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Soliton states in the quantum-chromodynamic effective Lagrangian

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The work of Skyrme has shown that the SU(2)$\times$SU(2) chiral model has nontrivial topological sectors which admit solitons for generic chiral Lagrangians. In this paper, we study such models in the presence of baryon fields. The baryon number and strangeness of the solitons, and the bound states of the nucleon to the soliton are investigated. It is found that long-lived levels with large baryon number B and strangeness ($\gtrsim 6$ in magnitude) and masses somewhere in the range 1.8 to 5.6 GeV must exist. Some of these levels have half-integral electric charge and exotic relation between B and spin s (e.g., even B and half-integer s). It is speculated that these levels may be related to the anomalous nuclei whose existence has been confirmed in cosmic-ray and LBL Bevalac experiments.

I. INTRODUCTION

Nowadays, it is widely understood by elementary-particle physicists that when a continuous symmetry of a Lagrangian is broken by an asymmetric vacuum, solitonic excitations may be created which are stable for topological reasons. The most widely known of such solitons are the monopoles[1] which arise due to the symmetry breakdowns in grand unified theories. These states are stable due to the properties of the fields at spatial infinity ($\equiv$the two-sphere S^2); the topological invariant characterizing them is derived from the second homotopy group π_2 of a certain differentiable manifold.

Less familiar than the monopoles are the solitons discovered by Skyrme[2–5] in the SU(2)$\times$SU(2) chiral model. In this model, finiteness of energy requires the field [which is a 2$\times$2 SU(2) matrix] to go to a constant value at ∞. Thus the fixed-time slices of Minkowski space are effectively compactified to the three-sphere S^3 and the fields u can be classified by the winding number of the map $S^3\rightarrow$space of values of the field, that is, $S^3\rightarrow S^3$. Skyrme and others[2–5] have studied the properties of these topological sectors, and it has been suggested that the ground states of the sectors with winding numbers ± 1 are in fact the nucleon and antinucleon.

In this paper, we attempt a detailed analysis of the solitons in the winding number ± 1 sectors adopting the point of view that they are new states in the chiral Lagrangian. The coupling of the baryon octet to these solitons, and possible soliton-nucleon bound states are also investigated. Two sets of states in the mass range 1.8 to 5.6 GeV, with high baryon number and strangeness ($\gtrsim 6$ in magnitude), are predicted. Some of these states are found to be exotic, with half-integer charge and "wrong" relation between baryon number and spin (for instance, even baryon number and half-integer spin).

Section II reviews previous work[2–5] on the solitons in the SU(2)$\times$SU(2) chiral model. The masses of the solitons in the lowest nontrivial topological sectors are also estimated to be somewhat in the range 1.9 to 5 GeV.

Section III contains further brief comments on Skyrme's model.

In Sec. IV, we study the baryon number B and strangeness S of the different topological sectors by coupling the chiral field to baryons and using techniques due to Goldstone and Wilczek.[6,7] We find that in the presence of the baryon octet, B and S are equal to $6t$ in the sector with topological charge t. Thus the $t=\pm 1$ solitons are likely to be characterized by high values of $|B|$ and $|S|$.

Section V studies the possible bound states of the nucleon to the $t=\pm 1$ solitons. It is found that the Dirac equation has (at least) one positive-energy normalizable solution in the presence of the $t=-1$ soliton, and one negative-energy normalizable solution in the presence of the $t=+1$ soliton. The latter is the antiparticle of the former. The mass of these two states is somewhere in the range 1.8—5.6 GeV.

In Secs. VI and VII, we interpret our results and discuss the experimental observation of these states. It is argued that either the solitons or these bound states are exotic, with half-integral electric charge, and "wrong" relation between baryon number B and spin s (for instance, even B and half-integer s). It is pointed out that the observation of these states is quite difficult because of their high |baryon number| and |strangeness|, relatively low mass, and other unique characteristics. We also find that this difficulty is further enhanced by the presence of barrier-penetration effects which inhibit t-violating processes by a factor of 10^{-4} or more.

Our results are summarized in Sec. VIII. It is speculated that the levels we find may be related to the anomalous nuclei whose existence seems to have been confirmed by cosmic-ray and LBL Bevalac experiments.[8] The long lifetimes of these nuclei and of the levels we find tend to support the conjecture.

Appendix A contains a proof that the equality of the baryon and topological numbers in the presence of a nucleon of mass m (and relations like $B=6t$ in the presence of a baryon octet) are valid to all orders in powers of $1/m$. It was originally shown[6] in the leading order in this expansion. Appendix B argues that in each solitonic sector, there are rotational excitations which fulfill the relation spin=isospin. The existence of these levels is suggested in older research; however, our reasonings are rather different. Appendix C briefly reviews the paper of Gipson and Tze[9] on the possibility of solitons of Skyrme's type in the Glashow-Salam-Weinberg model. It is pointed out that there are new features which govern these solitons as compared to the chiral solitons so that further work is required to extend the results of this paper to this model.

II. SKYRME'S SOLITON

It was pointed out by Skyrme[2–5] many years ago that the SU(2) nonlinear chiral model can admit static solutions characterized by nontrivial topology. In this section, we will outline Skyrme's considerations and estimate the mass of the lightest of these solitons.

The SU(2) chiral model is described by a Lagrangian of the form

$$L=-\tfrac{1}{2}f_\pi{}^2\mathrm{Tr}\partial_\mu u^\dagger\partial_\mu u + \frac{1}{32e^2}\mathrm{Tr}[\partial_\mu uu^\dagger,\partial_\nu uu^\dagger]^2+\cdots \equiv L_0+L_1+\ldots\,, \tag{2.1}$$

$$f_\pi\simeq 67\text{ MeV}\,, \tag{2.2}$$

$$u\to 1 \text{ as } r\equiv|\vec{x}|\to\infty\,, \tag{2.3}$$

where $u(x)$ is a 2×2 unitary matrix of determinant 1. The term L_0 in L is responsible for conventional current-algebra results and is well known. The term L_1 is crucial for the existence of static solitonic solutions as the work of Skyrme shows. The constant e will be estimated later. There can in general be many additional terms (represented by dots) in L (L being an effective Lagrangian); we shall assume in the rest of this paper that they do not seriously affect our conclusions. Finiteness of energy requires that $u\to$a constant matrix u_0 as $r\to\infty$. The boundary condition (2.3) follows from this observation since u_0 can be reduced to 1 by a chiral rotation.

The topology of constant-time surfaces in space-time is R^3. The boundary condition (2.3) effectively compactifies this R^3 to the three-sphere S^3. The field u maps this S^3 to SU(2) which too is topologically S^3. Since the homotopy group $\pi_3(S^3)$ is Z, we see immediately that this model has an infinite number of topological sectors characterized by an integer-valued winding number. Skyrme has shown that this winding number t can be written as the integral

$$t=\frac{1}{24\pi^2}\epsilon_{ijk}\int d^3x\,\mathrm{Tr}(\partial_i uu^\dagger\partial_j uu^\dagger\partial_k uu^\dagger)\,. \tag{2.4}$$

Associated with the "charge" t, there is also the conserved topological current

$$j_\mu=\frac{1}{24\pi^2}\epsilon_{\mu\nu\lambda\rho}\mathrm{Tr}(\partial_\nu uu^\dagger\partial_\lambda uu^\dagger\partial_\rho uu^\dagger)\,. \tag{2.5}$$

A static stable solution in the sector characterized by the number t can be interpreted as a soliton with topological number t.

It is not difficult to make a variational estimate of the soliton mass M for $|t|=1$. A suitable variational *Ansatz* is

$$u=\cos\theta(r)+i\tau_i\hat{x}_i\sin\theta(r)\,, \tag{2.6}$$

$$\cos\theta(r)=\frac{1-(r/R)^2}{1+(r/R)^2}\,,\qquad \sin\theta(r)=\frac{2(r/R)}{1+(r/R)^2}\,. \tag{2.7}$$

It is characterized by the variational parameter R which is a measure of the size of the soliton. The form (2.6) of u is characterized by a high degree of symmetry being invariant under combined spatial and isospin rotations. It can be shown[5] that the minimum of energy (for L_0+L_1) is in fact attained on this class of functions for $|t|=1$. A little algebra shows that the topological charge t in terms of θ is

$$t=\frac{1}{\pi}[\theta(0)-\theta(\infty)] \tag{2.8}$$

so that (2.7) gives $t=-1$. M is of course, independent of the sign of t. We have also checked that the value of M does not sensitively depend on the parametrization (2.7) of θ.

The energy integral for L_0+L_1 is

$$E(R)=\int d^3x\{\tfrac{1}{2}f_\pi^2\mathrm{Tr}\vec{\nabla}u^\dagger\vec{\nabla}u + \frac{1}{32e^2}\mathrm{Tr}[\partial_i uu^\dagger,\partial_j uu^\dagger]^2\} \tag{2.9}$$

if u is a static field. For our *Ansatz,* it takes the form

$$E(R)=\frac{2\pi^2}{e}f_\pi\left[\frac{I_1}{e(f_\pi R)}+2eI_2(f_\pi R)\right],$$

where

$$I_1=\frac{1}{\pi}\int_0^\infty dx\left[\frac{\sin^4\theta}{x^2}+2\sin^2\theta\left[\frac{d\theta}{dx}\right]^2\right]\cong 1.50\,, \tag{2.10}$$

$$I_2=\frac{1}{\pi}\int_0^\infty dx\left[\sin^2\theta+x^2\left[\frac{d\theta}{dx}\right]^2\right]\cong 2.98\,,$$

where $x=r/R$. Thus the minimum of $E(R)$ is when

$$R=R_0=\frac{0.5}{ef_\pi}\,. \tag{2.11}$$

The value of the minimum is

$$E(R_0)\equiv M=118\frac{f_\pi}{e}\,. \tag{2.12}$$

We need an estimate for e to get a number for M. For this, note that while L_1 does not contribute to S-wave scattering lengths, it does affect the P-wave π-π scattering length a_1^1; its contribution δa_1^1 to a_1^1 is a correction to the conventional current-algebra value 0.03.[10] A standard calculation using the parametrization

$$u=\frac{1+i\frac{1}{2\sqrt{2}f_\pi}\tau_i\pi_i}{1-i\frac{1}{2\sqrt{2}f_\pi}\tau_i\pi_i} \tag{2.13}$$

of u in terms of the pion field $\vec{\pi}$ gives

$$\delta a_1^1=\frac{1}{32\pi e^2}\left[\frac{m_\pi}{f_\pi}\right]^4, \tag{2.14}$$

where m_π is the pion mass. Experimentally, two conflicting numbers $\simeq 0.04$ and 0.1 have been reported for a_1^1.[10] This suggests the bounds

$$0.01\lesssim\frac{1}{32\pi e^2}\left[\frac{m_\pi}{f_\pi}\right]^2\lesssim 0.07 \tag{2.15}$$

or for $m_\pi\simeq 137$ MeV,

$$2.48\lesssim e^2\lesssim 17.39\,. \tag{2.16}$$

Thus, from (2.11) and (2.12),

$$0.25m_\pi^{-1}\lesssim R_0\lesssim 0.65m_\pi^{-1}\,,\quad 1.9\lesssim M\lesssim 5\ \text{GeV}\,. \tag{2.17}$$

For completeness, we may note here that a lower bound for the energy E can be derived in terms of t (Ref. 2) and that this bound is independent of the *Ansatz* for u. This follows from the inequality

$$\int d^3x\,\mathrm{Tr}\left\{\frac{f_\pi}{\sqrt{2}}\partial_i uu^\dagger\pm\frac{1}{8e}\epsilon_{ijk}[\partial_j uu^\dagger,\partial_k uu^\dagger]\right\}^2 \lesssim 0\,, \tag{2.18}$$

which implies that

$$E=-\int d^3x\left\{\frac{f_\pi^2}{2}\mathrm{Tr}(\partial_i uu^\dagger)^2 + \frac{1}{32e^2}\mathrm{Tr}[\partial_i uu^\dagger,\partial_j uu^\dagger]^2\right\} \gtrsim\frac{3\sqrt{2}\pi^2}{e}f_\pi|t|\,. \tag{2.19}$$

It is known, however, that this bound cannot be saturated.[4] Numerical work indicates that in the $|t|=1$ sector the variational estimate given before is closer to the actual static energy than the value given by (2.19).

III. FURTHER COMMENTS ON SKYRME'S SOLITONS (NO FERMIONS)

The *Ansatz* (2.6) is not invariant under separate isospin or spatial rotations, suggesting that the degrees of freedom associated with these geometric transformations will give rise to rotational excitations on quantization.[2,4] This seems in fact to be correct. We will have occasion to comment on these levels briefly in Appendix B.

The generalization of Skyrme's considerations to the flavor groups SU(N) ($N\geq 3$) brings in new features due to the fact that there are inequivalent maps of the compactified space S^3 into these groups. Thus for SU(3), we have the two subgroups SU(2) and SO(3) where this SU(2) acts *nontrivially* on the first two quarks (say) while SO(3) is the subgroup of real orthogonal matrices. The image of S^3 under u can cover either SU(2) or SO(3) many times, these two kinds of maps cannot in general be transformed into each other by a flavor rotation. [A general chiral rotation, which will not respect the boundary condition (2.3), is not relevant in this context.] For each such inequivalent subgroup [locally isomorphic to SU(2)] of SU(N) we will have associated solitons and their rotational excitations.

It will be worthwhile to estimate the mass of the lightest solitons when u has values in the SO(3) subgroup of SU(3), although we have not done so.

It may be remarked that there are no long-range forces between two solitons or a soliton and an antisoliton.[4] The reason is that the charge t on one such single soliton cannot be written as the integral of a field over a large surface surrounding the soliton; thus there is no necessity for a field falling off with the inverse square law from a soliton. There is therefore no compelling reason for a long-range force between pairs of these objects when they are well separated. Calculations confirm this argument, the force at large distances falls off faster than the inverse square of the distance d for *Ansätze* of the type (2.6), the power law for the force is $1/d^3$ for large d.

For $|t| \geq 2$, the minimum E_t of energy in the class of spherically symmetric *Ansätze* (2.6) is also an extremum for arbitrary (and not just spherically symmetric) variations of the field, as we know from general theorems.[11] However, the absolute minimum of energy in these sectors is not attained in this class since it is known that $E_t > |t| E_1$, thus the configuration with $|t|$ widely separated $|t|=1$ solitons have lower energy. This means that the $|t| \geq 2$ solitons are unstable and can decay into solitons with lower $|t|$. This decay proceeds by barrier penetration.

IV. THE BARYON NUMBER AND HYPERCHARGE OF SOLITONS

When u couples to baryons, the ground state of any soliton sector is changed, it consists now also of the filled Dirac sea. For such a system, the baryon number of the soliton is a meaningful concept, it is the expectation value of the baryon-number operator in the associated ground state. In this section, we will evaluate this number using known techniques.[6,7]

Let us first consider the simplified model where only the nucleon N couples to u. The conventional interaction of N and u is

$$L_I = -m\bar{N}UN = -m\bar{N}_R u N_L + \text{H.c.} , \tag{4.1}$$

$$U \equiv \cos\theta + i\tau_i \hat{x}_i \gamma_5 \sin\theta , \tag{4.2}$$

where m is the nucleon mass. If $|t\rangle$ is the ground state of the sector with topological number t, its baryon number B is defined by

$$B\langle t | t\rangle = \langle t | \int d^3x\, N^\dagger N | t\rangle . \tag{4.3}$$

In a recent paper, Goldstone and Wilczek[6] have given a method for evaluating the right-hand side in powers of $1/m$. The leading term in this expansion can be read off from their Eq. (6). [Note that their boundary conditions are different from (2.3).] The result to this order is

$$B = t . \tag{4.4}$$

We show in Appendix A that this equality is valid to all orders in powers of $1/m$.

This remarkable result, which claims that the soliton carries baryon number equal to t, has been conjectured before.[2,4] We can see however that it gets modified in the presence of more baryons. Thus the coupling of a baryon of isospin I to u can be

$$L_I = -m_I \bar{N}_R^{(I)} u_I N_L^{(I)} + \text{H.c.} , \tag{4.5}$$

where u_I is the representative of u in the representation with isospin I. For this coupling, the value of baryon number in the state $|t\rangle$ is not t, but rather,

$$\frac{1}{24\pi^2}\epsilon_{ijk} \int d^3x\, \text{Tr}(\partial_i u_I u_I^\dagger \partial_j u_I u_I^\dagger \partial_k u_I u_I^\dagger)$$
$$= \frac{1}{48\pi^2}\epsilon_{ijk} \int d^3x\, \text{Tr}\{\partial_i u_I u_I^\dagger [\partial_j u_I u^\dagger, \partial_k u_I u_I^\dagger]\} . \tag{4.6}$$

Now $\partial_j u_I u_I^\dagger$ and hence the commutator in (4.6) belong to the isospin I representation $L^{(I)}_{\text{SU}(2)}$ of the SU(2) Lie algebra $L_{\text{SU}(2)}$. The trace is thus over the product of two elements in $L^{(I)}_{\text{SU}(2)}$. If $x^{(I)}, y^{(I)}$ are the representatives of $x,y,\epsilon L_{\text{SU}(2)}$ in the representation with isospin I, we know that

$$\text{Tr}x^{(I)}y^{(I)} = \frac{(2I+1)I(I+1)}{[(2\times\frac{1}{2})+1]\frac{1}{2}(\frac{1}{2}+1)} \times \text{Tr}x^{(1/2)}y^{(1/2)} . \tag{4.7}$$

Hence the expression (4.6) is

$$\tfrac{2}{3}(2I+1)I(I+1)t \ . \tag{4.8}$$

When the baryon octet couples to the soliton, the baryon number of $|t\rangle$ is thus

$$B=(1+4+1)t \ =6t \ . \tag{4.9}$$

The coupling of additional fermions to u will increase this number.

The hypercharge Y of $|t\rangle$ can be calculated in a similar way. The operator for Y of the baryon octet is

$$\int d^3x(N^\dagger N-\Xi^\dagger\Xi) \ , \tag{4.10}$$

so that Y is zero for $|t\rangle$ and strangeness is $-6t$ in the presence of the baryon octet. [The relation $Y=0$ is not, however, maintained for an arbitrary SU(3) multiplet of baryons, as can be easily checked.]

Thus we see that these solitons are characterized by relatively high baryon number and strangeness ($\gtrsim 6$ in magnitude).

V. THE SOLITON-NUCLEON BOUND STATE

In this section, we want to show that there is a positive-energy bound state of the nucleon N to the $t=-1$ soliton (and a negative-energy bound state to the $t=+1$ soliton).

For the interaction (4.1), the bound states of N and u are governed by the Hamiltonian

$$H=\vec{\alpha}\cdot\vec{\mathrm{p}}+m\beta U \ . \tag{5.1}$$

This form of H is awkward for approximate computations of the spectrum due to the large factor m in the interaction. This difficulty can be circumvented by unitarily transforming H first:

$$H^{(1)}=VHV^\dagger \ =\vec{\alpha}\cdot\vec{\mathrm{P}}+\beta m \ , \tag{5.2}$$

$$V\equiv u\frac{1+\gamma_5}{2}+\underline{1}\frac{1-\gamma_5}{2} \ , \tag{5.3}$$

$$\vec{\mathrm{P}}\equiv\vec{\mathrm{p}}-iu\,\vec{\nabla}u^\dagger\frac{1+\gamma_5}{2} \ . \tag{5.4}$$

When the soliton size R_0 is large compared to m^{-1}, $H^{(1)}$ can be simplified by standard Foldy-Wouthuysen transformations.[12] The transformation which leads to an approximate Hamiltonian accurate up to leading terms in $1/m$ is generated by

$$W=\exp\left[\frac{\beta O_1}{2m}\right]\exp\left[\frac{\beta O_0}{2m}\right] \ , \tag{5.5}$$

$$O_0=\vec{\alpha}\cdot\vec{\mathrm{p}}-\frac{i}{2}\vec{\alpha}\cdot u\ \vec{\nabla}u^\dagger\frac{\gamma_5}{2} \ ,$$

$$O_1=\frac{\beta}{2m}[O_0,E_0]-\frac{1}{3m^2}O_0{}^3+O\left[\frac{1}{m^3}\right] \ ,$$

$$E_0=-i\vec{\alpha}\cdot u\ \vec{\nabla}u^\dagger\frac{\gamma_5}{2} \ . \tag{5.6}$$

The transformed Hamiltonian accurate to $O(1/m)$ is

$$\begin{aligned} H^{(2)}&=WH^{(1)}W^\dagger \\ &=\beta\left[\frac{1}{2m}(\vec{\alpha}\cdot\vec{\mathscr{P}})^2+m\right]-i\vec{\alpha}\cdot u\,\vec{\nabla}u^\dagger\frac{\gamma_5}{2} \ , \\ \vec{\mathscr{P}}&=\vec{\mathrm{p}}-iu\,\vec{\nabla}u^\dagger \ . \end{aligned} \tag{5.7}$$

For positive-energy eigenstates, the wave functions in the representation

$$\beta=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}, \quad \alpha_i=\begin{bmatrix}0 & \sigma_i\\ \sigma_i & 0\end{bmatrix} \tag{5.8}$$

are of the form

$$\begin{bmatrix}\phi\\ 0\end{bmatrix} , \tag{5.9}$$

where $\phi=(\phi_{a\alpha})$ $(a,\alpha=1,2)$ and a and α are isospin and spin labels. The eigenvalue problem for positive energies is thus

$$H^{(3)}\phi=\mu\phi, \ \ \mu>0 \ , \tag{5.10}$$

$$\begin{aligned} H^{(3)}\equiv m&+\frac{\mathscr{P}^2}{2m}+\frac{i}{2}\vec{\sigma}\cdot u\ \vec{\nabla}u^\dagger \\ &+\frac{i}{4m}\epsilon_{ijk}\sigma_i\mathscr{P}_j\mathscr{P}_k \ . \end{aligned} \tag{5.11}$$

We shall investigate only the ground state, this will be characterized by a high degree of symmetry. Since $H^{(3)}$ commutes with

$$\vec{\mathrm{r}}\times\vec{\mathrm{p}}+\frac{\vec{\sigma}}{2}+\frac{\vec{\tau}}{2}$$

(where $\vec{\sigma}$ acts on spin and $\vec{\tau}$ on isospin index), we can assume that

$$\left[\vec{\mathrm{r}}\times\vec{\mathrm{p}}+\frac{\vec{\sigma}}{2}+\frac{\vec{\tau}}{2}\right]\phi=0 \tag{5.12}$$

for the ground state. The general form of ϕ is thus

$$\phi=[f(r)+i\hat{x}\cdot\vec{\Sigma}g(r)]\Sigma_2 \,. \tag{5.13}$$

Here Σ_i are Pauli matrices the first index of which is acted on by the isospin matrices $\vec{\tau}$ and the second index by the spin matrices $\vec{\sigma}$. The wave function f corresponds to zero orbital angular momentum while g corresponds to orbital angular momentum 1. Substitution of (5.13) into (5.10) leads to the following radial equations:

$$\left[\frac{p_r^2}{2m}-\frac{\theta'}{2}+\frac{(\theta')^2}{8m}-\alpha\left[1+\frac{\theta}{2m}\right]\right]f-\beta\left[1+\frac{\theta'}{2m}\right]g-\frac{1}{4m}\left[\theta''+\frac{2\theta'}{r}+\frac{\alpha}{2mr}\right]g-\frac{\theta'g'}{2m}=(\mu-m)f \,, \tag{5.14}$$

$$\left[\frac{1}{2m}\left[p_r^2+\frac{2}{r^2}\right]-\frac{\theta'}{2}+\frac{(\theta')^2}{8m}+\alpha\left[1+\frac{\theta'}{2m}\right]-\frac{\beta}{mr}\right]g-\beta\left[1+\frac{\theta'}{2m}\right]f+\frac{1}{4m}\left[\theta''+\frac{2\theta'}{r}-\frac{2\alpha}{r}\right]g+\frac{\theta'f'}{2m}$$
$$=(\mu-m)g \,, \tag{5.15}$$

$$\alpha\equiv\frac{\sin\theta\cos\theta}{r} \,, \tag{5.16}$$

$$\beta\equiv\frac{\sin^2\theta}{r} \,. \tag{5.17}$$

Prime denotes differentiation on r.

The existence or otherwise of a bound state ($\mu-m<0$) is determined by the θ' term in the first equation. It acts as an attractive potential if θ increases with r, thus there is the possibility of a positive-energy bound state of the nucleon to the $t=-1$ soliton. An adequate variational *Ansatz* for the bound-state wave function is

$$f=ce^{-kr} \,, \tag{5.18}$$

$$g=0 \,, \tag{5.19}$$

where the variational parameter $1/k$ is a measure of the size of the bound state, and c is the normalization constant. For the parametrization (2.7), a numerical variational calculation gives

$$\mu\simeq m-3.4f_\pi e \,. \tag{5.20}$$

It corresponds to $k=k_0$ where

$$\frac{1}{k_0}\simeq\frac{0.17}{f_\pi e} \,. \tag{5.21}$$

[The value of μ is not sensitively dependent on the parametrization (2.7) of θ.] For the estimate (2.16), we thus find

$$-12 \text{ MeV}\lesssim\mu\lesssim 579 \text{ MeV} \,, \tag{5.22}$$

$$0.09m_\pi^{-1}\lesssim\frac{1}{k_0}\lesssim 0.22m_\pi^{-1} \,. \tag{5.23}$$

For e^2 as large as 17.39, $1/R_0m$ is of the order of 0.6 and is large while our approximation methods for μ are valid only if it is small. The lower limit for μ is thus unreliable. If we now replace θ by $-\theta$, the soliton with $t=-1$ becomes the soliton with $t=+1$. The antinucleon binds to this soliton, this bound state being the charge conjugate of the state found above. For this bound state, the upper two components of the wave function vanish while it is associated with the eigenvalue $-\mu$ of $H^{(3)}$.

Thus we find that corresponding to the two solitons of mass M and $t=\pm 1$, there are two bound states of mass $M+\mu$ where

$$1.8\lesssim M+\mu\lesssim 5.6 \text{ GeV} \,. \tag{5.24}$$

VI. INTERPRETATION

It is striking that at mass $M+\mu$, there is only one bound state of the nucleon field to the static $t=-1$ (or $t=+1$) background field u. This fact has remarkable consequences.

Thus let Ψ be the wave function of this level for a given u. We can expand N in terms of a complete set of eigenfunctions of H (Ref. 7):

$$N=a\Psi+\dots \,, \tag{6.1}$$

where $\cdots$ represents the contribution of all the remaining eigenfunctions. The operator a is the annihilation operator for the bound state so that

$$a^\dagger|-1\rangle \tag{6.2}$$

is the bound state. Note that if $|-1\rangle$ is k-fold degenerate, this level is also k-fold degenerate.

Now the fact that there is only one such annihilation operator a means that it has spin and isospin zero. Also, it carries baryon number -1 as we see from the expansion (6.1). Thus *the operator a is ex-*

otic, being bosonic in spin and carrying odd baryon number.[13] Since N obeys Fermi statistics, a also obeys Fermi statistics:

$$\{a,a\}=\{a^\dagger,a^\dagger\}=0\ , \tag{6.3}$$

$$\{a,a^\dagger\}=1\ , \tag{6.4}$$

$$\{A,B\}\equiv AB+BA\ . \tag{6.5}$$

Note also that *the electric charge of a is fractional, and equal to* $-\frac{1}{2}$.[13]

We cannot decide if $a^\dagger|-1\rangle$ has exotic properties without knowing the properties of $|-1\rangle$. It is difficult to determine the spin and isospin of solitons in the absence of an adequately precise quantum theory of solitons. (For suggestions regarding the quantum numbers of $|t\rangle$, see Refs. 2—4 and 9.) In any event, we see that either $|-1\rangle$ or $a^\dagger|-1\rangle$ is exotic.[13] Similar remarks apply to $|1\rangle$ and its partner. Thus we come to the following conclusion:

There are two or more (if $k\gtrsim 2$) degenerate exotic levels in the mass range 1.8—5.6 GeV which carry odd (even) baryon number and are bosonic (fermionic) in spin. Their baryon number and strangeness are large ($\gtrsim 6$) in magnitude. They may carry fractional (half-integral) electric charge.[14]

We may note here that the possibility of exotic states in the presence of solitons is well known in the literature.[7,6] Our analysis in fact borrows heavily from previous work which deals with such phenomena. What is perhaps novel in this paper is the exploration of these unusual phenomena in the chiral model which is so close to experiments in particle physics.

VII. PHENOMENOLOGICAL OBSERVATIONS

The production of these states is suppressed for several reasons as we shall see below. Their existence therefore probably does not contradict experimental results to date. They can of course be produced, although with a small cross section, in high-energy p-p or e^+-e^- reactions.

We shall now list some of the reasons why the production of these states is inhibited.

(1) The $t\neq 0$ levels are characterized by large baryon number B and strangeness (in magnitude). The decay of normal matter into these states is thus suppressed.

(2) The $|t|=1$ levels are relatively light, they also have large $|B|$ and |strangeness|. Quantum-number selection rules will require them to decay into several baryons (or antibaryons) and perhaps K's, but such processes are suppressed or forbidden due to their light mass.

(3) The states in the solitonic sectors are extended. Their pair production is thus likely to be suppressed by form-factor effects.

(4) There is a further suppression factor, of the order of 10^{-4} in rate, for any t-violating process.

We shall now explain the last point. Recall that Skyrme's topological charge is not conserved in the σ model if the σ field is allowed to fluctuate to zero. Thus if we formulate the latter model in terms of say a 2×2 matrix M with $M^\dagger M=\sigma^2$, then so long as $\sigma\neq 0$, we can define our u as M/σ without encountering singularities, and t is conserved. But if σ develops zeros, u is undefined at these zeros, and the excitation of such zeros can lead to t nonconservation. The suppression factor for rates for such t-violating processes is, in the semiclassical approximation,

$$[\exp(-S)]^2\ , \tag{7.1}$$

where S is the Euclidean action evaluated for a suitable field which interpolates between different values of the winding number t as time evolves from $-\infty$ to $+\infty$. For our purposes, it is sufficient to get a lower bound for S, it will give us an upper bound for the suppression factor. Now if we define

$$A_\mu=\tfrac{1}{2}\partial_\mu u u^\dagger\ , \tag{7.2}$$

then

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu-[A_\mu,A_\nu]=\tfrac{1}{4}[\partial_\mu u u^\dagger,\partial_\nu u u^\dagger] \tag{7.3}$$

and

$$S\gtrsim\int d^4x\,L_2=\frac{1}{2e^2}\int d^4x\,\mathrm{Tr}F^2\ . \tag{7.4}$$

By a well-known inequality,

$$\frac{1}{16\pi^2}\int d^4x\,\mathrm{Tr}F^2\gtrsim\frac{1}{16\pi^2}\left|\int d^4x\,\mathrm{Tr}F^*F\right|\ . \tag{7.5}$$

The right-hand side is just the magnitude of the change Δt of t from initial to final state. Thus

$$S\gtrsim\frac{8\pi^2}{e^2}\Delta t \tag{7.6}$$

and

$$\text{suppression factor}\lesssim\exp\left[-\frac{16\pi^2}{e^2}\right] \tag{7.7}$$

$$\simeq 10^{-4} \tag{7.8}$$

for $e^2 \simeq 2.48$.

A few comments are in order regarding this estimate: (a) No smooth u on Euclidean space-time will induce such a change of t (since t does not change under smooth deformations of u), u must necessarily have singularities. (b) Meron solutions can be written in the form (7.2) (Ref. 15) and a two-meron or a two-antimeron solution can lead to $|\Delta t| = +1$. These solutions are singular at isolated points, the positions of the singularities can be thought of as the zeros of σ. (c) A more detailed estimate of S using meron solutions leads to results which depend sensitively on the cutoffs used.

VIII. SUMMARY

Let us assume for definiteness that the states $|\pm 1\rangle$ are normal with no unusual relationship between baryon number and spin, and no fractional electric charge. Then our results can be summarized as follows.

(1) There are at least two states at each of the masses M and $M+\mu$ where

$$1.9 \lesssim M \lesssim 5 \text{ GeV} , \tag{8.1}$$

$$1.8 \lesssim M+\mu \lesssim 5.6 \text{ GeV} . \tag{8.2}$$

They correspond to $|\pm 1\rangle$ and $a^\dagger |\pm 1\rangle$. (If there are k states with $t=1$ or -1, there are $2k$ levels at each of these masses.) The magnitude of their baryon number and strangeness is large ($\gtrsim 6$).

(2) The more massive level (or levels) is exotic, with half-integer electric charge and "wrong" relation between baryon number and spin.

(3) These states are spatially extended. This property (which is expected to imply small form factors) in combination with their quantum numbers and mass values makes it difficult to observe them. There are further suppression factors of the order of 10^{-4} or more from barrier-penetration effects for t-violating processes which shield these states from decay into normal matter. Their lifetime for instance is thus expected to be much longer than what we would normally expect from their baryon number, strangeness, or mass.

Anomalous nuclei with unexpectedly large lifetimes and cross sections (≈ 10 times normal cross sections) have been reported in the literature.[8] It is conceivable that these are some of the levels we find. Although we have not found a mechanism to enhance the cross section of the $t \neq 0$ states on normal matter, these states are poorly understood and such an enhancement cannot be ruled out.

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APPENDIX A: EXACTNESS OF THE RELATION $B=t$

In the Goldstone-Wilczek derivation,[6] the result (4.4) is the leading approximation to B in powers of $1/m$. We shall now argue that it is in fact valid to all orders in powers of $1/m$.

The basic assumption we make is that the exact expression for B, just like the leading approximation (4.4), will be conserved for purely topological reasons whatever be the dynamics governing the time evolution of u.

The contribution to B proportional to $(1/m)^{n-3} (n \gtrsim 3)$ is of the form

$$B_n = \frac{1}{m^{n-3}} \int \epsilon_{\mu\nu\lambda\rho} j_n^\mu dx^\nu dx^\lambda dx^\rho$$

$$\equiv \frac{1}{m^{n-3}} \int \omega_n , \tag{A1}$$

$$B_3 \equiv t , \tag{A2}$$

where j_n^μ is independent of m and $j_3^\mu = j^\mu$ [Eq. (2.5)]. Further, by our assumption, the conservation of the current j_n^μ is an algebraic identity:

$$\partial_\mu j_n^\mu \equiv 0 . \tag{A3}$$

For dimensional reasons, j_n^μ will contain $n-3$ differentiations of u. Therefore,

$$j_n^\mu \neq \text{constant} \times j_3^\mu , \quad n \gtrsim 3 . \tag{A4}$$

However, it is a known theorem that the only algebraically conserved currents formed from u which give a nonzero charge are of the form constant $\times j_3^\mu$ (modulo terms which give zero charge). Thus

$$B_n = 0 \quad \text{for } n > 3 . \tag{A5}$$

We can also state this observation as follows. By our assumption, the differential forms ω_n are the pullbacks to space-time (via the map u) of closed differential forms on S^3. But the cohomology group H^3 (S^3) has one generator, and its pullback can be taken to be ω_3. Thus

$$\omega_n = \xi\omega_3 + \tilde{\omega}_n , \quad n > 3 \tag{A6}$$

$$\xi = \text{constant} , \tag{A7}$$

$$\int \tilde{\omega}_n = 0 \,. \tag{A8}$$

For dimensional reasons, ξ must be proportional to m^{n-3}. As ω_n is independent of m, it follows that

$$\xi = 0 \,, \tag{A9}$$

$$B_n = 0 \,. \tag{A10}$$

APPENDIX B: ROTATIONAL EXCITATIONS

In the text, we have made no attempt to develop a systematic quantum theory of the chiral solitons. In fact, it does not seem possible to develop such a theory since the model is not renormalizable. However, it is possible to make partial statements regarding the spectrum of quantum states by generalizing the elegant methods of Kerman and Klein, and Goldstone and Jackiw.[16] We shall now do so and argue that in each solitonic sector, there is a spectrum of rotational excitations with spin s=isospin I. Alternative arguments for the existence of these levels have also been given.[2,4]

Following Ref. 16, we shall assume below the existence of a complete set of single-particle states $|\Psi_n\rangle$ and the form of the matrix elements of the operator $\hat{u}$ (the quantum field associated with the classical chiral field) between these states in terms of the classical static solitonic solution u. We shall then show that any operator equation involving $\hat{u}$ and $\vec{\nabla}\hat{u}$ can be fulfilled provided that the corresponding classical equation involving u and $\vec{\nabla} u$ is fulfilled. We can thus show that the topological number of the state is in fact governed by the classical solution.

We have not, however, been able to generalize these considerations to expressions which involve the momenta conjugate to $\hat{u}$ in a useful way because of the nonrenormalizability of the theory.

The discussion ignores fermions, their inclusion will be commented on toward the end.

Let $\{D^{(\rho)}(g)\}$ denote the $(2\rho+1)$-dimensional unitary irreducible representation of $SU(2)=\{g\}$. If dg is the invariant SU(2) measure normalized to unity,

$$\int dg = 1 \,, \tag{B1}$$

we have the orthogonality and completeness relations

$$\int dg\, D^{(\rho)}_{ab}(g)^* D^{(\sigma)}_{cd}(g) = \frac{1}{2\rho+1}\delta_{\rho\sigma}\delta_{ac}\delta_{bd} \,, \tag{B2}$$

$$\sum_\rho (2\rho+1)\mathrm{Tr}[D^{(\rho)}(g')^\dagger D^{(\rho)}(g)] = \delta(g'^{-1}g) \,. \tag{B3}$$

We now postulate that in a solitonic sector with $t \neq 0$, there are a set of single-particle levels $|\vec{P}JMJ\alpha\rangle$ with three-momentum $\vec{P}$, spin and isospin J, and corresponding magnetic quantum numbers M and α (note that M here is *not* the soliton mass). The values of J are restricted to be either

$$0, 1, \cdots \tag{B4}$$

or

$$\tfrac{1}{2}, \tfrac{3}{2}, \cdots \,. \tag{B5}$$

The state normalization is

$$\langle \vec{P}'J'M'J'\alpha' | \vec{P}JMJ\alpha\rangle = \delta^3(P'-P)\delta_{J'J}\delta_{M'M}\delta_{\alpha'\alpha} \,. \tag{B6}$$

We also postulate the following form for the matrix elements of u:

$$\langle \vec{P}'J'M'J'\alpha' | \hat{u}_{ab}(0) | \vec{P}JMJ_\alpha\rangle = [(2J'+1)(2J+1)]^{1/2} \int \frac{d^3x}{(2\pi)^3} dg\, e^{-i(\vec{P}'-\vec{P})\cdot\vec{x}} \times [C^{(J')}D^{(J')}(g)]^*_{\alpha'M'}[C^{(J)}D^{(J)}(g)]_{\alpha M} u_{ab}(x,g) \,, \tag{B7}$$

$$C^{(J)} = e^{i\pi J_2^{(J)}} \,, \tag{B8}$$

$$u(\vec{x},g) = g[\cos\theta(r) + i\vec{\tau}\cdot\hat{x}\sin\theta(r)]g^\dagger \,. \tag{B9}$$

Note the following: (a) Here $u(\vec{x},g)$ is a flavor-rotated classical static solution. Because of the isospin invariance of the chiral dynamics, if $u(\vec{x},e) \equiv u(\vec{x})$ is a classical static solution, then so is $u(\vec{x},g)$ for all g. (b) In (B8), $J_2^{(J)}$ is the second spin-J angular momentum matrix and

$$C^{(J)}D^{(J)}(g)C^{(J)-1} = D^{(J)}(g)^* \,. \tag{B10}$$

The form of the right-hand side in (B7) has been adjusted so as to be compatible with the following identity fulfilled by the left-hand side:

$$\langle \vec{\mathrm{P}}\,'J'M'J'\alpha' \,|\, [g\hat{u}(0)g^{\dagger}]_{ab} \,|\, \vec{\mathrm{P}}JMJ\alpha \rangle = D^{(J')}(g)^{\dagger}_{\alpha'\beta'} \langle \vec{\mathrm{P}}\,'J'M'J'\beta' \,|\, \hat{u}(0)_{ab} \,|\, \vec{\mathrm{P}}JMJ\beta \rangle D^{(J)}(g)_{\beta\alpha}\,, \tag{B11}$$

$$[g\hat{u}(0)g^{\dagger}]_{ab} \equiv g_{aa'}\hat{u}(0)_{a'b'}g^{\dagger}_{b'b}\;. \tag{B12}$$

This identity follows from the existence of the isospin rotation operator $T(g)$ which fulfills

$$T(g)\hat{u}(0)_{ab}T(g)^{\dagger} = [g^{\dagger}\hat{u}(0)g]_{ab}\;. \tag{B13}$$

The matrix elements of $\hat{u}(x)$ are fixed by (B7) and translational invariance.

If $F(\hat{u}, \vec{\nabla}\hat{u})$ is any functional involving $\hat{u}$ and its gradient, then

$$\langle \vec{\mathrm{P}}\,'J'M'J'\alpha' \,|\, F(\hat{u}, \vec{\nabla}\hat{u}) \,|\, \vec{\mathrm{P}}JMJ\alpha \rangle$$

can be evaluated by saturating the intermediate states with states of the form $|\vec{\mathrm{Q}}KNK\beta\rangle$. The result is

$$\langle \vec{\mathrm{P}}\,'J'M'J'\alpha' \,|\, F(\hat{u}, \vec{\nabla}\hat{u}) \,|\, \vec{\mathrm{P}}JMJ\alpha \rangle$$

$$= [(2J'+1)(2J+1)]^{1/2} \int \frac{d^3x}{(2\pi)^3} dg\, e^{-i(\vec{\mathrm{P}}\,'-\vec{\mathrm{P}})\cdot\vec{x}}\, [C^{(J')}D^{(J')}(g)]^{*}_{\alpha'M'}[C^{(J)}D^{J}(g)]_{\alpha M}F(u, \vec{\nabla}u)\;. \tag{B14}$$

(The completeness and orthogonality relations of D's are required in the proof. The easiest way to convince oneself of the result is to check it for simple expressions like $\vec{\nabla}\hat{u}\hat{u}^{\dagger}$.)

It follows that we can consistently fulfill operator equations involving $\hat{u}$ and $\vec{\nabla}\hat{u}$ in any given topological sector using the *Ansatz* (B7) and the states $|\vec{\mathrm{P}}JMJ\alpha\rangle$. In particular, (a) the mean value of the soliton-number operator

$$\frac{1}{24\pi^2}\epsilon_{ijk} \int d^3x\, \mathrm{Tr}(\partial_i\hat{u}\hat{u}^{\dagger}\partial_j\hat{u}\hat{u}^{\dagger}\partial_k\hat{u}\hat{u}^{\dagger}) \tag{B15}$$

is t; (b) the static equation is fulfilled by $\hat{u}$ in the sense of mean values if u fulfills the static equation.

Our ability to at least partially saturate the operator equations consistently with the rotational levels $|\vec{\mathrm{P}}JMJ\alpha\rangle$ indicates that the full spectrum contains these levels.

It is a simple matter to generalize these considerations to include fermions, the necessary theoretical framework is available in previous work.[16] The details will be omitted here since they do not seem to suggest new predictions.

APPENDIX C: EXOTIC STATES IN THE STANDARD MODEL

The possibility of solitons of Skyrme's type in the Glashow-Salam-Weinberg model has been discussed by Gipson and Tze,[9] we shall briefly recall the arguments here. We will also remark on the baryon and lepton number of these solitons and on fermionic bound states to these solitons.

The possibility of Skyrme-type solitons in the $\mathrm{SU}(2)_W \times \mathrm{U}(1)$ model comes about as follows. Assuming for simplicity that there is only one Higgs doublet, its polar decomposition is

$$\Phi = S \begin{bmatrix} 0 \\ H \end{bmatrix} \equiv S\phi,\;\; S \in \mathrm{SU}(2)\;. \tag{C1}$$

Let us first make the following approximations.

(1) ϕ and H are frozen to their vacuum values:

$$\phi = \langle \phi \rangle = \begin{bmatrix} 0 \\ \langle H \rangle \end{bmatrix}\,. \tag{C1$'$}$$

(2) The gauge and fermion fields are absent. With such approximations, the Lagrangian reduces to the chiral form

$$\begin{aligned} L &= -\langle H \rangle^2 (\partial_\mu S^*)_{a2}(\partial_\mu S)_{a2} + \cdots \\ &\equiv -\tfrac{1}{2}\langle H \rangle^2 \mathrm{Tr}\partial_\mu S^{\dagger}\partial_\mu S + \cdots\;. \end{aligned} \tag{C2}$$

It allows for topological sectors as in the chiral model. The one-loop corrections to this L (Ref. 17) and the properties of the corresponding solitons have been described before.[9]

When the $\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge bosons B_μ are introduced, a few novel features are encountered: (1) In minimizing the energy, it is now possible to vary the field B_μ as well, so that the solitonic mass tends to get lowered from its value when B_μ is identically zero. (2) Skyrme's topological current

$$j_\mu = \frac{1}{24\pi^2}\epsilon_{\mu\nu\lambda\rho}\mathrm{Tr}(\partial_\nu SS^{\dagger}\partial_\lambda SS^{\dagger}\partial_\rho SS^{\dagger}) \tag{C3}$$

is conserved, but gauge variant. [The corresponding topological charge is a constant of motion, it is also integer valued and hence gauge invariant (for gauge transformations deformable to the identity).] There is also a gauge-invariant modification k_μ of the current j_μ which however has an anomaly[6]:

$$k_\mu = \frac{1}{24\pi^2}\epsilon_{\mu\nu\lambda\rho}\mathrm{Tr}(D_\nu SS^\dagger D_\lambda SS^\dagger D_\rho SS^\dagger - \tfrac{3}{2}D_\nu SS^\dagger F_{\lambda\rho}) , \tag{C4}$$

$$\partial_\mu k_\mu = \frac{1}{32\pi^2}\epsilon_{\mu\nu\lambda\rho}\mathrm{Tr}(F_{\mu\nu}F_{\lambda\rho}) . \tag{C5}$$

Now in QCD we encounter a similar situation for axial-vector U(1) transformations: there is a gauge variant but conserved current and a gauge invariant current with anomaly. The Kogut-Susskind mechanism for the generation of the η' mass requires these features. It is conceivable that under suitable physical conditions, there is a similar Kogut-Susskind mechanism associated with these topological currents.

When the fermions are introduced, we can try to study the values of the baryon and lepton currents in the presence of the background field S. Such a calculation has not been done, it is not a naïve transcription of the calculation in the chiral model due to the altered structure of the fermion—Higgs-boson interaction. Thus even for only one generation, this interaction for quarks is

$$-m_u\bar{q}_L^{(a)}S_{a1}u_R - m_d\bar{q}_L^{(a)}S_{a2}d_R + \mathrm{H.c.} , \tag{C6}$$

$$q = \begin{bmatrix} q^{(1)}\equiv u \\ q^{(2)}\equiv d \end{bmatrix} , \tag{C7}$$

which in the U gauge defined by

$$q_L' = S^\dagger q_L = \begin{bmatrix} u_L' \\ d_L' \end{bmatrix} , \tag{C8}$$

$$q_R' = q_R = \begin{bmatrix} u_R' \\ d_R' \end{bmatrix} \tag{C9}$$

becomes

$$-m_u\bar{u}_L' u_R - m_d\bar{d}_L' d_R + \mathrm{H.c.} \tag{C10}$$

Since $m_u \neq m_d$ the mass matrix here does not commute with covariant differentiation on the quark fields, a feature which can modify the previous calculations. Further, now m_α $(\alpha = u,d)$ are much smaller than $R_0{}^{-1}$ where R_0 is the soliton radius, so that expansion in powers of $1/R_0$ is not valid.

When $m_\alpha \to 0$, it is evident from (C10) that the quarks do not couple to the Higgs field directly. Thus if there are bound states in the Dirac equation of quarks or leptons to the solitons for sufficiently large m_α, they become unbound in the zero-mass limit.

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