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Abstract

A research study was conducted at an undergraduate college, comparing student experiences and successes in a mathematics course offered fully online, blended and face-to-face. In online courses, students enjoy the flexibility of learning at their own pace, not having to travel to school, as well as having consistent access to courses through a web browser. However, such conveniences do not automatically produce positive results. Some students lack the discipline, enthusiasm and sometimes feel socially isolated from their peers when learning online. Despite these challenges, online courses continue to be developed in response to the demand for online learning opportunities. However, there is a need to determine the impact of these opportunities on student learning. The majority of the participants in this study were satisfied with their learning experience. Furthermore, the results analysis did not find enough evidence to confirm that there were any significant differences in the achievement, as measured by students' final grades and exam scores based on whether the course was taught online, as a blended course, or face-to-face format. The researchers, therefore, concluded that students can attain the same level of academic achievement through online, blended, or face-to-face courses (measuring the final exam and course final grades).

Introduction

An undergraduate college located in Queens, New York recently approved one of its mathematics courses, *Computer Algebra Systems*, to be offered fully online. Computer Algebra Systems is a one-credit course (co-requisite to pre-calculus and pre-requisite to calculus 1) that traditionally meets 50 minutes per week face-to-face in a computer lab for which students receive a course grade. The course introduces students to a computer algebra system, *Mathematica*, which teaches them basic functions such as performing calculations, solving equations, graphing functions, transforming functions and more. Moreover, the course explores the use of these functions in relation to various pre-calculus topics. Students in this course also investigate real-world problem-solving scenarios related to pre-calculus topics. In addition to the course being offered fully online, as well as face-to-face, some of the sections used blended learning, as the class would meet online for some sessions and face-to-face for others.

Offering the course fully online, as well as blended, created more flexibility and made learning more convenient for students, enabling them to learn somewhat at their own pace. Students were able to access the online course content, such as videos, discussions, e-books, and others, more than once. This benefits the students as it allows them to access the lessons more than once and at any time. In the traditionally conducted Computer Algebra Systems course that runs only 50 minutes long, when teachers tried to get students to explore the content, recognize patterns, and interpret outputs, the lesson always felt rushed due to the time constraint. In the online and blended setting, students who take the courses cannot sit back with a blank face and not contribute to lessons. Instead, they have to be responsible and put in an effort. They are forced to participate in discussions and express their ideas in ways that others can understand. Students are also able to network with their classmates by journaling and chatting in order to better understand concepts and form conclusions. Discussing mathematical content can be challenging and intimidating for students. An instructor's role is important in facilitating the online component of a course, providing avenues for successful learning experiences and positive outcomes.

The change to the Computer Algebra Systems course in its being offered fully online and blended (in addition to the traditional approach) presented an immediate need to determine its impact on student learning. This study seeks to determine this impact by investigating students' experiences and comparing their successful performance in the Computer Algebra Systems course that is offered online, blended, and face-to-face.

Literature Review

The past decade has seen incredible growth in the development of the internet and increased use of technology in classrooms of all educational levels (kindergarten to college and beyond). The variety of available information, as well as advanced web-based tools, has caught the eye of the educational community, presenting teachers with vast amounts of enhanced methods for delivering knowledge to distance learners (Shachar & Neumann, 2010). Distance learning in higher education was viewed with skepticism in the past but has clearly become more accepted as a legitimate form of delivery in recent years (Shachar & Neumann, 2010). Online learning supports both real-time and asynchronous communication between instructors and learners and offers rich educational resources through multiple media (Means, et al., 2010). A rising number of higher educational establishments have adopted online learning (Council for Higher Education Accreditation, 2002), and now offer a variety of courses partially or fully online. Online course delivery "has the potential to change and revolutionize teaching and learning at every level of education" (Mills & Raju, 2011, p. 1).

Comparing Online Learning to traditional Methods

A vast amount of studies have compared online learning to traditional learning styles. Some studies report no significant difference with respect to the two settings (Jones & Long, 2013; Lenzen, 2013), while other studies have found that online learning outperforms traditional approaches (Love, et al., 2006; Hughes, et al., 2007). According to a study that was performed by the U.S. Department of Education (DOE) in 2009, which compared online learning to traditional learning for over 1,000 cases (from 1990 to 2009), "online learning has been modestly more effective, on average, than the traditional face-to-face instruction with which it has been compared" (Means et al., 2009, p. 14). However, several studies (Figlio, et al., 2010; Jaggars & Bailey, 2010) have criticized the DOE 2009 study and argued that some of the online learning cases reflected better performance compared to the traditional environment due to the blending of online instruction with in-person assistance. In 2010, the U.S. DOE updated its findings and found that after comparing online, face-to-face, and blended environments, online learning appeared to be just as effective as traditional classes. The DOE found that blended instruction was more effective than pure online or pure face-to-face instruction.

Clark (2012) feels that the main advantage of blended learning is that the instructors have the opportunity to provide students with web-based, interactive self-study and self-paced tools. In addition, follow-up face-to-face sessions reinforce what students learn. In a blended learning environment, online meetings "sometimes force students to be more prepared and to participate more actively in the learning process than they might while sitting in the classroom" (Chen & Jones, 2007, p. 12). As per Silver in the Yearbook of the National of Teachers of Mathematics, "student verbalization [publicly] can not only help teachers to gain insight into the knowledge and thinking of their students, but also furnish a powerful way for students to learn from each other" (Martinovic, 2004, p. 2).

Is Online Learning Right for Everyone?

In Kearsley's opinion, online learning is not a suitable learning environment for every student, teacher, or even for every subject area, and that "students without the necessary self-discipline and study skills find [this type of] teaching medium frustrating" (in Engelbrecht & Harding, 2005, p. 271). Toch (2010) has stated that "the younger and more disadvantaged students are, the more they need school to be a place rather than merely a process, the more they need to be connected to a network of adults supporting them in many different ways every day" (p. 1). Allen (2001) furthermore believes that "a key ingredient to the success of the online course is a clientele that has very good reading skills" (p. 5). He, as well as Burton & Goldsmith (2002), pointed out the greater need for students to be self-disciplined and self-motivated if are going to be successful in their online courses.

Engelbrecht & Harding (2005) previously observed that "excellent classroom teachers do not necessarily make good online instructors and furthermore institutions should not insist that all instructors teach online" (p. 271).

McQuiggan (2012) has observed that most faculty have no experience teaching online, having spent most of their years learning with the traditional form of teaching, and further recommends that teachers pursue professional development to learn to teach online effectively.

Learning Mathematics Online

As a direct result of the growth in technological capabilities and the growing popularity of both online and blended courses, online mathematics courses are being developed to meet the increase in student needs (Jones & Long, 2013; Engelbrecht & Harding, 2005; Lenzen, 2013). According to Xu & Jaggars (2013), despite the issues, "online learning is an important strategy to improve course access and flexibility in higher education, especially in community colleges, with benefits from both the student perspective and the institutional perspective". However, a challenge that presents itself to the mathematics education community is the lack of experiential results and discussion regarding the efficacy of these types of courses. For this reason, the study seeks to contribute to the research in investigating student experiences learning mathematics online.

Purpose of the Study

This study sought to compare student's learning and other success outcomes in a mathematics course offered in three different learning environments - online, blended, and face-to-face. Additionally, it investigates students' satisfaction levels with their learning environments because students' levels of satisfaction, as well as motivation levels are significantly related to learning outcomes (Bryant et al. 2005, Lee, 2014). When students are satisfied with an online learning experience, they tend to be more motivated to learn the content of the course (Graham & Scarborough, 2001). The research questions (RQ) that guided this investigation are as follows:

RQ1. What are student perceptions of their prior online learning experiences?
RQ2. What are students' attitudes/opinions of learning math online? What are their overall levels of satisfaction, and which obstacles do they encounter when learning online?
RQ3. In what ways are students in these courses engaged in the learning of mathematics?
RQ4. How, if at all, does this engagement differ from the traditional setting?
RQ5. Are there differences in student achievement, measured according to course final grades and final exam scores, based on whether the course is taught in an online, blended, or traditional format? If so, what are these differences?

Methodology

Sample Selection

For two semesters, data was collected from six sections of the Computer Algebra Systems (CAS) course, for which a researcher (Assistant Professor) taught two sections online and two sections blended. Another faculty member (Adjunct Lecturer - PhD student) taught two of the traditional face-to-face sections. Both researchers received training, had experience teaching online, and had taught the CAS course for over three years prior to this study.

A total of 95 students from these sections agreed to participate in the study. The online section had 20 participants; the blended had 46, and the face-to-face section had 27 participants. A majority of the students were freshman majoring in scientific fields (math, biology, computer science, health science, and more) which did not surprise the researchers, as this course was a co-requisite for a pre-calculus course and prerequisite for calculus 1. This course was often among the first courses students took in math if they were pursuing a STEM-related field.

Course Structure

The purpose of the CAS course was for students to be introduced to the software Mathematica (a CAS), and for them to explore pre-calculus topics through the use of the software. As mentioned in the introduction of this paper, some of the topics included graphing with Mathematica; obtaining information from the graph of a function; transforming functions, including reflection; and modeling our world with functions and more. Students were given access to a free copy of the software to install on their personal computers in order for them to practice (outside of class) what was being taught, complete their assignments, and explore the software further on their own. The college also had the software installed on computers in laboratories for students to use if they wanted to. Both instructors visited each other's classes (the traditional course instructor was given access to the online and blended course through Blackboard) to take field notes, describe their experience and report on student engagement and the differences between the classes.

All course sections used a common syllabus, according to which both instructors taught the same topics each week. The researcher (Assistant Professor) designed all of the learning modules for each of the course sections. The learning material aides were posted on Blackboard. The researcher also developed assessment tools, such as problem sets/activities that were used for both online and traditional classes so that students could be assessed similarly. Grading rubrics were developed and used so that the assessments would be graded the same way as well.

Traditional Face-to-face Environment

The traditional course section met for 50 minutes per week in a computer laboratory. Students were provided with a computer to work on individually and logged into Blackboard to access their lessons. The instructor would project the lesson through an overhead display, and students explored pre-calculus topics like graphing functions with Mathematica and shifting them vertically. If this were the lesson topic for the class session, for example, the instructor would review how to graph using Mathematica. Next, students would discover what happens to the graph of a basic function when one adds a positive constant or subtracts a positive constant from in order to learn about vertical shifting.

Students from the traditional class benefited very much from their setting because the lesson was always taught in real time. Any of the students' questions or comments received immediate feedback from the instructor or another student. Teaching in real time also allowed the instructor to ask questions based on the class' responses. The students' facial expressions offered signals and sometimes caused the instructor to overemphasize a part of the content instruction that they might not be understanding. For example, some students struggled to get the right output in Mathematica because of the strict syntax rules. When plotting functions, if they forgot to capitalize the "P" in Plot or did not include a semicolon or square bracket where it was needed, the function would not be graphed, and sometimes students had trouble interpreting the software feedback as to why they did not get the correct output. In these cases, for example, the instructor would emphasize parts of the lesson and display various examples of content to help students learn.

The drawback in the traditional setting reported by the instructor was that the lesson always felt rushed due to the fact that the course only met for 50 minutes (per week). By the time the students and the instructor settled into the classroom, almost 10 minutes would have passed since the start of class because students would have to boot up the computer, sign in with their network ID, sign into Blackboard, and then download their lesson for the day. This took away a great deal of class time. Getting students to connect with the lesson, explore the content, recognize patterns, and interpret outputs also presented challenges. Students were sometimes distracted by the computer in front of them and tempted to browse through their email or visit YouTube. Some had these applications up on their screen along with the lesson they were looking at. This presented a distraction from their learning.

Online Environment

Students in the online setting learned completely online with the use of personal videos, e-books, PowerPoint slides and Mathematica tutorial files, which the researcher (the instructor) created herself. Web resources and YouTube videos were also used as learning aids in these sections. The researcher chose to create her own videos to make the class more personal so that the students would hear her voice and not just a random (but relevant) person on YouTube presenting and reviewing the material. If students had problems understanding a concept or

instruction on how to input a command in Mathematica, another video was sometimes created to address the misunderstanding. However, because the researcher had taught the CAS course for so many years, she was able to highlight in her personal videos common misunderstandings students experience when learning Mathematica and exploring math. She believed this helped the students learn better.

The advantage to the online section was that students could access their lessons over and over again for the duration of the semester. Just about every week, the students were presented with a new topic. Within that week, they learned a concept through the lesson materials online. Additionally, throughout the week they were engaged in activities to complete, such as a dialogue with the class as whole or in groups answering specific discussion questions (DQ). Students would have to read and respond to others' posts, and provide substantive feedback rather than just writing "I agree." An example of a DQ used in this class is as follows: "*Describe to me as if I were your friend how to go about finding the domain and range of a function. Provide 3 example functions. State the domain and range of each of your examples along with the graphs of each*." This type of DQ not only encouraged the students to dig deeper into the content and talk about it in meaningful ways, but it also gave the instructor an avenue through which to connect the discussion topic with previous topics learned or topics that were forthcoming. In addition, it was critical that the instructor provide clear, substantive, and constructive feedback to the students in a reasonable amount of time. It was important for the instructor to be present in the forum. Discussing mathematical content can be challenging and intimidating for some students to do, especially those who do not have a solid math background. These instances on the online forums presented teachable moments. The instructor's role was important in facilitating the online component, providing avenues for students to have a successful learning experience and reach positive outcomes. The online forums were where the instructor gained some type of notion of whether or not students were engaged in learning the lesson topic and how they were engaged.

Students from the online setting benefited from having ample time to discuss the content that was presented for the week. They were given days to respond to a DQ, which gave them more time to learn the content before posting. However, the drawback was that students had to wait for their teacher or classmates to respond to their posts. You can see from the student's open-ended responses and interview results later in this article (see the Results section, RQ2) how students felt when their classmates did not post their work at all or took time to add to the discussion, even when there were strict deadlines by which students should post.

Blended Environment

Students from the blended setting met 33% online and 67% face-to-face. The face-to-face meetings mimicked the traditional setting, and the class met for 50 minutes (during the face-to-face sessions). They were taught in real time during the online meeting times and went through the lessons that were posted on Blackboard. Students were able to follow what was being taught, and when exploring content, the teacher was right there to provide immediate feedback. When the blended course met face-to-face, they did not have access to the teacher-made videos, as those videos were only made available during the online lessons.

Students in the blended setting benefited from the online meetings as they did not have to travel to school to learn. They were able to access the lessons online and had ample time to respond to the DQs, which were posted for weeklong durations, not just 50 minutes. However, unlike the online setting, the blended sections sometimes met face-to-face for students to ask the instructor their questions and get immediate feedback. Some students would use the face-to-face time to get clarification on what was learned during the online lessons.

Data Collection

Several instruments were used to collect data for this study. In order to answer RQs 1 and 2, two surveys were administered to the participants. The first survey aimed to gain information about the student's background experience learning online or in a blended setting. The second survey collected students' opinions, attitudes, obstacles, and overall satisfaction levels toward learning math online. The surveys had multiple choice questions, Likert-type questions, as well as open-ended questions. A select amount of students were interviewed and given a chance to elaborate on their answers to the surveys.

Both instructor researchers visited each other's class (the traditional course instructor was given access to the online and blended course on Blackboard) and took field notes to gain a better understand of the students'

learning experiences and report on student engagement and differences between the classes. This information was used to answer RQs 3 and 4.

The semester ended with in-person final exam that took place on campus for all of the course sections. The final exam was similar for all of the sections and graded by both researchers using a rubric scale. The results of the final exam as well as the final course grade were used to answer RQ 5.

Data Analysis

Fathom software was used to examine the data collected from both surveys. Data results were divided into tables, including overall percentages and means. Open-ended responses from the survey instruments were categorized and coded separately by both researchers based on inter-rater reliability (Kottner, et al., 2011), and Cohen's Kappa (Cohen, 1960) was calculated for a demonstration of the overall level of agreement between the two researchers. Interview question results were transcribed and reported verbatim and in narrative form.

The Kruskal Wallis (Weaver, et al., 2017) test was used to determine whether there were differences in students' levels of achievement, measured according to their course final grade and final examination test scores in the three different settings (online, blended and face-to-face). The null hypothesis was determined to be zero, and for statistical testing, any p-value below 0.05 reflected significance. Before using the Kruskal Wallis test, researchers tested the data for the normality and homogeneity of variance using boxplots, the Shapiro-Wilks test, and the one-way Anova test.

Results

The results from Survey 1 (S1) and Survey 2 (S2) were used to answer RQs 1 and 2.

RQ1: What are student perceptions of their prior online learning experiences?

At the beginning of the semester, students were given a background survey (S1) on their prior experiences learning online in order to answer RQ1. The survey contained multiple choice questions, a Likert-type question, as well as open-ended question. Although 95 participants signed up for the study, only 64 participants' data was used to answer RQ1 (Online – 17; Blended – 25; face-to-face – 22). If participants simply did not answer the survey questions, or gave invalid responses, their feedback was not counted in the answer of this research question.

The survey questions asked the participants how many online courses and blended courses they were currently taking/have taken in the past. It also asked them to identify their two top reasons for taking the online and/or blended course, with eight choices for them to select from. Students were also able to elaborate (open-ended) on their top reasons if they picked "none of the reasons above" for Q2 and Q3, and for Q5 and Q6. Lastly, students were asked to identify how confident they felt about taking an online/blended course.

S1.Q1: How many online courses are you currently taking/have taken in the past?

Table 1. S1.Q1 Descriptive results

	Learning Environment		
Choices	Online	Blended	Face-to-face (f2f)
0 (Never)	0.0%	60.0%	68.2%
1	47.1%	32.0%	13.6%
2	23.5%	4.0%	13.6%
3	11.8%	0.0%	0.0%
4 or more	17.7%	4.0%	4.6%

Table 1 indicates that nearly half (47.1%) of the students in the online setting admitted to experiencing online learning for the first time, while the other portion (52.9%) of students admitted that they were currently taking/have taken two or more online courses in the past. As for the blended and face-to-face (f2f) settings, a

majority of the students (60% in the blended setting and 68.2% in the online setting) disclosed that they were not currently taking an online course nor had they taken one in the past.

S1.Q2 and S1.Q3: Identify the top reason and second top reason you are currently taking (or have taken) an online course.

More than half (70.6%) of the students in the online setting admitted their top reason for taking an online course was "convenience and/or flexibility" (see Table 2), with their second top reason being that "they wanted to experience learning online" (picked by 41.2% of the students). However, the top reason of convenience and/or flexibility did not explain the students who were in the blended and in the f2f settings, because a majority of them (60% of the students in the blended and 68.2% in the f2f) (see Table 2) admitted to not having experience learning online. As for those in the blended and f2f settings that had experience learning online, 20% in the blended mostly indicated "face-to-face or blended wasn't an option or did not fit their schedules" as their top reason, and 13.6% of students in the f2f setting mostly picked "wanted to experience learning online" (see Table 2). The second top reason picked by students in the blended settings was "convenience/flexibility" (20%), and for the f2f students, the two second top reasons were "convenience/flexibility" (9.1%) and "cost-effectiveness" (9.1%) equally (see Table 2). Remarkably, none of the students selected "learn best online" or "cost effectiveness" as their top reasons for taking an online course (see Table 2).

Table 2. S1.Q2 & S1.Q3 Descriptive results

Choices	Learning Environment					
	Online		Blended		Face-to-face (f2f)	
	Top Reason	2nd Top Reason	Top Reason	2nd Top Reason	Top Reason	2nd Top Reason
Convenience/Flexibility	70.6%	23.5%	8.0%	20.0%	9.1%	9.1%
Cost – effectiveness	0.0%	5.9%	0.0%	4.0%	0.0%	9.1%
Face-to-face or Blended was not an option or did not fit schedules	17.7%	17.7%	20.0%	0.0%	0.0%	4.6%
You learn best in a blended learning environment	0.0%	5.9%	0.0%	4.0%	0.0%	0.0%
You wanted to experience learning online	11.8%	41.2%	8.0%	12.0%	13.6%	4.6%
Familiarity with the instructor	0.0%	5.9%	4.0%	0.0%	4.6%	0.0%
None of the reasons above	0.0%	0.0%	0.0%	0.0%	4.6%	4.6%
N/A (Implying you have never taken an online course before)	0.0%	0.0%	60.0%	60.0%	68.2%	68.2%

S1.Q4: If you picked "None of the above" (for the previous two questions), please elaborate here on the top reason(s) you are currently taking (or have taken in the past) an online course.

All students from the online and blended settings made their first and second choices from the list of different options. Only one f2f student picked "None of the above," and noted that her reason was that the course is required for her/his major.

S1.Q5: Number of blended courses currently taking/have taken

Table 3. S1.Q5 Descriptive results

Options	Learning Environment		
	Online	Blended	Face-to-face (f2f)
0 (Never)	58.8%	0.0%	50.0%
1	11.8%	76.0%	22.7%
2	17.7%	12.0%	13.6%
3	0.0%	8.0%	0.0%
4 or more	11.8%	4.0%	13.6%

While a majority (58.8%) of students in the online setting (see Table 3) claimed they were not currently taking nor had they taken taken in the past a blended course, a portion of them (41.2%) admitted that they were currently taking or have taken a blended course at least once at the time of survey completion. Similarly, half the amount of students in the f2f setting disclosed that they had not learned in a blended setting in the past (or current semester), while the other half admitted to having taken one or more blended courses in the past or current semester. A majority of the students (76%) in the blended section expressed that they were experiencing blended learning for the first time, while 24% of the students in this setting noted that they were currently taking or had taken two or more blended courses in the past.

S1.Q6 – S1Q7: Identify the top reason and second top reason you are currently taking (or have taken) an online course.

At least half of the students in the online and in the f2f settings admitted to never having taken a blended course before. As for those who were currently taking a blended course, or had done so in the past (see Table 4), "convenience/flexibility" was the top reason in the online setting (29.4%) as well as the f2f setting (13.6%). The "none of the above" choice was also picked 13.6% of those participating in the f2f setting. A majority of the students in the blended sections also mostly picked "convenience/flexibility" (36%) as their top reasons. Interestingly, none of students from the blended and f2f settings picked "cost-effectiveness" as their first top reason. However, other than "convenience/flexibility," "cost-effectiveness" was the only other option picked as the top reason in the online setting.

The second top reason picked by the online students (17.7%) and picked 24% of the time by students in the blended section was that "face-to-face or totally online wasn't an option or did not fit schedule" (see Table 4). Interestingly, the reason **"***None of the reasons above"* was second top pick in the blended (32%) and f2f (18.2%) settings.

Table 4. S1.Q6 & S1.Q7 Descriptive results

Choices	Learning Environment					
	Online		Blended		Face-to-face	
	Top Reason	2nd Top Reason	Top Reason	2nd Top Reason	Top Reason	2nd Top Reason
Convenience/Flexibility	29.4%	5.9%	36.0%	16.0%	13.6%	9.1%
Cost – effectiveness	11.8%	0.0%	0.0%	0.0%	0.0%	4.6%
Face-to-face or Blended wasn't an option or did not fit schedules	0.0%	17.7%	16.0%	24.0%	4.6%	0.0%
You learn best in a blended learning environment	0.0%	11.8%	12.0%	20.0%	4.6%	4.6%
You wanted to experience learning online	0.0%	5.9%	4.0%	0.0%	9.1%	4.6%
Familiarity with the instructor	0.0%	0.0%	8.0%	8.0%	4.6%	9.1%
None of the reason above	0.0%	0.0%	24.0%	32.0%	13.6%	18.2%
N/A (Implying you have never taken an online course before)	58.8%	58.8%	0.0%	0.0%	50.0%	50.0%

S1.Q8: If you picked "None of the above" (for the previous two questions), please elaborate here on the top reason(s) you are currently taking (or have taken) an online course.

Three students (two from the blended and one from the f2f sections) responded that they did not even know they were signed up for a blended course when they took it. In another response (from the blended setting), a student said that he or she was "advised to be open-minded and challenge myself with a hybrid course." From the f2f setting, another said that his or her reason(s) for taking a blended course was that he or she wanted to see what learning online would be like and still have the opportunity to learn from a professor. Lastly, three students (two from the blended and one from the f2f) implied that they were taking the course because it was a requirement to fulfil their major.

S1.Q9: Overall, how confident you felt taking a class online

A majority of students (76.4%) in the online setting and in the blended setting (64%) felt some level of confidence taking an online course (see Table 5). Although 68.2% (a majority) of the students in the f2f setting admitted to never experiencing this environment, 18.2% in this setting admitted to feeling some level of confidence taking an online course. Less than 5% of students in the f2f setting as well as in the blended felt fearful of taking an online course. The student who picked the option "N/A" from the online setting may have misread the question or simply picked that option in error because all of the online students were currently taking an online course. However, this response did not change the overall results of this survey question.

Table 5. S1.Q9 Descriptive results

	Learning Environment		
Options	Online	Blended	Face-to-face
Very confident	52.9%	32.0%	4.6%
Somewhat confident	23.5%	32.0%	13.6%
Neutral	11.8%	20.0%	9.1%
Somewhat fearful	5.9%	12.0%	0.0%
Very fearful	0.0%	4.0%	4.6%
N/A (Implying you have never taken a class online/blended before)	5.9%	0.0%	68.2%

RQ2: What are students' attitudes/opinions toward learning math online? What are their overall levels of satisfaction, and which obstacles do they encounter when learning online?

Toward the end of the semester, students in the online and blended settings were issued the second survey (S2), which aimed to collect their overall opinions and attitudes learning math online as well as learn about the obstacles encountered. Fourteen participants in the online setting and 35 in the blended setting responded to the second survey, which contained 18 Likert-type (quantitative) questions and four open-ended (qualitative) questions. The Likert-type questions presented five attributes that students had to choose from, indicated by numbers such as: 5-Strongly agree, 4-Agree, 3-Neutral, 2-Disagree, and 1-Strongly disagree. The responses were analyzed using basic descriptive and inferential statistics. Data from the open-ended questions was considered and analyzed in hopes that the results might help the researchers fully interpret students' overall satisfaction with learning math online.

Six participants from the online/blended settings were interviewed to further elaborate on their opinions, attitudes, and obstacles with regard to learning math online. The interviews were conducted on a voluntary bases, and it took a minimum of 10 and maximum of 30 minutes to complete. The interviewees were paid in cash $10 after completing the interview. The responses were then transcribed and analyzed in order for researchers to answer this research question.

S2.Q1 – Q18 Likert-type-question results

Based on a general examination of the percentages in Tables 6 and 7, it can be seen that all of the Likert-type questions received the rating of mostly "strongly agree" or "agree" in both the online and blended settings. In the online setting, none of the questions were rated "disagree" nor "strongly disagree," but this was not the case in the blended setting. These students disagreed mostly with the statement about finding it easy to communicate with other students online (Q4) and disagreed that technologies required for the course were available and easy to download (Q5). Furthermore, out of all of the ratings in the blended settings, Q4 and Q5 received the lowest average ratings (3.71 and 3.91, respectively). The mean score ratings were greater than or equal to four for the online setting, and greater than four for the blended setting (all but for Q4 and Q5).

Table 6. S2.Q1 – Q18 Descriptive results for online setting ($n = 14$)

Survey 2 Questions (S2.Q1-Q18)	Mean	SA	A	N	D	SD	Missing
Q1: This course met my expectations.	4.42	50%	42.9%	7.1%	0.0%	0.0%	0.0%
Q2: My individual learning needs were met.	4.50	57.1%	35.7%	7.1%	0.0%	0.0%	0.0%
Q3: The instructor provided a clear clarification of assignment details, due dates and grading scheme.	5.00	92.9%	0.0%	0.0%	0.0%	0.0%	7.1%
Q4: I found it easy to communicate online with other students in my class.	4.07	35.7%	35.7%	28.6%	0.0%	0.0%	0.0%
Q5: The technologies required for the course were available and easy to download.	4.50	57.1%	35.7%	7.1%	0.0%	0.0%	0.0%
Q6: There were clear descriptions of technical support offered.	4.21	35.7%	50%	14.3%	0.0%	0.0%	0.0%
Q7: The online portion provided sufficient resources for the course assignments.	4.50	64.3%	21.4%	14.3%	0.0%	0.0%	0.0%
Q8: The content videos and other online materials were relevant to learning objectives of the course.	4.79	78.6%	21.4%	0.0%	0.0%	0.0%	0.0%
Q9: The course assignments were not too difficult.	4.00	14.3%	71.4%	14.3%	0.0%	0.0%	0.0%
Q10: Students have to be self-motivated to be successful in an online program.	4.58	14.3%	50%	35.7%	0.0%	0.0%	0.0%
Q11: My instructor guided and provided valuable feedback for discussion questions.	4.62	57.1%	35.7%	0.0%	0.0%	0.0%	7.1%
Q12: My instructor provided valuable feedback for individual assignments and exams.	4.57	57.1%	42.9%	0.0%	0.0%	0.0%	0.0%
Q13: My instructor provided prompt feedback to my concerns/Questions.	4.64	71.4%	21.4%	7.1%	0.0%	0.0%	0.0%
Q14: The grading criteria/rubric was made available to me.	4.64	64.3%	28.6%	7.1%	0.0%	0.0%	0.0%
Q15: The grading criteria/rubric was clear.	4.57	64.3%	28.6%	7.1%	0.0%	0.0%	0.0%
Q16: The instructor provided a timely feedback for assessments turned in.	4.64	64.3%	35.7%	0.0%	0.0%	0.0%	0.0%
Q17: I was overall satisfied with my online course.	4.64	64.3%	35.7%	0.0%	0.0%	0.0%	0.0%
Q18: I would recommend this type of online class to others.	4.71	71.4%	28.6%	0.0%	0.0%	0.0%	0.0%

5- SA (Strongly agree), 4- A (Agree), 3- N (Neutral), 2- D (Disagree), 1- SD (Strongly disagree), Missing (Unanswered)

Table 7. S2.Q1 – Q18 Descriptive results for blended setting ($n = 35$)

Survey 2 Questions (S2.Q1-Q18)	Mean	SA	A	N	D	SD	Missing
Q1: This course met my expectations.	4.31	40.0%	51.4%	8.6%	0.0%	0.0%	0.0%
Q2: My individual learning needs were met.	4.31	37.1%	57.1%	5.7%	0.0%	0.0%	0.0%
Q3: The instructor provided a clear clarification of assignment details, due dates and grading scheme.	4.63	68.6%	25.7%	5.7%	0.0%	0.0%	0.0%
Q4: I found it easy to communicate online with other students in my class.	3.71	25.7%	40.0%	20%	8.6%	5.7%	0.0%
Q5: The technologies required for the course were available and easy to download.	3.91	37.1%	37.1%	8.6%	14.3%	2.9%	0.0%
Q6: There were clear descriptions of technical support offered.	4.06	34.3%	34.3%	28.6%	0.0%	0.0%	2.9%
Q7: The online portion provided sufficient resources for the course assignments.	4.31	51.0%	34.3%	11.4%	0.0%	2.9%	0.0%
Q8: The content videos and other online materials were relevant to learning objectives of the course.	4.57	68.6%	22.9%	5.7%	2.9%	0.0%	0.0%
Q9: The course assignments were not too difficult.	4.09	31.4%	45.7%	22.9%	0.0%	0.0%	0.0%
Q10: Students have to be self-motivated to be successful in an online program.	4.23	37.1%	51.4%	8.6%	2.9%	0.0%	0.0%
Q11: My instructor guided and provided valuable feedback for discussion questions.	4.57	60.0%	37.1%	2.9%	0.0%	0.0%	0.0%
Q12: My instructor provided valuable feedback for individual assignments and exams.	4.59	62.9%	28.6%	5.7%	0.0%	0.0%	2.9%
Q13: My instructor provided prompt feedback to my concerns/Questions.	4.66	68.6%	28.6%	2.9%	0.0%	0.0%	0.0%
Q14: The grading criteria/rubric was made available to me.	4.59	60.0%	34.3%	2.9%	0.0%	0.0%	2.9%
Q15: The grading criteria/rubric was clear.	4.40	48.6%	42.9%	8.6%	0.0%	0.0%	0.0%
Q16: The instructor provided a timely feedback for assessments turned in.	4.51	57.1%	37.1%	5.7%	0.0%	0.0%	0.0%
Q17: I was overall satisfied with my online course.	4.23	40%	48.6%	8.6%	0%	2.9%	0%
Q18: I would recommend this type of online class to others.	4.43	48.6%	45.7%	5.7%	0%	0%	0%

5- SA (Strongly agree), 4- A (Agree), 3- N (Neutral), 2- D (Disagree), 1- SD (Strongly disagree), Missing (Unanswered)

S2.Q19 – Q22 Open-ended-type-question results

Regarding the students' open-ended responses, where they were given a chance to elaborate on difficulties they may have experienced learning math online (see the actual questions in Table 8, Q19 - 22), both researchers read through and coded the responses independently. They transformed chunks of data into smaller phrases using the inter-rater approach (Kottner, et al., 2011). The inter-rater analysis resulted in a Kappa constantly above .84 ($p < 0.001$) for all of the question results, which established a reliability of coding, measuring outstanding agreement between the raters (Landis & Koch, 1977). The students did not report any difficulties with navigating the interface of the course. However, they did admit to having difficulties using the Mathematica software. For example, they had problems inputting the correct command to get the right answer. They had problems understanding polynomial/piecewise functions and graphing in Mathematica. Students also reported difficulties working in groups due to group members not completing their parts. Aside from these points, the students also experienced problems understanding discussion questions. For a complete list of difficulties with respect to the open-ended question and the results, see Table 8.

Table 8. S2.Q19 – Q22 Results for Survey 2

	Students' Responses	
Questions	Online	Blended
Q19: List any difficulties you experienced with the course interface (navigation of the course)	-None	-None
Q20: List any difficulties you had learning concepts that were presented in the online settings.	-graphing in *Mathematica* -understanding polynomial functions -inputting the correct expression in *Mathematica* to get the right answer	-applying certain formulas to the problems given -understanding piecewise functions -inputting the correct expression in *Mathematica* to get the right answer -understanding pre-assigned functions in *Mathematica* -the online videos were fast-paced
Q21: List any difficulties you had completing the discussion questions/individual assignments/exams.	-understanding and/or comprehending the discussion questions -not able to get immediate clarification from the professor when needing help -problems with group work due to group members not completing their parts	-understanding and/or comprehending the discussion questions -applying what was taught -keeping up with the due dates -group members did not participant -communicating with other group members -not able to get immediate clarification from the professor when needing help
Q22: List any difficulties you had with the online course.	-problems with downloading *Mathematica* -problems with group work due to group members not completing their parts	-completing the tasks on time -keeping up with the due dates -using *Mathematica* properly -the assignments sometimes did not correlate with the online lesson -working in group discussions -submitting work in Blackboard.

Interview Question Results

The interview results were transcribed and reported in Table 9 & Table 10 verbatim and narratively. The researchers did not report the responses that did not address the question(s) that were asked. For example, when asked "how does the amount of coursework in your online education program compare with traditional in-class instruction," one of the interviewees' responses referred to the difference in the settings of learning online as compared to face-to-face. He or she did not answer the intended question.

Overall, none of the interviewees reported disliking their online learning experiences, but expressed the pros and cons of learning online versus in the traditional classroom. The students felt they could address their own opinions through group discussions in the online class, but admitted that they sometimes missed face-to-face interaction with their teacher as well as being able to chat with their classmates in person (Table 9, Q1).

None of the student responses indicated any communication problems between themselves and the instructor. They felt their professor responded quickly to their concerns (Table 9, Q2), and that the feedback was constructive and given in a timely manner (Table 9, Q3). However, the communication difficulties they admitted to having were between them and their classmates. The main complaint made was that when having to complete an assignment in groups, not all group members completed their work on time, or they did not

communicate with the group until the last minute, which held back the assignment from being completed (Table 9, Q2 & Table 10, Q6–Q7).

Regarding how students resolved their technical issues, students reported that they received help from their professor, the college technical staff, Blackboard specialist, or themselves (Table 9, Q4). They believed that the online environment (the navigation of the course, layout, graphics) was clear and easy to use (Table 9, Q5), and that the coursework in the online and in the traditional settings were the same (Table 10, Q6). For a complete list of interview responses, see Table 9 & Table 10.

Table 9. Interview results (Q1-Q5)

	Responses from 6 Interviewees
Questions	
Q1: As a student, how does your experience of online educational programs compare with traditional in-class instruction? How do you like or dislike it?	(1) Online learning is better environment for the introverted student than going to the traditional classroom because they can get their "true thoughts online [in] the class discussion". (2) "You get more out of the traditional setting because you're required to sit there and solve [problems] and take the information in opposed to reading it on your own and having your own misinterpretation of the information." (3) "I liked the online course because it saved me a lot of time." However, sometimes it took over a day to get a response" from the professor, but in the traditional class, "you can just get an answer after asking right away." (4) I liked learning online because it allowed me to "do [my] work on the go when I am travelling." (5) The online experience was a lot easier for me. I liked the format of it. (6) I like the online environment, but miss being able to see my professor's facial expression when they were teaching, as well as being able to chat with my classmates in person. (6) The downfall being online is that sometimes you might end up in an online class where students "are not participating equally or engaged in the work [which] makes the experience much more difficult and less enjoyable."
Q2: In your online class, how do you feel about the communication between yourself and the instructor? Also, between you and other students in the class?	(2) Every problem or concern I had, she had gotten back to me within a couple of hours. (3) The communication "between me and the instructor, it was good", but "between me and the other students, it was not good and hard to talk to other students in the class online because many students did their assignments the last minute." (4) "It as easy for me to talk to my instructor, but when it came to the other students, I wasn't really getting the feedback from them. I would constantly email them but wouldn't hear from them until the assignment was due. So by that time I had to do the whole assignment by myself." (5) When it came to needing the professor, the professor got back to me quickly. (5) In the traditional classes, you don't get one to one face-to-face communication not unless you go to the professor's office. There is more convenience for the teacher to reach back to students in an online course than the traditional. (5) Group work was hard because not every student was on the same page with each other. (6)"My professor was extremely responsive, she responded within a matter of hours". (6) "As for my classmates, it was difficult to engage with [them], they were not very disciplined"
Q3: As a student, how do you view the feedback from the instructor? Is it in a timely manner? Constructive? Provide examples.	(1) "The feedback from the instructor was constructive; it was [given] in a good timely manner, maybe not as fast as I would like it, that is why I prefer f2f." (2) The instruction got back to me in a timely manner. "Whenever I did something wrong, she would get back to me and let me know what I did wrong and I would fix it." (3) "The feedback was amazing! She would show us where examples are. If we had a problem, she wouldn't tell us the answer but tell us where to look for the answer and she did it quickly" (4) "When I did get feedback, it was very clear. I understood if I did something right or if I needed to work on something in an area. I was able to ask the professor questions if I didn't quite understand what they [the email] was saying. Even though it was through email, I was able to get the feedback I needed." (5) The instructor provided feedback right away, so I knew what to do for my assignments. (6) My professor has been "really responsive and have given generous feedback and clear guidance.
Q4: How do you resolve your technical issues? Was the university technical support helpful? Who helped you mostly to resolve technical issues?	(1) The school help desk helped me. (2) "I was able to figure it out on my own." (3) "Mostly our professor helped us with technical issues." (5) The Blackboard specialist helped resolve my issues.
Q5: How did you view the online environment? For example: the navigation of the course, layout, graphics, user friendliness, etc.	(2) "The navigation and layout of Blackboard is convenient and user friendly." (4) "I think the interface was really clear. The feedback from the instructor and the videos helped me to see things clearly. (5) "It was easy. My class had information like how to go through Blackboard, where to find the assignments and lessons. It was clear for the class.

(1) – interviewee one; (2) – interviewee two; (3) – interviewee three; (4) – interviewee four; (5) – interviewee five; (6) – interviewee six

Note: Some interview questions adapted from "Students' perceptions towards the quality of online education: A qualitative approach", by Y. Yang, & L. F. Cornelius, 2004, Association for Educational Communications and Technology, 27, p. 861–877.

Table 10. Interview results continued (Q6-Q8)

Questions	Responses from 6 Interviewees
Q6: How does the amount of coursework in your online education program compare with traditional in class instruction?	(2) "I think it's pretty much the same." (3) "The amount of coursework for online and traditional was the same amount". (4) "I think it's pretty much the same [the online platform] considering you are not sitting in a classroom." (5) I have more time to complete my work when the class is online.
Q7: List any difficulties you had completing the discussions questions, individual assignment and exams.	(1) "With group assignments it was problems with group members that held back the group by them not posting on time". (2) No they [the discussions, individual assignment and exams] was all pretty standard. (3) The exams were fair, and the discussions were easy. However, with group work, "some student's didn't reply for a long time" and it was the entire group's responsibility for discussing. (5) There were problems with group work. The group members were not all on the same page because they were not online the same time. That posed problems.
Q8: List any other difficulties you had with the online course.	(1) There weren't any more problems. (3) Navigating through Mathematica. (4) Most of the difficulties what when having to complete group work. I didn't get a lot of feedback from group members. I had to prepare most of the work before the due date just in case the other group members didn't post. (5) I didn't have any other problems.

(1) – interviewee one; (2) – interviewee two; (3) – interviewee three; (4) – interviewee four; (5) – interviewee five; (6) – interviewee six
Note: Some interview questions adapted from "Students' perceptions towards the quality of online education: A qualitative approach", by Y. Yang, & L. F. Cornelius, 2004, Association for Educational Communications and Technology, 27, p. 861–877.

RQ 3: In what ways are students in the courses engaged in the learning of mathematics?

Engagement can be defined as evidence of students actively participating in a course, expressing interest and willingness to explore, and practicing concepts in order to attain quality learning. Research supports the fact that true engagement leads to student achievement, and learning improves when students are actively engaged in the course itself (Martin, 2012; Michael, 2006).

Students in the online setting engaged differently in learning math compared to the face-to-face sections. When developing the online portion of the course, and how students would actually learn, researchers were challenged to design the course delivery strategies so that the absence of face-to-face real-time learning would not hinder opportunities for students to engage in the lesson.

Students in the online setting learned the content asynchronously by watching prepared videos, reading eBooks, lecture notes, textbook pages, and other online materials that were uploaded to their class space on Blackboard. Students had ongoing access to these manipulatives throughout the semester, which gave them an advantage. The instructor's evidence of their engagement in the content came through their interactions on class discussion boards and group discussions. Several researchers favor the usage of discussion boards when trying to promote student engagement (Riggs & Linder, 2016; Petty & Farinde, 2013; Mills & Raju, 2011). It was important for the instructor to provide discussion topics that would get the students to reflect on what was learned, and to apply their knowledge of the content in meaningful ways. The discussion questions were open ended, requiring students to think critically and problem-solve. The teachable moments came when the instructor realized what level the students were at in understanding the content, and guided students in exploring it in order to gain a better perspective. These discussion topics also gave the instructor an avenue to connect the discussion topic with previous topics learned or topics that were forthcoming.

In an online course, the risk of students feeling isolated and disconnected is of great concern (Engelbrecht & Harding, 2005; Harmon, et al., 2014; Roberts & McInnerney, 2007), as it often leads to students dropping out of the course (Dai, 2007; Ali & Smith, 2015). For this reason, as recommended by Dixon (2010), the online sections included not just student-to-teacher communications, but also student-to-student interaction to help prevent any student from feeling secluded. This was done through the use of group work activities, during which students were asked, for example, to solve math problems and explore content together. It was quite essential that the instructor's presence was seen oftentimes in all discussion forums, making substantive comments to students' posts throughout the duration of the assignment and not just at the end of it (Lowenthal & Parscal, 2008). The instructor was also expected to answer students' questions in a proper amount of time, whether it was via the discussion board or an email. In Dixon's (2010) view, a teacher's "social presence [in an online course] is the phenomenon that helps translate virtual activities into impressions of 'real' people" and can help promote engagement. (P. 2)

Figure 1 below provides an example of a discussion activity, "*Try My Transformations*!" that was used in the online and blended sections. Students had to complete this activity individually. This assignment came after a lesson on transforming functions when students learned how to shift functions vertically and horizontally.

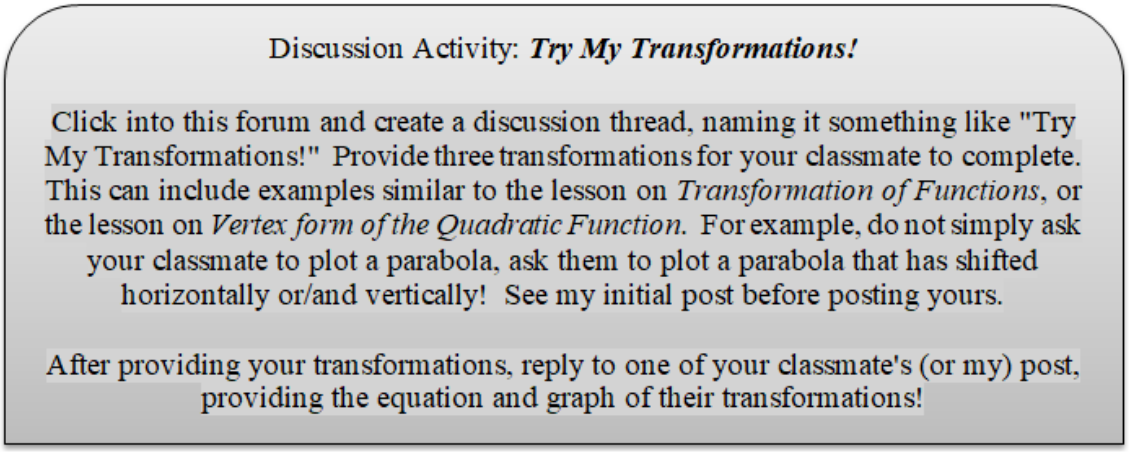


Figure 1. Example of a discussion activity

Figure 2 below displays three transformations that were posted by a student in the online class for his or her classmates to answer.

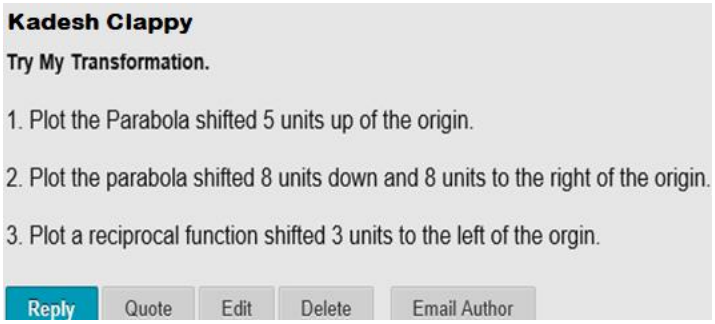


Figure 2. Response to discussion activity

The instructor's responsibility was to provide feedback and let students know whether they were correct or incorrect in their responses. An example of effective feedback could include the following statements:

> *Nice job Caleb! Your answer is quite correct. It's good you adding five to the outside of x^2 to make the function go upwards five units from the origin.*
>
> *Great job Keiko! Thanks for your efforts, however, your parabola shifted downwards five units from the origin and not upward as Kadesh asked, because you subtracted 5 from x^2. Look at the eBook and video again on vertical transformations to review how to shift functions upwards! Make this correction to your work and post again!*
>
> *Nice job Kayla! You shifted the function upward by five units. However, you did not plot a parabola. You plotted a cubic function. Please correct this and post again☺.*

At the end of the assignment, the entire class had a pool of examples of transforming functions. They were able to see each other's posts, make comments and learn from each other's mistakes.

Students in the online and blended sections also engaged with the content by working in small groups to answer discovery questions. Figure 3 below displays an example of this. In this activity, each student in the group was assigned a particular question to answer to in their group discussion area.

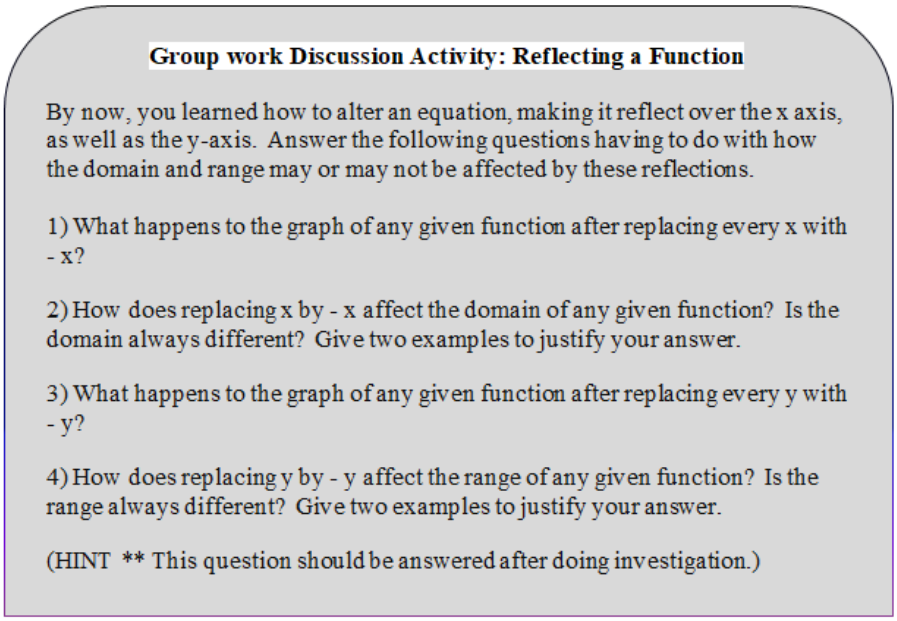


Figure 3. Example of group work

The instructor added to the discussion often throughout the assignment to make sure the responses were moving in the right direction. After the assignment ended, main key points were posted to the class as a whole by the instructor with the answers highlighted.

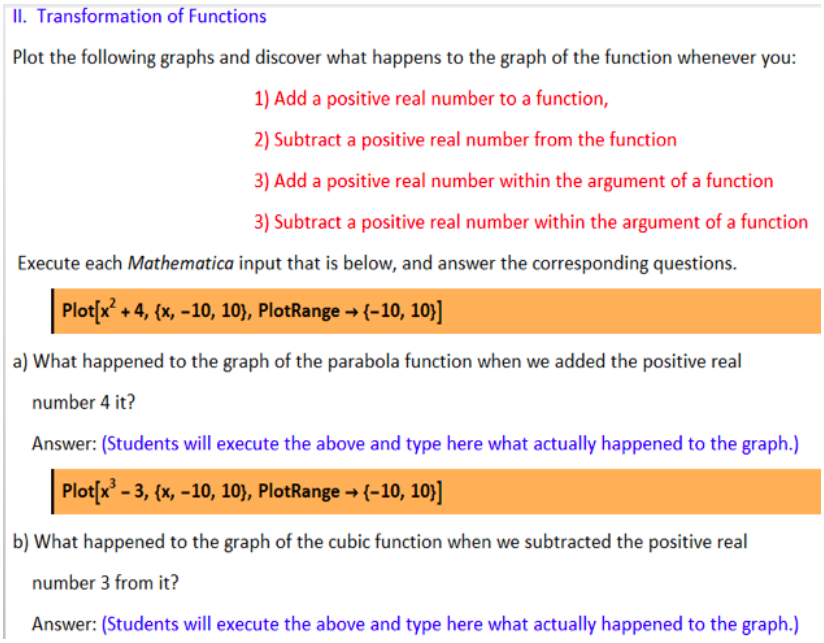
As part of another group activity also used in the online and blended course that promoted student engagement, the instructor provided the entire class with one sheet filled with problems on it pertaining to the lesson at hand. Each group was assigned the same amount of problems to answer from the sheet. They had one week to discuss the answers to the questions assigned to them and had to post their responses to the class discussion section (at the end of the week) so that the entire class could see their answers. The instructor did not let the groups know if their answers were correct or incorrect while they were working on the assignment. The instructor only answered general questions about how to apply what was learned to the questions. At the end of the assignment, all of the questions were answered from the sheet, which was graded by the instructor. This type of group work forced students to collaborate and learn from each other.

When students from the blended setting met online, they engaged in learning math the same way as in the online setting. When they met face-to-face, they mirrored the same experience as the traditional setting. The traditional course met face-to-face in a computer lab where each student worked from a PC. The lesson objectives were the same and the assignments were similar when compared to the online setting; however, the delivery method was different. In the traditional setting, the teacher taught in front of the classroom using lesson files that were projected from an overhead display. Students were expected to follow the lesson from their computer, as it was posted on Blackboard for them to access. The instructor facilitated student engagement by getting students to explore the content instead of simply listing rules to follow. For example, when teaching students about horizontal and vertical shifting (the same topic explained in the Methodology section), they were guided through a discovery of what happens to any given function f(x), whenever:

a positive real number "k" is added to it	f(x) + k
a positive real number "k" is subtracted from it	f(x) – k
a positive number "k" is added within the argument of a function	f(x + k)
a positive number "k" is subtracted within the argument of a function	f(x - k)

A glimpse of the classwork can be seen below in Figure 4. Students were asked to use the *Mathematica* code, and also discover the transformation function. They were asked questions such as:

> *What happened to the graph of the parabola function x^2 when we added the positive real number 4? What happened to the graph of the cubic function x^3 when we subtracted the positive real number 3 from it?*


II. Transformation of Functions

Plot the following graphs and discover what happens to the graph of the function whenever you:

1) Add a positive real number to a function,

2) Subtract a positive real number from the function

3) Add a positive real number within the argument of a function

3) Subtract a positive real number within the argument of a function

Execute each *Mathematica* input that is below, and answer the corresponding questions.

```
Plot[x^2 + 4, {x, -10, 10}, PlotRange → {-10, 10}]
```

a) What happened to the graph of the parabola function when we added the positive real number 4 it?

Answer: (Students will execute the above and type here what actually happened to the graph.)

```
Plot[x^3 - 3, {x, -10, 10}, PlotRange → {-10, 10}]
```

b) What happened to the graph of the cubic function when we subtracted the positive real number 3 from it?

Answer: (Students will execute the above and type here what actually happened to the graph.)

Figure 4. Lesson on transformation of function in the traditional class

At the end of this lesson, students were able to do transformations, such as the following, on their own:

> *Plot a parabola function that is shifted downward 3 units from the origin.*
> *Plot an absolute value function shifted to the right by 5 units and upward 20 units from the origin.*
> *Plot a reciprocal function that is shifted upward 2 units from the origin.*

Based on this lesson, the instructor was also able to introduce the concepts of domain, range, and asymptotes. The benefit of this face-to-face setting was that the instructor was there in real time to guide student engagement in learning. The instructors could see if the students were following the lesson and could change their approach according to the student responses. Students received immediate feedback from the instructor as well as from their classmates, which encouraged them and motivated their learning.

RQ 4: How, if at all, does this engagement differ from the traditional setting?

Students in the traditional setting engaged with the content differently than students in the online setting did in that they did not have the option to discuss the content online (through discussions/group activities) throughout the entire week with the class. They did not learn based on instructor-made videos. Some of the students (in the traditional setting) may have wanted to have that option as a way of refreshing their mind of what was taught. The traditional instructor promoted student engagement differently compared to the online setting, guiding students in exploring content, asking appropriate questions, and allowing students to work with each other to come up with solutions in real time. The blended setting engaged in the content the same way as the online setting, when the students met online. When they met face-to-face, they engaged in the lesson the same way as the traditional setting. However, students in the blended setting, unlike those in the online one, had the opportunity to ask follow-up questions during the weeks when they met online. That was a plus for them. Students in the blended setting also had the opportunity to engage with content throughout an entire week when they met online. The traditional setting did not have this option.

RQ 5: Are there differences in student achievement, measured according to course final grades and final exam scores, based on whether the course is taught in an online, blended, or traditional format? If so, what are these differences?

The semester ended with a face-to-face, in-class final examination, which students in all course settings (online, blended and f2f) took inside of a computer laboratory (at difference times). The test comprised of pre-calculus questions, and students had to use Mathematica to solve them. The exam was similar for all settings, and was

graded by both research instructors using a grading rubric. Students also received a course final grade at the end of the semester, and that grade represented their overall understanding and accomplishments during the course. The final exam grade and course final grade were used for the researchers to determine whether there were differences in the students' achievement based on whether the course was taught online, in a blended format, or face-to-face.

The following sections describe the course final grade and final exam data in terms of skewness, kurtosis, normality, and homogeneity assumptions. The Kruskal Wallis test was used to determine whether there were differences in achievement with respect to the settings. The null hypothesis was set to zero, and any p-value below 0.05 reflected significance. This next section specifies the results of this test.

Data Description, Normality & Homogeneity Assumptions:

Although 95 participants (online – 20, blended – 46, face-to-face – 29) signed up for the study, seven (online – 2, blended -3, face-to-face – 2) students withdrew from the course or did not take the final exam. As a result, they were not included in the analysis of this research question. It can be seen in Table 11 that the mean score for the online setting's final exam was greater than that of the blended and face-to-face settings.

Table 11. Final exam descriptive results

Variables	N	Mean	Standard Deviation
Online Final	18	74.72	13.58
Blended Final	43	62.56	20.26
F2f Final	27	66.04	25.50

The lowest final exam average was found in the blended setting. However, the standard deviations were quite large for all three settings, which meant the test scores were spread out vastly from the mean. The course final grade mean (see Table 12) was the highest in the online setting ($\sim B^{-}$) compared to the blended ($\sim C$) and face-to-face ($\sim C^{+}$), with similar standard deviation values.

Table 12. Course final grade descriptive results

Variables	N	Mean	Standard Deviation
Online Final	18	2.83	1.33
Blended Final	43	2.13	1.35
F2f Final	27	2.26	1.43

The final exam data (see Figure 5) from both online and blended settings was normally distributed, while the face-to-face setting scores seemed to be skewed to the left. These visual inspections were confirmed by the Shapiro-Wilk's test (Sharpiro & Wilk, 1965; Razali & Wah, 2011) for normality, which revealed the data from the online and blended settings did reflect some skewness and kurtosis, but did not differ significantly from normality. For example, data from the online setting had a skewness of -.165 (SE = .536) and kurtosis equal to -1.156 (SE = 1.038). The computation of the z value .553/.616, which was approximately -.90 (it does fall between ± 1.96), proved the setting approximately normal. Similarly, the blended setting final's exam data (using this same test) proved to be almost normal. However, the face-to-face final exam scores did not reflect a normal distribution. When the Sharpiro-Wilks test was used to test the normality for the course final grade data set, it revealed that the entire course final grades (for all three settings) was not normal.

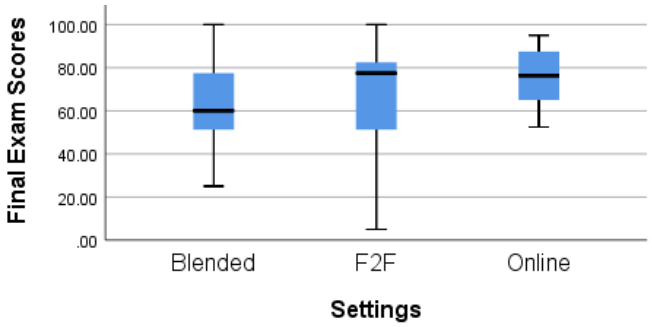


Figure 5 – Simple boxplot of final exam scores by settings

A one-way ANOVA test for homogeneity of variance was used to determine whether the final exam and the course final grade data for all three settings had similar distributions of scores. The test concluded that the assumption of equal variances for the course final exam scores ($F(2,85)=1.701$, $p=.189$) and course final grade ($F(2,85)=1.347$, $p=.265$) held for all three settings.

Kruskal Wallis

The results of the Kruskal Wallis test concluded that there was not enough evidence at a 5% level of significance to conclude that the final exam scores for the online, blended, and face-to-face course settings were different ($\chi^2(2) = 4.942, p = .085$). The Kruskal Wallis test also concluded the same result of no difference in the course final grade data distributions for the three settings ($\chi^2(2) = 3.704, p = .157$).

Discussion

The goal of this study was to compare student experiences and success in an undergraduate mathematics course, *Computer Algebra Systems*, which was taught online, in a blended format, and face-to-face. A majority of students in all three settings noted that they had not experienced taking an online nor blended course before taking the course involved in this study. The most picked reason for them taking (or have taken) an online or blended course was convenience/flexibility, which, according to other researchers, is one of the most listed reasons for students taking online/blended courses (Noel-Levitz, 2009; Harris & Martin, 2012).

A majority of students in the online and blended settings were satisfied with their learning experience. The researchers were pleased with this result because as implied by Graham & Scarborough (2001), when students are satisfied with their online learning experience, they tend to be more motivated to learn the content of the course. Specifically, all of the satisfaction statements (from survey 2) received an average ranking greater than or equal to four from the online setting. All but two of the satisfaction statements from the blended setting were ranked with an average score greater than four. The lowest ranked statement, receiving M=3.71 in the blended setting, was "I found it easy to communicate online with other students in my class." This did not surprise the researchers because the open-ended and interview responses from the students expressed students' dissatisfaction with working in groups online. Students in both the online and blended settings reported generally (in the open-ended and interview responses) that when they had to work in groups to complete a task, other group members did not participate well, causing, for instance, an extra task to fall on one person in the group. This reminded the researchers just how important it is for facilitators of online and blended courses to monitor group work closely and make sure issues are addressed quickly and appropriately. When students have problems working in groups, this promotes frustration and hinders learning. The feedback from the students encouraged the researchers to seek other ways to manage group work online, such as requiring inactive students to meet separately with the instructor, not only to explain their lack of attendance and lack of engagement, but to further explore the discussion question (or missed activity). This additional action might encourage students to participate better in group work. It may also help clarify concepts that a student may not have understood from a lesson.

Students also revealed (from the opened-ended and interview responses) that there were not any problems communicating with their instructor online. They generally reported that she got back to them quickly, providing clarification and clear directions. However, they did miss the face-to-face, real-time interaction with their teacher and classmates. The researcher wishes to improve the online and blended course by adding synchronized meetings that would give students opportunities to get immediate feedback from the professor, and provide opportunities for them to engage with other students in real time. Synchronized meetings can also be used to address issues students are faced with when using Mathematica to learn and explore pre-calculus topics, as this was another matter reported by the students as a difficulty.

This study also sought to determine whether there were differences in student achievement as measured by the course final grade and final exam scores based on whether the course was taught online, in a blended format, or face-to-face. The results contradicted the vast amount of studies that found differences in achievement with respect to these settings (Bain, 2012; Ashby et al., 2011, Gibson, 2008; Campbell et al., 2002; Carpinelli et al., 2006). The Kruskal Wallis test results in this study did not find enough evidence for researchers to determine that there were any significant differences in the achievement in the three settings (with respect to the course final grade and final exam scores). Based on these findings, the researchers believe that regardless of the setting,

students can achieve equal academic success across online, blended, and face-to-face courses (Jones & Long, 2013; Larson, & Sung, 2009; Vilardi, 2013).

Limitations and Recommendations

A clear limitation to this study is that the sample size was relatively small. The sample size was affected by freshman students that were under 18 years old and enrolled in the course. Students under 18 years old were not permitted to participate in the study, as per the college Institutional Research Board (IRB). The sample size was also affected by students that agreed to participate in the study but did not take time to complete the surveys. Additionally, as for the students who gave invalid responses to Survey 1, their replies were not used when RQ1 was analyzed. Another limitation was that although the courses were taught during two semesters, they were taught by the same professors. This study would have benefited from courses being taught by multiple instructors.

Conclusion

The development of online and blended courses in undergraduate colleges over the past decade has grown tremendously. Although they are still viewed with skepticism by many institutions and educators, these online courses continue to develop. It is important for researchers to continue assessing students' overall experiences, satisfaction levels, and learning outcomes to make sure quality learning is achieved.

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