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COLOURED MONOPOLES FROM FRACTIONAL CHARGES IN OCD

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In a recent paper, Slansky et al. have suggested that $SU(3)_C$ may be broken to $SO(3) \times Z_3$ in order to explain the possible existence of fractionally charged states. We point out that there are low mass monopoles associated with such a symmetry breakdown and discuss their properties. Such states may be observable at LEP if not at PETRA energies.

1. Introduction

There is some evidence that particles of fractional electric charge exist in nature [1].

In a recent paper [2], a model has been proposed within the general framework of QCD which permits the existence of such states. In this model, the colour group $G = SU(3)_C$ is dynamically broken to the subgroup $H = SO(3) \times Z_3$. Since SO(3) is unbroken, the usual dogma that only SO(3) singlets appear as asymptotic states is accepted. They need not, however, be G or Z_3 singlets. This allows the possibility of fractionally charged states, formed for example from two quarks.

Whenever there is a breakdown of a gauged symmetry, there is the possibility of monopole sectors in the model. The existence or otherwise of such sectors is governed by the structure of a homotopy group which in our case is $\pi_2(G/H)$ [3, 4]. In this note, we observe that this group is Z_2. Thus there is a multiplicative, conserved topological charge Q in the model which can take values ± 1. We also estimate the mass μ_S of the ground state S of the $Q = -1$ sector and briefly explore the properties of other states in this sector. The mass μ_S is quite low, of the order of 33 MeV or less, so that S is the lightest stable hadronic state for this model. [We ignore the unlikely possibility that diquark states are even less massive.]

An effective lagrangian approach is used in this paper to estimate μ_S. Thus a suitable order parameter Φ is introduced to describe the symmetry breakdown $G \to H$ and the Bogomol'ny bound [4] is generalized to bound the energy in the $Q = -1$ sector. In this way, we are able to bound μ_S in terms of the QCD coupling constant and the mass of the gluons associated with the broken generators.

Our work borrows extensively from the review article of Goddard and Olive [4] and in particular from their discussion of the symmetry breakdown $SU(3) \to SO(3)$.

According to them, Sato is responsible for the identification of the appropriate order parameter for this symmetry breakdown.

Sects. 2 through 6 develop the necessary formalism to describe the monopoles for the symmetry breakdown $G \to H$. The proofs of the requisite geometrical results are sketched and suitable spherically symmetric ansätze for the order parameter Φ and the glue field W_μ are written down. In sect. 7, a bound is derived for μ_S from these ansätze. In sect. 8, the properties of the ground and excited states in the monopole sector are described. It is pointed out that the presence of these monopole states or diquarks may cause a broad rise in the R-value, and that these exotics may be observable at LEP if not at PETRA energies. We also estimate the density of such states in the present universe.

2. The group $\pi_2[SU(3)_C/SO(3)\times Z_3]$

In the model of ref. [2], the colour group

$$G = SU(3)_C \tag{2.1}$$

is dynamically broken to the subgroup

$$H = SO(3)\times Z_3 . \tag{2.2}$$

In the defining representation of $SU(3)_C$, this subgroup SO(3) can be taken to be any of the groups

$$g SO(3)_R g^{-1}, \qquad g \in G, \tag{2.3}$$

where $SO(3)_R$ is the group of real orthogonal matrices with determinant 1. The discrete group Z_3 is the centre of $SU(3)_C$. It cannot be broken without breaking baryon number.

The properties of the quantum numbers for the monopoles from such a symmetry breakdown are governed by the homotopy group $\pi_2(G/H)$ [3, 4]. In this section, we prove that

$$\pi_2(G/H) = Z_2 . \tag{2.4}$$

In other words, the topological charge Q is multiplicative in this instance and has values 1 and -1. The proof, which is standard, is formulated in such a way that the notation and ideas introduced are useful in later sections.

Let λ_α be the Gell-Mann matrices, let

$$\Gamma(G) = \{D(g)\}_{g\in G} \tag{2.5}$$

denote the adjoint representation of G,

$$g\lambda_\alpha g^{-1} = D(g)_{\beta\alpha}\lambda_\beta, \tag{2.6}$$

and let

$$\Gamma(\mathrm{K}) = \{D(k)\}_{k\in\mathrm{K}} \tag{2.7}$$

for any subgroup K. Consider the coset space

$$\Gamma(\mathrm{G})/\Gamma(\mathrm{H}) = \{D(g)\Gamma(\mathrm{H})\}_{g\in\mathrm{G}}. \tag{2.8}$$

We want to first prove that this is the same as G/H.

The proof proceeds by first defining the following G action on Γ(G)/Γ(H):

$$D(g)\Gamma(\mathrm{H}) \to D(g')D(g)\Gamma(\mathrm{H}), \qquad g' \in \mathrm{G}. \tag{2.9}$$

This action is transitive. The stability group at Γ(H) consists of all those $g \in \mathrm{G}$ which fulfill

$$D(g)\Gamma(\mathrm{H}) = \Gamma(\mathrm{H}). \tag{2.10}$$

That is,

$$D(g) \in \Gamma(\mathrm{H}). \tag{2.11}$$

In other words, the stability group is precisely H. Hence Γ(G)/Γ(H) = G/H.

According to [3, 4],

$$\pi_2[\Gamma(\mathrm{G})/\Gamma(\mathrm{H})] = \pi_1[\Gamma(\mathrm{H})]_{\Gamma(\mathrm{G})}, \tag{2.12}$$

where the right-hand side is the homotopy group of all closed paths in Γ(H) which are trivial in Γ(G). Now

$$\Gamma(\mathrm{H}) = \Gamma[\mathrm{SO}(3)] \tag{2.13}$$

since $\Gamma(\mathrm{Z}_3)$ is trivial. Also since Γ[SO(3)] is a faithful representation of SO(3) and is topologically identical to SO(3),

$$\begin{aligned}\pi_1[\Gamma(\mathrm{H})] &= \pi_1[\mathrm{SO}(3)]\\ &= \mathrm{Z}_2.\end{aligned} \tag{2.14}$$

On the other hand,

$$\pi_1[\Gamma(\mathrm{G})] = \mathrm{Z}_3. \tag{2.15}$$

Thus $\pi_1[\Gamma(H)]_{\Gamma(G)}$ is the kernel of the homomorphism

$$Z_2 \to Z_3 . \tag{2.16}$$

Since Z_3 has no Z_2 subgroup, this kernel is Z_2 itself. The conclusion (2.4) follows.

3. The order parameter

In this section, we parametrise G/H; that is, specify the asymptotic form of the order parameter Φ at spatial infinity. We also give the form and transformation properties of Φ for all spatial points $\boldsymbol{x} = (x^1, x^2, x^3)$.

As $r \equiv |\boldsymbol{x}| \to \infty$,

$$\Phi \to \Phi^{(\infty)}, \tag{3.1}$$

where $\Phi^{(\infty)}$ is a function on the two sphere S^2_∞ at $r = \infty$ with values in G/H. We claim that $\Phi^{(\infty)}$ can be constructed as follows: Let $M^{(\infty)}$ be a 3×3 matrix valued function on S^2_∞ subject to the constraints

$$M^{(\infty)\dagger}(p) M^{(\infty)}(p) = 1, \tag{3.2}$$

$$\det M^{(\infty)}(p) = 1, \tag{3.3}$$

$$M^{(\infty)\mathrm{T}}(p) = M^{(\infty)}(p), \tag{3.4}$$

at any point $p \in S^2_\infty$. $M^{(\infty)}(p)$ is thus a symmetric SU(3) matrix. We allow $\mathrm{SU(3)_C} = \{g\}$ to act on $M^{(\infty)}$ according to the rule

$$M^{(\infty)}(p) \to g M^{(\infty)}(p) g^{\mathrm{T}}. \tag{3.5}$$

Then

$$M^{(\infty)}(p) \lambda_\alpha M^{(\infty)-1}(p) = \Phi^{(\infty)}_{\beta\alpha}(p) \lambda_\beta \tag{3.6}$$

gives $\Phi^{(\infty)}$.

The group action on $\Phi^{(\infty)}$ follows from (2.6) and (3.6). It is

$$\Phi^{(\infty)}(p) \to D(g) \Phi^{(\infty)}(p) D(g^{\mathrm{T}}). \tag{3.7}$$

As the first step in substantiating the claim, we show that (a) the action of G on the space of matrices $\{M^{(\infty)}(p)\}$ is transitive, and (b) the stability group is SO(3).

The proof of (a) relies on a theorem of Schur [5] according to which $M^{(\infty)}(p)$ can be diagonalized by the transformation (3.5). In the diagonal form as well, $M^{(\infty)}(p)$ is an SU(3) matrix. Thus we can reduce $M^{(\infty)}(p)$ to the unit matrix by a further transformation of the form (3.5). This proves (a).

The proof of (b) is now trivial since the stability group of the unit matrix is $SO(3)_R$.

The transformation which brings $M^{(\infty)}(p)$ to the unit matrix also brings $\Phi^{(\infty)}(p)$ to the unit matrix. Thus $\{\Phi^{(\infty)}(p)\}$ is a coset space. The stability group for this space is defined by all those $g \in SU(3)_C$ with the property

$$D(g)D(g^T) = 1, \tag{3.8}$$

or

$$gg^T \equiv z \in Z_3. \tag{3.9}$$

Now we can write $z = (z^2)^2$, $z^2 \in Z_3$. Then since $g(z^2)^{-1} \in SO(3)_R$, such g's form the subgroup $SO(3)_R \times Z_3$ which means in turn that $\{\Phi^{(\infty)}(p)\} = G/H$.

Let $\hat{n} = (0,0,1)$ denote the north pole of the two sphere at infinity. In the sequel, without loss of generality, we will impose the boundary conditions

$$M^{(\infty)}(\hat{n}) = 1, \tag{3.10}$$

$$\Phi^{(\infty)}(\hat{n}) = 1. \tag{3.11}$$

The asymptotic form $\Phi^{(\infty)}$ does not uniquely determine Φ for all $\boldsymbol{x}$. We will assume that Φ is an 8×8 matrix for all $\boldsymbol{x}$ with the transformation property

$$\Phi \to D(g)\Phi D(g^T). \tag{3.12}$$

This choice of Φ allows for the possibility that $\Phi(\boldsymbol{x})$ is a vector in the $\underline{27}$ representation of $SU(3)_C$ [2].

4. Spherical symmetry for Φ

We shall look for spherically symmetric monopoles. More precisely, for Φ, we shall assume that

$$[-i(\boldsymbol{x} \times \nabla)_a + d(t_a)\circ]\Phi = 0. \tag{4.1}$$

Here

$$t_1 = \tfrac{1}{2}\Sigma_3, \qquad t_2 = \tfrac{1}{2}\Sigma_1, \qquad t_3 = \tfrac{1}{2}\Sigma_2,$$

$$\Sigma_a = \begin{bmatrix} \sigma_a & 0 \\ 0 & 0 \end{bmatrix}, \qquad \sigma_a = \text{Pauli matrices}. \tag{4.2}$$

Thus t_a generate an SU(2) subgroup $SU(2)_t$ of $SU(3)_C$. The representatives of t_a in the representation Γ(G) are denoted by $d(t_a)$ while

$$d(t_a)\circ\Phi \equiv d(t_a)\Phi + \Phi d(t_a^{\mathrm{T}}). \tag{4.3}$$

The particular relation between t_a and $\frac{1}{2}\Sigma_a$ in (4.2) is dictated by the following [4]: (4.1) implies

$$\hat{x}^a d(t_a)\circ\Phi = 0, \tag{4.4}$$

where $\hat{x}^a = x^a/r$. Or as $r\to\infty$ and at the north pole $[\hat{x} = \hat{n} = (0,0,1)]$,

$$d(t_3)\circ\Phi^{(\infty)}(\hat{n}) = 0. \tag{4.5}$$

In view of (3.11), t_3 must therefore be an element of the Lie algebra $\underline{\mathrm{SO(3)}_{\mathrm{R}}}$ of $\mathrm{SO(3)_R}$. The identification in (4.2) of t_3 is compatible with this requirement.

The forms of $M^{(\infty)}$ and $\Phi^{(\infty)}$ consistent with (4.1) are easily found. Let

$$\Lambda_2 = \begin{bmatrix} i\sigma_2 & 0 \\ 0 & 1 \end{bmatrix}. \tag{4.6}$$

It is an element of $\mathrm{SU(3)}_C$. Further

$$t_a^{\mathrm{T}}\Lambda_2 = -\Lambda_2 t_a, \tag{4.7}$$

and hence

$$d(t_a^{\mathrm{T}})D(\Lambda_2) = -D(\Lambda_2)d(t_a). \tag{4.8}$$

If we define

$$N^{(\infty)} = M^{(\infty)}\Lambda_2, \tag{4.9}$$

$$\Psi = \Phi D(\Lambda_2), \tag{4.10}$$

$$\Psi^{(\infty)} = \Phi^{(\infty)}D(\Lambda_2), \tag{4.11}$$

spherical symmetry requires

$$[-i(\boldsymbol{x}\times\nabla)_a + t_a *]N^{(\infty)} = 0, \tag{4.12}$$

$$[-i(\boldsymbol{x}\times\nabla)_a + d(t_a) *]\Psi = 0, \tag{4.13}$$

$$[-i(\boldsymbol{x}\times\nabla)_a + d(t_a) *]\Psi^{(\infty)} = 0. \tag{4.14}$$

where $x * y = [x, y]$. The solution for $N^{(\infty)}$ consistent with (4.12) and (3.11) is

$$N^{(\infty)} = \tfrac{1}{3} - \sqrt{\tfrac{1}{3}}\,\lambda_8 + 2i\hat{x}_a t_a . \tag{4.15}$$

This determines $M^{(\infty)}$ and hence $\Phi^{(\infty)}$ via (3.6).

Let us make the simplifying assumption that $M^{(\infty)}$ is the asymptotic value of a 3×3 matrix M [not necessarily fulfilling (3.2)–(3.4)] with the transformation law

$$M \to gMg^{\mathrm{T}}, \tag{4.16}$$

such that

$$M\lambda_\alpha M^{-1} = \Phi_{\beta\alpha}\lambda_\beta . \tag{4.17}$$

Then the spherical symmetry

$$\left[-i(\boldsymbol{x} \times \nabla)_a + t_a *\right] N = 0 \tag{4.18}$$

of

$$N = M\Lambda_2 \tag{4.19}$$

and the boundary condition (4.15) restrict it to the form

$$N = \tfrac{1}{3}\alpha(r) - \sqrt{\tfrac{1}{3}}\,\beta(r)\lambda_8 + 2i\gamma(r)\hat{x}_a t_a , \tag{4.20}$$

$$\alpha(\infty) = \beta(\infty) = \gamma(\infty) = 1 . \tag{4.21}$$

[The functions α, β, γ must, of course, be such that M is invertible.] This equation leads to a simplified ansatz for Φ, which, however, is not the most general one possible. [We will not require this form of Φ in the subsequent analysis.]

5. Spherical symmetry for W

The gauge field for $\mathrm{SU}(3)_{\mathrm{C}}$ is

$$W_\mu = i\tfrac{1}{2}\lambda_a W_\mu^a . \tag{5.1}$$

We shall work in the gauge

$$\hat{x}_i W_i = 0 . \tag{5.2}$$

In our problem, spherical symmetry for W_i is the statement

$$[J_a + d(t_a)*]W_i = 0, \tag{5.3}$$

where the angular momentum J_a contains the usual orbital and spin parts.

The most general such spherically symmetric W_i is

$$W_i = \frac{1}{ir^2}\varepsilon_{ijk}x_j t_k \delta(r) + i(\delta_{ia} - \hat{x}_i\hat{x}_a)t_a\varepsilon(r) + a(r)t_i + b(r)x_i\lambda_8. \tag{5.4}$$

The gauge (5.2) reduces this to

$$W_i = \frac{1}{ir^2}\varepsilon_{ijk}x_j t_k \delta(r) + i(\delta_{ia} - \hat{x}_i\hat{x}_a)t_a\varepsilon(r) \tag{5.5}$$

so that for all $\boldsymbol{x}$, W_i has values in the Lie algebra $\underline{\mathrm{SU}(2)}_t$ of $\mathrm{SU}(2)_t$.

There are boundary conditions on $\delta(r)$ from finiteness of energy which, as usual, will impose the condition

$$-i\varepsilon_{ijk}x_j D_k \Psi^{(\infty)} = 0, \tag{5.6}$$

or, equivalently, the condition

$$-i\varepsilon_{ijk}x_j D_k N^{(\infty)} = 0 \tag{5.7}$$

as $r \to \infty$. Here D_i denotes covariant differentiation. The difference of (4.18) (with $r \to \infty$) and (5.7) leads to

$$\left[(\delta(r) - 1)t_i + \varepsilon(r)\varepsilon_{ijk}x_j t_k, N^{(\infty)}\right] = 0 \tag{5.8}$$

as $r \to \infty$ and we have used (4.4). At the north pole, where $N^{(\infty)} = \Lambda_2$, this implies an inconsistency unless

$$\delta(r) \to 1, \qquad \varepsilon(r) \to 0 \tag{5.9}$$

as $r \to \infty$.

The asymptotic form of the magnetic field

$$F_{ij} = \partial_i W_j - \partial_j W_i + [W_i, W_j] \tag{5.10}$$

calculated from (5.5) and (5.9) is

$$\frac{i}{r^2}\varepsilon_{ijk}\hat{x}_k(\hat{x}\cdot t). \tag{5.11}$$

6. Topology of the ansätze

We can now show that the ansätze (4.15) and (5.5) have non-trivial topology, so that they can in fact describe states in the monopole sector of the theory.

According to [4], the closed curve in H that we have to examine is

$$h(s) = \exp[it_3 4\pi s], \qquad 0 \leq s \leq 1. \tag{6.1}$$

Now since t_3 has half-integer eigenvalues, the canonically normalized angular momentum generator (with integral eigenvalues) for $SO(3)_R$ is not t_3, but $2t_3$. Thus the curve (6.1) is the set of rotations between 0 and 2π around a fixed axis in $SO(3)_R$. It is well known that this curve cannot be deformed to a point if during this deformation, it stays within $SO(3)_R$. It follows that our ansätze are appropriate for the monopole sector.

7. The Bogomol'ny bound

We use essentially standard methods [4] to derive this bound. We start from the lower bound

$$\mu_S \geq \int d^3x \left[\tfrac{1}{2} f^2 \operatorname{Tr} D_i\Phi^\dagger D_i\Phi - \frac{1}{2e_A^2} \operatorname{Tr} d(*F_i) d(*F_i) \right] \tag{7.1}$$

for the ground-state energy μ_S in the monopole sector. [S stands for soliton.] Here (a) f is a parameter with the dimension of energy, (b) the definition of $*F_i$ is

$$*F_i = \sqrt{\tfrac{1}{2}}\, \varepsilon_{ijk} F_{jk}, \tag{7.2}$$

(c) $d(*F_i)$ is the representative of $*F_i$ in the adjoint representation,

$$*F_i = *F_i^\alpha d\left(\tfrac{1}{2} i\lambda_\alpha\right) \tag{7.3}$$

and both the traces are over 8×8 matrices, (d) because the second trace is in the adjoint representation, e_A is not the conventional QCD coupling constant e, but rather

$$\frac{1}{e^2} = \frac{1}{e_A^2} \frac{\operatorname{Tr} d(\lambda_\alpha) d(\lambda_\alpha)}{\operatorname{Tr}(\lambda_\alpha \lambda_\alpha)} = \frac{6}{e_A^2}. \tag{7.4}$$

Replacing Φ by Ψ and using

$$\operatorname{Tr}\left[\sqrt{\tfrac{1}{2}}\, f D_i\Psi \pm \frac{1}{\sqrt{2}\, e_A} d(*F_i)\right]^\dagger \left[\sqrt{\tfrac{1}{2}}\, f D_i\Psi \pm \frac{1}{\sqrt{2}\, e_A} d(*F_i)\right] \geq 0 \tag{7.5}$$

and the antihermiticity of $*F_i$, we find

$$\mu_S \geqslant \left| \frac{f}{2\sqrt{2}\, e_A} \int d^3x\, \varepsilon_{ijk} \mathrm{Tr}\left(D_i\Psi - D_i\Psi^\dagger \right) d\left(F_{jk} \right) \right|. \tag{7.6}$$

Now since Ψ transforms by conjugation under $SU(2)_t$ and W_i has values in $\underline{SU(2)}_t$ [eq. (5.5)],

$$D_i\Psi - D_i\Psi^\dagger = \partial_i(\Psi - \Psi^\dagger) + \left[W_i, \Psi - \Psi^\dagger \right]. \tag{7.7}$$

Using the Bianchi identity, we can thus write

$$\varepsilon_{ijk} \mathrm{Tr}\left(D_i\Psi - D_i\Psi^\dagger \right) d\left(F_{jk} \right) = \varepsilon_{ijk} \partial_i \mathrm{Tr}(\Psi - \Psi^\dagger) d\left(F_{jk} \right), \tag{7.8}$$

$$\mu_S \geqslant \left| \frac{f}{2\sqrt{2}\, e_A} \int_{S^2_\infty} \mathrm{Tr}\left[\hat{x}_a d(t_a) \right]\left[\Psi^{(\infty)} - \Psi^{(\infty)\dagger} \right] \frac{\hat{x}_i}{r^2} \varepsilon_{ijk} dx^j \wedge dx^k \right|, \tag{7.9}$$

where we have used (5.11).

Since $\hat{x}_a d(t_a)$ and $\Psi^{(\infty)} - \Psi^{(\infty)\dagger}$ are both spherically symmetrical,

$$(\boldsymbol{x} \times \nabla)_i \mathrm{Tr}\left[\hat{x}_a d(t_a) \right]\left[\Psi^{(\infty)} - \Psi^{(\infty)\dagger} \right] = 0, \tag{7.10}$$

which means that the value of the trace is its value at the north pole:

$$\mathrm{Tr}\left[\hat{x}_a d(t_a) \right]\left[\Psi^{(\infty)} - \Psi^{(\infty)\dagger} \right] = \mathrm{Tr}\, d(t_3)\left[D(\Lambda_2) - D(\Lambda_2)^\dagger \right]. \tag{7.11}$$

In view of the definition (2.6) and the identities $\lambda^\dagger_\alpha = \lambda_\alpha, \mathrm{Tr}\,\lambda_\alpha\lambda_\beta = 2\delta_{\alpha\beta}$, $D(\Lambda_2)$ is real orthogonal and $d(t_3)$ is antisymmetric. Thus the right-hand side is $2\,\mathrm{Tr}\, d(t_3) D(\Lambda_2)$ and

$$\mu_S \geqslant \left| \frac{4\sqrt{2}\,\pi f}{e_A} \mathrm{Tr}\, d(t_3) D(\Lambda_2) \right|. \tag{7.12}$$

To evaluate the trace, note that

$$\Gamma[SU(2)_t] = \underline{3} + \underline{2} + \underline{2} + \underline{1}, \tag{7.13}$$

where the numbers stand for the dimensions of the $SU(2)_t$ irreducible representations. Since Λ_2 is the rotation by π around the $\frac{1}{2}\sigma_2$ axis, we can write the corresponding direct sum decomposition

$$D(\Lambda_2) = \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \oplus i\sigma_2 \oplus i\sigma_2 \oplus 1 \tag{7.14}$$

in a suitable basis, while for $d(t_3)$,

$$d(t_3) = -i\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \oplus \tfrac{1}{2}\sigma_2 \oplus \tfrac{1}{2}\sigma_2 \oplus 0. \tag{7.15}$$

The trace in (7.12) is thus $2i$ and

$$\mu_S \geqslant 8\pi f/\sqrt{3}\, e. \tag{7.16}$$

The mass μ_G of the gluons associated with the broken generators is given in this model by

$$\mu_G^2 = 12 e^2 f^2. \tag{7.17}$$

Thus

$$\mu_S \geqslant \mu_G/3\alpha, \qquad \alpha = e^2/4\pi. \tag{7.18}$$

A static solution similar to the one found by Prasad and Sommerfield [4] is expected to exist and saturate this bound. Such a solution will have $D_0\Phi = F_{0i} = 0$ so that the angular momenta of the H group for this solution calculated from Noether's theorem are identically zero. This indicates that the corresponding quantum state is an H singlet. Since the uniqueness of the ground state in the $Q = -1$ sector will also require this result, we can conclude with some confidence that this state is indeed an H singlet and is therefore experimentally observable.

8. Discussion

Ref. [2] has already described several interesting physical consequences implied by the symmetry breakdown $SU(3)_C \to SO(3) \times Z_3$. The presence of a monopole sector for such a symmetry breakdown opens up the possibility for a further range of novel phenomena which we now briefly discuss.

We denote the ground state of the monopole sector by S.

(1) The mass μ_S of S is very low. If the gluon mass μ_G is about 20 MeV [2] and α is about 0.2 [6], (7.18) gives the estimate

$$\mu_S \approx 33 \text{ MeV}. \tag{8.1}$$

Since this value of α is at 10 GeV^2 and α is supposed to grow with decreasing energy, (8.1) may be (a perhaps gross) overestimate. There is unfortunately no really good way of estimating α at low energies. Thus if α is continued using the one-loop renormalization group equation, for four flavours, it rises to the value $\simeq 0.4$ at

0.25 GeV^2 and to the value $\simeq 2.4$ at 0.01 GeV^2. For such values of α, however, perturbation theory is suspect.

(2) In so far as $SO(3) \times Z_3$ is an exact symmetry, S is absolutely stable due to its topological properties. It is thus the lightest stable hadronic state. (The fractionally charged states are expected to be more massive [2].)

(3) The spin of S is uncertain. While the naive expectation would be that S is spinless, it is known [7] for example that in the presence of suitable external probes, the system S + probe may behave like an electric charge + Dirac magnetic monopole system; in such a situation, it could be as though S is a fermion. These considerations, however, are semiclassical, and it has been suggested at least for the Bogomol'ny-Prasad-Sommerfield model [8, 4] that a full quantum treatment will reveal the spin of S to be 1.

(4) The state S has the topological quantum number $Q = -1$. This quantum number is multiplicative. Thus only an even number of S's can be produced in collisions of ordinary particles. The state S has strong interactions.

(5) The state S is the ground state of the monopole sector. It is natural to assume that it is a singlet under the flavour group $U(N_f) \times U(N_f)$ since in the effective lagrangian approach, there is already another order parameter to treat the breakdown of the chiral flavour group. Thus S is electrically neutral.

There can, however, be excitations above the ground state in the monopole sector which transform non-trivially under the flavour group and have weak and electromagnetic interactions. A suitable description of these excitations may be provided by an order parameter χ which transforms like

$$\bar{q}(1+\gamma_5)q\bar{q}(1-\gamma_5)q, \tag{8.2}$$

where the flavour and colour indices have been suppressed. In the ground state of the monopole sector, it will be Φ times a flavour singlet factor [the colour indices in $\bar{q}(1 \pm \gamma_5)q$ can of course be combined to an $\underline{8}$]. Its deviations $\delta\chi$ from Φ need not of course be flavour singlets or electrically neutral. [The observable states must be SO(3) singlets.] These electric charges are integral.

In view of the small characteristic mass scales μ_G and μ_S of the problem, the masses of such low-lying excitations are expected to be of the order of fractions of a pion mass. They have strong interactions. Their topological quantum number is -1. They can thus decay into S by strong or weak/electromagnetic processes. A pair of them, of course, has zero topological charge.

(6) The production of $Q = -1$ states is inhibited for the same reasons that the production of diquarks is inhibited [2]. Thus there is a potential between these states which rises to a height of 200 GeV/μ_G at a distance of 200 fm/μ_G, μ_G being in MeV. The potential falls off at larger distances. These exotic states can therefore be liberated only if they are able either to tunnel through the potential barrier or are sufficiently energetic.

These charged monopole states can be distinguished experimentally from muons because of their strong interactions. The cross section for their pair production in e^+e^- collisions at sufficiently high energies ($\geqslant$ the height of the potential barrier) is expected to be about (monopole charge)$^2 \times$ cross section for $e^+e^- \to \mu^+\mu^-$ (modulo form factors). These states if they exist may thus be observed at LEP if not at PETRA energies.

If E in GeV is the c.m. energy of two of these exotic states (diquark or monopoles), the tunnelling probability is governed by the well-known factor

$$\exp\left[-\int_{r_0}^{r_1} p\, \mathrm{d}x\right], \tag{8.3}$$

where r_0 and r_1 are the turning points of the potential, and the momentum p in the relativistic approximation is roughly $|200\ \mathrm{GeV}/\mu_G - E|$. Using a linear potential for distances less than $200\ \mathrm{fm}/\mu_G$ and approximating the potential at larger distances also by a linear one, we find this factor to be

$$\exp\left\{-\left(\frac{200}{\mu_G} - E\right)^2\right\}. \tag{8.4}$$

This is appreciable over a broad range of E, say $200\ \mathrm{GeV}/\mu_G \pm 1$ GeV, suggesting that the production of these states may cause a broad rise in the R value before it levels off. This effect is likely to be enhanced and the final plateau of R will be higher if, as one expects, there are several closely packed states (with mass differences of the order of 100 MeV or less) in the monopole sector. However, monopole form factors may suppress this effect.

(7) Like so many other species of particles, the monopoles S will have been abundantly produced in the early universe. Since they are absolutely stable except for pair annihilation, there is a chance of a measurable density of these particles surviving to the present epoch. We now estimate this density. Our calculations follow the well-known estimates for the U(1) monopoles [9].

Let $n(T)$ be the number density of monopoles at temperature T and let $r(T) = n(T)/T^3$. The rate equation for $n(T)$ can be solved to yield

$$\frac{1}{r(T)} = \frac{1}{r(T_i)} + \frac{A}{p-1}\frac{Cm_P}{\mu_S}\left[\left(\frac{\mu_S}{T}\right)^{p-1} - \left(\frac{\mu_S}{T_i}\right)^{p-1}\right], \tag{8.5}$$

where T_i denotes the temperature below which monopole production is negligible ($\simeq 20$ GeV in our case.) m_P is the Planck mass $\simeq 1.22 \times 10^{19}$ GeV. The explanation of the other quantities in (8.5) and their estimates are as follows:

(a) Estimate of $r(T_i)$: We can assume that at T_i, the monopoles are separated by about the capture radius r_c. A potential of the form $V \simeq \mathrm{constant} \cdot e^{-\mu_G r}/r$ for far separated monopoles gives $r_c \lesssim \mu_G^{-1}$ (on using the virial theorem to estimate the two monopole bound-state energy, cf. [9]). Thus

$$r(T_i) \simeq \frac{1}{r_c^3 T_i^3} \simeq \left(\frac{\mu_G}{T_i}\right)^3 \simeq 10^{-9}. \tag{8.6}$$

(b) Estimate of the second term in (8.5): A and p are defined in terms of the annihilation rate D by

$$D = \langle \sigma v \rangle = \frac{A}{\mu_S^2}\left(\frac{\mu_S}{T}\right)^p. \tag{8.7}$$

σ is the capture cross section and v is the r.m.s. velocity. As an estimate, $\sigma \simeq \pi r_C^2 \simeq \pi/\mu_G^2$. This gives $A = \pi\mu_S^2/\mu_G^2 \simeq 2.72\pi$ and $p = -\frac{1}{2}$. Also $C = 0.6/\sqrt{N}$, where N is the effective number of spin degrees of freedom of particles which are light compared to the temperature. We take $C \simeq \frac{1}{20}$ [9]. These values give at the present epoch ($T = 2.7$ K),

$$\text{second term in (8.5)} \simeq \frac{2A}{3}\frac{Cm_P}{\mu_S}\left(\frac{T_i}{\mu_S}\right)^{3/2} \simeq 15 \times 10^{23}. \tag{8.8}$$

$1/r(T_i)$ is negligible compared to this. We thus have

$$r(T) \simeq 7 \times 10^{-25},$$

or

$$n(T) \simeq 10^{-6}\,(\text{km})^{-3}. \tag{8.9}$$

Unfortunately this is too small a density to be detected in cosmic ray experiments.

There may be long-range forces between monopoles [10]. Crude estimates suggest that such forces do not change the value of $n(T)$ to an observable level. Note also that barrier effects which may cut off annihilation at $\simeq 19$ GeV increase $n(T)$ only by $\simeq 10^2$.

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