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Peccei-Quinn symmetry as flavor symmetry and grand unification

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We consider the possibility of identifying the Peccei-Quinn (PQ) symmetry as also the flavor symmetry in multigenerational grand unification schemes. The essential ingredient, a global, axial U(1) symmetry in the PQ mechanism to avoid the strong *CP*-violation problem provides useful constraints on the fermion—Higgs-boson couplings in the theory, thereby leading to identical "canonical" forms for fermion mass matrices in both the charged sectors. These forms are the conjectured Fritzsch-type matrices exhibiting the "nearest-neighbor" interactions in generation space. From among the popular schemes for grand unification, SO(10) emerges as one which has several advantages over the others for constructing multigenerational grand unification models. Reasonable assumptions regarding the quark masses lead to unique PQ quantum-number assignments for the fermionic generations. These quantum numbers combined with the hierarchy in quark masses lead to a picture in which the lighter generations are composite in nature. One can then show qualitatively that the hierarchy is such that log$m$ varies linearly with respect to the generation index.

I. INTRODUCTION

The Peccei-Quinn (PQ) mechanism[1] was originally introduced at the standard $SU(3)\times SU(2)\times U(1)$ level with the primary purpose of solving the strong *CP*-violation problem.[2] A global Abelian degree of freedom was invoked to rotate away the potentially dangerous $\theta F\tilde{F}$ term. However, in doing so, it was quite necessary to assume that the corresponding global U(1) symmetry was axial, leading to color anomalies, and to enlarge the Higgs system so that at least two scalar doublets with opposite PQ assignments are included. With the above ingredients, the strong *CP*-violation problem is essentially solved. But one is faced with the axion problem[3] and the necessity to explain its experimental invisibility. The recent proposal[4] of Dine, Fischler, and Srednicki (DFS) does exactly that by including a complex Higgs singlet in addition to the two scalar doublets. The axion mass and its coupling strength to normal matter are both inversely proportional to the vacuum expectation value (VEV) of the additional singlet; and if this VEV is large enough, the DFS axion is practically invisible. It is then only a phantom, harmless axion.

While the DFS scenario may by far be the best solution to the strong *CP*-violation problem, it is highly artificial within the $SU(3)\times SU(2)\times U(1)$ framework to assign an arbitrarily large VEV to one of the Higgs multiplets in the theory. This, however, is not the case when one contemplates the PQ symmetry along with grand unification. The strong *CP*-violation problem then can be embedded within the more general gauge-hierarchy problem. Indeed, several grand unified versions of the DFS idea have been proposed.[5] In this paper, *we attempt to tie the axiality of $U(1)_{PQ}$ together with the complex irreducible nature of the fermionic family.* From our point of view, SO(10) and E(6) are preferable to SU(5). It turns out that the same physics which dictates the axiality of $U(1)_{PQ}$ also requires the desired pair structure of the Higgs doublets. We thus argue that the essential ingredients of the PQ mechanism are actually a signature of proper grand unification.

Recently $U(1)_{PQ}$ symmetry within the framework of grand unification was suspected to lead to difficulties with constraints from astrophysics.[6] However, more recently several authors have proposed variations of the basic DFS mechanism which can avoid this problem.[7] A soft breaking of $U(1)_{PQ}$ by Higgs-boson mass terms may be simplest. Grand unified models combined with $U(1)_{PQ}$ thus remain very attractive.

Now, one of the most striking features associated with the idea of grand unification is its possible realization already at the so-called single-generation level. This leaves aside the overall flavor problem.[8] Moreover, the uniqueness[9] of the possible candidate theories when one considers some requirements on the fundamental fermions, along with the existence of general no-go theorems,[11] which practically forbid simple multigenerational grand unification, undoubtedly signify the special role played by theories such as SU(5) (Ref. 11), SO(10) (Ref. 12), and E(6) (Ref. 13), despite the "superfluous replication"[14] or the generation problem[14] they are not capable of dealing with. This is where the PQ symmetry may play an extra important role. *Namely, $U(1)_{PQ}$ symmetry can be successfully utilized*[15] *as also the horizontal flavor symmetry.* Such an idea is strongly supported by the one-to-one correlation[16] between the axial character of the horizontal group factor and the canonical structure[17] of the fermion mass matrix. Indeed, we demonstrate how horizontal $U(1)_{PQ}$ leads to a variety of fermionic mass relations and allows us to express the generalized Cabibbo-Kobayashi-Maskawa mixing angles in terms of quark-mass ratios. The relative deformation of $m(u)$ vs $m(d)$, which is known[16] to accompany *local* $U(1)_A$ horizontal models, is neatly avoided.

An extremely interesting further result that emerges is the fact that the PQ assignments of all the particles which make their appearance in the theory are uniquely determined up to an overall arbitrary scale. In particular, the fermionic generations exhibit the quantized PQ assignments

$$1,-3,5,-7,\ldots ,$$

respecting some special discrete subgroup of $U(1)_{PQ}$. We interpret the uniqueness and the special characteristics of the multigenerational extension of the PQ mechanism as a signature of "horizontal compositeness." Only one family of fermions and its associated Yukawa-interacting scalars need be regarded as fundamental constituents. Other fermionic generations can be viewed as composites of the basic family and appropriately coupled Higgs doublets of the theory. The associated fermion mass matrix and the iterative structure of the theory then tell us that *the lighter the family the more composite it is,* in the sense that more scalars go into making it. From this point of view, the muon is more elementary than the electron. Thus, the picture of compositeness that emerges from our considerations is very different from those that are currently described in the literature.

To proceed further and understand the full generation structure one needs dynamics. At present, we have no detailed dynamical scheme. Nevertheless, we show by analyzing the dominant effective Feynman graphs that the combination of $U(1)_{PQ}$ and the horizontal compositeness idea leads to a qualitative understanding of the fermionic mass hierarchy. In fact, in a very crude approximation, log*m varies linearly with respect to the generation index.* We use quite strongly the facts that only symmetrically coupled Higgs bosons can trigger the mass-generating process, and that only real scalars under the gauge group allow for a composite family structure. Consequently, apart from severely restricting the Higgs system, $U(1)_{PQ}$ also acquires the power of choosing its grand-unifying group partner. $G=SO(10)$ emerges as the only tenable candidate.

Altogether, we attempt to provide in this paper a link among various physical phenomena, our major observation being that the strong *CP*-violation problem, minimal grand unification, the generation puzzle, and even the conjectured horizontal compositeness are very tightly correlated by means of the PQ symmetry.

This paper is organized as follows: In Sec. II, we discuss PQ symmetry in the context of single-generation grand unification schemes SU(5), SO(10), and E(6). In Sec. III, we first consider a U(1) horizontal symmetry in the generation space and show how a simple requirement, namely, that the U(1) assignments distinguish the different generations, leads to severe restrictions on the forms of the mass matrices. We argue that in order to have nondegenerate, nonzero eigenvalues for the quark masses, a minimum of two Higgs doublets are necessary and thus link the U(1) symmetry with the U(1) symmetry necessary to implement the PQ mechanism. We then examine this symmetry in conjunction with grand unification schemes. Section IV is devoted to some phenomenological aspects such as the mixing angles, mass hierarchies, and the idea of "horizontal" compositeness. We show, albeit qualitatively, that the mass hierarchy is one where log$m$ varies linearly with the generation index.

II. PECCEI-QUINN SYMMETRY AND SINGLE-GENERATION GRAND UNIFICATION SCHEMES

Consider a single generation of fermions. In addition to the customary gauge transformations of $SU(3)\times SU(2)\times U(1)$, let the Lagrangian be symmetric under a global continuous U(1) symmetry. The most general transformation laws of the quarks under the latter symmetry are given by

$$\begin{bmatrix} u \\ d \end{bmatrix}_L \rightarrow e^{iax_q}\begin{bmatrix} u \\ d \end{bmatrix}_L , \qquad u_R\rightarrow e^{iaxu}u_R \;,\quad d_R\rightarrow e^{iaxd}d_R \;, \tag{2.1}$$

where a is an arbitrary parameter and x_q, x_u, and x_d are the U(1) hypercharges. The global U(1) symmetry is to be identified with the PQ symmetry required to solve the strong *CP* problem, in which case it has to be axial. Hence,

$$x_q=-x_u=-x_d=x\neq 0 \;. \tag{2.2a}$$

From the above relation it follows that the color anomaly associated with the $U(1)_{PQ}$, namely,

$$2x_q-x_u-x_d=4x\neq 0 \;. \tag{2.2b}$$

Thus, the postulated, global U(1) symmetry has the required anomaly to remove the masslessness of the associated Goldstone boson once the symmetry is broken. Were the U(1) symmetry a local symmetry, the nonvanishing anomaly would spoil the renormalizability of the theory.

The axial nature of the $U(1)_{PQ}$ symmetry also leads to the requirement that there be at least two *distinguishable* Higgs doublets if both the up and down quarks are to acquire tree-level masses. To see this, we write the fermion—Higgs-boson coupling terms in the Lagrangian

$$\mathscr{L}_Y=\Gamma_u\bar{q}_L\phi_u^\dagger u_R+\Gamma_d\bar{q}_L\phi_d d_R+\text{H.c.} \tag{2.3}$$

and observe that, while ϕ_u and ϕ_d have identical $SU(2)\times U(1)$ quantum numbers, their U(1) hypercharges h_u,h_d must satisfy

$$h_u=-x_q+x_u=-2x \;,\quad h_d=x_q-x_u=2x \;. \tag{2.4}$$

Therefore, they carry *opposite* PQ assignments and hence are distinguishable. The need for two Higgs doublets thus originates from quark-mass considerations.

While the PQ mechanism solves the strong *CP*-violation problem, it creates the well-known axion problem. Within the original framework, the pseudo-Goldstone boson associated with the breaking of PQ symmetry leads[3] to a tiny-mass particle, the axion, which should have been seen experimentally if it existed. As noted earlier, the latter difficulty can be avoided if we modify the original PQ framework and introduce, in addition to the two Higgs doublets, a complex Higgs singlet with a sufficiently large vacuum expectation value.[4] By this modification one can suppress to any desired degree the couplings of the axion to ordinary matter, making it an "invisible" or "phantom" axion. However, the required huge magnitude of the VEV of the singlet makes no sense at the $SU(3)\times SU(2)\times U(1)$ level. But within the framework of grand unification schemes, the presence of a singlet with exactly the required properties is necessary for other reasons. Thus, if we temporarily ignore the hierarchy and fine-tuning problems with which the grand unified theories are beset, we are no worse off than before[5] with the added advantage of having eliminated the strong CP-violation problem.

Therefore, we proceed now to discuss in some detail single-generation grand unification schemes based on $G\times U(1)_{PQ}$, where the symmetry group G can be one of the currently popular grand unification groups, $G=SU(15)$, SO(10), or E(6). Our principal aim is to see which one provides the required features of $U(1)_{PQ}$ being axial, the pairing of Higgs doublets, the suppression of the axion couplings in the most natural way possible once we accept the above group structure.

A. $G=SU(5)$

The basic set of fermions ψ_L, characterized by their left-handed helicity states, belong to a reducible combination of two representations $\underline{10}$ and $\underline{5}^*$ of SU(5), $\psi_L=\psi_{10}+\psi_{5^*}$. *The PQ assignments of* ψ_{10} *need not be the same as those of* ψ_5. Consequently the axiality of $U(1)_{PQ}$ is guaranteed only for the up quarks. *For other flavors it has to be imposed.* The up quarks are special since both u_L and u_L^C belong to the same irreducible representation $\underline{10}$ and consequently carry the same PQ assignments. u_L and u_R then in turn must carry opposite PQ hypercharges.

The minimal fermion—Higgs-boson couplings in SU(5) theory is given by

$$\mathscr{L}_Y=\Gamma_u\psi_{10}^T C\phi_u\psi_{10}+\Gamma_d\psi_{10}^T C\phi_d^\dagger\psi_{5^*}\ , \tag{2.5}$$

where C is the antisymmetric charge-conjugation operator, ϕ_u and ϕ_d are two independent scalar multiplets which transform as $\underline{5}$ or $\underline{45}$. Note that ϕ_u has to be a $\underline{5}$, otherwise $m_u=0$ as a consequence of the antisymmetric nature of the ϕ_{45} couplings. As in (2.3), ϕ_u and ϕ_d have to have opposite PQ assignments once we let ψ_{10}, ψ_{5^*} have the same assignments in order that $U(1)_{PQ}$ be axial.

Thus, neither the axial nature of $U(1)_{PQ}$ nor the two Higgs multiplets with opposite PQ assignments follow naturally in the case of SU(5). However, it does provide the necessary framework for suppressing the axion couplings along with its mass in a natural way. The $SU(2)\times U(1)$-singlet scalar with a large VEV, assumed in the DFS mechanism, *is* present in the theory to begin with. It is an element of the Higgs multiplet ϕ_{24}, complexified to accommodate $U(1)_{PQ}$, which is responsible for the spontaneous breakdown of SU(5) into its maximal $SU(3)\times SU(2)\times U(1)$.

B. $G=SO(10)$

Grand unification based on $G=SO(10)$ has the following three main features that distinguishes it from SU(5): (i) The theory is automatically anomaly free. (ii) It allows for more than one way in which the symmetry can be broken down to $SU(3)\times SU(2)\times U(1)$. It allows, for instance, an intermediate left-right-symmetric substructure. (iii) The single-generation fermionic states belong to a *single* irreducible complex representation.

It is mainly the last feature which provides a link between grand unification and the strong CP-violation problem. It also allows, as we shall see in Sec. III, a unique multigenerational extension. Since f_L and f_L^c both are members of the same complex irreducible representation ψ_{16_L}, they must transform alike under any additional direct-product symmetry. If such an extra symmetry happens to be a global $U(1)_{PQ}$, f_L and f_R must carry opposite PQ assignments. This conclusion is valid for any arbitrary representation. *Thus, the axiality of* $U(1)_{PQ}$ *is not a free choice. It is dictated by the pure generation structure of flavor-chiral SO(10).* The PQ assignments can then be thought of as the common "*family name*" for all quarks and leptons which belong to the same family.

Let us next consider the fermion—Higgs-boson couplings which have the compact form

$$\mathscr{L}_Y=\Gamma\widetilde{\psi}_{16}C\phi^\dagger\psi_{16}\ , \tag{2.6}$$

where ϕ is either $\underline{16}$, $\underline{120}$, or $\underline{126}$. If more than one Higgs multiplet contributes, they all must have the same PQ assignments. Now under the decomposition of SO(10) into $SU(5)\times U(1)$ [with U(1) being the local $T_{3R}+\frac{3}{4}(B-L)$], we discover the desired pair structure in the Higgs system, namely,

$$\begin{aligned}&\underline{10}=\underline{5}+\underline{5}^*\ ,\\&\underline{120}=(\underline{5}+\underline{5}^*)+\underline{10}+\underline{10}^*+\underline{45}+\underline{45}^*\ ,\\&\underline{126}=1+(\underline{5}^*+\underline{45})+\underline{10}+\underline{15}^*+\underline{50}\ .\end{aligned} \tag{2.7}$$

We observe that for each ϕ_d ($\underline{5}$ representation) there exists ϕ_u (another $\underline{5}$) with exactly opposite PQ assignments, because in each of the above irreducible representations, which are the only ones that couple to the ψ_{16_L}, there is a $(\phi_d+\phi_u^*)$ combination. This property exhibits a natural link between SO(10) and $U(1)_{PQ}$.

C. $G=E(6)$

Finally, we will examine briefly $G=E(6)$. As in SO(10), E(6) admits a single lowest-dimensional complex irreducible representation, namely $\underline{27}$, to which the basic set of fermions belong. Under the decomposition of E(6) with respect to SO(10), $\underline{27}=\underline{16}+\underline{10}+\underline{1}$. It has the disadvantage, of course, that just to start with it contains more than the known low-energy fermions and symmetry breaking has to be invoked in such a manner that they acquire heavy enough masses to have escaped observations. Further the relevant Higgs multiplets that can couple to the fermions and give masses to them are $\underline{27}$, $\underline{351}$, and $\underline{351}'$. In contrast to SO(10), where some of the relevant Higgs multiplets are real under the gauge group, all the above multiplets of E(6) are *complex*. This property turns out to be detrimental to multigenerational extension of $U(1)_{PQ}$, leading to SO(10) as the most preferable candidate for G.

III. PECCEI-QUINN SYMMETRY AS HORIZONTAL FLAVOR SYMMETRY

In this section we shall consider some features of an axial U(1) symmetry acting as a horizontal symmetry on the generation. We shall see that it has all the characteristics of a PQ symmetry. The identification of the two can thus be made. The initial part of the discussion can be made at the $SU(3)\times SU(2)\times U(1)$ level.

Let the left-handed spin-$\frac{1}{2}$ chiral fields ψ_L^i belonging to the ith generation transform as

$$\psi_L^i \to e^{iax_i}\psi_L^i , \tag{3.1}$$

under some global, axial symmetry $U(1)_A$. To distinguish the different generations and thereby avoid the "superfluous" replication, we impose the condition that the axial charges x_i satisfy

$$x_i \neq x_j , \quad i\neq j , \tag{3.2}$$

and consider its implications on the form of the mass matrix. For this purpose it is sufficient to consider a specific quark sector with charge $\frac{2}{3}$ or $-\frac{1}{3}$ at the $SU(2)\times U(1)$ level. In what follows we shall consider the down-quark sector.

We note that if a Higgs doublet ϕ, which transforms as

$$\phi \to e^{iah}\phi , \tag{3.3}$$

is to be coupled to quark fields transforming as (3.1), a Yukawa-type coupling $\bar{q}_L^i \phi q_R^i$ is allowed if and only if

$$x_i + x_j = h . \tag{3.4}$$

The above condition which is symmetric between i and j is a direct consequence of the axial nature of the assumed U(1) symmetry. Were the U(1) symmetry vectorial, we would have the condition $x_i - x_j = h$.

It is convenient for the following discussion to introduce a symmetric matrix X given by

$$X_{ij} = x_i + x_j . \tag{3.5}$$

We note that the trivial-looking condition (3.2) imposes severe restrictions on the mass matrix. Thus, at most one element which equals h can appear in any given row, column, and along the principal diagonal of X and the result is that a given Higgs doublet can have at most N different entries in an N-dimensional mass matrix. For $N=3$, as an example, the matrix X has the form

$$X = \begin{bmatrix} \cdot & h & \cdot \\ h & \cdot & \cdot \\ \cdot & \cdot & h \end{bmatrix} \tag{3.6}$$

with the dots representing entries different from h.

If there were only one Higgs doublet ϕ with $U(1)_A$ hypercharge h, the corresponding mass matrix $m(d)$ corresponding to (3.6) will have, at the tree level, two eigenvalues numerically equal. And in the general N-dimensional case, the mass matrix will have at most $[(N+1)/2]$ different eigenvalues. Our present knowledge regarding the quark masses requires that the mass matrix has N nondegenerate, nonzero eigenvalues. This can be achieved by having more than one Higgs multiplet; the minimal requirement is two. If we have two Higgs doublets ϕ and ϕ', which transform under U(1) as

$$\phi \to e^{iah}\phi , \quad \phi' \to e^{iah'}\phi' , \tag{3.7}$$

the matrix X has the form

$$X = \begin{bmatrix} \ddots & h & \\ h & & h' \\ & h' & h' \end{bmatrix} . \tag{3.8}$$

The condition (3.2), to avoid superfluous application, implies that $h \neq h'$. With two Higgs doublets, the form (3.8) for X is unique up to permutations in generation space. The corresponding mass matrix then has the "canonical form"

$$m(d) = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & \sim we^{i\beta} & 0 & 0 \\ \cdot & \sim we^{i\beta} & 0 & \sim ve^{i\alpha} & 0 \\ \cdot & 0 & \sim ve^{i\alpha} & 0 & \sim we^{i\beta} \\ \cdot & 0 & 0 & \sim we^{i\beta} & \sim ve^{i\alpha} \end{bmatrix} , \tag{3.9}$$

and for general v, w, α, and β it will have in general N nondegenerate, nonzero eigenvalues. Further, the above form leads to N inhomogeneous equations for the N unknown $U(1)_A$ quantum numbers x_i, $i=1,2,\ldots,N$ in terms of h and h',

$$\begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & \\ 0 & 1 & 1 & 0 & \cdots & \\ & & & & & \\ 0 & 0 & 0 & 0 & \cdots & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} h \\ h' \\ \vdots \\ h' \\ h \end{bmatrix} , \tag{3.10}$$

leading to a unique solution,

$$x_k = \tfrac{1}{4}(h+h') + \tfrac{1}{4}(2N-2k+1)(-1)^{N-k}(h-h') , \tag{3.11}$$

and the explicit form for X,

$$X = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 4h'-3h & h' & 3h'-2h & 2h'-h \\ \cdot & h' & 3h-2h' & h & 2h-h' \\ \cdot & 3h'-2h & h & 2h'-h & h' \\ \cdot & 2h'-h & 2h-h' & h' & h \end{bmatrix} . \tag{3.12}$$

Note that so far we have used only the axial nature of $U(1)_A$ symmetry. It could be global or local without affecting the form of the mass matrices or the determination of the quantum numbers x_k according to (3.11). The difference between global and local $U(1)_A$ becomes apparent, however, when we consider the mass matrix in the charge $(\frac{2}{3})$–sector (up sector). We observe that if $x_i + x_j = h$ or h' is the criterion for determining the nonvanishing matrix elements in $m(d)$, the corresponding criterion for the up-quark mass matrix $m(u)$ is

$$x_i+x_j=-h \text{ or } -h' . \tag{3.13}$$

Since x_i,x_j are determined uniquely by specifying $m(d)$, it should be possible to determine the structure of $m(u)$ from (3.12) and examine whether, by suitably restricting h and h', one can obtain $m(u)$ with all the desirable features, namely, with nondegenerate, nonzero eigenvalues. Indeed, if we go back to X in (3.12), identify some of its elements as $-h$ and $-h'$, and check whether the corresponding mass matrix $m(u)$ has nondegenerate, nonzero eigenvalues, we find, after some algebra, that there are *only* two possibilities:

(a) The local solution:

$$h=0 \ (N \text{ odd}) , \quad h'=0 \ (N \text{ even}) . \tag{3.14}$$

In this case,

$$\sum x_i=\sum x_i^3=0 , \tag{3.15}$$

implying the absence of triangular anomalies. The associated $U(1)_A$ symmetry has to be a local gauge symmetry to avoid a true Goldstone boson. It can then be looked upon as a purely horizontal symmetry factor in flavor-unifying attempts with gauge groups larger than SU(5), SO(10), or E(6). Such a possibility has been examined before; the accompanying mass matrix $m(u)$ has the desired features, but it does not possess a canonical structure identical to the one for $m(d)$.[16] The nonvanishing elements in $m(u)$ are shifted around relative to those in $m(d)$.

(b) The global solution:

$$h+h'=0 . \tag{3.16}$$

The associated $U(1)_A$ symmetry has anomalies; it must be global in order that the theory be renormalizable. It can be identified as the global, axial PQ symmetry with the $U(1)_{PQ}$ quantum numbers determined uniquely in terms of $h=-h'\equiv 2x$. The matrix X has the form

$$X=\begin{vmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -7h & -h & -5h & -3h \\ \cdot & -h & 5h & h & 3h \\ \cdot & -5h & h & -3h & -h \\ \cdot & -3h & 3h & -h & h \end{vmatrix} \tag{3.17}$$

and

$$x_N=x , \quad x_{N-1}=-3x , \quad x_{N-2}=5x , \quad \cdots , \qquad x_1=-(-1)^N(2N-1)x . \tag{3.18}$$

For $N=3$, the two mass matrices are given by

$$m(d)=\begin{bmatrix} 0 & d_1ve^{+i\alpha} & 0 \\ d_2ve^{+i\alpha} & 0 & d_3we^{i\beta} \\ 0 & d_2we^{i\beta} & d_5ve^{i\alpha} \end{bmatrix} , \tag{3.19a}$$

$$m(u)=\begin{bmatrix} 0 & u_1we^{-i\beta} & 0 \\ u_2we^{-i\beta} & 0 & u_3ve^{-i\alpha} \\ 0 & u_4ve^{-i\alpha} & u_5we^{-i\beta} \end{bmatrix} , \tag{3.19b}$$

where $d_1,d_2,\ldots,d_5$ and $u_1,u_2,\ldots,u_5$ are arbitrary Yukawa couplings and $\langle\phi\rangle=ve^{i\alpha}$, $\langle\phi'\rangle=we^{i\beta}$. Note that both $m(d)$ and $m(u)$ have an identical structure; they are not independent, but correlated with the following correspondence:

$$m(d)_{ij}\sim ve^{i\alpha} \text{ or } we^{i\beta}\leftrightarrow m(u)_{ij}\sim we^{-i\beta} \text{ or } ve^{-i\alpha} . \tag{3.20}$$

These interesting features of the mass matrices along with a unique set of quantum numbers for the fermionic families at the $SU(3)\times SU(2)\times U(1)$ level leads us to consider the global $U(1)_{PQ}$ symmetry as also the horizontal flavor symmetry in the context of grand unification. Let us consider the symmetry group $G\times U(1)_{PQ}$, where G is the single-generation, grand unification group, and ask whether the additional $U(1)_{PQ}$-symmetry requirements leading to the desirable characteristics of the mass matrices place any restrictions on the choice of G. The case of $G=SU(5)$ has been discussed in Sec. II A. Even at the single-generation level, it fails to provide a natural setting for $U(1)_{PQ}$. Next if we consider SO(10) and a single Higgs-scalar representation ϕ, the Yukawa couplings which give rise to the canonical form for the mass matrices (3.19a) and (3.19b) can be written as

$$L_Y=G_1\psi^T(x)C[\phi(2x)]^\dagger\psi(x)+G_2\psi^T(x)C[\phi(2x)]\psi(-3x)+G_2'\psi^T(-3x)C\phi(2x)\psi(x) +G_3\psi^T(-3x)C[\phi(2x)]^\dagger\psi(5x)+G_3'\psi^T(5x)C[\phi(2x)]^\dagger\psi(-3x)+\cdots . \tag{3.21}$$

Note the alternating of $\phi^\dagger(2x)$ and $\phi(2x)$ in the above form, which follows from the requirement of $U(1)_{PQ}$ symmetry. If now $\phi(2x)$ is real, it will decompose pairwise,

$$\phi(2x)=\phi_d(2x)+\phi_u^\dagger(2x)+\cdots ,$$

giving rise to the mass matrices that have the correlated forms (3.19a), (3.19b). If it is complex, the correlation is in general lost. In the case of SO(10), the Higgs representations that can occur in (3.21) are $\underline{10}$, $\underline{120}$, and $\underline{126}$. We known that $\underline{10}$ and $\underline{120}$ are both real. Further, $\underline{10}$ is symmetric while $\underline{120}$ is antisymmetric. The $\underline{126}$ representation is complex and symmetric. Hence, it follows that for $\phi\equiv\underline{10}$,

$$G_2=G_2' , \quad G_3=G_3' , \quad \cdots ,$$

leading to the desired forms (3.19a) and (3.19b) for m_d and m_u, respectively. For $\phi\equiv\underline{120}$,

$$G_1=0 , \quad G_2=-G_2' , \quad G_3=-G_3' .$$

These lead to mass matrices with pair degeneracy in the mass eigenvalues. For $\phi = \underline{126}$,

$$G_1 = G_3 = G_3' = \cdots = 0\ ,\quad G_2 = G_2'\ ,\quad G_4 = G_4' = \cdots$$

or

$$G_2 = G_2' = G_4 = G_4' \cdots = 0\ ,\quad G_3 = G_3' \cdots\ .$$

This also leads to a pair degeneracy in the mass eigenvalues. Thus, in a minimal picture, where only one Higgs field couples to fermions, the Higgs representation must be $\underline{10}$. ϕ_{120} or ϕ_{126} alone would lead to a pair degeneracy in the mass eigenvalues, due to in the first case, the antisymmetric and in the second case, the complex nature of the representations. In general, unless a minimality condition is imposed on the Higgs system, the existence of more than one Higgs multiplet is permissible without affecting the canonical structure of $m(d)$ and $m(u)$.

If $G \equiv$ E(6), the allowed Higgs representations $\underline{27}$, $\underline{351}$, and $\underline{351}'$ are all complex. Hence, none of these representations has a pairwise decomposition $\phi = \phi_d + \phi_u^\dagger + \cdots$ as in the case of $\underline{10}$ of SO(10). Consequently $m(d)$ and $m(u)$ are not correlated, that is, the same expectation values do not enter these mass matrices. To the extent that this is a desirable feature, SO(10) emerges as a preferred candidate for G. Besides, as noted in Sec. II C, E(6) has other problems even at the single-generation level. We shall relegate the general features of the preferred $\mathrm{SO(10)}\times\mathrm{U(1)_{PQ}}$ model, as well as numerical details concerning mass matrices and mixing angles, in a sequel to this paper, and conclude this section by summarizing the salient results discussed so far.

An axial $\mathrm{U(1)}_A$ horizontal symmetry leads to the canonical form of the mass matrices. The U(1) quantum numbers are completely determined. If $\mathrm{U(1)}_A$ is global, the symmetry can be identified with $\mathrm{U(1)_{PQ}}$. Further, if the accompanying single-generation grand unification group is SO(10), the opposite quantum numbers of ϕ_u and ϕ_d, the scale of $\mathrm{U(1)_{PQ}}$ breaking, and the correlation of the up and down mass matrices are automatic. The number of generations unfortunately is still arbitrary.

IV. PHENOMENOLOGICAL ASPECTS, MASS HIERARCHIES, AND "HORIZONTAL" COMPOSITENESS

From Secs. II and III, it should be evident that $\mathrm{U(1)_{PQ}}$ provides, both at the $\mathrm{SU(3)}\times\mathrm{SU(2)}\times\mathrm{U(1)}$ and at the grand unification level, constraints that restrict the forms of the mass matrices, and hence lead to phenomenological consequences. Of particular interest are the canonical forms (3.19a) and (3.19b) for $m(d)$ and $m(u)$ generated by a *single real* Higgs representation. The canonical form assures nondegenerate, nonzero eigenvalues for the masses, and the single real representation provides a simple correlation between the up- and down-sector quark mass matrices. This situation, realizable only in the $\mathrm{SO(10)}\times\mathrm{U(1)_{PQ}}$ grand unification scheme due to the existence of a real representation $\underline{10}$ that can couple to $\underline{16}$ representation of the fermions, makes it an attractive minimal scheme. But is this minimal scheme a satisfactory one? The following brief and qualitative comments are in order concerning this question:

(1) With some reasonable assumptions concerning the quark mass hierarchies (which we will discuss a bit later), the canonical structure leads to the well-known formula

$$\tan\theta_C \simeq \left[\frac{m_d}{m_s}\right]^{1/2}, \tag{4.1}$$

for the Cabibbo angle θ_C. The current quark-mass values of $m_d \sim 7.2$ MeV and $m_s \sim 150$ MeV, which are the presently accepted values for these masses, lead to a value of θ_C which is in excellent agreement with experiments. More generally, the canonical form enables one to eliminate the VEV's and the Yukawa-type couplings in favor of quark masses leading to expressions for the generalized Cabibbo-type mixing angles in the Kobayashi-Maskawa matrix. We will discuss this in more detail in a sequel to this paper. But to the extent that the mixing angles are directly related to the quark masses, the canonical forms provide a predictive framework which can be tested against phenomenological analyses.

(2) In the minimal scheme with mass matrices completely correlated, the Cabibbo-Kobayashi-Maskawa matrix U_C is real and orthogonal. Therefore, the conventional celebrated phase δ vanishes leading to no weak *CP* violation in the charged-current sector. The situation remains unaltered even if we make the Yukawa-type couplings complex. Weak *CP* violation has to be then relegated to the Higgs sector. This in itself is an attractive property of the $\mathrm{U(1)_{PQ}}$ scheme. The need for two distinguishable Higgs doublets [at the $\mathrm{SU(3)}\times\mathrm{SU(2)}\times\mathrm{U(1)}$ level] leads to flavor-changing neutral currents mediated by Higgs scalars. There is a good possibility of correlating and explaining the smallness and the superweak character of weak *CP* violation and the smallness of flavor-changing neutral currents in a more satisfactory way than currently possible. These features can be attributed to the relatively heavy masses of the mediating Higgs scalars.

(3) In the minimal scheme, there is no satisfactory explanation for the smallness of the left-handed neutrino masses.

(4) There are at most $[\frac{1}{2}(N+1)]$ arbitrary Yukawa coupling constants in the minimal version of $\mathrm{SO(10)}\times\mathrm{U(1)_{PQ}}$. This means, renormalization effects and higher-order loop corrections aside, there are $(N-1)$ mass relations between N up quarks and N down quarks, in an N-generation model. These are in addition to the obviously unsatisfactory mass relation $m(e) = m(d)$. Examination of the relations between up- and down-quark masses in the case of three generations shows that they are not realistic, some of them being violated rather badly.

The above considerations suggest that the minimal scheme of $\mathrm{SO(10)}\times\mathrm{U(1)_{PQ}}$ with only the $\underline{10}$ representation for the Higgs bosons generating the quark and lepton mass matrices is not adequate to explain the known features of the relevant masses and weak *CP* violation. It is necessary to enlarge the Higgs sector by adding $\underline{120}$ or $\underline{126}$ or both. The addition of these representations does not affect the canonical forms, but releases us from the tight correlation between the up-quark, down-quark, and lepton masses. As stated before, we shall take up these considerations in a sequel to this paper.

We shall now turn our attention to the question of mass hierarchy or the gradation of masses from the first to the third (and perhaps the Nth) generation in the canonical form. We shall speculate on how the mass hierarchy can arise. Undoubtedly, computations of any predictive value are still very far, but a number of qualitative arguments can be made.

We shall start by considering how the mass eigenvalues change when we go from n generations to $(n+1)$ generations. Consider first $n=2$, and a mass matrix of the form

$$\begin{bmatrix} 0 & B \\ B & A \end{bmatrix} \tag{4.2}$$

as an example. The eigenvalues are

$$\begin{aligned} &A, B^2/A \quad \text{for } A \gg B \, , \\ &\sim A, \sim A \quad \text{for } A \sim B \, , \\ &\sim B, \sim B \quad \text{for } A \ll B \, . \end{aligned} \tag{4.3}$$

To avoid near degeneracy of masses we have to choose $A \gg B$. This feature of the last diagonal term in the mass matrix being dominant should persist for n generations to give the desired hierarchy of eigenvalues. Notice that this term corresponds to the heaviest generation. In terms of PQ quantum numbers, this is the generation with the lowest quantum number (magnitude only). The lighter generations must carry higher PQ numbers. Starting with a basic unit x for the heaviest generation, these quantum numbers, as we noted earlier, go like $-3x$, $5x$, $-7x$, etc., with the Higgs fields carrying $2x$. As we go up in PQ quantum number towards lighter generation the terms in the mass matrix become smaller and smaller. In fact, if $B/A \simeq \epsilon$, for n generations, the terms go as $1, \epsilon, \epsilon^2 \cdots$ in units of A. Such a mass matrix is consistent with the experimentally observed hierarchy of quark masses. We shall return to a more detailed discussion of this point after introducing the idea of horizontal compositeness which, as we shall see, helps to understand in a simple way how such a hierarchy might arise.

Now the Higgs field can couple nearest-neighbor generations. The additivity of the PQ quantum number suggests that perhaps one could think of the higher, lighter generations as bound states of a fundamental generation, which is the heaviest, and the Higgs field Φ. Copies of Φ or Φ^* are sequentially added to the fundamental generation to get the increase of PQ quantum number in steps of $2x$.

The idea that the lighter generations are more composite than the heavier generation certainly seems counter to intuition but does not in any way contradict experimental information. Experimental bounds on compositeness from the anomalous magnetic moment of the electron and muon or the $\mu \to e$ transition rates only tell us that the scale of the postulated binding should be beyond TeV's.

We now turn from $\mathrm{U}(1)_{\mathrm{PQ}}$ and check whether this idea of compositeness is consistent with the group theory of grand unification. The basic diagonal Yukawa coupling term is

$$\begin{aligned} \mathscr{L}_Y = G_1[\, & \psi^T(x) C \phi^\dagger(2x) \psi(x) \\ & + \psi^\dagger(x) C \phi(2x) \psi^*(x)] \cdots \, . \end{aligned} \tag{4.4}$$

To second order this produces a term like

$$\psi^T(x) \phi^\dagger(2x) \psi^*(x) \phi(2x) \, .$$

We can identify this with a term like $\psi^T(x) C \phi(2x) \psi(-3x)$ provided $\psi^*(x)\phi^\dagger(2x)$ is bound to $\psi(-3x)$. The grand unification group theory to be checked is whether the product representation of ψ^* with $\phi^\dagger$ contains the basic fermion representation. For SO(10), $\phi = \underline{10}, \underline{120}, \underline{126}$. $\psi = \underline{16}$ and

$$\begin{aligned} &\underline{16}^* \times \underline{10} = \underline{16} + \underline{144} \, , \\ &\underline{120} = \underline{16} + \underline{144} + \underline{560} + \underline{1200} \, , \\ &\underline{126} = \underline{144} + \underline{672} + \underline{1200} \, . \end{aligned}$$

In SO(10), we can use $\underline{10}$ or $\underline{120}$ to form the bound states; $\underline{10}$ as we noted earlier is preferred on other grounds. The reality of $\underline{10}$ and $\underline{120}$ under SO(10) is crucial in forming bound states. For between the Yukawa term and the binding combination $\psi^* \phi^\dagger$ there is a conjugation of ψ. For ϕ, which are real under SO(10), the group theory is unchanged and we get a $\underline{16}$ in the final state for the same reason that the coupling $\psi^\dagger(x)\phi(2x)\psi^*(x)$ is allowed. For E(6), all the Higgs scalars are in complex representations and the compositeness picture does not work. Again for SU(5), we can form bound states but one has superfluous replication. For instance, $\psi_{10_L}(-3x)$ can be formed in two ways:

$$\epsilon^{\alpha\beta\gamma\delta\theta} (\psi_{\overline{10}_R})_{\gamma\delta} (\phi_d^\dagger)_\theta \quad \text{or} \quad \psi_{5R}^\alpha \phi_u^\beta - \psi_{5R}^\beta \phi_u^\alpha \, .$$

There are two generations with the same PQ number. Thus, to avoid superfluous replication we should choose SO(10) as the grand unification group. Even in the SO(10) scheme we cannot allow both $\underline{10}$ and $\underline{120}$ in the Higgs sector since this would lead to a doubling of all generations due to binding of $\underline{10}$ and $\underline{120}$. In the decomposition of the product representation, there are higher-dimensional representations. We have to rule out these either on the general principle that lower-dimensional representations tend to be lower in energy or on some principle similar to the maximal-attractive-channel criterion used in hypercolor-type theories. It is anyway a detailed question of dynamics.

Thus, the idea of horizontal compositeness along with the other principles discussed earlier leads to a unique picture of grand unification, viz., an $\mathrm{SO}(10) \times \mathrm{U}(1)_{\mathrm{PQ}}$ model with the Yukawa-type Higgs fields being in $\underline{10}$ and $\underline{126}$.

To go beyond these qualitative results, one needs more detailed information about the dynamics. While this is admittedly very difficult, we shall make some estimates for the various terms in the mass matrix which will help to understand the starting question to this discussion, viz., the hierarchy of masses. The general higher-order term which can contribute to the mass matrix is of the form

$$\frac{1}{\Lambda^{\Delta-1}} \frac{\psi\phi^{p-1}}{\Lambda^{p-1}} \frac{\psi\phi^{q-1}}{\Lambda^{q-1}} \phi^\Delta \, , \tag{4.5}$$

where Λ is a mass scale introduced on dimensional grounds. Interpreting $\psi\phi^{p-1}/\Lambda^{p-1}$ as the bound-state fermion corresponding to the pth generation ($p=1$ is the heaviest generation), this term gives a contribution to the (p,q) element of the mass matrix. Λ is fixed in terms of p,q by the $U(1)_{PQ}$ symmetry. ψ^p and ψ^q give a PQ charge x_p+x_q at the vertex; since each scalar can cancel only $\pm 2x$, the minimum value of Δ is

$$\Delta(p,q)=\left|\frac{x_p+x_q}{2x}\right| = \begin{cases} p+q-1\ , & p+q \text{ even}\ ,\\ p-q\ , & p+q \text{ odd}\ . \end{cases} \tag{4.6}$$

The tree-level mass terms are given by $\Delta=0$. The formation of a bound state has a mass scale Λ associated with it. Introducing a normalization factor ϵ for the composite operator representing the bound state, viz., $\psi\phi/\Lambda\sim\epsilon\psi(-3x)$, $\psi\phi^2/\Lambda^2\sim\epsilon^2\psi(5x)$, etc., we can easily see that the tree-level mass matrix is of the form

$$\begin{bmatrix} \cdot & & \epsilon^2 & \\ & \cdot & & \epsilon \\ \epsilon^2 & & \cdot & \\ & \epsilon & & 1 \end{bmatrix} \tag{4.7}$$

up to an overall normalization. Phenomenologically, $\epsilon=|m_{12}/m_{11}|$. Let us include $\Delta\neq 0$ terms. Replacing the ϕ's by their VEV's, we get a factor $\eta\Lambda$ for each ϕ, where η is a combination of coupling constants and other normalization factors. We should have $\eta\ll\epsilon<1$, otherwise perturbation theory would break down. The (p,q) element of the mass matrix then looks like $\sim\epsilon^{p+q-2}\eta^{\Lambda-1}$, i.e.,

$$m\simeq\begin{bmatrix} \cdot & \cdots & & & & \cdot \\ \vdots & & & & & \vdots \\ \cdot & & & & \cdot & \epsilon^3\eta^2 \\ \cdot & & & \cdot & \epsilon^2 & \epsilon^2\eta^2 \\ \cdot & & \cdot & \epsilon^2 & \epsilon^2\eta^2 & \epsilon \\ \cdot & \cdot & \epsilon^3\eta^2 & \epsilon^2\eta^2 & \epsilon & 1 \end{bmatrix}\ . \tag{4.8}$$

Each term m_{pq} can have further corrections $\sim\eta^2 m_{pq}$.

Going back to the mass matrix (4.7) at the tree level we can extract the rough pattern of eigenvalues. One can construct symmetric polynomials of the eigenvalues as

$$(\lambda_1+\lambda_2+\cdots)=\mathrm{tr}\mathscr{M}=1\ ,$$

$$(\lambda_1\lambda_2+\lambda_2\lambda_3+\cdots)=\tfrac{1}{2}(\mathrm{tr}\mathscr{M})^2-\mathrm{tr}\mathscr{M}^2] =\epsilon^2+G(\epsilon^4)\ ,$$

etc. Ignoring interference effects we then get

$$\lambda_k\sim\epsilon^{2(k-1)}\ .$$

Thus, log$m$ is a linear function of the generation index. We may note here that Bjorken[18] was the first to suggest the use of log$m$ as a smooth function of k; the tree-level pattern (4.7) has also been discussed by Fritzsch.[17]

V. CONCLUDING REMARKS

The "superfluous" replication of families of particles continues to plague grand unification schemes. Various attempts to incorporate this feature, predict the number of generations, and derive restrictions on the form of mass matrices have all had limited successes so far. If we consider quarks and leptons as fundamental constituents all the way to and beyond the grand unification mass scale, there does not appear to be any simple way to incorporate the generation structure.

In this paper we give up the idea of being able to predict the number of generations. Instead, given the number of generations, we show how a global, axial U(1) symmetry will severely restrict the forms of quark mass matrices leading to testable predictions. The idea becomes even more attractive when it is realized that the assumed U(1) symmetry can be the same as the celebrated $U(1)_{PQ}$ symmetry through which one can avoid the strong *CP*-violation problem. Further, during the course of determining the mass matrices with certain desirable properties, we also determine the $U(1)_{PQ}$ assignments which is suggestive of a new interpretation of the generation puzzle. It is sufficient to begin with one family of fermions and Higgs scalars. The other families can be regarded as composites of fermions and scalars. The quantum numbers and the mass hierarchy indicate that the lighter the generation, the more composite it is. Of course, one needs dynamics to make this idea concrete, but qualitatively it appears to be a very attractive and novel way to understand the generation puzzle.

We have shown, mostly from qualitative considerations, that the most attractive possibility for a multigenerational grand unified scheme is $SO(10)\times U(1)_{PQ}$. In a sequel we will examine such a model in detail and study its consequences.

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