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## DIFFERENTIAL EVOLUTION ALGORITHM (DE) TO ESTIMATE THE COEFFICIENTS OF UNIFORMITY OF WATER DISTRIBUTION IN SPRINKLER IRRIGATION

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Iran, has caused most of the water used and as much as possible to avoid losses. One of the important parameters in agriculture is water distribution uniformity coefficient (CU) in sprinkler irrigation. CU amount of water sprinkler operating depends on different pressure heads (P), riser height (RH), distance between sprinklers on lateral pipes (SI) and the distance between lateral pipes (Sm). The best combination of the above parameters for maximum CU, is still unknown for applicators. In this research, CU quantities of zb model sprinkler (made in Iran) were measured at Hashemabad cotton research station of Gorgan under 3 different pressure heads (2.5, 3 and 3.5 atm), 2 riser heads (60 and 100 cm) and 7 sprinkler (SI×Sm including 9×12, 9×15, 12×12, 15×12, 12×18, 15×15, 15×18m) arrangements. By using differential evolution algorithm (DE), CU equation was optimized and the best optimized coefficients obtained. In this algorithm, the coefficients F and CR equal to 2 and 0.5, respectively, with a population of 100 members and 1000 number of generations (iterations), provides the best results. Absolute error between the results of this algorithm with the measured results is 2.2%. As well as values Wilmot (d) and the root-mean square error (RMSE), equal to 0.919 and 2.126, respectively. This results show that this algorithm has high accuracy to estimate water distribution uniformity.

**Keywords:** Uniformity Coefficient, Water Distribution, Sprinkler Irrigation, Differential Evolution Algorithm (DE).

### INTRODUCTION

The uniformity of water application in a sprinkler irrigation system is an important aspect of the system performance. That be defined with water distribution uniformity coefficient (CU). The first study of sprinkler irrigation uniformity has been done by Christiansen [3] in California, that led to the Christiansen uniformity coefficient is presented (Eq. 1).

$$CU = 1 - \sum_{i=1}^n \frac{|x_i - \bar{x}|}{n \cdot \bar{x}} \quad (1)$$

In the above formula, CU is Christiansen uniformity coefficient,  $x_i$  is depth of water collected in each can of water (mm),  $\bar{x}$  is the average depth of water in cans (mm) and n is the number of cans to collect water.

In sprinkler irrigation systems, are very common to use Christiansen uniformity coefficient. Many researchers in the field of water distribution uniformity coefficient in fixed sprinkler systems have been worked. Other research such as Hart and Reynolds [5], Karmeli [8], Vories and Bernuth [12], Dabbous [4], Heerman [6], Keller and Bliesner [9], Carrion et al. [2], Montero et al. [10] and Bavi et al. [1] have investigated different aspects of water distribution uniformity coefficient.

A sprinkler water distribution pattern depends on the system design parameters such as: the sprinkler spacing, operating pressure, nozzle diameter, and environmental variables such as: wind speed and direction [9]. The sprinkler irrigation distribution patterns have been characterized by various statistical uniformity coefficients [8] and various coefficients of uniformity (CUs) have been developed over the past decades. Hart and Reynolds [5] proposed “distribution efficiency”,  $DE_{pa}$ , a value based on numerical integrations of the normal distribution function while  $DE_{pa}$  is determined by first selecting a target CU and a target “percent area adequately irrigated”.

Due to the importance of understanding the uniformity coefficient, this coefficient using the results of a single sprinkler according to the overlapping neighboring sprinklers are measured.

As stated previously, different researchers have used various concepts to express the coefficients of uniformity, hence the equations lead to different results in the expression of the distributed water uniformity in the same fields. In this study, evaluate different uniformity coefficients with Differential Evolution Algorithm (DE) to propose the best and optimized equation for CU.

Problems which involve global optimization over continuous spaces are ubiquitous throughout the scientific community. In general, the task is to optimize certain properties of a system by pertinently choosing the system parameters. So in this study, CU has been estimated by DE algorithm, and funded the best and optimizes coefficients in CU equation.

## MATERIALS AND METHODS

### The field experiments data

The field experiments were conducted on farmland located in Hashem Abad Agricultural Research Station of Gorgan Cotton Research Institute, about 11 kilometers northwest from Gorgan. The lands were irrigated by solid set sprinkler irrigation systems. The sprinkler uniformity tests were conducted using rain-gauge for uniformity coefficients measuring. The model of sprinkler is zb that made in Iran.

Table 1. Christiansen uniformity coefficient distribution (%) in different treatments pressure, height and spacing of sprinkler [7].

Pressure	Sprinkler Height	Sprinkler Spacing( $S_m \times S_l$ )						
		15×18	15×15	12×18	15×12	12×12	9×15	9×12
3.5	60	80.2	82.5	85	85.8	91	86.2	87.5
	100	79.5	83.5	86.3	85.8	91.6	86.5	91.1
3	60	81.2	84.1	84.9	86.1	87.5	87.6	90.2
	100	84.8	84.6	86.7	87.1	89.9	89.6	92.2
2.5	60	73.7	79.5	74.5	81.1	85.8	82.9	85.3
	100	77	80.7	82.6	83.5	86.4	84.7	86

In this study, the coefficient values of water CU for zb model in three different treatments of water working pressure (2.5, 3 and 3.5 atm), two sprinkler height treatments (60 and 100 cm)

and seven treatments sprinklers arrangement network ( $S_m \times S_l$ ) Includes  $12 \times 9$ ,  $15 \times 9$ ,  $12 \times 12$ ,  $12 \times 15$ ,  $18 \times 12$ ,  $15 \times 15$ ,  $18 \times 15$  m were measured at Cotton Research Station, Gorgan, Iran [7]. To measure Christiansen uniformity coefficient, equation 1 was used. The results of this study are shown in Table 1.

### Differential evolution algorithm

Differential Evolution (DE) algorithm is a branch of evolutionary programming developed by Rainer Storn and Kenneth Price [11] for optimization problems over continuous domains. In DE, each variable's value is represented by a real number. The advantages of DE are its simple structure, ease of use, speed and robustness. DE is one of the best genetic type algorithms for solving problems with the real valued variables. Differential Evolution is a design tool of great utility that is immediately accessible for practical applications. DE has been used in several science and engineering applications to discover effective solutions to nearly intractable problems without appealing to expert knowledge or complex design algorithms. Differential Evolution uses mutation as a search mechanism and selection to direct the search toward the prospective regions in the feasible region. Genetic Algorithms generate a sequence of populations by using selection mechanisms. Genetic Algorithms use crossover and mutation as search mechanisms. The principal difference between Genetic Algorithms and Differential Evolution is that Genetic Algorithms rely on crossover, a mechanism of probabilistic and useful exchange of information among solutions to locate better solutions, while evolutionary strategies use mutation as the primary search mechanism.

Differential Evolution (DE) is a parallel direct search method which utilizes NP D-dimensional parameter vectors.

$$x_{i,G}, \quad i = 1, 2, \dots, NP \quad (2)$$

As a population for each generation G. NP does not change during the minimization process. The initial vector population is chosen randomly and should cover the entire parameter space. As a rule, we will assume a uniform probability distribution for all random decisions unless otherwise stated. In case a preliminary solution is available, the initial population might be generated by adding normally distributed random deviations to the nominal solution  $x_{nom,0}$ . DE generates new parameter vectors by adding the weighted difference between two population vectors to a third vector. Let this operation be called mutation. The mutated vector's parameters are then mixed with the parameters of another predetermined vector, the target vector, to yield the so-called trial vector. Parameter mixing is often referred to as "crossover" in the ES-community and will be explained later in more detail. If the trial vector yields a lower cost function value than the target vector, the trial vector replaces the target vector in the following generation. This last operation is called selection. Each population vector has to serve once as the target vector so that NP competitions take place in one generation. More specifically DE's basic strategy can be described as follows:

#### Mutation

For each target vector  $x_{i,G}$ ,  $i = 1, 2, \dots, NP$ , a mutant vector is generated according to:

$$V_{i,G+1} = x_{r_1,G} + F \times (x_{r_2,G} - x_{r_3,G}) \quad (3)$$

With random indexes  $r_1, r_2, r_3 \in \{1, 2, \dots, NP\}$  integer, mutually different and  $F > 0$ . The randomly chosen integers  $r_1, r_2$  and  $r_3$  are also chosen to be different from the running index  $i$ , so that NP must be greater or equal to four to allow for this condition.  $F$  is a real and constant factor  $\in [0, 2]$  which controls the amplification of the differential variation  $(x_{r_2,G} - x_{r_3,G})$ . Fig.1

shows a two-dimensional example that illustrates the different vectors which play a part in the generation of  $V_{i,G+1}$ .

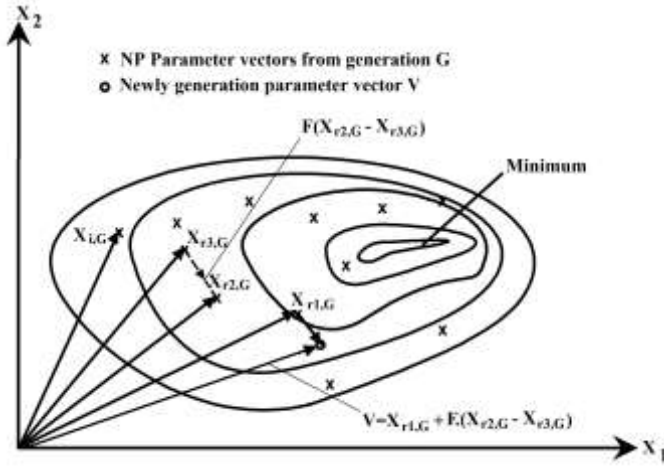


Figure 1. An example of a two-dimensional cost function showing its contour lines and the process for generating  $V_{i,G+1}$ .

#### Crossover

In order to increase the diversity of the perturbed parameter vectors, crossover is introduced. To this end, the trial vector:

$$u_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, \dots, u_{Di,G+1}) \quad (4)$$

Is formed, where:

$$u_{ji,G+1} = \begin{cases} V_{ji,G+1} & \text{if } randb(j) \leq CR \text{ or } j = ranbr(i) \\ x_{ji,G} & \text{otherwise} \end{cases} \quad (5)$$

$$j = 1, 2, \dots, D.$$

In Eq. (5),  $randb(j)$  is the  $j$ th evaluation of a uniform random number generator with outcome  $\in [0; 1]$ . CR is the crossover constant  $\in [0; 1]$  which has to be determined by the user.  $ranbr(i)$  is a randomly chosen index  $\in 1, 2, \dots, D$  which ensures that  $u_{i,G+1}$  gets at least one parameter from  $V_{i,G+1}$ .

#### Selection

To decide whether or not it should become a member of generation  $G+1$ , the trial vector  $u_{i,G+1}$  is compared to the target vector  $x_{i,G}$  using the greedy criterion. If vector  $u_{i,G+1}$  yields a smaller cost function value than  $x_{i,G}$ , then  $x_{i,G+1}$  is set to  $u_{i,G+1}$ ; otherwise, the old value  $x_{i,G}$  is retained.

In this paper, to achieve a nonlinear relationship, that can be related the uniformity coefficient to the parameters listed, the sum of squared error objective function should be used as follows:

$$\phi(s) = \sum_{i=1}^n (CU_i(m) - CU_i(s))^2 \quad (6)$$

In the above equation, m and s, are measurements and estimated index, respectively. In this research from all experimental data, for the estimation of model, 70 percent of the experimental data was randomly selected and the last 30 percent used for validation of the obtained equations.

According to research Hezarjaribi et al. [7] Eq. 7, has shown good accuracy than other equations to estimate the Christiansen uniformity coefficient for working pressure of the sprinkler, sprinkler height, distance between sprinklers on the pipes side and distance between side pipes.

$$CU = k_1 P^{k_2} RH^{k_3} S_l^{k_4} S_m^{k_5} \quad (7)$$

Where P is the pressure, RH is the height of Sprinkler, S<sub>l</sub> and S<sub>m</sub>, is distance between sprinklers on the pipes side and distance between side pipes, respectively. k<sub>1</sub>, k<sub>2</sub>, k<sub>3</sub>, k<sub>4</sub> and k<sub>5</sub> are the fixed numbers that will optimize with the differential evolution algorithm.

In this research program has been written in Matlab for using differential evolution algorithm and non-linear equation for different values of F, CR, different populations (NP) and different number of generations (NG) were studied.

For verifying the fitted model against experimental results, has been used relative error, absolute error, the root-mean square error (RMSE), mean absolute error (MAE), coefficient of determination (R<sup>2</sup>) and the parameter d [13].

## RESULTS AND DISCUSSION

In the first step, to obtain the best conditions for algorithm that provide the most optimal and does not local optimum problem, 10 combinations of different modes for the coefficients F and CR were examined. After finding the best combination of coefficients values F and CR, algorithms for solving the independent populations were examined, to this purpose, the population of 25, 50, 100, 500 and 1000 members were studied. Finally, the best combination of coefficients and population were used to examine the effect of the number of generations, so three generations of the 500, 1000 and 10000 were studied. Totally, the algorithm was run 90 times for various conditions and obtained the best case.

To reach a minimum value of eq. (6), the number of generations 1000, an initial population of 1000, the parameter (F) = 2 and the parameter (CR) 0.5 were considered. And the results were converging and a good agreement with experimental data was observed.

According to Eq. 7 on 70% of the measured data the optimal coefficients are obtained as the eq. 8 by the differential evolution algorithm. Uniformity coefficient of this optimal equation is revealed in Table 2.

Table 2. Christiansen uniformity coefficient distribution from DE algorithm (%)

Pressure	Sprinkler Height	Sprinkler Spacing(S <sub>m</sub> ×S <sub>l</sub> )						
		15×18	15×15	12×18	15×12	12×12	9×15	9×12
3.5	60	81.57	83.74	83.27	86.48	88.29	87.79	90.67
	100	83.30	85.52	85.04	88.32	90.16	89.66	92.59
3	60	79.73	81.85	81.39	84.53	86.29	85.81	88.62
	100	81.42	83.59	83.11	86.32	88.12	87.63	90.50
2.5	60	77.60	79.67	79.22	82.28	83.99	83.52	86.25
	100	79.25	81.36	80.90	84.02	85.77	85.29	88.08

$$CU = 111.61P^{0.1483} RH^{0.0411} S_l^{-0.0925} S_m^{-0.1443} \quad (8)$$

To evaluate the goodness of the optimal equation, the equation was used to estimate Christiansen uniformity coefficient distribution of the 70% of the experimental data (Fig. 2).

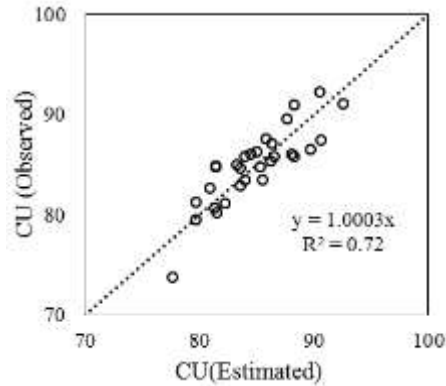


Figure 2. Measured results are compared with the estimated results( 70 percent of the data).

Then, the model obtained from 70% of the data was verified with remaining 30% of the measured data. To evaluate the goodness of the optimal equation, the equation was used to estimate Christiansen uniformity coefficient distribution of the 30% of the experimental data (Fig. 3).

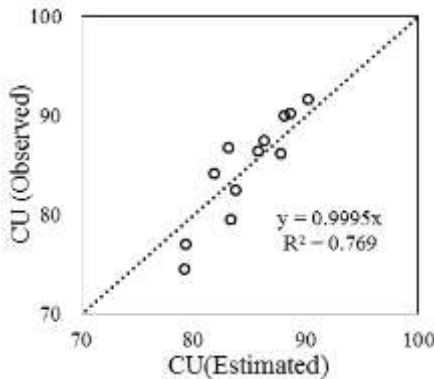


Figure 3. Measured results are compared with the estimated results( 30 percent of the data).

The estimated Christiansen uniformity coefficient distribution using Eq. 8 have been compared with the 30% and 70% of the observed value and good agreement was observed (Table 3).

Table 3. Statistical measure for the comparison of the estimated with the observed values.

Data	RMSE	MAE	R <sup>2</sup>	Absolute Error	Relative Error	d (Willmot)
70% Data	1.970	0.045	0.720	2.02%	0.02%	0.922
30% Data	2.472	0.094	0.769	2.63%	0.26%	0.915
All Data	2.126	0.005	0.741	2.2%	0.06%	0.919

It was revealed that the maximum absolute error was less than 3%, this error was for 30% data. Also the statistical parameters Wilmot (d) reveal that the optimal coefficients that obtained with DE algorithm are very good.

## CONCLUSION

In this study nonlinear equation uniformity coefficients in sprinkler irrigation has been optimal by using differential evolution algorithm. The best results obtained in F and CR equal to 2 and 0.5, respectively. Also the number of generations 1000, an initial population of 1000, have shown a good agreement between experimental data and estimated data.

Another result of this study is that differential evolution algorithm is a very high rate of convergence to find optimal point in nonlinear equations. Coefficients of uniformity equation in sprinkler irrigation was optimized very well by differential evolution algorithm

The estimated Christiansen uniformity coefficient distribution have been compared with the 30% and 70% of the observed value and good agreement was observed. It was revealed that the maximum absolute error was less than 3% and the mean square of the sum of squared differences between the data ( $\eta(s) = \frac{\sqrt{\phi(s)}}{n}$ ) is equal 0.328 by this algorithm.

Generally, can be said that differential evolution (DE) algorithm, for optimizing nonlinear functions is very good, and compare to other algorithms has a much higher rate of convergence, Whereas dose not local optimal problems.

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