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Catalysis of Flavor Nonconservation by Monopoles

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A simple argument based on current algebra is given to show how monopoles can lead to baryon decay and other flavor-changing processes.

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The recent papers by Rubakov,[1] Callan,[2] and Wilczek[3] suggest the possibility that monopoles can catalyze baryon number nonconservation and in particular proton decay. The proton is expected to decay typically at strong-interaction rates. However, since the calculations are fairly long and involved, a number of questions have remained unclear. In this paper I construct a simple argument based on current algebra to show how this effect may take place.

We shall consider quantum electrodynamics with Dirac-type monopoles. The structure of grand unification is not relevant to the discussion. The fermion sector will have the proton and the electron. The Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} - \bar{p}[\gamma\cdot(\partial - ieA) + m_p]p - \bar{e}[\gamma\cdot(\partial - ieA) + m_e]e, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{1}$$

The electric, axial, and baryon currents are defined by

$$J_\mu{}^Q = ie(\bar{p}\gamma_\mu p + \bar{e}\gamma_\mu e), \quad J_\mu{}^5 = i(\bar{p}\gamma_\mu\gamma_5 p + \bar{e}\gamma_\mu\gamma_5 e), \quad J_\mu{}^B = i\bar{p}\gamma_\mu p. \tag{2}$$

The starting point is the well-known result that the naive algebra of these currents is modified by anomalies. The modified current algebra is[4]

$$\begin{aligned}&[J_0{}^Q(\vec{x},t), J_0{}^5(\vec{y},t)] = (ie^2/\pi^2)\tilde{F}^{0j}(y)\partial_j\delta(\vec{x}-\vec{y}), \quad [J_0{}^Q(\vec{x},t), J_i{}^5(\vec{y},t)] = (ie^2/2\pi^2)\tilde{F}^{ij}(y)\partial_j\delta(\vec{x}-\vec{y}),\\ &[J_i{}^Q(\vec{x},t), J_0{}^5(\vec{y},t)] = -(ie^2/2\pi^2)\partial_j\tilde{F}^{ij}(x)\delta(\vec{x}-\vec{y}) + (ie^2/2\pi^2)\tilde{F}^{ij}(y)\partial_j\delta(\vec{x}-\vec{y});\end{aligned} \tag{3}$$

$$\begin{aligned}&[J_0{}^B(\vec{x},t), J_0{}^5(\vec{y},t)] = (ie/2\pi^2)\tilde{F}^{0j}(y)\partial_j\delta(\vec{x}-\vec{y}), \quad [J_0{}^B(\vec{x},t), J_i{}^5(\vec{y},t)] = (ie/4\pi^2)\tilde{F}^{ij}(y)\partial_j\delta(\vec{x}-\vec{y}),\\ &[J_i{}^B(\vec{x},t), J_0{}^5(\vec{y},t)] = -(ie/4\pi^2)\partial_j F^{ij}(x)\delta(\vec{x}-\vec{y}) + (ie/4\pi^2)\tilde{F}^{ij}(y)\partial_j\delta(\vec{x}-\vec{y}),\end{aligned} \tag{4}$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$ and $\partial_j\delta(\vec{x}-\vec{y}) = (\partial/\partial x_j)\delta(\vec{x}-\vec{y})$. We shall not need other commutators. These equal-time commutators are derived by use of the Bjorken-Johnson-Low method.[4] I.e., one picks up the coefficient $1/p_0$ as $p_0 \to \infty$ in $T[J_\mu(x)J_\nu{}^5(y)]$ after Fourier transformation. The function may have terms which go as p_0 or constant at large p_0. In defining the covariant T^* product using the equal-time commutators we may have to add noncovariant seagulls.[5]

We shall now go through the standard way of defining the T^* product of currents and relating it to anomalies. Consider $J_\mu{}^Q$, $J_\mu{}^5$, and the algebra (3) first. The T^* product is defined by

$$T^*[J_\mu{}^Q(x)J_\nu{}^5(y)] = T[J_\mu{}^Q(x)J_\nu{}^5(y)] + \tau_{\mu\nu}(x,y). \tag{5}$$

$\tau_{\mu\nu}(x,y)$ is the seagull term. I take the divergence of this equation on μ to get

$$(\partial/\partial x^\mu)T^*[J_\mu{}^Q(x)J_\nu{}^5(y)] = T^*[\partial_\mu J_\mu{}^Q(x)J_\nu{}^5(y)] + [J_0(\vec{x},t), J_\nu{}^5(\vec{y},t)]\delta(t_x - t_y) + (\partial/\partial x^\mu)\tau^{\mu\nu}. \tag{6}$$

By choosing the seagull term to cancel the equal-time commutator we can get conservation of the vector current. The fact that we can move $(\partial/\partial x^\mu)$ through the T^* product ensures the validity of the naive Ward identities. From Eqs. (3) we see that the appropriate choice of $\tau_{\mu\nu}$ is

$$\tau^{00} = 0; \quad \tau^{i0} = (ie^2/\pi^2)\tilde{F}^{i0}(y)\delta^4(x-y); \quad \tau^{0i} = 0; \quad \tau^{ij} = (ie^2/2\pi^2)\tilde{F}^{ij}(y)\delta^4(x-y). \tag{7}$$

We now take the divergence of (5) on ν and use Eq. (7) for $\tau_{\mu\nu}$. This will lead us to the axial anomaly. Care should be taken to keep terms of the form $\partial_\mu\tilde{F}_{\mu\nu}$ which are not zero in the monopole sector. The result is

$$(\partial/\partial y^\nu)T^*[J_\mu{}^Q(x)J_\nu{}^5(y)] = T^*[J_\mu{}^Q(x)\partial_\nu J_\nu{}^5(y)] + \tau_\mu{}'(x,y) + (ie^2/\pi^2)\tilde{F}_{\mu\nu}\partial_\nu\delta^4(x-y) + \frac{ie^2}{\pi^2}\begin{cases}0 & \text{for } \mu = 0\\ (\partial_\nu\tilde{F}_{\nu i})\delta^4(x-y) & \text{for } \mu = i\end{cases}, \tag{8}$$

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where I have added a seagull term $\tau_\mu'(x,y)$ for the definition of the T^* product of $J_\mu{}^Q(x)\partial_\nu J_\nu{}^5(y)$. This seagull term can be shown to be,[6]

$$\tau_i'=0;\quad \tau_0'=(ie^2/\pi^2)(\partial_i\tilde{F}_{i0})(y)\delta^4(x-y). \tag{9}$$

Equation (8) thus becomes the fully covariant equation

$$(\partial/\partial y^\nu)T^*[J_\mu{}^Q(x)J_\nu{}^5(y)]=T^*[J_\mu{}^Q(x)\partial_\nu J_\nu{}^5(y)]+(ie^2/\pi^2)\tilde{F}_{\mu\nu}(y)\partial_\nu\delta^4(x-y)+(ie^2/\pi^2)\partial_\nu\tilde{F}_{\nu\mu}(y)\delta^4(x-y). \tag{10}$$

The new term in Eq. (10) is the last one depending on the magnetic current density. This term tells us that the axial anomaly cannot be gauge invariant. To see this more clearly we multiply (10) by $\eta(y)\partial_\mu\Lambda(x)$ and integrate over x and y. The current J_μ can be written as $\partial/\partial A_\mu(x)$ acting on an effective action. Keeping in mind that $\int\partial_\mu\Lambda(\partial/\partial A_\mu)d^4x$ generates a gauge transformation we have,

$$\delta_\Lambda G^5(\eta)=(ie^2/\pi^2)\int\partial_\mu\Lambda\partial_\nu\eta\tilde{F}_{\mu\nu}d^4x, \tag{11}$$

where $G^5(\eta)$ is the axial anomaly, viz.

$$G^5(\eta)=\int\eta(y)\partial_\mu J_\mu{}^5(y)d^4y. \tag{12}$$

$\delta_\Lambda G^5(\eta)$ denotes the variation in $G^5(\eta)$ under gauge transformation by Λ. The right-hand side of Eq. (11) vanishes in the usual (no monopole) sector but is nonzero with monopoles.

The first result is that the axial anomaly has a term proportional to $\partial_\nu\tilde{F}_{\nu\mu}$ which is not gauge invariant. From Eq. (10) or by integrating Eq. (11) one can see that this is of the form $A_\mu\partial_\nu\tilde{F}_{\nu\mu}$.

The vector current equation (6) with (7) and after folding in the test functions η,Λ becomes

$$\delta_\eta G^Q(\Lambda)=0, \tag{13}$$

where $G^Q(\Lambda)$, the electric current anomaly, is zero. We can shift anomalies from $J_\mu{}^5$ to $J_\mu{}^Q$ and back by adding appropriate covariant seagull terms to Eq. (5). In the more general case, instead of Eqs. (11) and (13), what we get would be

$$\delta_\Lambda G^5(\eta)-\delta_\eta G^Q(\Lambda)=(ie^2/\pi^2)\int\partial_\mu\Lambda\partial_\nu\eta\tilde{F}_{\mu\nu}d^4x. \tag{14}$$

This equation captures in a succinct way the properties of the current algebra (3) that are relevant to the monopole sector. This is also the Wess-Zumino type consistency condition on the anomalies pertinent to the algebra (3).[7]

In the functional formalism, the shift of anomalies is achieved by adding counterterms to the action which are made up of the gauge fields and whose variations produce the necessary shift in the expression for the anomalies. This corresponds to the addition of seagulls. In fact if S_c is the counter term, the seagull for $T^*(J_\mu J_\nu{}^5)$ is

$$\tau_{\mu\nu}'=\frac{\partial}{\partial A_\mu(x)}\frac{\partial}{\delta P_\nu(y)}S_c, \tag{15}$$

where P_ν is a dummy gauge field for $J_\nu{}^5$ which can be set equal to zero after $\tau_{\mu\nu}$ is evaluated.

Since we cannot interpret gauge-variant terms in any simple way, we shall make $G^5(\eta)$ gauge invariant by adding a counterterm. This will give a vector anomaly which we shall analyze. The counter term to be added is

$$S_c=(ie^2/\pi^2)\int\theta A_\nu(\partial_\mu\tilde{F}_{\mu\nu})d^4x \tag{16}$$

$$=-(ie^2/\pi^2)\int P_\alpha(y)(\partial/\partial y^\alpha)G(x-y)A_\nu(x)\partial_\mu\tilde{F}_{\mu\nu}(x)d^4xd^4y, \tag{17}$$

where $P=\partial_\alpha\theta$ and $\partial_\alpha\partial_\alpha G(x-y)=\delta^{(4)}(x-y)$. Directly at the level of currents we can think of this as a seagull

$$\tau_{\mu\nu}=-(ie^2/\pi^2)(\partial/\partial y^\nu)G(x-y)(\partial_\alpha\tilde{F}_{\alpha\mu})(x). \tag{18}$$

This produces the vector current anomaly

$$\partial_\mu J_\mu{}^Q=(ie^2/\pi^2)\partial_\nu\theta(\partial_\mu\tilde{F}_{\mu\nu}). \tag{19}$$

The field θ is not in general zero; it corresponds to the fluctuation around the monopole introduced by Rubakov and Callan.[8] Electric current as originally defined in the theory is not conserved. We can define a conserved electric charge by redefining the current as

$$\mathcal{J}_\mu{}^Q=J_\mu{}^Q-(ie^2/\pi^2)\theta(\partial_\nu\tilde{F}_{\nu\mu}). \tag{20}$$

This redefinition of the current is natural since with the counterterm (16), it is $\mathcal{J}_\mu{}^Q$ which couples to A_μ.

We can interpret these as follows. When particles come in and scatter off the condensate around the monopoles, there can be "charge changing" processes which deposit charge on the core of the monopole. The extra term in Eq. (20) measures this charge. The action (16) measures the electrical energy of this charge. Charge-changing processes of the type $p + M \to M^{++} + \bar{p}$ are possible but would be suppressed by the change in the energy associated with the process, i.e., by the change in the action (16).[9]

Consider now the algebra of baryon current (4). Proceeding exactly as in the case of the electric current, we get the relation

$$\delta_\epsilon G^5(\eta) - \delta_\eta G^B(\epsilon) = (ie/2\pi^2)\int \partial_\mu \epsilon \partial_\nu \eta \tilde{F}_{\mu\nu} d^4x. \quad (21)$$

$\delta_\epsilon G^5(\eta)$ denotes the variation of $G^5(\eta)$ under a gauge transformation corresponding to baryon number. $G^5(\eta)$ is a function of the gauge fields in the problem and since there are no gauge fields associated with baryon number, $\delta_\epsilon G^5(\eta) = 0$. Equation (21) then shows that we must necessarily have a baryon anomaly given by

$$\partial_\mu J_\mu{}^B = (ie/2\pi^2)\partial_\nu \theta (\partial_\mu \tilde{F}_{\mu\nu}). \quad (22)$$

We could define a conserved current in this case also but it is not very natural since there are no gauge fields associated with baryon number. There is no action like (16) which naturally leads to such a redefinition. For the same reason there will be no extra action suppression for baryon number nonconserving processes. Since S_c of Eq. (16) is unchanged it has no effect on charge conserving baryon number nonconserving processes.

A few remarks are in order. Although I used baryon number as an example, it is clear that all global quantum numbers which have anomalous current algebra of the form (4) will not be conserved. Examples are muon number, lepton number, quark flavors, etc. For local symmetries, there will be action terms of the form (16) which will suppress processes which change the associated quantum number.

For a point monopole $\partial_\mu J_\mu{}^B \sim \partial_t \theta(r=0)\delta^3(x)$. The nonconservation occurs at the site of the monopole and it is crucial to have $\partial_t \theta(r=0) \neq 0$. The explicit calculation shows the same pattern. More details can be found in Ref. 8.

I want to qualify the sense in which these global quantum numbers are not conserved. Although not natural we could go ahead and redefine the baryon number (and other quantum numbers) identifying the extra term $\sim \theta \partial_\mu \tilde{F}_{\mu\nu}$ as the intrinsic baryon current of the monopole. If the global quantum number was conserved without monopoles, then this is at least a consistent procedure. However, for a grand unified theory for which there is no baryon number conservation even without monopoles we cannot attribute intrinsic baryon number to the monopole and we have a genuine nonconservation of the quantum number. From the experimental point of view, there is no distinction between these cases unless we have creation and annihilation of monopoles.

Finally, although I discussed only electrodynamics I never used the point monopole field expressions. The result does not seem to be an artifact of the point monopole limit.

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[1]V. Rubakov Pis'ma Zh. Eksp. Teor. Fiz. 33, 658 (1981) [JETP Lett. 33, 644 (1981)], and Nucl. Phys. B203, 311 (1982).

[2]C. G. Callan, Phys. Rev. D 25, 2141 (1982), and 2058 (1982).

[3]F. Wilczek, Phys. Rev. Lett. 48, 1144 (1982).

[4]R. Jackiw and K. Johnson, Phys. Rev. 182, 1459 (1969); S. L. Adler and D. G. Boulware, Phys. Rev. 184, 1740 (1969); R. Jackiw, in Lecture on *Current Algebra and Its Applications*, edited by S. B. Treiman, R. Jackiw, and D. J. Gross (Princeton Univ. Press, Princeton, N.J., 1972).

[5]M. Chanowitz, Phys. Rev. D 2, 3016 (1970).

[6]Equation (14) is verified by direct computation in V. P. Nair, Syracuse University Report No. C00-3533-248, 1983 (to be published). Diagrammatic verification of the seagull term (9) starts from that calculation and retraces the present arguments from Eq. (14) back to Eq. (8). A direct computation of $T[J_\mu{}^Q(x)\,\partial_\nu J_\nu{}^5(y)]$ also involves the same diagram with the same result.

[7]J. Wess and B. Zumino, Phys. Lett. 37B, 95 (1971); A. P. Balachandran, G. Marmo, V. P. Nair, and C. G. Trahern, Phys. Rev. D 25, 2713 (1982); A. P. Balachandran, V. P. Nair, and C. G. Trahern, Phys. Rev. D 27, 1369 (1983).

[8]Nair, Ref. 6.

[9]A. P. Balachandran and J. Schechter, Syracuse University Report No. C00-3533-249, 1983 (to be published), introduce essentially the Lagrangian of Eq. (16) but interpret it as an effective Lagrangian for the J_0, $J_0{}^5$ algebra.