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Mode Discrimination in Dissipatively Coupled Laser Arrays

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We show that dissipative coupling between an array of passive optical resonators creates a ladder of decay rates in the complex eigenfrequencies. This effect promotes mode discrimination in laser arrays, while, the lowest and highest-order modes exhibit the highest and lowest lasing thresholds, respectively. The array supermodes and their corresponding eigenfrequencies are calculated analytically through a tight-binding model, and the single-mode operation range is derived. The results are exemplified through finite element simulation of an array of transversely coupled semiconductor laser cavities. © 2019 Optical Society of America

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The peculiar dynamics of light in photonic lattices has been a subject of interest in optics for several decades. On the one hand, photonic lattices have been largely utilized to explore fundamental linear and nonlinear wave phenomena through controllable experiments [1, 2]. On the other hand, waveguide array configurations have been widely used in photonics for a variety of applications such as, integrated signal processing [3], wavelength-division multiplexing [4, 5], and mode-division multiplexing [6], while, coupled resonator arrays have shown promise for slowing down light [7]. In addition, in semiconductor lasers, array geometries have been long considered as a feasible route for scaling up the output power without crossing the intensity limits of individual lasers [8].

A common property in the above mentioned examples of optical waveguide/resonator arrays is the evanescent coupling between nearest neighbor elements, where, light can tunnel between two elements through the evanescent tail of their localized fields with negligible leakage of energy to the surrounding environment. One can refer to this process as dispersive coupling given that it results in frequency splitting of the eigenfrequencies of coupled resonators. On the other hand, two optical resonators can interact through dissipative coupling, where, the transfer of energy from one resonator to the other is accompanied with loss of energy to the surrounding environment. In this case, dissipation happens as an inevitable part of the transfer of energy from one cavity to the other. This can happen when two resonators are coupled via their radiative fields or through a secondary element which absorbs or scatters light outside the

system. As illustrated schematically in Fig. 1(a), the dispersive coupling of an array of ideal optical resonators results in the splitting of the eigenfrequencies of the supermodes along the real frequency axis. Alternatively, the resonators can be coupled through a scattering element, which, instead results in eigenfrequencies splitting along the imaginary frequency axis. The latter mechanism can thus discriminate the supermodes of an array of coupled resonators in terms of their decay rates, which is highly desirable for applications such as in mode filtering in laser arrays.

Fig. 1. An array of resonators interacting with (a) dispersive coupling, and (b) dissipative coupling. The associated eigenfrequencies are distributed along the real and imaginary axes, respectively.

Often resulting in undesired effects such as poor output beam qualities and temporal instabilities, multimode operation is considered a challenge in laser arrays [8]. For many applications, it is of interest to prevent certain modes from lasing in favor of, ideally, one lasing mode. A natural solution to this problem is to utilize loss for selectively filtering certain modes. To mention a few examples, previous approaches involve spatial filtering in an external cavity [9, 10], and utilizing radiation losses in antiguides [11]. In more recent years, interest has been raised to selectively filter some of the eigenmodes of an intrinsically multimode laser cavity by incorporating lossy counterparts in the system and by creating coupling between the active and lossy cavities. In particular, it is shown that selective parity-time symmetry breaking can result in single mode lasing [12–14]. In addition, it is shown that by creating phase-matching between certain number of modes, through the so-called optical supersymmetry, one can prevent the lasing of some modes through a lossy partner array [15–17].

In this letter, we introduce a novel array configuration which

allows for discriminating modes and promoting single-mode operation based on the concept of dissipative coupling. Consider an array of identical passive resonators with uniform dissipative coupling between nearest neighbors as shown schematically in Fig. 1(b). Here, we assume each resonator being at resonance at the frequency of ω_0 with an intrinsic decay rate of κ_ℓ , when considered in isolation from the other resonators. The rate of dissipative coupling between two neighbor resonators is assumed to be κ_e . Therefore, energy conservation demands an external decay rate for each resonator in addition to the internal losses. The external loss is equal to κ_e for the two resonators located at the edges of the lattice, and $2\kappa_e$ for all other resonators. Therefore, the coupled mode equations governing the modal amplitudes of the array can be written as:

$$\begin{aligned} \frac{da_1}{dt} &= (-i\omega_0 - \kappa_\ell - \kappa_e) a_1 - \kappa_e a_2, \\ &\vdots \\ \frac{da_n}{dt} &= (-i\omega_0 - \kappa_\ell - 2\kappa_e) a_n - \kappa_e (a_{n-1} + a_{n+1}), \\ &\vdots \\ \frac{da_N}{dt} &= (-i\omega_0 - \kappa_\ell - \kappa_e) a_N - \kappa_e a_{N-1}, \end{aligned} \quad (1)$$

where, in these relations, the subscript n represent the site number.

By defining $\mathbf{a}(t) = (a_1(t), a_2(t), \dots, a_N(t))^t$, equations (1) can be written in a matrix form as $d\mathbf{a}/dt = -iH\mathbf{a}$, where, H is a tridiagonal matrix with off-diagonal elements $H_{n,n+1} = H_{n,n-1} = -i\kappa_e$ and diagonal elements $H_{n,n} = \omega_0 - i\kappa_\ell - i2\kappa_e$ with the exception of the first and last elements which are $H_{1,1} = H_{N,N} = \omega_0 - i\kappa_\ell - i\kappa_e$. The supermodes of this lattice, defined as $\mathbf{a}(t) = \mathbf{v} \exp(-i\Omega t)$, satisfy the following eigenvalue equation for the eigenfrequencies Ω_m and their eigenvectors $\mathbf{v}_m$ ($m = 1, 2, \dots, N$):

$$H\mathbf{v}_m = \Omega_m \mathbf{v}_m, \quad (2)$$

This eigenvalue equation can be solved analytically as explained in the following. First, we consider the ansatz of $v(n) = \sin(\alpha n + \beta)$ for the eigenvectors, where, α and β represent two constants. It is straightforward to show that this ansatz satisfies Eq. (2) in the bulk lattice, i.e., for $n = 2, \dots, N-1$, as long as the eigenfrequencies are chosen as $\Omega = \omega_0 - i\kappa_\ell - i2\kappa_e (1 + \cos(\alpha))$. The unknown constants α and β can be found by imposing proper boundary conditions at the edges of the lattice, i.e., at $n = 1$ and $n = N$. Equivalently, one can consider two fictitious lattice sites at $n = 0$ and at $n = N + 1$, and fix their amplitudes, a_0 and a_{N+1} , such that the amplitudes at the edge sites, a_1 and a_N , follow the same equation as in the sites located in the bulk. The bulk evolution equation can be extended to the first, $n = 1$, and last, $n = N$, sites as long as $a_0 = -a_1$ and $a_{N+1} = -a_N$. By enforcing these two boundary conditions, one finds $\alpha = m\pi/N$ and $\beta = -\alpha/2$. Therefore, the complex eigenfrequencies $\Omega_m = \omega_m - i\kappa_m$ ($m = 1, 2, \dots, N$) are found to be:

$$\Omega_m = \omega_0 - i\kappa_\ell - i2\kappa_e \left(1 + \cos\left(\frac{m\pi}{N}\right)\right). \quad (3)$$

In addition, the eigenvectors $\mathbf{v}_m$ ($m = 1, 2, \dots, N$) are found as a function of channel number ($n = 1, 2, \dots, N$) according to:

$$v_m(n) = \sin\left(\left(n - \frac{1}{2}\right) \frac{m\pi}{N}\right). \quad (4)$$

According to relation (3), by increasing the mode number, the modal loss decreases from $\kappa = \kappa_\ell + 2\kappa_e (1 + \cos(\pi/N))$ for

$m = 1$ to $\kappa = \kappa_\ell$ for $m = N$, thus, creating a ladder of complex eigenfrequencies with different decay rates. Quite interestingly, the highest-order mode, $m = N$, is free of external losses. The associated eigenvector is found to exhibit uniform intensity across the array while flipping sign at every channel, i.e., $\mathbf{v}_N = (+1, -1, +1, -1, \dots)^t$. In fact, the change of sign of this eigenvector at each resonator creates destructive interference of the fields of all pairs of neighbor resonators in the region between them, which, instead protects this mode from leakage to the outside environment. Figure 2(a), shows the eigenfrequencies of a lattice of six elements in the complex plane. The corresponding eigenvectors are plotted in Fig. 2(b).

Fig. 2. (a) The eigenfrequencies of a lattice of six dissipatively coupled resonators in the complex frequency domain, and (b) the eigenvectors of the lattice (dressed with Gaussian profiles).

It should be noted that in practice achieving pure dissipative coupling between two resonators is difficult given that direct evanescent coupling is involved. Therefore, in general, the coupling should be considered as a combination of dispersive and dissipative processes. In this case, the evolution equations (1) should be modified by changing the coupling coefficient according to $-\kappa_e \rightarrow -\kappa_e + i\mu$. The presence of the dispersive coupling, disperses the frequencies of the supermodes along the real frequency axis. However, it does not prevent the mode discrimination along the imaginary axis. The insertion of the dispersive coupling in the tight-binding model also results in a finite external loss for the highest order mode. This is indeed the case in a realistic scenario due to the finite overlap of the highest order mode with the intermediate medium. However, one can prevent this by using a proper external loss mechanism such as radiative leakage via a single mode channel which is prevented for overlapping fields with an anti-symmetric profile thus creating conditions near ideal dissipative coupling. In the following, we discuss an example of a realistic design of a dissipatively coupled array.

Here, we consider a configuration of an array of coupled distributed Bragg reflector (DBR) laser cavities[18]. In such cavities, dielectric loading through a ridge waveguide geometry imposes transverse localization in the active region. On the other hand, the two Bragg reflectors in the front and the back facets create longitudinal localization. In this structure, partial dissipative coupling can be achieved by loading the regions between each two channels with metal strips. In our simulations, we consider an array of six elements with parameters similar to those re-

ported in Ref. [16]. The thickness and height of each channel is assumed to be $1\mu\text{m}$ and 500nm , respectively, while the height of the active layer is taken to be 300nm . At the central wavelength of $\lambda = 1.55\mu\text{m}$, the refractive indices of the cladding and active regions are assumed to be $n_c = 3.14$ and $n_g = 3.40$, respectively, while the complex index of the lossy region is taken to be $n_l = 3.67 + i4.18$. For a given length of the channels, one can find the complex eigenfrequencies of the array through numerical methods. This can be done through full-wave simulation of the three-dimensional structure or by reducing it to an effective two-dimensional structure via the effective index method [18]. Alternatively, given that the coupling occurs in the transverse direction, here, we focus on the transverse eigenmodes of the array, assuming that the structure is uniform along the longitude. In this manner, one can find the effective propagation indices of the array supermodes, which, are associated with a set of longitudinal modes when considering the longitudinal confinement due to the end mirrors and assuming that the longitudinal mode spacing is much larger than the transverse.

Fig. 3. (a) The effective propagation indices of an array of dissipatively coupled waveguides in the complex plane, and (b) the transverse electric field intensity profiles of the array eigenmodes sorted from the lowest (top) to the highest (bottom) order mode. In (a), the circles show the results from direct finite element simulation (FEM) of the array, while the triangles show the effective indices obtained from coupled mode theory (CMT).

Figure 3(a) shows the real and imaginary parts of the complex propagation indices $n_{\text{eff}} = n_R + in_I$ of an array of six coupled waveguides obtained from finite element simulations. As expected, the effective indices are sorted in the complex plane from low to high attenuation constants. The electric field intensity profiles of the associated eigenmodes are shown in Fig. 3(b). As clearly indicated in this figure, the highest-order mode has the minimum overlap with the regions between the waveguides, thus is better protected from coupling losses. On the other hand, any other mode with index $m = 1, 2, \dots, N - 1$ shows $m - 1$ nodes, thus experiences considerable overlap with $N - m$ lossy regions. Therefore, the highest order mode exhibits the lowest

lasing threshold in an active configuration.

According to Fig. 3(a), the effective mode indices of the array are also separated along the real index axis, which, indicates the presence of dispersive coupling. To explore this effect, we consider a spatial counterpart of the temporal coupled mode equations (1) for the array of coupled waveguides simulated here. The complex coupling constant $-\kappa'_e + i\mu'$, where, we use the primed parameters in order to distinguish the spatial coupling constants from the temporal coupling coefficients of Eq. (1), can be obtained by considering two coupled channels. From the finite element simulations, the real and imaginary parts of the coupling are found to be $-\kappa'_e = -0.0021/\lambda_0$, and $\mu' = 0.0123/\lambda_0$ respectively. By using these coupling coefficients, one can obtain the complex propagation indices, shown with triangles in Fig. 3(b), which are in good agreement with those calculated from numerical simulations of the entire array.

Given that the lasing threshold in an active array is proportional to the level of linear losses, the ladder of losses in the dissipatively coupled array can be utilized for selectively filtering modes in laser arrays. In this system, the higher-order modes exhibit a lower lasing threshold. In the framework of the tight-binding model utilized here, and assuming that all modes have the same linear gain g_0 , the gain level for single mode operation of a laser array is bound to $\kappa_N \leq g_0 < \kappa_{N-1}$, where, κ_N and κ_{N-1} respectively represent the loss of the N 'th and $(N - 1)$ 'th mode. Therefore, the gain margin for single mode operation is $\Delta g = \kappa_{N-1} - \kappa_N = 2\kappa_e [1 - \cos(\pi/N)]$. According to this relation, the gain margin for single mode operation Δg increases linearly with the level of dissipative coupling while it decreases rapidly with the size of the array. On the other hand, even for small margins of single mode operation, the dissipatively coupled array can suppress a large number of lower-order modes. Considering large arrays, the mode selectivity can be better quantified through a density of states as a function of the linear losses. One can define the density of states as $D(\kappa) = dN(\kappa)/d\kappa$, where, $N(\kappa)$ represents the number of modes with decay rates smaller than κ . The function N can be calculated directly from Eq. (3), which, results in the following density of states for an array of N resonators:

$$D(\kappa) = \frac{N}{\pi} \frac{1/2\kappa_e}{\sqrt{1 - ((\kappa - \kappa_\ell)/2\kappa_e - 1)^2}} \quad (5)$$

As shown in Fig. 4(a), the density of states is symmetric with respect to a central value which is the sum of the intrinsic and external losses of each resonator $\kappa_c = \kappa_\ell + 2\kappa_e$. The minimum density of states occurs at the central value, while it becomes singular at the two extreme decay rates of $\kappa_{\min} = \kappa_\ell$ and $\kappa_{\max} = \kappa_\ell + 4\kappa_e$. Considering an active gain medium creating a linear gain g , and assuming that the modal gain is equal for all modes, the number of lasing modes can be calculated as $N_0 = \int_{\kappa_\ell}^g D(\kappa) d\kappa$. The ratio of the number of lasing modes to the total number of array modes is thus found to be $N_0/N = 1 - \frac{1}{\pi} \cos^{-1} [(g - \kappa_\ell)/2\kappa_e - 1]$. Figure 4(b) depicts the ratio of lasing modes as a function of the gain in a dissipatively coupled array. According to this figure, for the gain value of $g = \kappa_\ell + \kappa_e/10$, as an example, approximately 10% of the modes are above their oscillation threshold.

It should be noted that operating in higher-order modes can generally change the far-field radiation intensity pattern from a main normally-emitting lobe to side lobes emitting at oblique angles [19]. Assuming that all elements are identical, the far-field radiation intensity of the array $I(\theta)$, can be written

Fig. 4. (a) The density of states as a function of the decay rates $D(\kappa)$ in a one-dimensional dissipatively coupled resonator array. (b) The ratio of the number of lasing modes to the total number of modes.

as $I(\theta) = I_0(\theta)A(\theta)$, where, $I_0(\theta)$ is the normalized far-field intensity of a single element, $A(\theta)$ represents the array intensity factor, and θ is the polar angle in the plane of radiation and measured from the array axis as shown in the inset of Fig. 5 [20]. For a given eigenmode, the array intensity factor can be written as $A(\theta) = \left| \sum_{n=1}^N v_m(n) \exp(i2\pi(d/\lambda_0)\cos(\theta)) \right|^2$, where, d is the center-to-center separation of the array elements. In the dissipatively coupled array, the highest-order eigenmode is $v_N(n) = (-1)^n$ which results in $A(\theta) = |\sin(N\psi/2)/\sin(\psi/2)|^2$, where, $\psi = 2\pi(d/\lambda_0)\cos(\theta) + \pi$. According to this relation, the angle of maximum intensity, $\theta_{\max}$, depends critically on the ratio of the distance between neighbor elements to wavelength, d/λ . For $d > \lambda/2$, the peak intensity of the array factor reaches N^2 due to cooperative emission of all elements as in superradiance [21]. This, however, does not result in an N^2 scaling of the array intensity given that the peak intensity of the array factor $\theta_{\max}$ does not happen at the peak of a single element, $\theta = \pi/2$. Considering a Gaussian beam profile for a single element, its normalized far-field intensity can be written as $I_0 \approx \exp[-2(\pi^2 w_0^2/\lambda_0^2)(\theta - \pi/2)^2]$, where, $2w_0$ is the beam waist. Therefore, the peak radiation intensity of the array is $I_{\max} = I_0(\theta_{\max})A(\theta_{\max}) = N^2 \exp[-2(\pi^2 w_0^2/\lambda_0^2)(\theta_{\max} - \pi/2)^2]$, which, can be much larger than unity. Figure 5 depicts the normalized far-field intensity of a single element $I_0(\theta)$, the array factor $A(\theta)$, and the far-field radiation intensity $I(\theta)$ of an array of six elements with parameters similar to those in Fig. 3. In this case, the peak intensity is increased by a factor of 10, compared to a single emitter.

In conclusion, we introduced a novel photonic lattice with dissipative coupling between nearest neighbors. The non-conservative interaction among an array of resonators results in a ladder of decay rates in the complex frequency domain. This mechanism sorts modes in terms of their attenuation rates, thus creates mode discrimination in favor of higher-order modes which are more prone from the external losses. Therefore, dissipative coupling can be utilized to promote single-mode operation in laser arrays. The proposed method is versatile and applicable to a wide range of laser cavities, while it remains to systematically investigate the design criteria for creating maxi-

Fig. 5. The far-field radiation intensity pattern of an array of six elements; the normalized intensity of a single emitter $I_0(\theta)$ (red), the array intensity factor $A(\theta)$ (blue), and the far-field intensity of the array $I(\theta) = I_0(\theta)A(\theta)$ (orange). The inset depicts a top view of the array and the angle θ in the plane of radiation. Here, we considered the example of Fig. 3 with the center-to-center separation of the emitters being $d = 2\mu\text{m}$ and the Gaussian beam width of each emitter is taken to be $w_0 = 1\mu\text{m}$.

mal dissipative coupling between two resonators.

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