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# Breakdown of Flavor Conservation in a Monopole Background

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We give a simple derivation of the catalysis of baryon decay and other flavor-changing processes by monopoles. The role of the axial anomaly is clarified. A general current-algebraic argument which clarifies the nature of flavor violation is also given.

## I. INTRODUCTION

The recent papers by Rubakov,[1] Callan,[2] and Wilczek[3] have raised the exciting possibility that monopoles can catalyze baryon-number violation, in particular the decay of the proton. The proton is expected to decay typically at strong-interaction rates. The calculations which have been done so far have not clarified a number of questions. Are the rates really strong-interaction rates or are there suppression factors due to the masses of $W$ bosons?[3] To what extent is the $(V-A)$ nature of SU(5) grand unified theory significant? What is the role of grand unification? If we do find monopoles and they do catalyze proton decay, does it tell us anything about the structure of grand unified theories? Finally, one would like to have a simple calculational technique for rates and branching ratios, which can take into account, at least in a systematic perturbation theory, quark masses and fermionic modes of nonzero angular momentum.

In this paper, we attempt a simple four-dimensional field-theoretical reconstruction of the arguments of Rubakov, Callan, and Wilczek. The basic observation is that the extra radial angular momentum of fermions in a monopole background helps to convert axial-type interactions into vector-type interactions for zero-angular-momentum modes. This leads to a very simple derivation of the effective Lagrangian of Rubakov and Callan in a way that clarifies the role of monopoles and the axial anomaly. Mass effects and nonzero-angular-momentum modes can be incorporated. It is argued that grand unified theories are not relevant at all, even to the extent of the algebraic structure of fermion representations. They are required only to have finite-energy monopole solutions. We do the calculations for Abelian Dirac-type monopoles although they can be easily repeated for extended monopoles of the 't Hooft-Polyakov type. This also makes it evident that the scale of the monopole mass is irrelevant to the question of proton decay. Proton decay does happen at typical strong-interaction rates. Monopoles can also catalyze violation of other global quantum numbers; muon-electron transition is an example.

In Sec. II we derive the effective Lagrangian of Rubakov and Callan. Section III shows how this can be applied to compute expectation values in a monopole background. Breakdown of chiral symmetry is demonstrated. Proton decay and violation of other global quantum numbers are discussed. We repeat some known results in this section for the sake of a comprehensive discussion. In Sec. IV we consider the case of several flavors and also give an appraisal of fermion-mass terms. Section V gives a general argument to show how violation of global quantum numbers can occur. This paper is concluded with a short discussion.

## II. THE QUANTUM STATE OF THE MONOPOLE

We consider quantum electrodynamics with Dirac-type monopoles. Initially, we consider only one flavor of fermions. Eventually we shall need more flavors, but the generalization is quite straightforward. The Lagrangian is thus

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}-\bar{\psi}[\gamma\cdot(\partial-ieA)+m]\psi\ ,\qquad F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\ . \tag{1}$$

The monopole field configuration is given by

$$A_0=0,\quad A_i=-\frac{g}{4\pi}\frac{(\hat{r}\times\hat{z})_i}{r(1-\hat{r}\cdot\hat{z})}\ . \tag{2}$$

Or in terms of the magnetic field,

$$B_i=\frac{g}{4\pi}\frac{\hat{r}_i}{r^2}\ . \tag{3}$$

$\hat{r}$ is the unit vector along the radial direction, $\hat{z}$ is the unit vector along the $z$ axis. The Dirac quantization rule is given in our notation by $eg/4\pi=n/2$. We shall consider a single monopole background $(n=1)$ mostly.

It is well known[4] that the angular momentum operator of fermions in this monopole background is given by

$$\vec{J}=\vec{r}\times\vec{\pi}+\frac{\vec{\Sigma}}{2}-q\hat{r}\ , \tag{4}$$

where

$$q=\frac{eg}{4\pi}=\frac{n}{2},\quad \pi=p-eA=-i(\partial-ieA)\ ,$$
$$\Sigma_i=\frac{1}{4i}\epsilon_{ijk}[\gamma_j,\gamma_k]\ .$$

For a single monopole background, $J=0$ is the lowest mode of angular momentum. The electric- and axial-charge densities are defined by

 

$$J_0(x)=e\psi^{\dagger}(x+\epsilon/2)\psi(x-\epsilon/2)\;, \tag{5}$$

$$J_0^5(y)=\psi^{\dagger}(x+\epsilon/2)\gamma_5\psi(x-\epsilon/2)\;. \tag{6}$$

Using the standard Bjorken-Johnson-Low method, one gets the equal-time commutation rule[5]

$$[J_0(\vec{x}),J_0^5(\vec{y})]=+\frac{ie^2}{2\pi^2}\vec{B}\cdot\vec{\nabla}_x\delta(\vec{x}-\vec{y})\;. \tag{7}$$

Upon integration over $\vec{x}$ and $\vec{y}$, we get in the presence of a magnetic monopole

$$[Q,Q^5]=2\frac{ieq}{\pi}\;. \tag{8}$$

This simple result shows that spectra of the electric charge $Q$ and axial charge $Q^5$ are continuous and unbounded. (They are like the $\vec{x}$ and $\vec{p}$ operators in quantum mechanics.) The fact that the electric charge is not quantized in the monopole sector is, of course, well known after Witten's work[6] whose formula for the charge of the monopole embodies this fact. Physically we are interested in monopoles of definite electric charge. Because of Eq. (8), this implies that the axial charge cannot be diagonalized. This also shows that chiral selection rules are not meaningful for the monopole sector. Equations (7) and (8) require a more detailed discussion since there are several fine points involved. We shall return to them in Sec. V.

We are thus interested in defining a $Q$-diagonal state of the monopole which includes fluctuations of $Q^5$. To do this, we introduce fluctuations around the classical monopole field configuration $A_\mu^M$ of Eq. (2), writing

$$A_\mu=A_\mu^M+\xi_\mu\;, \tag{9}$$

where

$$\xi_0=\frac{\partial\phi}{\partial r},\;\;\xi_i=-\hat{r}_i\frac{\partial\phi}{\partial t},\;\;\phi=\phi(r,t)\;. \tag{10}$$

The corresponding electric and magnetic fields are given by

$$\begin{aligned}E_i&=\hat{r}_i\left[\frac{\partial^2\phi}{\partial t^2}-\frac{\partial^2\phi}{\partial r^2}\right]=\hat{r}_i(\Box\phi)\;,\\ B_i&=\frac{g}{4\pi}\frac{\hat{r}_i}{r^2}\;.\end{aligned} \tag{11}$$

Callan and Rubakov[1,2] have given arguments to justify considering $\xi_\mu$ of this form. The following calculations will show why these are the relevant fluctuations.

The contribution of the fermions to the functional integral can be written as

$$e^{iW(A^M,\phi)}=\int[d\psi\,d\bar{\psi}]\exp\left[i\int[\mathscr{L}_{\rm KE}(A^M,\phi)-m\bar{\psi}\psi]d^4x\right]\;, \tag{12}$$

where

$$\mathscr{L}_{\rm KE}(A^M,\phi)=-\bar{\psi}\gamma\cdot(\partial-ieA^M-ie\xi)\psi\;. \tag{13}$$

To facilitate integration, we introduce the change of variables

$$\psi=e^{i\gamma_5\alpha}Q,\;\;\bar{\psi}=\bar{Q}e^{i\gamma_5\alpha}\;. \tag{14}$$

The Lagrangian becomes

$$\mathscr{L}=-\bar{Q}\gamma\cdot[\partial-ie(A^M+\xi)+i\gamma_5\partial\alpha]Q-m\bar{Q}e^{2i\gamma_5\alpha}Q\;. \tag{15}$$

From Eq. (4) for the angular momentum we have

$$1=\frac{\hat{r}\cdot\Sigma}{2q}-\frac{\hat{r}\cdot J}{q}\;. \tag{16}$$

Using this equation and the identity

$$\gamma_\mu[\gamma_\alpha,\gamma_\beta]=2(\delta_{\mu\alpha}\gamma_\beta-\delta_{\mu\beta}\gamma_\alpha)+2\epsilon_{\nu\mu\alpha\beta}\gamma_\nu\gamma_5\;, \tag{17}$$

the axial-vector term $i\gamma_\mu\gamma_5\partial_\mu\alpha$ in Eq. (15) can be reduced as

$$i\gamma_\mu\gamma_5\partial_\mu\alpha=\frac{1}{2q}(\gamma_0\partial_r\alpha+i\gamma\cdot\hat{r}\partial_t\alpha)-\frac{i}{q}\gamma_\mu\gamma_5\partial_\mu\alpha\hat{r}\cdot J\;. \tag{18}$$

We have a vector interaction which is of the right form to be absorbed into the fluctuation field $\phi$. We can now rewrite Eq. (15) as

$$\begin{aligned}\mathscr{L}=&-\bar{Q}\gamma\cdot[\partial-ie(A^M+\xi')]Q\\&+\frac{i}{q}\bar{Q}\gamma_\mu\gamma_5\hat{r}\cdot JQ\partial_\mu\alpha-m\bar{Q}e^{2i\gamma_5\alpha}Q\;,\end{aligned} \tag{19}$$

where

$$\begin{aligned}\xi_0'&=\xi_0+\frac{1}{2eq}\partial_r\alpha\;,\\ \xi_i'&=\xi_i-\frac{\hat{r}_i}{2eq}\partial_t\alpha\;,\end{aligned} \tag{20}$$

or

$$\phi'=\phi+\alpha/2eq\;.$$

The identity (16) is crucial to this reduction and that in turn depends on the extra radial angular momentum $-q\hat{r}$.

The change of variables (14) is, however, anomalous. The anomaly can be thought of, in the language of functional integrals, as a change of fermion measure[7]

$$[d\psi\,d\bar{\psi}]=[dQ\,d\bar{Q}]\exp[2i\,{\rm Tr}(\gamma_5\alpha)]\;, \tag{21}$$

where the axial anomaly ${\rm Tr}(\gamma_5\alpha)$ can be easily computed as[7,8]

$${\rm Tr}(\gamma_5\alpha)=-\frac{e^2}{4\pi^2}\int\alpha\vec{E}\cdot\vec{B}\,d^4x\;. \tag{22}$$

We can now rewrite the functional integral (12) as

$$e^{iW(\phi)}=e^{2i\,{\rm Tr}(\gamma_5\alpha)}\int[dQ\,d\bar{Q}]\exp\left[i\int\mathscr{L}_{\rm KE}\left[\phi+\frac{\alpha}{2eq}\right]d^4x\right]\exp\left[i\int\mathscr{L}'d^4x\right]\;, \tag{23}$$

where

$$\mathscr{L}'=\frac{i}{q}\bar{Q}\gamma_\mu\gamma_5\hat{r}\cdot JQ\partial_\mu\alpha-m\bar{Q}e^{2i\gamma_5\alpha}Q\ . \tag{24}$$

The special role of the $J=0$ mode for zero-mass fermions is now clear. In this case $\mathscr{L}'$ vanishes and we get the simple equation

$$W(\phi)=-\frac{e^2}{2\pi^2}\int \alpha\vec{\mathrm{E}}\cdot\vec{\mathrm{B}}\,d^4x+W(\phi+\alpha/2eq)\ , \tag{25}$$

which can be solved exactly for $W(\phi)$. Our strategy will therefore be to treat $\mathscr{L}'$ as a perturbation and expand Eq. (23) in powers of $\mathscr{L}'$. The lowest-order result is Eq. (25), which we write for small $\alpha$ as

$$\delta W=-\frac{e^3q}{\pi^2}\int \vec{\mathrm{E}}\cdot\vec{\mathrm{B}}(\delta\phi)d^4x$$

$$=-\frac{e^2q(eg)}{\pi^2}\int \Box\phi(\delta\phi)dr\,dt \tag{26}$$

using Eq. (11). Integrating we get

$$W(\phi)=-\frac{e^2}{2\pi}\int \phi\Box\phi\,dr\,dt+W(\phi=0) \tag{27}$$

for a single-monopole-background field (i.e., $n=1$ or $q=\frac{1}{2}$).

A few remarks are in order at this point. First we note that $W(\phi=0)$ is the fermion functional integral in a pure monopole background with no fluctuations. It can thus be computed with the well-known partial-wave analyses in a monopole background.[9,10] Now a minor question about why we did not use the finite-difference equation (25): That equation is strictly valid only for infinitesimal $\alpha$. Since $E_i$ depends on $\phi$ (or $\alpha$), the anomaly term should strictly be interpreted as

$$-\frac{e^2}{2\pi^2}\int_0^\alpha d\alpha\,\vec{\mathrm{E}}(\alpha)\cdot\vec{\mathrm{B}}\,d^4x\ .$$

With this interpretation, Eq. (25) will give the same result, viz., Eq. (27). This integration procedure is the same as finding an effective Lagrangian for the anomaly in the manner of Wess and Zumino.[11]

In Eq. (27), there is a possible "constant" of integration of the form $\omega\int\Box\phi\,dr\,dt$ where $\omega$ is a constant. This is of the form $\theta\int F_{\mu\nu}F_{\alpha\beta}\epsilon_{\mu\nu\alpha\beta}d^4x$, which we shall add to the action to take account of $\theta$ vacuums. Thus, we do not write it in Eq. (27), absorbing it into the definition of $\theta$.

The action for the gauge field gives

$$S(A)=\int 2\pi r^2(\Box\phi)^2dr\,dt+S_{\mathrm{mon}}\ , \tag{28}$$

where $S_{\mathrm{mon}}$ is the action for the monopole, i.e.,

$$S_{\mathrm{mon}}=\tfrac{1}{2}\int B^2d^4x\ . \tag{29}$$

Combining Eqs. (27) and (28) we get an effective action for $\phi$:

$$S(\phi)=\int\left[r^2(\Box\phi)^2-\frac{e^2}{4\pi^2}\phi\Box\phi+e\theta\frac{\Box\phi}{(32\pi^3)^{1/2}}\right]dr\,dt\ , \tag{30}$$

where we have rescaled $\phi$ as $\phi\to\phi/\sqrt{2\pi}$ and also included the

$$\theta\int\epsilon_{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}d^4x$$

term with the correct normalization.

The equation of motion for $\phi$ is

$$\Box(r^2\Box\phi)-\frac{e^2}{4\pi^2}\phi\Box\phi=0\ , \tag{31}$$

which can be solved by

$$\phi=f-h,\quad \Box f=0,\quad \Box h-\frac{e^2}{4\pi^2r^2}h=0\ . \tag{32}$$

The electric charge of the system is given by

$$Q=\oint\vec{\mathrm{E}}\cdot d\vec{\mathrm{S}}=\left.\frac{4\pi(r^2\Box\phi)}{\sqrt{2\pi}}\right|_{r\to\infty}$$

$$=-\left.\frac{4\pi(r^2\Box h)}{\sqrt{2\pi}}\right|_{r\to\infty}=-\frac{e^2}{\pi}\frac{h(\infty)}{\sqrt{2\pi}}$$

$$=\frac{e^2}{\pi}\frac{\phi(\infty)}{\sqrt{2\pi}}\ , \tag{33}$$

where we used Eq. (32) and assumed $f(\infty)=0$. The boundary value of $\phi$ is thus the electric charge (up to proportionality factors). To construct $Q$-diagonal states, we should therefore do a second quantization of the field theory (30) and construct $\phi$-diagonal states fixing the boundary value $\phi(r=\infty)$ to be numerically equal to $\pi\sqrt{2\pi}/e^2$ times the charge we would like to have on the monopole. $Q_5$, which will be associated with the conjugate momentum to $\phi$, will not be diagonal on these states. It is now clear that fluctuations of the type (10) are the relevant ones producing $\phi$-diagonal (and $Q$-diagonal) states for the monopole.

Going back to Eq. (30) we note that the $\theta$-dependent term can be absorbed by a linear shift of $\phi$:

$$r^2(\Box\phi)^2-\frac{e^2}{4\pi^2}\phi(\Box\phi)+e\theta\frac{\Box\phi}{(32\pi^3)^{1/2}}=r^2\left\{\Box\left[\phi-\frac{\theta}{e}\left[\frac{\pi}{2}\right]^{1/2}\right]\right\}^2-\frac{e^2}{4\pi^2}\left[\phi-\frac{\theta}{e}\left[\frac{\pi}{2}\right]^{1/2}\right]\Box\left[\phi-\frac{\theta}{e}\left[\frac{\pi}{2}\right]^{1/2}\right]\ .$$

The boundary value of $\phi$ can thus be identified as $(\theta/e)\sqrt{\pi/2}$. Use of this in Eq. (33) yields the Witten formula for the charge of a monopole in a $\theta$-vacuum state, viz., $Q=e\theta/2\pi$ (Ref. 6). If we have zero-mass fermions, the $\theta$ term can be "rotated" away, i.e., we can choose $\phi(\infty)=\theta/2e=0$. This is the idea of "disappearing" dyons."[2] When all fermions are massive, there is no freedom of global axial transformations and the dyons persist.

The limit $m \to 0$ is expected to be discontinuous.[12]

## III. THE FUNCTIONAL INTEGRAL FOR THE MONOPOLE

The monopole state can be labeled by $|A^M, \phi(\vec{x}), \phi(\infty)\rangle$, i.e., by the classical field configuration $A^M$, the configuration of the field $\phi$, and in particular its boundary value $\phi(\infty)$, which measures the charge of the system. At $t=-\infty$, we have the monopole state $|A^M, \phi(\infty)\rangle$. There are no static solutions to the field equation (31) except $\phi = \phi(\infty) =$ constant everywhere so that we do not have to worry about internal states for the monopole associated with static modes of $\phi$. At $t=+\infty$ we have the same field configuration. The interpolating field configurations appearing in the functional integral are all functions $\phi(r,t)$ which respect the boundary condition

$$\phi(r,t)\,|_{r\to\infty} = \phi(\infty)\ .$$

Since the physical dyons are obtained by a simple shift of $\phi$, we shall consider neutral monopoles hereafter, setting $\phi(\infty)=0$ (corresponding to $\theta=0$).

The time-ordered expectation value of an operator $\mathcal{G}$ in the monopole background is given by

$$\langle \text{mon}\,|\,T\mathcal{G}\,|\,\text{mon}\rangle = \frac{1}{Z}\int [d\phi][dQ\,d\bar{Q}]\mathcal{G}\exp\left[i\left\{S(\phi) - \int d^4x\,[\bar{Q}\gamma\cdot(\partial - ieA^M)Q + \mathcal{L}']\right\}\right], \tag{34}$$

where $S(\phi)$ is given by Eq. (30). Notice that the fermion kinetic energy term has a pure monopole background in accordance with the remarks following Eq. (27). $1/Z$ represents division by the same integral with $\mathcal{G}=1$. We have not written the monopole action $-\frac{1}{2}\int B^2 d^4x$, which is independent of $\phi$ and cancels between numerator and denominator. $\mathcal{L}'$ can be written (with the rescaling $\phi \to \phi/\sqrt{2\pi}$) as

$$\begin{aligned}\mathcal{L}' = {} & ie\left[\frac{2}{\pi}\right]^{1/2}\bar{Q}\gamma_\mu\gamma_5\hat{r}\cdot JQ\partial_\mu\phi \\ & - m\bar{Q}\exp\left[ie\left[\frac{2}{\pi}\right]^{1/2}\gamma_5\phi\right]Q\ .\end{aligned} \tag{35}$$

Since the Lagrangian of Eq. (30) is not quadratic in the time derivatives, it may not be immediately obvious that we can use Eq. (34) to compute Green's functions and expectation values. One can set up a Hamiltonian formulation for $\phi$ using Ostrogradski's method for higher-derivative Lagrangians and check that Eq. (34) is indeed true.[13]

If mass terms are neglected, the action is quadratic in $\phi$ and so the $\phi$ integration can be done. This involves only the computation of the propagator for $\phi$. It is easier to go to Euclidean space at this stage so that the Feynman prescription is automatically taken care of. The Euclidean-space version of Eq. (34) is given by

$$\langle \text{mon}\,|\,\mathcal{G}\,|\,\text{mon}\rangle = \frac{1}{Z}\int [d\phi][dQ\,d\bar{Q}^\dagger]\mathcal{G}e^{-S(\phi,Q,Q^\dagger)}\ , \tag{36}$$

where

$$\begin{aligned}S(\phi,Q,Q^\dagger) = {} & \int dr\,dt\left[r^2(\Box\phi)^2 + \frac{e^2}{4\pi^2}\phi\Box\phi\right] \\ & + \int d^4x\left\{Q^\dagger\gamma\cdot(\partial - ieA^M)Q - ie\left[\frac{2}{\pi}\right]^{1/2}Q^\dagger\gamma_\mu\gamma_5\hat{r}\cdot JQ\partial_\mu\phi + mQ^\dagger\exp\left[e\left[\frac{2}{\pi}\right]^{1/2}\gamma_5\phi\right]Q\right\}\end{aligned}$$

and $\Box = \partial_t{}^2 + \partial_r{}^2$.

We shall shortly be looking at proton decay and chiral symmetry breaking which requires several flavors. The effective (Euclidean) action for $\phi$ for $2N$ flavors is (for zero mass, $J=0$ modes)

$$S_{\text{eff}} = \left[r^2(\Box\phi)^2 + \frac{e^2(2N)}{4\pi^2}\phi\Box\phi\right]dr\,dt\ . \tag{37}$$

We shall first concentrate on the case of two flavors, a proton-positron system. In this case $N=1$, and the propagator for the $\phi$ field satisfies the equation

$$\left[\Box(r^2\Box) + \frac{e^2}{2\pi^2}\Box\right]G(rt,r't') = \delta(r-r')\delta(t-t') \tag{38}$$

with $\langle\phi(x)\phi(y)\rangle = -\frac{1}{2}G(x,y)$. Equation (38) has the "solution"

$$G(x,x') = F(x,x') - H(x,x')\ ,$$

where

$$\begin{aligned}&\Box F(x,x') = \frac{2\pi^2}{e^2}\delta(r-r')\delta(t-t')\ ,\\ &\left[\Box + \frac{e^2}{2\pi^2 r^2}\right]H(x,x') = \frac{2\pi^2}{e^2}\delta(r-r')\delta(t-t')\ .\end{aligned} \tag{39}$$

The fluctuation field $\phi$ gives the electrostatic potential $\partial\phi/\partial r$ in the Dirac equation. In order to avoid too singular a behavior at the origin, we shall impose the boundary condition $\partial\phi/\partial r = 0$ at $r=0$. This implies that we should take $\partial G/\partial r = 0$ at $r=0$. The solutions to Eq. (39), with this boundary condition, are[1,2]

$$F(x,x')=\frac{\pi}{2e^2}\left[\ln\frac{(r-r')^2+(t-t')^2}{R^2}+\ln\frac{(r+r')^2+(t-t')^2}{R^2}\right],$$
$$H(x,x')=-\frac{\pi}{e^2}Q_{d(\kappa)}\left[1+\frac{(r-r')^2+(t-t')^2}{2rr'}\right],\tag{40}$$

where $d(\kappa)=(\kappa+\frac{1}{4})^{1/2}-\frac{1}{2}$, $\kappa=e^2/2\pi^2$. $R$ is an infrared cutoff. $Q_m(z)$ is the modified Legendre function of order $m$.

We shall need the small- and large-distance behavior of $G(x,x')$. We have

$$H(x,x')=\frac{\pi}{2e^2}\ln\frac{(r-r')^2+(t-t')^2}{4r^2}+\frac{\pi}{e^2}\{\psi[d(\kappa)+1]-\psi(1)\}+\cdots \quad \text{for small } (r-r')^2+(t-t')^2$$

and

$$H(x,x')\to 0 \text{ as } |t-t'|\to\infty .$$

$\psi$ is the digamma function. This gives, for $G(x,x')$,

$$G(rt,rt)=\frac{2\pi}{e^2}\ln\left[\frac{2r}{R}\right]-\frac{\pi}{e^2}\{\psi[d(\kappa)+1]-\psi(1)\}+\cdots ,$$
$$G(rt,r't')=\frac{2\pi}{e^2}\ln\frac{|t-t'|}{R} \quad \text{as } |t-t'|\to\infty .\tag{41}$$

One of the characteristics of this solution is that $\partial G/\partial t\neq 0$ at $r=0$. This is evident from Eq. (41) for large $|t-t'|$. The asymptotic behavior $\ln|t-t'|$ is crucial to getting chiral symmetry breaking and proton decay. The importance of having $\partial\phi/\partial t\neq 0$ at $r=0$ will be evident from another point of view in Sec. V.

With this formalism, we can now investigate the question of chiral symmetry breaking and proton decay. The operator

$$\bar{\psi}_L\psi_R=\bar{Q}_L Q_R e^{ie\sqrt{2/\pi}\phi}$$

has the Euclidean version

$$Q^\dagger\left[\frac{1-\gamma_5}{2}\right]Qe^{-e\sqrt{2/\pi}\phi} ,$$

while $\bar{\psi}_R\psi_L$ goes over to

$$Q^\dagger\left[\frac{1+\gamma_5}{2}\right]Qe^{e\sqrt{2/\pi}\phi} .$$

We thus have

$$\langle\bar{\psi}_L\psi_R(x)\bar{\psi}_R\psi_L(x')\rangle=\left\langle\exp\left[e\left[\frac{2}{\pi}\right]^{1/2}[\phi(x')-\phi(x)]\right]\right\rangle\left\langle Q^\dagger\frac{1-\gamma_5}{2}Q(x)Q^\dagger\frac{1+\gamma_5}{2}Q(x')\right\rangle$$
$$=-\exp\left[-\frac{e^2}{2\pi}G(x',x')-\frac{e^2}{2\pi}G(x,x)+\frac{e^2}{\pi}G(x,y)\right]\mathrm{Tr}\left[\frac{1-\gamma_5}{2}S(x,x')\frac{1+\gamma_5}{2}S(x',x)\right],\tag{42}$$

where $S(x,y)$ is the fermion propagator in the pure monopole background:

$$\gamma\cdot(\partial-ieA^M)S(x,y)=-\delta(x-y) .\tag{43}$$

For a monopole background one has the result[9,10]

$$-i\gamma_0\gamma_i(p-eA)_iQ=-\frac{\gamma_0\gamma_r}{r}\frac{\partial}{\partial r}(rQ)+\frac{\gamma_0\gamma_r}{r^2}(\Sigma\cdot l+1)Q ,$$
$$\vec{l}=\vec{r}\times(\vec{p}-e\vec{A}),\quad \gamma_r=\gamma\cdot\hat{r} ,\tag{44}$$

and

$$(\Sigma\cdot l+1)^2=J^2 .\tag{45}$$

In a mode decomposition of $Q(x)$, if we neglect $J\neq 0$ modes, the Lagrangian is, using the above two equations,

$$\mathscr{L}=-\frac{1}{r^2}\chi^\dagger\left[\gamma_0\frac{\partial}{\partial x^4}-\gamma_r\frac{\partial}{\partial r}\right]\chi ,$$
$$S=\int\mathscr{L}\,d^4x$$
$$=-4\pi\int dr\,dt\,\chi^\dagger\bar{\gamma}\cdot\partial\chi ,\tag{46}$$

where $Q=\chi/r$, $\bar{\gamma}_0=\gamma_0$, and $\bar{\gamma}_1=\gamma\cdot\hat{r}$. The $J=0$ mode in a monopole background behaves as a free two-dimensional fermion (with the half-line restriction on $r$). In the spirit of treating the $J\neq 0$ modes as a perturbation, we can saturate the propagator by the $J=0$ mode ($Q=\chi/r$) to obtain the lowest-order result

$$\mathrm{Tr}\left[\frac{1-\gamma_5}{2}S(x,x')\frac{1+\gamma_5}{2}S(x',x)\right]=-\frac{1}{32\pi^4}\frac{1}{r^2}\frac{1}{r'^2}\frac{1}{(r-r')^2+(t-t')^2} .\tag{47}$$

Notice that Eq. (47) is simply the canonical scaling result since $\chi$ has dimension $\frac{1}{2}$. There is subtlety in handling the one-dimensional fermion because the Hamiltonian following from Eq. (46) is not self-adjoint.[9,10,12] This does not affect Eq. (47). Using this result and Eq. (41) for $G(rt,r't')$ we get

$$\langle\bar{\psi}_L\psi_R(x)\bar{\psi}_R\psi_L(x')\rangle=\frac{1}{128\pi^4}\frac{1}{r^3}\frac{1}{r'^3}\left[\frac{(t-t')^2}{(r-r')^2+(t-t')^2}\right]\times\exp(\{\psi[d(\kappa)+1]-\psi(1)\})\tag{48}$$

for large $(t-t')$, and

$$\langle \bar{\psi}_L \psi_R(x) \bar{\psi}_R \psi_L(x') \rangle = \left[ \frac{C}{8\sqrt{2}\pi^2 r^3} \right] \left[ \frac{C}{8\sqrt{2}\pi^2 r'^3} \right] \text{ as } |t-t'| \to \infty ,$$

where $C = \exp(\frac{1}{2}\{\psi[d(\kappa)+1] - \psi(1)\})$. Notice that the two powers of $|t-t'|$ from the $\phi$ propagator are quite crucial in obtaining a nonzero result. For large Euclidean-time separation we expect the expectation value to obey cluster decomposition, giving

$$\langle \bar{\psi}_L \psi_R(x) \rangle = e^{i\delta} \frac{C}{8\sqrt{2}\pi^2 r^3} . \tag{49}$$

$\delta$ is an arbitrary phase.

The result is that chiral symmetry is spontaneously broken in the presence of monopoles. The Goldstone-type mode associated with this is the fluctuation field $\phi$. Further, the fields $\psi_L$ and $\psi_R$ can be of any flavor. Thus, we can have matrix elements of the form $\langle \bar{e}_L e_R \rangle$, $\langle \bar{p}_L p_R \rangle$, and $\langle \bar{p}_L e_R \rangle$, and their conjugates for the proton-positron system. The proton can decay into a positron.

## IV. SEVERAL FLAVORS AND MASS EFFECTS

Let us now consider $2N$ flavors. The propagator is as given in Eq. (41) with a multiplicative factor of $1/N$. If we now repeat the computation of the previous section we would get zero since powers of $|t-t'|$ in the numerator and denominator of Eq. (48) do not match. In fact, it is easy to see that only operators with $2N$ fermion fields of maximal chiral charge have nonzero expectation values, i.e., operators with $N\bar{\psi}_L$'s and $N\psi_R$'s or their conjugates. The specific flavors of the $\psi_L$'s and $\psi_R$'s do not matter. For an operator $\mathcal{G}$ of this type,

$$\langle \mathcal{G}(x) \mathcal{G}^\dagger(y) \rangle = \frac{e^{-N\ln r/R}}{r^{2N}} \frac{e^{-N\ln r'/R}}{r'^{2N}} \frac{e^{2N\ln|t-t'|/R}}{(x-y)^{2N}} \tag{50}$$

as $|t-t'| \to \infty$, implying by cluster decomposition

$$\langle \mathcal{G}(x) \rangle \simeq \frac{C}{r^{3N}} \tag{51}$$

(again a pure scaling behavior).

Let us now return to proton decay. The matrix element $\langle \bar{p}_L e_R \rangle$ implies the process $p \to e^+$. (Since we have a monopole background, the momentum need not be conserved and we can have this process.[14]) One can think of this process as the scattering of the incoming proton off the condensate $\langle \bar{p}_L e_R \rangle$ surrounding the monopole. Since the matrix element does not depend on any coupling constant, the decay rate will be completely determined by the energy of the proton.

Addition of more flavors like the muon, charm, and strange quarks will change this result, since only matrix elements with several fermion fields are nonzero. Proton decay would seem to be eliminated because of the need to have heavier than proton fermion in the final state. However, the introduction of fermion masses and threshold effects also gives us matrix elements with less numbers of fermion fields. Although we are doing the calculation for Dirac monopoles, we think of them as the low-energy description of extended monopoles of the 't Hooft-Polyakov type. The remnant of this structure is that the number of flavors coupling to the monopole is always even. Thus, to go beyond the proton-positron system let us consider the addition of muons and $\tau$ leptons (say). Treating the masses as a perturbation, we can write

$$\langle \bar{p}_L e_R(x) \rangle = -i \int \langle \bar{p}_L e_R(x) [\bar{p}_L p_R(y) m_p + m_\mu \bar{\mu}_L \mu_R(y) + \cdots ] \rangle d^4y . \tag{52}$$

While we cannot calculate the matrix elements $\langle \bar{p}_L p_R(x) \bar{p}_L p_R(y) \rangle$, etc., exactly since $\bar{p}_L e_R$ and $\bar{p}_L p_R$ are at different points, the arguments given so far show that they are nonzero. We can still have direct proton-positron transitions. One can also have $p \to e^+\gamma$, $p \to e^+\pi^0$, etc., whose rates will depend on the probability of bremsstrahlung of photons and pions.

The addition of more flavors also brings in several flavor-changing matrix elements giving processes like $\mu \to e$, $p \to \mu$, $\tau \to p$, etc. The corresponding rates will depend on mass factors through Eq. (52). The question is how many such mass factors should we have?

To analyze this question, we should estimate the effect of mass terms. There are two aspects to the introduction of mass terms. The first concerns the modification of the equations of motion for $\phi$. The contribution of the mass term to the effective Lagrangian can be estimated as

$$m \cos\left[ e \left[ \frac{2}{\pi} \right]^{1/2} \phi \right] \langle \mathrm{Tr} Q\bar{Q} \rangle \simeq \frac{m}{r^2} \cos\left[ e \left[ \frac{2}{\pi} \right]^{1/2} \phi \right] \langle \mathrm{Tr} \chi\bar{\chi} \rangle , \tag{53}$$

using Eq. (46). We have the trace of the $(1+1)$-dimensional free fermion field which is a logarithmically divergent quantity. However, $\chi$ satisfies a free-field equation only for pointlike monopole field configurations. The logarithmic divergence ceases to be valid once we are inside the core of the monopole. It is therefore natural to introduce a cutoff $M_x$ which is characteristic of the size of the monopole. The mass term thus gives a term

$$\frac{m^2}{r^2} \ln\left[ \frac{M_x}{m} \right] \cos\left[ e \left[ \frac{2}{\pi} \right]^{1/2} \phi \right]$$

to the effective Lagrangian for $\phi$. The equation of motion for $\phi$ becomes

$$\Box(r^2 \Box\phi) - \frac{e^2}{4\pi^2} \Box\phi - m^2 e \left[ \frac{2}{\pi} \right]^{1/2} \ln\left[ \frac{M_x}{m} \right] \sin\left[ e \left[ \frac{2}{\pi} \right]^{1/2} \phi \right] = 0 . \tag{54}$$

The $\ln|t-t'|$ behavior will certainly be preserved for $r \lesssim 1/m$; in fact, one can extend the long-range correlation effect even beyond $r \sim 1/m$, since the effect of the mass term in Eq. (54) drops off as $1/r^2$.

Mass terms affect the condensates again through fermion propagators. In computing $\bar{\psi}_L \psi_R(x) \bar{\psi}_R \psi_L(x')$, we have

$$\mathrm{Tr}\left[S(x,x')\frac{1+\gamma_5}{2}S(x',x)\frac{1-\gamma_5}{2}\right].$$

The fermion propagators now carry masses and the matrix element, instead of the pure scaling behavior $1/|x-x'|^2$, will have an exponential falloff $e^{-m|x-x'|}$. We can interpret these results as saying that the monopole thus has a condensate of fermions around it which falls off as a power of $r$ within a radius $\sim 1/m$ and then falls off exponentially.[14] The decay and transition rates considered as a function of energy will be cut off below the masses of the participating particles.

The discussion shows that the number of flavors to be included in the effective Lagrangian is determined by the energy of the incoming particle. The incoming particle cannot see the effects of condensates of fermions of mass greater than its energy. The number of flavors is thus effectively the number of light fermion flavors compared to the energy of the incoming particle. In calculational terms fermions of higher mass do not contribute significantly to the anomaly. Or rather, for them, the mass term in the Lagrangian dominates over their contribution to the anomaly and hence we have to use a different expansion with the mass term included at the zeroth order and treat the anomaly as a perturbation.

More detailed questions about mass effects and branching ratios will be taken up in a possible future paper.

## V. VIOLATION OF GLOBAL QUANTUM NUMBERS: A GENERAL ARGUMENT

We now give a simple argument to see why monopoles lead to violation of global quantum numbers.[15] This can be illustrated by the proton-positron system. The electric and axial-vector currents are defined by

$$\begin{aligned} J_\mu^Q &= ie(\bar{p}\gamma_\mu p+\bar{e}\gamma_\mu e)\ ,\\ J_\mu^5 &= i(\bar{p}\gamma_\mu\gamma_5 p+\bar{e}\gamma_\mu\gamma_5 e)\ . \end{aligned} \tag{55}$$

The baryonic current is defined to be

$$J_\mu^B = i\bar{p}\gamma_\mu p\ . \tag{56}$$

Analogous to the derivation of the anomalous commutation rule (7), one can derive the full anomalous current algebra.[5] We have the results

$$\begin{aligned} [J_0^Q(\vec{x},t),J_0^5(\vec{y},t)] &= \frac{ie^2}{\pi^2}\tilde{F}^{0j}(y)\partial_j\delta(\vec{x}-\vec{y})\ ,\\ [J_0^Q(\vec{x},t),J_i^5(\vec{y},t)] &= \frac{ie^2}{2\pi^2}\tilde{F}^{ij}(y)\partial_j\delta(\vec{x}-\vec{y})\ , \end{aligned} \tag{57}$$

$$\begin{aligned} [J_i^Q(\vec{x},t),J_0^5(\vec{y},t)] &= -\frac{ie^2}{2\pi^2}\partial_j\tilde{F}^{ij}(x)\delta(\vec{x}-\vec{y})\\ &\quad +\frac{ie^2}{2\pi^2}\tilde{F}^{ij}(y)\partial_j\delta(x-y)\ ,\\ [J_0^B(\vec{x},t),J_0^5(\vec{y},t)] &= \frac{ie}{2\pi^2}\tilde{F}^{0j}(y)\partial_j\delta(x-y)\ ,\\ [J_0^B(\vec{x},t),J_i^5(\vec{y},t)] &= \frac{ie}{4\pi^2}\tilde{F}^{ij}(y)\partial_j\delta(x-y)\ ,\\ [J_i^B(\vec{x},t),J_0^5(\vec{y},t)] &= -\frac{ie}{4\pi^2}\partial_j\tilde{F}^{ij}(x)\delta(\vec{x}-\vec{y})\\ &\quad +\frac{ie^2}{4\pi^2}\tilde{F}^{ij}(y)\partial_j\delta(\vec{x}-\vec{y})\ , \end{aligned} \tag{58}$$

where $\tilde{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$.

Consider anomalies in the baryon, electric, and axial-vector currents. They should satisfy the Wess-Zumino consistency conditions which are a (covariant) representation of the current algebra in terms of functional differential operators. They can also be thought of as integrability conditions for the existence of an effective action reproducing the anomalies or as an expression of the group property of the associated phase transformations.[8,11]

Wess-Zumino—type consistency conditions associated with the current algebra (57) and (58) are

$$\delta_\Lambda G^5(\eta)-\delta_\eta G^Q(\Lambda)=\frac{ie^2}{\pi^2}\int\partial_\mu\Lambda\partial_\nu\eta\tilde{F}_{\mu\nu}d^4x\ , \tag{59}$$

$$\delta_\epsilon G^5(\eta)-\delta_\eta G^B(\epsilon)=\frac{ie}{2\pi^2}\int\partial_\mu\epsilon\partial_\nu\eta\tilde{F}_{\mu\nu}d^4x\ , \tag{60}$$

where $G^5$, $G^Q$, and $G^B$ are the anomalies associated with the currents $J_\mu^5$, $J_\mu^Q$, and $J_\mu^B$, respectively. For example,

$$G^5(\eta)=\int\eta(x)\partial_\mu J_\mu^5(x)d^4x\ . \tag{61}$$

$\eta$, $\Lambda$, and $\epsilon$ are the corresponding test functions. $G$'s are functionals of the gauge fields of the theory. $\delta_\Lambda G^5(\eta)$ denotes the variation of $G^5(\eta)$ when the gauge fields in $G^5(\eta)$ undergo a gauge transformation by the function $\Lambda$.

The arguments which lead us from the anomalous current algebra to Eqs. (59) and (60) have been given in Ref. 15. Instead of repeating those arguments here, we shall derive Eqs. (59) and (60) by direct computation of the anomalies. Again we consider one flavor to begin with. The vector and axial-vector currents, including phase factors appropriate for gauge invariance, are

$$\begin{aligned} J_\mu(x) &= i\bar{\psi}(x+\epsilon/2)\gamma_\mu\\ &\quad\times\exp\left[ie\int_{x-\epsilon/2}^{x+\epsilon/2}A\cdot dx\right]\psi(x-\epsilon/2)\ ,\\ J_\mu^5(x) &= i\bar{\psi}(x+\epsilon/2)\gamma_\mu\gamma_5\\ &\quad\times\exp\left[ie\int_{x-\epsilon/2}^{x+\epsilon/2}A\cdot dx\right]\psi(x-\epsilon/2)\ . \end{aligned} \tag{62}$$

We need the result

$$\lim_{\epsilon\to 0}\epsilon_\nu\langle\bar{\psi}(x+\epsilon/2)\gamma_\mu\gamma_5\psi(x-\epsilon/2)\rangle=-\frac{ie}{8\pi^2}\tilde{F}_{\mu\nu}\ , \tag{63}$$

and also the expression for $\psi$ in terms of $Q$ and $\phi$, viz.,

$$\psi=e^{-ie\gamma_5\sqrt{2/\pi}\phi}Q\ . \tag{64}$$

The expectation values of the currents can then be easily computed as

$$\begin{aligned} \langle J_\mu(x)\rangle &= \frac{ie^2}{8\pi^2}\frac{\partial_\nu\phi}{\sqrt{2\pi}}\tilde{F}_{\mu\nu}\ ,\\ \langle J_\mu^5(x)\rangle &= \frac{ie^2}{8\pi^2}A_\nu\tilde{F}_{\mu\nu}\ . \end{aligned} \tag{65}$$

Equations (65) give the induced vector and axial-vector currents in a monopole background. The identity (63), which is crucial to this derivation, can be checked in diagrammatic perturbation theory. It is an exact result in the sense of the Adler-Bardeen theorem. It can also be checked for the extended monopole case taking the point-monopole limit afterwards so that questions of defining the fermion propagator with the attendant difficulties of self-adjointness do not affect its validity. [For the extended monopole case Eq. (63) and its consequences in Eq. (64) are essentially contained in Ref. 2).

From Eq. (64) we get the anomalies

$$\begin{aligned} G^{Q}(\Lambda) &= \frac{ie^2}{8\pi^2}\int \partial_\mu\Lambda \frac{\partial_\nu\phi}{\sqrt{2\pi}}\tilde{F}_{\mu\nu}d^4x\ , \\ G^5(\eta) &= \frac{ie^2}{16\pi^2}\int \eta F_{\mu\nu}\tilde{F}_{\mu\nu}d^4x \\ &\quad + \frac{ie^2}{8\pi^2}\int \eta A_\nu\cdot\partial_\mu\tilde{F}_{\mu\nu}d^4x\ . \end{aligned} \tag{66}$$

These give

$$\begin{aligned} \delta_\eta G^{Q}(\Lambda) &= -\frac{ie^2}{8\pi^2}\int \partial_\mu\Lambda\partial_\nu\eta\tilde{F}_{\mu\nu}d^4x\ , \\ \delta_\Lambda G^5(\eta) &= \frac{ie^2}{8\pi^2}\int \partial_\mu\Lambda\partial_\nu\eta\tilde{F}_{\mu\nu}d^4x\ , \end{aligned} \tag{67}$$

giving

$$\delta_\Lambda G^5(\eta) - \delta_\eta G^{Q}(\Lambda) = \frac{ie^2}{4\pi^2}\int \partial_\mu\Lambda\partial_\nu\eta\tilde{F}_{\mu\nu}d^4x\ . \tag{68}$$

This is the result for one flavor. For two flavors we get a factor of 2 for $G^{Q}(\Lambda)$, 2 for $G^5(\eta)$, and 1 for $G^B(\epsilon)$, leading to Eqs. (59) and (60). Incidentally, by integrating the Wess-Zumino condition (59) we can get an effective action which reproduces the anomalies. This action is exactly identical to the one given by Eq. (27).

Consider Eq. (59). If the expression $G^5(\eta)$ is gauge invariant, e.g.,

$$G^5(\eta) = \frac{ie^2}{8\pi^2}\int \eta F\tilde{F}\,d^4x\ ,$$

then $\delta_\Lambda G^5(\eta)=0$. Equation (59) then indicates that the electric current necessarily has anomalies, i.e., $G^{Q}(\Lambda)$ cannot be zero. The expressions for the anomalies are in general not unique but depend on the regularization scheme. One can go from one scheme to another by adding counterterms to the action which are made up of the gauge fields and whose variations produce the necessary shift in the expression for the anomalies.[8,16] We could thus choose a scheme which gives electric charge conservation, but then $G^5(\eta)$ cannot be gauge invariant. [It is clear from Eq. (59) that this peculiarity occurs only in the monopole sector, i.e., only if $\partial_\mu\tilde{F}_{\mu\nu}$ is nonzero.] Since we cannot interpret gauge-variant results meaningfully, we shall use a gauge-invariant $G^5(\eta)$ and $\partial_\mu J_\mu^Q \neq 0$. This lack of electric-current conservation is not catastrophic since we are doing a background-field calculation. We have an open system and there can be processes which deposit charge on the monopole. To make this point clearer we look at the anomaly for $J_\mu^Q$ implied by Eq. (59). It is easily seen to be

$$\partial_\mu J_\mu^Q = \frac{ie^3}{\pi^2}\frac{\partial_\nu\phi}{\sqrt{2\pi}}(\partial_\mu\tilde{F}_{\mu\nu})\ . \tag{69}$$

The amount of violation of the quantum number is proportional to the magnetic-current density $\partial_\mu\tilde{F}_{\mu\nu}$. Notice that we can define a conserved electric charge by redefining the current as

$$\mathscr{J}_\mu^Q = J_\mu^Q - \frac{ie^3}{\pi^2\sqrt{2\pi}}\phi(\partial_\nu\tilde{F}_{\nu\mu})\ . \tag{70}$$

Indeed, this redefinition naturally occurs since to go from $G^{Q}(\Lambda)=0\delta_\Lambda G^5(\eta)\neq 0$ to $\delta_\Lambda G^5(\eta)=0$ we have to add to the action the counterterm[17]

$$S_c = \frac{ie^3}{\pi^2\sqrt{2\pi}}\int \phi(\partial_\mu\tilde{F}_{\mu\nu})A_\nu d^4x\ . \tag{71}$$

This has the effect of giving an anomaly to the original current $J_\mu^Q$, but the redefined conserved current $\mathscr{J}_\mu^Q$ is what couples to $A_\mu$. The extra current has the natural interpretation as the charge deposited on the monopole in scattering processes. The action (71) measures the electrostatic energy associated with the charge on the monopole. Thus, "charge-changing" processes of the type $p+M \to M^{++}+\bar{p}$ are possible but would be suppressed by the extra energy of Eq. (71). $S_c=0$ for $A_0=0$ at $r=0$, i.e., for $\partial\phi/\partial r=0$ at $r=0$. With this boundary condition for $\phi$ we cannot generate charge-changing processes. To do so, we should introduce a background $A_0$ which would then give $S_c\neq 0$ and the suppression effect.

Consider now Eq. (60). $\delta_\epsilon$ denotes the gauge transformation corresponding to baryon number. Since $G^5(\eta)$ is a function of the gauge fields and since there are no gauge fields corresponding to baryon number, $\delta_\epsilon G^5(\eta)=0$. Thus,

$$\delta_\eta G^B(\epsilon) = -\frac{ie^2}{2\pi^2}\int \partial_\mu\epsilon\partial_\nu\eta\tilde{F}_{\mu\nu}d^4x\ .$$

Baryon number necessarily has anomalies. It is this anomaly which leads to baryon-number violation. One can estimate the violation as

$$\partial_\mu J_\mu^B = \frac{ie^2}{2\pi^2\sqrt{2\pi}}\partial_\nu\phi(\partial_\mu\tilde{F}_{\mu\nu})\ . \tag{72}$$

Although we could redefine the current again to get conservation, it is not very natural since there are no gauge fields associated with $J_\mu^B$. For the same reason baryon-number-violating processes do not have extra action suppressions [as in Eq. (71)]. For a point monopole we have

$$\partial_\mu J_\mu^B \sim \partial_t\phi(r=0)\delta(\vec{x})\ . \tag{73}$$

It is of key importance to have $\partial_t\phi(r=0)\neq 0$. (The monopole is "sucking in" baryon number.) This is in accordance with the remarks following Eq. (41) that the $\ln|t-t'|$, crucial to having nonzero $\langle p_L e_R\rangle$, also has $(\partial\phi/\partial t)(r=0)\neq 0$.

The argument we have outlined applies essentially to all global quantum numbers which have anomalous current algebra of the form (58), e.g., lepton number, muon number, quark flavors, etc.

We want to qualify the sense in which these quantum numbers are violated. Even though there is no gauge field associated with baryon number, we can redefine the baryon number identifying the extra term $\sim\phi\partial_\mu\widetilde{F}_{\mu\nu}$ as the baryon number of the monopole. (This would not be derivable by Noether variation from a Lagrangian.) If baryon number were conserved without monopoles, this would be a consistent procedure. We would then have overall baryon-number conservation taking into account the baryon number that disappeared into the monopole. However, for a grand unified theory for which there is no conservation even without monopoles, we cannot define intrinsic baryon number for the monopole; there is a genuine violation.

Can we distinguish these two cases? Experimentally, there is no distinction unless we have creation and annihilation of monopoles. At low energies when we scatter particles off a monopole, we should see baryon-number and flavor-changing processes irrespective of the grand unification structure. Notice in particular that we can put into a monopole or take away any amount of a global quantum number. Theoretically, a full quantum theory of monopoles would be required to formulate the distinction precisely.[18]

An analogy with the violation of baryon number (and other global quantum numbers) by black holes would be helpful at this point. Baryons can disappear into the black hole. We can attempt to preserve the conservation law by attributing a certain baryon number to the black hole. This is not very natural since the baryonic charge of the black hole is not detectable in any way outside the event horizon. (By contrast, it would be if baryon number was coupled to a gauge field.) Nevertheless, we do get conservation. It is only when we consider creation and annihilation processes like the evaporation of the black hole that a distinction between baryons disappearing into the black hole and nonconservation can be made. The Hawking radiation, because it is thermal, is symmetric with respect to baryons and antibaryons.[19] The evaporation of the black hole by this process reveals the impossibility of defining an intrinsic baryon number for the black hole. Similarly, the question of how meaningful intrinsic quantum numbers are for a monopole requires processes which create and annihilate monopoles.

Finally, we close this section with a remark about boundary conditions. The field $\phi$ was introduced as electrical fluctuations around the monopole. As such it is dynamical and is insensitive to flavor. It leads to anomalies as in Eq. (73). The question of boundary conditions on $\phi$ at $r=0$ becomes important. Although $\phi$ satisfies an equation quartic in the derivatives, we have only freedom in choosing $\phi(0)$ and $\partial\phi/\partial r\,|_{r=0}$, since $\phi=f-h$ and boundary conditions on $f$ have been fixed. [Compare Eq. (32).] If we choose $\phi(0)=$ constant so that $\partial_t\phi(0)=0$, the catalysis effect is killed. The condition $\partial\phi/\partial r=0$ at $r=0$ leads to $\partial_t\phi(0)\neq 0$ and we have the effect. Which boundary condition is correct depends on the extended-monopole theory. Following Eq. (39), we have indicated that the preferred boundary condition is $\partial\phi/\partial r=0$ at $r=0$, leading to the catalysis effect.[20] Since $\phi$ is also the axial-vector phase of the fermions, the freezing $[\phi(0)=0]$ or freedom $[\partial_t\phi(0)\neq 0]$ of $\phi$ can also be written as a condition on $\psi_L$ and $\psi_R$ at the origin.

## VI. DISCUSSION

Our arguments show, in agreement with Refs. 1—3, that monopoles do catalyze proton decay at rates comparable to strong-interaction rates. Other quantum numbers can also be violated. It is clear that grand unification and the $(V-A)$ nature of SU(5) are not relevant. But of course we do need grand unification to the extent of having finite-energy monopoles. The Dirac-type monopoles, like classical charged particles, have infinite self-energy. For charged particles this is no catastrophe since the renormalization structure of the quantum theory tells us that their energy is a free parameter. This is not expected to happen for monopoles, hence we should start out with finite-energy monopoles of the 't Hooft-Polyakov type. Nevertheless, since grand unification monopoles have a very small size $(\sim 1/M_x)$, our point-monopole approximation seems reasonable. One can, of course, repeat all the calculations presented here for the extended monopole also. We expect that additions to the point-monopole theory, like the extra action of Eq. (71), will emerge naturally in the theory of extended monopoles.

In conclusion, our result is that the formation of the fermion condensate around the monopole is governed completely by low-energy dynamics. The condensate helps to transfer flavor from the incoming particles onto the monopole, leading to flavor-changing effects in scattering processes. If the overall theory conserves flavor, the flavor defect in scattering can be thought of as an intrinsic flavor of the monopole which it will give up when annihilated. One has flavor conservation, but this still differs from standard flavor-changing effects in that one can put into the monopole or take out any (even unquantized) amount of a global quantum number. If the theory does not conserve a certain quantum number (e.g., baryon number in GUT's), there is no meaning to the intrinsic quantum number of the monopole and there is a genuine breakdown of flavor conservation.

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