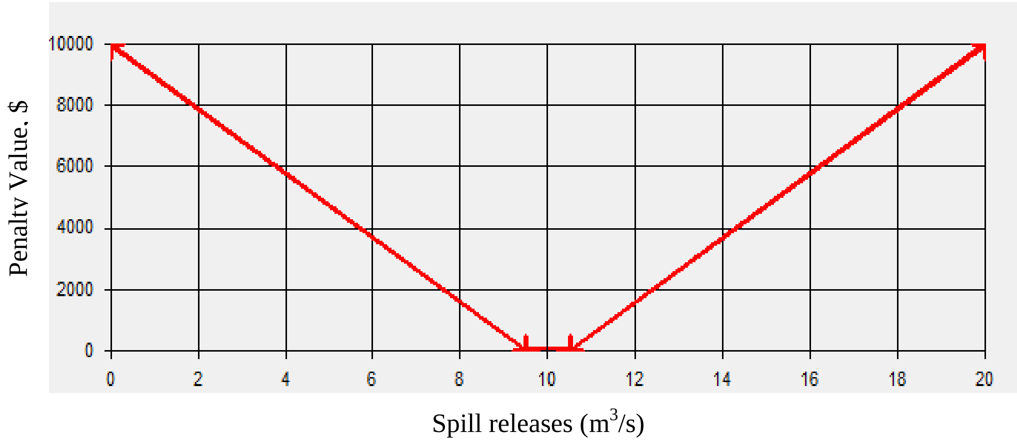

Figure 2: Example of a penalty function for spillway releases

The third term of the objective function refers to the maximization of the revenue from power generation. Due to the negative sign of this term, the model performs a maximization of the revenue even though the objective function as a whole is being minimized. In other words, because the model tries to make this term as negative as possible, the absolute value of the revenue will also be the highest possible. This revenue is computed for each sub-time step, multiplying the power generation, the number of hours in each sub-time step (Δt* *priceZoneFraction*) and the corresponding price.

The weighting coefficients Wv, W_S and W_R are input parameters whose values should be based in the priorities of the decision makers. Nevertheless, it is a common practice in this optimization method to run the model several times varying these coefficients until the set of non-inferior solutions is generated (Revelle et al. [1]).

Model Constraints

The constraints in the OPT model are used to fulfill the following purposes:

- Satisfy the continuity equation (reservoir mass balance).
- Set limits to the optimization variables.
- Relate the total turbine release per time step (*turbQp,t*) with the turbine release in the sub-time steps (*turbQZonep,t,z*).
- Calculation of the power generation through Generation Production Functions.

The Generation Production Functions (GPFs) are a family of tridimensional surfaces that provides the maximum power generation as a function of forebay elevation, turbine discharge and turbine availability. Shawwash [2] developed a procedure to build the GPFs for the different hydropower generation plants in the BC Hydro system.

USE OF CHANCE CONSTRAINTS IN THE OPT MODEL

The penalty functions used in the objective function are piecewise linear functions that allow the user to set targets for the forebay elevation and the spillway releases; nevertheless, these targets might be violated depending on the trade-off between the different objectives, which in turn is conditioned by the inflow scenario, the weight coefficients and the penalty values assigned to the targets. Penalty functions and piecewise linear functions have been used by different authors in reservoirs operations. Sigvaldason [3] developed a flow network model for the Trent River System in Ontario. He used penalty functions in the objective function in order to consider the "operation perception of optimal operation". Can and Houck [4] proposed two optimization models for a multi-reservoir system in the Green River Basin, Kentucky. The first model made use of piecewise linear penalty functions, similar to the OPT model, while the second model used a preemptive Goal Programming approach. Oliviera and Loucks [5] developed a Generic algorithm (GA)-based methodology which identifies the system release rule and the reservoir balancing functions as piecewise linear functions.

The use of penalty functions in the OPT model presents some limitations: first, it is not an easy task for the user to assign the penalty values for the different targets. The x-axis of the two type of penalty functions use different units (e.g., meters for the forebay elevation, cubic meters per second for the spillway releases); therefore, in order to have a balanced trade-off in the optimization process, it requires a comprehensive analysis to decide which slope should be assigned to each segment of the piecewise linear functions. Second, the optimization method that is used in the model requires the multiplication of the objective terms by some weight coefficients. The selection of these coefficients might also require a detailed analysis. Marler and Arora [6] carried out a survey of different approaches used to determine these weights, but they concluded that even varying the weights consistently and continuously may not result in an even and complete representation of the Pareto optimal set.

Therefore, it is desirable to find an alternative formulation for the OPT model that could provide the benefits of the penalty functions and at the same time overcome the limitations previously described. We have investigated the chance constraints method which could be a suitable alternative. Chance constraints act is a similar way to the penalty functions and they can be used as "soft constraints" allowing the establishment of targets but also considering that under certain conditions these targets may not be satisfied. They present the advantage that it can be easier for the user to establish reliability levels for the chance constraints compared to the construction of the penalty functions. An alternate formulation for the OPT model using Chance Constraints is proposed in Equations 2 to 6:

Maximize: $$\sum_{p,t,z} P_{p,t,z} * 24 * priceZoneFraction_{t,z} * price_{t,z} \tag{2}$$

Subject to:

$$P\left[V_{p,t} \geq VTarget\_low_{p,t}\right] \geq \gamma_{p,t} \tag{3}$$

$$P\left[V_{p,t} \leq VTarget\_up_{p,t}\right] \geq \delta_{p,t} \tag{4}$$

$$P\left[spilQ_{p,t} \geq spilQTarget\_low_{p,t}\right] \geq \alpha_{p,t} \tag{5}$$

$$P\left[spilQ_{p,t}. \leq spilQTarget\_up_{p,t}\right] \geq \beta_{p,t} \tag{6}$$

Where *VTarget_low*, *VTarget_up*, *spilQTarget_low* and *spilQTarget_up* are the lower and upper limits for the preferred volume and spillway release regimes. It can observed that in contrast to the objective function of the original model presented in Equation 1, the objective function using chance constraints consists of a single objective, which is the maximization of revenue from power generation. This change converts the OPT model into a single LP problem and the use of weight coefficients is no longer required. Therefore, the second limitation from the use of penalty function is also eliminated using chance constraints.

However, there are two main challenges that arise when using chance constraints. First, it is necessary to find a deterministic equivalent for constraints 3 to 6 and this requires the definition of probability distribution for the random variables used in the chance constraints. Since these are dependent on an operation policy, both the probability distributions of *V* and *spilQ* are unknown and they must be defined in terms of another random variable with a known distribution (Loucks and Dorfman [7]). This can be achieved through the use of *linear decision rules* (LDR) which defines the volume and spill releases in terms of the inflow, another random variable whose probability distribution can be constructed based on historical records. Then a deterministic equivalent formulation can be derived and used in the optimization model. The second challenge is to determine the highest possible reliability levels of meeting the preferred volume and spillway releases regimes. This can be accomplished running the model several times for different reliability levels, each time with a higher reliability that in the previous run, until an infeasible operation is encountered. The increments in the reliability levels must be small enough in order to accurately find the highest possible level. The multiple running of the model in order to find these reliability levels is equivalent to the multiple runs required by the variation of the weight coefficients in the original OPT formulation. Nevertheless, it is easier for the user to increase the reliability levels than to take decisions about the variation of the weight coefficients.

Definition of Linear Decision Rules

Linear decision rules have been used in reservoir system optimization to determine optimal operation policy rules in LP applications. Basically, they define volume and spillway releases in terms of the inflow and a deterministic parameter. Loucks and Dorfman [7] proposed the following general syntax for a LDR:

$$spilQ_{p,t} = (1-\lambda) * Inflow_{p,t} + V_{p,t-1} - b_t \tag{7}$$

Where b is an unknown determinist variable defined for each time step of the study period and λ is a parameter with values between 0 and 1 that indicates how much of the inflow will be considered in the spillway operation rule. In the original linear decision proposed by Revelle et al. [8] the λ parameter was equal to 1, and therefore the spillway releases relied only on storage during the previous time step. Loucks [9] found that this assumption yielded conservative results, and hence he proposed a value of 0 for λ. Sreenivasan and Vedula [10] also used the general LDR with λ equal to 0, but in addition, they incorporated the turbine releases as an additional deterministic variable in the rule. Several other authors have proposed different linear decision rules, but in this paper we perform that analysis using the general decision rule proposed by Loucks and Dorfman incorporating the turbine releases as suggested by Sreenivasan and Vedula as outlined in Equation 8:

$$spilQ_{p,t} = (1-\lambda) * Inflow_{p,t} + V_{p,t-1} - turbQ_{p,t} - b_t \tag{8}$$

Development of Linear Decision Rules for a multi-reservoir system

Before the LDR can be used in the chance constraints, the spillway release must be defined in terms of deterministic variables or random variables with known distributions. This means that equation 8 must be modified in order to eliminate the dependence of *spilQp,t* over *Vp,t-1*. This can be accomplished replacing the LDR into the equation of storage continuity. Equation 9 presents the continuity relationship for a single reservoir:

$$V_{p,t} = V_{p,t-1} + Inflow_{p,t} - turbQ_{p,t} - spilQ_{p,t} \tag{9}$$

Using a λ value of 0 and substituting equation 8 into the continuity equation yields:

$$V_{p,t} = b_{p,t} \tag{10}$$

Therefore, storage is set to the deterministic variable *b*. This applies to all the time steps of the study period, thus the spillway releases in the LDR can be expressed in terms of deterministic variables and random variables with known distributions:

$$spilQ_{p,t} = Inflow_{p,t} - turbQ_{p,t} + b_{p,t-1} - b_t \tag{11}$$

Equations 10 and 11 can now be substituted into the chance constraints of the model presented in equations 2 to 6. Nevertheless, in a multi-reservoir system, the total inflow that enters a reservoir is the sum of the local inflow, the turbine discharge and spillway releases from upstream reservoirs. Equation 12 presents an extension of the LDR for a multi-reservoir system:

$$spilQ_{p,t} = Inflow_{p,t} + \sum_{j=1}^{p-1} Link1_{j,p} * \left[inflow_{j,t} - turbQ_{j,t} + b_{j,t-1} - b_{j,t}\right] + \sum_{j=1}^{p-1} Link2_{j,p} * turbQ_{j,t} - turbQ_{p,t} + b_{p,t-1} - b_{p,t} \tag{12}$$

Where the parameters *Link1* and *Link2* are flags that indicate if there are some spillway and turbine connections between the reservoirs. These connections will be described with an example in the following section. After replacing Equations 10 and 12 into the original chance constraints, the deterministic equivalent constraints can be defined. In Equations 3 and 4 the storage random variable is replaced by the deterministic variable *b*, therefore the probability can be eliminated as it is shown in Equations 13 and 14.

$$b_{p,t} \geq VTarget\_low_{p,t} \tag{13}$$

$$b_{p,t} \leq VTarget\_up_{p,t} \tag{14}$$

In Equations 5 and 6, after replacing the spill variable with the linear decision rule shown in Equation 12, the deterministic equivalent can be found moving all the terms inside the probability to the right, except the inflow, and applying the inverse cumulative distribution function of the inflow to both sides of the external inequality. The resulting constraints are shown in Equations 15 and 16.

$$F^{-1}_{\Sigma Inflow}\left[1 - \alpha_{p,t}\right] + \sum_{j=1}^{p-1} Link1_{j,p} * \left[inflow_{j,t} - turbQ_{j,t} + b_{j,t-1} - b_{j,t}\right] + \sum_{j=1}^{p-1} Link2_{j,p} * turbQ_{j,t} - turbQ_{p,t} + b_{p,t-1} - b_{p,t} \geq spilQTarget\_low_{p,t} \tag{15}$$

$$F^{-1}_{\Sigma Inflow}\left[\beta_{p,t}\right] + \sum_{j=1}^{p-1} Link1_{j,p} * \left[inflow_{j,t} - turbQ_{j,t} + b_{j,t-1} - b_{j,t}\right] + \sum_{j=1}^{p-1} Link2_{j,p} * turbQ_{j,t} - turbQ_{p,t} + b_{p,t-1} - b_{p,t} \leq spilQTarget\_up_{p,t} \tag{16}$$

Where $F^{-1}_{\Sigma Inflow}[.]$ is the inverse cumulative distribution function of reservoir p inflows. It can be observed that the storage and the spillway releases are no longer variables in the new deterministic constraints. Instead, power generation, turbine releases and the deterministic parameter b would be the new outputs of the model.

Application and Results

The proposed model formulation was tested for a case study using the Stave Falls hydropower projects located near Mission, British Columbia. Figure 3 presents a simplified schematic of the Stave River system. Table 1 presents the values for the parameters Link1 and Link2 for the Stave Falls project. If the spillway from one reservoir is able to reach another reservoir going through the spillways of the intermediate reservoirs, the Link1 flag between them is equal to 1. A similar criterion applies for the turbine discharge and Link2. If there are no intermediate reservoirs, Link1 and Link2 will be 1 if the spillway and turbine releases can flow from one reservoir into the other. The Link1 and Link2 parameters are used to define the deterministic equivalents of the chance constraints, using equations 13 to 16.

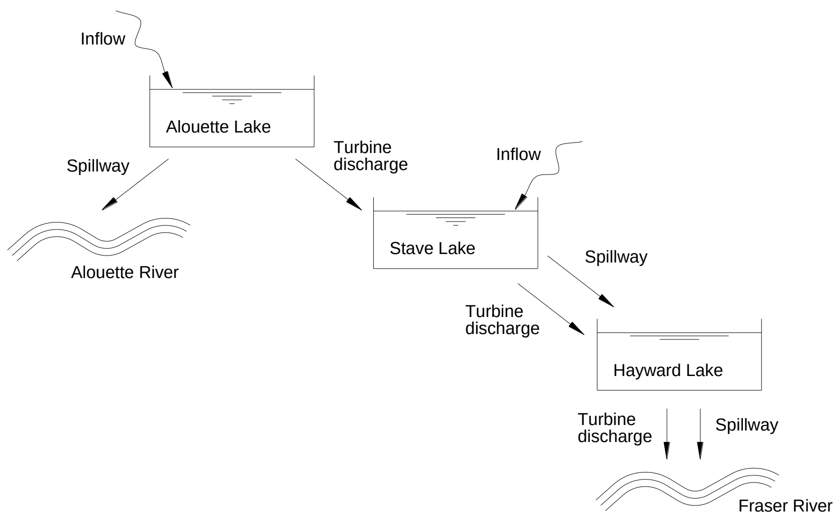

Figure 2: Hydraulic configuration of the Stave Falls project.

Table 1: Link1 and Link2 values for the Stave Falls project.

	Link1 flag (spillway discharge)			Link2 flag (turbine discharge)		
	Alouette L.	Stave L.	Hayward L.	Alouette L.	Stave L.	Hayward L.
Alouette L.	0	0	0	0	1	1
Stave L.	0	0	1	0	0	1
Hayward L.	0	0	0	0	0	0

Results and Discussion

After defining the preferred operating regimes considering different demands such as flood control, recreation and environmental protection, the optimization model was run repeatedly by gradually increasing the reliability levels for equations 15 and 16 in each run. The selected LDR conditioned the model to keep the elevation within the preferred values throughout the optimization. Table 3 presents the results for two different alternatives. Although it is possible to specify different reliability levels during the length of the study period, in this case they were kept constant. In Alternative 1, the reliability levels were increased as much as possible, thereby giving priority to constraint 16. In Alternative 2 the reliability levels were relaxed, with the exception of the reliability level for the minimum spill of Alouette Lake. It can be observed that the annually generation increases when the reliability levels are decreased.

Table 3: Model results

	Alternative 1			Alternative 2		
	α	β	Annual Generation (MWh)	α	β	Annual Generation (MWh)
Alouette L.	0.45	0.80	45,000	0.60	0.60	53,200
Stave L.	0.60	0.85	193,100	0.60	0.60	197,900
Hayward L.	0.60	0.85	239,500	0.60	0.60	245,800

REFERENCES

[1] Revelle C., Whitlatch E. and Wright J. "*Civil and Environmental Systems Engineering*", 2nd edition, Pearson Prentice Hall, (2004)

[2] Shawwash Z., "*A Decision Support System for Real-Time Hydropower Scheduling in A Competitive Power Market Environment*", University of British Columbia, (2000).

[3] Sigvaldason O., "A Simulation Model for Operating a Multipurpose Multireservoir System", *Water Resources Research*, Vol. 12, No. 2 (1976), pp 263-278.

[4] Can E. and Houck M., "Real-time Reservoir Operation by Goal Programming", *Journal of Water Resources Planning and Management*, Vol. 110, No. 3 (1984), pp 297-309.

[5] Oliveira R. and Loucks D., "Operating Rules for Multireservoir Systems", *Water Resources Research*, Vol. 33, No. 4 (1997), pp 839-852.

[6] Marler R. T. and Arora J.S., "Survey of multi-objective optimization methods for engineering", *Structural Multidisciplinary Optimization*, Vol. 26, (2004), pp 369-395.

[7] Loucks D.P. and Dorfman P. J., "An Evaluation of Some Linear Decision rules in Chance-Constrained Models for Reservoir Planning and Operation", Water Resources Research, Vol. 11, No. 6, (1975), pp 777-782.

[8] Revelle C., Joeres E., Kirby W., "The Linear Decision Rule in Reservoir Management and Design. 1, Development of the Stochastic Model", Water Resources Research, Vol. 5, No. 4, (1969), pp 767-777.

[9] Loucks D., "Some Comments on Linear Decision Rules and Chance Constraints", Water Resources Research, Vol. 6, No. 2, (1970), pp 668-671.

[10] Sreenivasan K. R. and Vedula S., "Reservoir operation for hydropower optimization: A chance-constrained approach", Sadhana, Vol. 21, Part 4, (1996), pp 503-510.