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QCD Vacuum as a Chromomagnetic Superconductor: Microscopic Physics

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The compactness of the QCD gauge group leads to the existence of chromomagnetic monopole configurations. The interactions of these monopoles are studied using electric-magnetic duality as a guide. It is shown that the gluon-monopole interaction acquires an extra minus sign which tends to destabilize the monopole configurations. A model is presented which exploits this observation. We present a scenario where the monopoles condense, giving chromomagnetic superconductivity, via a first-order phase transition at a scale where the electric and magnetic charges are equal, i.e., $\alpha=\frac{1}{2}$. In this vacuum medium gluons propagate like $1/k^4$ which is now seen to be the dual equivalent of the statement that the vacuum is a chromomagnetic superconductor.

I. INTRODUCTION

Theoretical prejudices and some supportive experimental facts have made quantum chromodynamics (QCD) the favored theory of strong interactions. The most important feature of strong interactions is the confinement of quarks and gluons. Despite universal belief and experimental confirmation, confinement defies proof as a consequence of the principles of continuum QCD.

It has long been recognized that the true vacuum state of QCD cannot be the naive vacuum accessible to and probed by perturbation theory. The vacuum state must incorporate complicated nonperturbative effects. It is becoming evident that a proper treatment of these effects will come about by understanding the vacuum as a special QCD medium with nontrivial properties. A quark-antiquark pair embedded in such a medium is not easily described as interacting via gluon exchange. A better description involves strings or bags. The detailed properties of the medium determine the appropriate macroscopic constructs which are useful. Ideally the vacuum medium can be given a microscopic description based on the QCD Lagrangian. A phenomenologically useful macroscopic picture can then be abstracted.

The development of our current understanding of superconductivity provides a hopeful parallel for QCD. The crucial theoretical task in superconductivity was finding the proper ground state. This was achieved with the development of the Bardeen-Cooper-Schrieffer (BCS) theory.[1] Before this, however, significant insight was acheived via macroscopic models such as London's equations and the Ginzburg-Landau theory.[1]

Superconductivity is useful not just as a historical guide but it also provides a vivid model for the actual confinement mechanism.[2–4] A superconductor shows zero resistance to electric currents but more spectacular is its ability to exclude magnetic fields, viz., the Meissner effect. The relevance for confinement is seen if we perform the *Gedankenexperiment* of attempting to place a magnetic monopole inside the superconductor. Meissner effects forbid this. A monopole-antimonopole pair can, however, exist in a superconductor. In a type-II superconductor the mediating magnetic flux would be squeezed into a thin flux tube or vortex. The $m\bar{m}$ pair will be bound by a linear potential. In a type-I superconductor the pair will be squeezed into a small volume (as small as is allowed by their kinetic energy). Inside this volume superconductivity is destroyed and there is local restoration of normal phase. The $m\bar{m}$ exist in a bubble or bag.

The correspondence with the chromodynamic situation of string and bag models for hadrons and confining potentials is obvious. There are two significant distinctions. In QCD, particles carrying color electric charges as opposed to magnetic charges are confined. Second, the color fields and fluxes are non-Abelian in contrast to the case of ordinary, Abelian superconductivity. The appropriate analogy would thus be a chromomagnetic superconductor with chromoelectric Meissner effect and confinement of color electric charges, a dual version of ordinary superconductivity. A state of magnetic superconductivity would be produced by the condensation of magnetic monopoles. In ordinary superconductivity the attractive phonon-mediated electron-electron interaction is crucial to the formation of Cooper pairs and their condensation. For chromomagnetic monopoles, the non-Abelian nature of the forces should be the crucial ingredient for condensation.

In this paper we elaborate on the analogy with superconductivity. We feel that the description of the QCD vacuum as a chromomagnetic superconducting medium is compelling and fruitful, and will ultimately prove to be true. Its derivation from first principles is far from complete. Nevertheless we organize this paper in a logical mode starting from a microscopic picture. We offer what we feel is a plausible and attractive scenario for the emergence of the confining mechanism from first principles. Our model highlights some novel aspects of the dynamics that seem to play key roles in confinement. In a sequel we shall discuss the macroscopic consequences of the chromomagnetic superconductor.

Confinement naturally involves three levels of description: topological questions, microscopic dynamics, and macroscopic dynamics. Since the gauge group SU(3) is compact QCD naturally has chromomagnetic monopoles. The description of these configurations and their importance are considered in Sec. II. In Sec. III we consider the interactions of monopoles based on electromagnetic duality. We concern ourselves with an Abelian theory containing both electric and magnetic charges. Even though we assume a linear description we find that we are able to uncover much of the physics of confinement. Electromagnetic duality is vital for this discussion. In a magnetic superconductor it is shown that the photon (or gluon) propagator goes like $1/k^4$ for small k. Such propagation is equivalent to superconductivity and hence $1/k^4$ propagation implies confinement via a Meissner effect. Duality of Higgs and confinement phases also emerges. In Sec. IV we show how the chromomagnetic monopoles condense to give a state of magnetic superconductivity. Some relevant topological and group-theoretical questions are discussed. Section V gives an explicit calculation showing the condensation of monopoles. QCD is considered as a system of chromomagnetic monopoles, charged gluons, and "photons" (Abelian gauge fields) with respect to a fixed but arbitrary Abelian projection. The charged-gluon—monopole interaction tends to cause, via radiative corrections, the monopole condensation. The dynamics is similar to that of the Abelian-Higgs model which is well known to be the relativistic generalization of the Ginzburg-Landau theory of superconductivity.[2] In QCD superconductivity emerges via a first-order phase transition at a scale where the fundamental electric charge is equal to the magnetic charge ($\alpha_s=\frac{1}{2}$). Since this value of α is so close to the perturbative regime it provides an understanding of precocious scaling.

II. CHROMOMAGNETIC MONOPOLES

Most QCD calculations rely on quarks and gluons as the fundamental variables. While this is justified for perturbative calculations, in the nonperturbative regimes close to confinement, where the topological aspects of the theory are important, the proper choice of variables is a more subtle matter.[5,6] Topologically the most important difference between a non-Abelian gauge theory and a set of Abelian (QED type) gauge fields is the compactness of the non-Abelian gauge group. Thus, in QCD, because SU(3) is compact, the color electric charges defined with respect to any maximal Abelian subgroup are quantized. This in turn implies that we can write down gauge field configurations that asymptotically look like magnetic monopoles of any chosen Abelian direction, i.e., there are configurations that look like

$$F_{ij}^a=\frac{\epsilon_{ijk}\hat{x}_k}{r^2}G^a\ , \tag{2.1}$$

where G^a are the magnetic charges.[7] Charge quantization assures that we can consistently satisfy the Dirac-type quantization rule

$$\exp(4\pi ieG)=1\ . \tag{2.2}$$

The possible values of G^a are given by the vectors of the lattice reciprocal to the weight lattice of the gauge group, i.e., the lattice characteristic of the dual group of the gauge group in the sense of Goddard, Nuyts, and Olive.[7] However, in contrast to Ref. 7, we do not have a real breakdown of SU(3) to any subgroup H. Thus we need the group dual to the full gauge group. For QCD without fermions the gauge group is strictly $SU(3)/Z_3$. The dual group is SU(3). Nonsingular monopoles correspond to closed curves in the Abelian subgroups (or projection) which are homotopically trivial in the full group, namely, SU(3). These are octets.

Monopole configurations are of the type (2.1) only at large distances. As we move in from infinity to within a characteristic radius r_0, the field strength can be taken to decrease, becoming zero (say) at $r=0$. The singularity of (2.1) at $r=0$ can be avoided. We thus have monopole configurations of finite energy. Of course such configurations will not be solutions of the equations of motion. This is not necessary for them to influence the dynamics.

Monopoles are the way by which the global aspects of the gauge group manifest themselves at the level of particle interactions. This is their great role in non-Abelian gauge theories. Indeed one can argue that topologically a non-Abelian gauge theory is equivalent to a set of Abelian gauge theories supplemented by monopoles.[5]

Since they are not solutions of the static equations of motion, these monopole configurations can expand, contract, or disappear. They can be made explicit as reasonably localized particle states by, for example, gauge-fixing conditions. 't Hooft has shown how these monopole configurations can be constructed as 't Hooft-Polyakov-type monopoles associated with the singularities of the gauge-fixing condition.[5] The gauge fixing is done in terms of a Higgs-type scalar field which may be a composite operator constructed from the gauge fields, e.g., $\Phi=F_{\mu\nu}F_{\mu\nu}$ considered as a matrix in SU(3) indices. The condition $\Phi(x)=\Phi_0(x)$, where Φ_0 is a specific chosen function (which is also a 3×3 matrix), can be used as a gauge-fixing condition. If Φ transforms as the adjoint representation, which happens if Φ is the composite $F_{\mu\nu}F_{\mu\nu}$, the choice $\Phi=\Phi_0$ fixes the gauge for all gluons except those corresponding to the maximal Abelian subgroup $U(1)\times U(1)$. In fact $\Phi=\Phi_0(x)$ mimics the breakdown $SU(3)\to U(1)\times U(1)$. The monopoles corresponding to the two U(1) directions can then be constructed as 't Hooft-Polyakov-type monopoles, Φ playing the role of the Higgs field.

We shall consider a slightly different but related approach. Our point of view is that since the monopole configurations are implicit in the theory what is required is a regularization scheme which will make these manifest. We claim that as far as the infrared properties of the theory are concerned such a (nonperturbative) regularization is provided by the addition of a scalar field Φ transforming as the adjoint representation of the gauge group. The field Φ is given a nonvanishing vacuum expectation value of magnitude v. This gives spontaneous breakdown of the SU(3) symmetry down to $U(1)\times U(1)$.

The spectrum now consists of massive gluons (of mass $M \simeq ev$, e being the gauge coupling), which are charged with respect to the two U(1) directions, two massless gluons, two massive neutral components of Φ (of mass $\sim v$) and stable monopoles of mass $\sim M/e^2$ and radius $\sim 1/M$. We now assume that e gets large as we move to lower energies and look at the low-energy behavior of the theory ($q^2 < M^2$). As we move to low energies, the effective e increases. The monopoles become less and less massive and at these energies they are essentially pointlike. Their contribution to low-energy physics becomes more and more important. Eventually the monopoles undergo condensation. This is the confinement phase. The scalar fields (some of which are the longitudinal components of the massive gluons) have all decoupled by now. Thus it is clear that Φ only played the role of a regulator.

A necessary condition for e to increase for decreasing q^2 is that the theory be asymptotically free. The addition of the Φ field will not destroy the asymptotic freedom of QCD and so we expect e^2, at least initially, to increase as q^2 decreases. If the β function stays sufficiently negative (as is usually assumed in discussions of confinement) e can become sufficiently large to justify the picture discussed above.

The picture presented above is certainly not a mathematically well-defined regularization. Nevertheless we feel that it does provide, in broad outline, a heuristic formulation of how to introduce and manipulate the all important monopole configurations.

Let us also consider the case where the addition of the scalar field destroys asymptotic freedom. (This does not happen for QCD.) In this case e becomes small as we move to low energies. The monopoles become very heavy and decouple. We have genuine spontaneous breakdown of symmetry or Higgs phase. It should be emphasized that a Higgs phase is possible only if there are enough matter fields as to render the theory not asymptotically free. Otherwise, the theory would be in a confinement phase.

To recapitulate briefly: There are monopole field configurations in any non-Abelian gauge theory. These are of importance in determining the nonperturbative dynamics especially the phase structure of the theory. To probe the phase structure of the theory we can add a scalar field in the adjoint representation so long as this does not change the nature of the flow of the coupling constant with energy. For asymptotically free theories the low-energy behavior is dominated by monopoles of almost zero mass which are almost pointlike.

The latter point is of significance since the interactions of pointlike monopoles with gluons and charged particles can be studied as a dual analog of point charged-particle interactions. Over the next few sections we shall use this to show that the monopole vacuum becomes nontrivial and that the monopoles undergo condensation.

III. ELECTROMAGNETIC DUALITY AND MONOPOLE INTERACTIONS

In this section we deduce some of the properties of magnetic-monopole interactions treating monopoles as the electromagnetic dual of charged particles.[8] Most of the discussion can be done in terms of Abelian gauge fields (referred to as electromagnetism or the photon). We first consider the effective action in a translationally invariant source-free medium,

$$S = -\tfrac{1}{4}\int F_{\mu\nu}(x)\epsilon(x-y)F_{\mu\nu}(y)d^4x\, d^4y \ ,$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ . \tag{3.1}$$

$\epsilon(x-y)$ is the dielectric constant. For a free theory or in the lowest order of perturbation theory $\epsilon(x-y)=\delta(x-y)$. The complicated microscopic interactions involving charged particles, monopoles, etc., are summed up in the functional form of $\epsilon(x-y)$. Of course, the action (3.1) is not the most general effective action possible; however higher nonlinear terms are not relevant for our limited purpose.

Although the assumed, quasilinear, form (3.1) may seem too simple and naive to account for the complicated and inherently non-Abelian phenomena of confinement we shall see that it can, in fact, describe the essentials of confinement. The reason for this is that we will allow the dynamics to simultaneously incorporate electric *and* magnetic charges. Non-Abelian confinement of electric charge is related to the linear, Abelian theory of *electric* charge confinement in a *magnetic* superconductor. *A posteriori* we will then be justified in using an effective Lagrangian such as (3.1) for modeling confinement in QCD.

The action (3.1) leads to the equations of motion

$$\partial_\mu \tilde{G}_{\mu\nu} = 0 \ , \tag{3.2a}$$

$$\partial_\mu \tilde{F}_{\mu\nu} = 0 \ , \tag{3.2b}$$

where

$$\tilde{G}_{\mu\nu} = \int \epsilon(x-y)F_{\mu\nu}(y)d^4y \ , \tag{3.3}$$

$$\tilde{F}_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta} \ . \tag{3.4}$$

We can equally well describe the physics of Eqs. (3.2) in terms of the magnetic potential B_μ dual to A_μ with

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \ . \tag{3.5}$$

The action (3.1) can be rewritten in terms of dual variables as

$$S = -\tfrac{1}{4}\int G_{\mu\nu}(x)\mu(x-y)G_{\mu\nu}(y)d^4x\, d^4y \tag{3.6}$$

with μ, the magnetic permeability, defined by

$$\int \epsilon(x-y)\mu(y-z)d^4y = \delta(x-z) \ . \tag{3.7}$$

The corresponding equations of motion are

$$\partial_\mu \tilde{G}_{\mu\nu} = 0 \ , \tag{3.8a}$$

$$\partial_\mu \tilde{F}_{\mu\nu} = 0 \ , \tag{3.8b}$$

with

$$\tilde{F}_{\mu\nu} = \int \mu(x-y)G_{\mu\nu}(y)d^4y \ , \tag{3.9}$$

$$\tilde{G}_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}G_{\alpha\beta} \ . \tag{3.10}$$

In version (3.1), (3.2b) is the Bianchi identity, while in version (3.6), (3.8a) is the Bianchi identity. Electromagnetic duality is the operation $G_{\mu\nu} \leftrightarrow F_{\mu\nu}$.

In the presence of sources, the equations of motion are modified. An electric source (charged particles) is best treated by adding $J_\mu^E A_\mu$ to (3.1) which, upon variation with respect to A_μ, gives

$$\partial_\mu \widetilde{G}_{\mu\nu} = J_\nu^E, \quad \partial_\mu \widetilde{F}_{\mu\nu} = 0 \; . \tag{3.11}$$

Magnetic sources (monopoles) are best treated by adding $J_\mu^M B_\mu$ to (3.6) giving

$$\partial_\mu \widetilde{G}_{\mu\nu} = 0, \quad \partial_\mu \widetilde{F}_{\mu\nu} = J_\nu^M \; . \tag{3.12}$$

The theory can be formulated in terms of A_μ or B_μ. Which formulation is easier depends on the type of sources present.

Electric currents are those which couple to A_μ. The electric current correlations are thus given by

$$\langle J_\mu^E \rangle = \frac{\delta\Gamma}{\delta A_\mu} \; , \qquad \langle J_\mu^E(x) J_\nu^E(y) \rangle = \frac{\delta^2\Gamma}{\delta A_\nu(y)\delta A_\mu(x)} \; . \tag{3.13}$$

Likewise the magnetic current correlations are given by

$$\langle J_\mu^M \rangle = \frac{\delta\Gamma}{\delta B_\mu} \; , \qquad \langle J_\mu^M(x) J_\nu^M(y) \rangle = \frac{\delta^2\Gamma}{\delta B_\nu(y)\delta B_\mu(x)} \; . \tag{3.14}$$

Γ is the effective action of the theory equal to S in our case. Using these and Eqs. (3.1) and (3.6) we get *for the same medium*

$$\langle J_\mu^E(x) J_\nu^E(y) \rangle = -\int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} (k^2\delta_{\mu\nu} - k_\mu k_\nu)\epsilon(k^2) \; , \tag{3.15}$$

$$\langle J_\mu^M(x) J_\nu^M(y) \rangle = -\int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} (k^2\delta_{\mu\nu} - k_\mu k_\nu)\mu(k^2) \; , \tag{3.16}$$

where $\epsilon(k^2)$ is the Fourier transform of $\epsilon(x-y)$ and $\epsilon\mu = 1$.

As we remarked earlier $\epsilon(k^2) = 1 = \mu(k^2)$ is free electromagnetism in the vacuum. In perturbation theory, the deviation of ϵ and μ from 1 can be interpreted as the vacuum polarization due to charged-particle or monopole loops. We then have, for perturbatively small $\chi(k^2)$ [for monopole loops we write $\chi'(k^2)$],

$$\epsilon(k^2) = 1 + \chi(k^2) \Longleftrightarrow \mu(k^2) \simeq 1 - \chi(k^2) \; , \tag{3.17}$$

$$\epsilon(k^2) \simeq 1 - \chi'(k^2) \Longleftrightarrow \mu(k^2) = 1 + \chi'(k^2) \; . \tag{3.18}$$

From these equations we see that charged particles $[\chi(k^2) > 0]$ produce screening effects for the A_μ propagator ($\epsilon > 1$) while they produce antiscreening for the B_μ propagator. Likewise monopole loops produce screening for $B_\mu (\mu > 1)$ and antiscreening for A_μ, i.e., any particle screens its own direct potential (to which it minimally couples) and antiscreens the dual potential.

This antiscreening effect is of great significance. It answers several questions about gauge theories.[8] In Sec. II, essentially following 't Hooft, we have reduced the non-Abelian gauge theory (QCD) to an Abelian theory with monopoles. A natural question would then be: what happens to the asymptotic freedom of the non-Abelian theory in the Abelian version? The antiscreening effect shows that monopoles can maintain asymptotic freedom.

In ordinary electrodynamics, charged particles lead to the Landau singularity. Since monopoles contribute with the opposite sign to the A_μ propagator, the addition of monopoles to standard QED should quench the singularity to some extent. Because of this role of monopoles, they should be the dominant component in determining the physics near the Landau singularity; the best description in such energy regimes is provided by the dual variables, viz., B_μ and monopoles. In QCD, because of asymptotic freedom, the Landau singularity is in the infrared regime. Thus the most convenient microscopic description of low energy QCD is provided by the chromomagnetic monopoles, reinforcing our arguments of the last section about the significance of monopoles in QCD.

To lowest order it is clear that the antiscreening effect is equivalent to the following prescription:[8] the magnetic-photon (B_μ)—charged-particle vertex is identical to the A_μ—charged-particle vertex with the coupling constant e replaced by ie. We emphasize that this prescription of the coupling of a gauge particle to its dual charge must be used only when all the dual charges appear in loops. A simplified effective theory can arise only when all *external* particles are of the same charge variety (i.e., all electric or all magnetic quantities). Likewise the A_μ-monopole vertex is obtained by replacing the coupling g ($=2\pi n/e$) by ig in the B_μ-monopole vertex. We shall now find further support for this prescription to all orders in perturbation theory. We shall ignore contact interactions of charged particles such as arising from $\lambda(\phi^*\phi)^2$-type interactions and concentrate on minimal gauge couplings. The Lagrangian is thus typically

$$\mathscr{L} = |(\partial - ieA)\phi|^2 + m^2\phi^*\phi \; , \tag{3.19}$$

where ϕ is the field of the *charged* particle. The effect of ϕ's is contained in one-loop graphs with photons as external lines. Higher-ϕ-loop graphs are obtained by composing such one-ϕ-loop graphs by photon contractions. Thus it is sufficient to consider one-ϕ-loop graphs. The one-ϕ-loop graphs together give the effective action for A, say $\Gamma(A)_\phi$. $\Gamma(A)_\phi$ being a Lorentz scalar can be written as a series in powers of $F_{\mu\nu}F_{\mu\nu}$ and $F_{\mu\nu}\widetilde{F}_{\mu\nu}$:

$$\Gamma(A)_\phi = \sum_{nm} C_{nm} e^{2n+2m} (F_{\mu\nu}F_{\mu\nu})^n (F_{\alpha\beta}\widetilde{F}_{\alpha\beta})^m \; . \tag{3.20}$$

We think of this as being written in momentum space with C's being functions of momenta so that derivatives on $F_{\mu\nu}$ do not have to be explicitly written. Let us now do a duality transformation on (3.20): $\vec{E} \to \vec{B}, \vec{B} \to -\vec{E}$, $e \to g$. Under this both $F_{\mu\nu}F_{\mu\nu}$ and $F_{\mu\nu}\widetilde{F}_{\mu\nu}$ change sign. The charged particle circulating in the loop is replaced by a monopole. Thus for the monopole contribution we get

$$\Gamma(A)_{\text{monopole}} = \sum_{n,m} C_{nm} g^{2(n+m)}(-1)^{n+m}(F_{\mu\nu}F_{\mu\nu})^n(F_{\alpha\beta}\widetilde{F}_{\alpha\beta})^m . \tag{3.21}$$

We see that the monopole-A_μ contribution is given by $e^2 \to -g^2$ in the charged-particle—A_μ vertices. A similar argument shows magnetic photons (B_μ) couple to charges with $-e^2$. Contact interactions of the monopole field as well as nonminimal couplings are not given by this prescription. In the presence of such interactions the monopole is no longer the exact dual of a charged particle. If in a theory all the monopole interactions are given by our prescription, we shall say that it is saturated by duality. Whether or not QCD is saturated by duality is a separate question to which we return in a couple of paragraphs.

It should be mentioned that the $e \to ie$ prescription is implicitly contained in some previous discussions of monopole interactions.[9] So long as we are interested only in monopole loops, we are always in the zero-monopole-charge sector and an electrodynamics can be constructed without the necessity of a fixed spacelike vector n_μ which appears in earlier formulations of electrodynamics with monopoles.[10] Such a formulation gives results in agreement with our prescription.

We will eventually be concerned with diagrams with monopoles attached to charged-gluon loops [Fig. 1(b)]. Such couplings can be determined by first considering the coupling of a magnetic photon B_μ to the charged-gluon loop [Fig. 1(a)]. The arguments presented above specify a coupling constant of $-e^2$. By dual gauge invariance (i.e., gauge invariance in the magnetic sector) this coupling is the same for magnetically charged partners of B_μ. Once we have established how gluon loops couple to magnetically charged, electrically neutral particles gauge invariance fixes this coupling for all other magnetically charged, electrically neutral particles. The desired coupling [Fig. 1(b)] is $-e^2$.

We have not yet exhausted the usefulness of Eqs. (3.15) and (3.16). Let us apply them to the case of superconductivity. ϵ and μ in this case include fully nonperturbative effects; they are no longer nearly 1. Simplified discussions of superconductivity assume $\mu=0$. This rigidly excludes magnetic fields from inside the superconductor in conformity with the strict Meissner effect. But we know that magnetic fields can penetrate into a superconductor up to the London penetration depth λ_L. The proper statement is that, for small k^2,

$$\mu = k^2/m_L{}^2, \quad m_L = \frac{1}{\lambda_L} \tag{3.22}$$

and so

$$\epsilon = \frac{m_L{}^2}{k^2} . \tag{3.23}$$

Equation (3.15) now gives

$$\langle J_\mu^E(x) J_\nu^E(y) \rangle = -\int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right] m_L{}^2 . \tag{3.24}$$

The photon (A_μ) thus acquires a mass m_L (through summation of bubbles). This is in agreement with the fact that superconductivity is the Higgs phase of electrodynamics.

Consider now the dual variable B_μ. Equation (3.22) gives for the effective action (3.6)

$$S = \tfrac{1}{4} \int G_{\mu\nu} \frac{\Box}{m_L{}^2} G_{\mu\nu} d^4x , \tag{3.25}$$

showing that B_μ has a propagator that goes like $1/k^4$ (Ref. 8). Thus we have

electric superconductivity $\Longleftrightarrow$ { massive photon (A_μ); $\frac{1}{k^4}$ propagator for dual potential B_μ .

The $1/k^4$ behavior of the potential B_μ shows that a monopole-antimonopole pair in an electric superconductor would be bound and confined by a linear potential. (The description of such an effect would still be highly nonperturbative as the spread-out linear potential is squeezed into flux tubes by the Higgs field.) We thus see that the Higgs phase in terms of A_μ and charged particles is the confinement phase in the dual description.

In a magnetic superconductor we have $\epsilon = k^2/m_L{}^2$. The B_μ field becomes massive while A_μ has a propagator that goes like $1/k^4$ (Ref. 11). Electrically charged particle pairs would then be confined by a linear potential. Again we have the duality of Higgs phase and confinement.

The duality of Higgs phase and confinement suggests that duality should be a strong guide to the description of confinement, i.e., at least for low k^2, the interactions of (chromomagnetic) monopoles can be saturated by duality.

We conclude this section by summarizing the salient points of our analysis. "Mixed mode" interactions (i.e., charged particle with B_μ or monopole with A_μ) are obtained from the corresponding direct interactions by multiplying the coupling constant by a factor of i. The in-

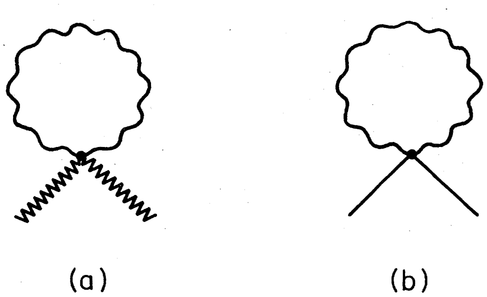

FIG. 1. Wavy lines are gluons, saw-toothed lines are magnetic photons (B_μ), and solid lines are magnetic monopoles. Thus in (a) we have the gluon-loop correction to the B propagator. This is evaluated via duality arguments in Sec. III. (b) is the gluon-loop correction to the monopole propagator, related to (a) by gauge symmetry. The coupling in both diagrams is $-e^2$.

teractions of chromomagnetic monopoles are dominated by those dictated by duality at low k^2.

IV. CONDENSATION OF MONOPOLES AND CONFINEMENT

By now we have all the basic physical principles to understand the confinement mechanism in QCD. In this section, we discuss some of the specifics of this mechanism.

To begin, we have monopole-field configurations in QCD (or any compact non-Abelian gauge theory). The existence of these configurations depends on a key topological fact, compactness of the gauge group. To facilitate the discussion of these configurations we stabilize them by the addition of a scalar field in the adjoint representation of the gauge group. This is not strictly necessary, but it allows us to begin a definite discussion. This field acts as a regulator for the theory. At this stage, the theory has two massless gluons [corresponding to the two U(1) directions] denoted A_μ, six charged massive gluons W_μ, and monopoles. Since the interaction of monopoles is more easily discussed in terms of the dual formalism, we use the dual potential B_μ. We have monopoles coupled minimally to the massless B_μ and electrically charged particles W_μ when these later particles appear in loops. As we argued in the last section it is reasonable for low-energy QCD to saturate monopole interactions by duality. The monopoles can condense if their mass squared is driven negative by radiative corrections. Now, because of minimal coupling, the radiative corrections due to B_μ give a positive renormalization to m^2 of the monopole. The question then is what do the W_μ do? Notice that although we reduced the original SU(3) gauge theory to two U(1) gauge theories, the choice of the two U(1) directions is arbitrary. In other words, there is still an SU(3) symmetry in the theory which connects the W_μ's and A_μ's and dictates the coupling of W_μ to monopoles. [Once we have made the Abelian projection, this SU(3) symmetry is not unitarily realizable on the states but is retained as a symmetry of the operator equations of motion.] The coupling of A_μ to monopoles contains an extra factor of i as given by duality. The W_μ-monopole coupling also carries this factor of i. Thus radiative corrections due to the W_μ's give a negative contribution to the m^2 of the monopoles. The dominance of this negative contribution happens at low energies (as we show in a model calculation in the next section), leading to the condensation of monopoles. Thus we get a state of magnetic superconductivity and confinement.[12] The state of magnetic superconductivity is a Higgs phase in terms of dual variables and there are Abrikosov-Nielsen-Olesen-type vortices or flux tubes[13] which bind the electrically charged particle pairs.

The non-Abelian nature of the gauge group is quite crucial to this mechanism. A compact Abelian gauge symmetry [U(1): standard electrodynamics with monopoles] already gives us the necessary topological element, viz., existence of monopole-field configurations. However, this may not yield condensation of monopoles. In the non-Abelian case we further have charged gluons W_μ whose interactions are determined by symmetry and which then lead to instability of the monopoles potential and their condensation.

There are also indirect signals for the condensation of monopoles. Several authors have argued that with a suitable truncation of the Schwinger-Dyson equation of QCD the gluon propagator goes like $1/k^4$ (Ref. 11). As we showed in the last section, this is equivalent to the condensation of monopoles and a resultant state of chromomagnetic superconductivity.[12]

Since the monopoles are octets the relevant symmetry group for monopole states in $SU(3)/Z_3$ which is triply connected, i.e., $\Pi_1\,[SU(3)/Z_3]=Z_3$. Thus a (Higgs-type) complete breakdown of this symmetry leads to vortex solutions. These are the vortices of quantized chromoelectric flux which bind the quarks into color-singlet states.[13] (Three of them can annihilate into topologically trivial objects, e.g., gluons.)

V. CONDENSATION OF MONOPOLES: A MODEL CALCULATION

In this section we give the promised model calculation to show the emergence of a nontrivial magnetic vacuum. For the sake of simplicity we consider an SU(2) gauge theory. Going to the Abelian projection, the relevant degrees of freedom are two massive gluons $W_\mu^\pm$, a U(1) gluon, and a magnetic monopole which we take to be a scalar (represented by the complex scalar ϕ). The U(1) gauge boson is represented by the dual potential B_μ. The Lagrangian for B_μ and the monopole is just standard scalar electrodynamics,

$$\begin{aligned} \mathscr{L} &= -\tfrac{1}{4}H_{\mu\nu}{}^2-\tfrac{1}{2}\,|D_\mu\phi\,|^2\,, \\ H_{\mu\nu} &= \partial_\mu B_\nu-\partial_\nu B_\mu\,, \\ D_\mu &= \partial_\mu - igB_\mu\,. \end{aligned} \tag{5.1}$$

g is the magnetic coupling constant, essentially the reciprocal of e, the gauge coupling constant. As we argued in Sec. II, the monopole is essentially massless in the energy regime of interest.

Including a mass term proportional to M^2/e^4 will change our quantitative results, but will not affect our qualitative conclusions. Further, in accordance with the discussion in Sec. III, the monopole interactions are saturated by duality and so no self-interactions of the monopole field are included. As is well known superconductivity, and hence confinement, can be achieved in scalar electrodynamics if a Higgs-type mechanism arises.[1,2] It will be our goal to show that this happens for the monopoles and magnetic photons.

For our purposes (5.1) is only an effective theory valid for energies less than some scale typical of the monopole size or equivalently the mass M of the massive gluons $W_\mu^\pm$. As the discussion of Sec. II shows, at higher energies the Abelian projection does not give the most convenient choice of variables. Perturbation theory of massless (electric) gluons must be used. The Lagrangian (5.1) thus carries with it a physical cutoff $\simeq M$. To this Lagrangian we must also add the "mixed mode" interactions, viz., coupling of $W_\mu^\pm$ to B_μ and ϕ. The basic vertices which are relevant to mass renormalization are

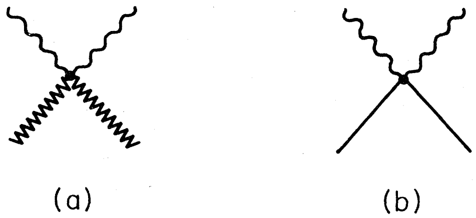

FIG. 2. The basic interaction vertices: (a) gluon—magnetic-photon vertex and (b) gluon-monopole vertex.

shown in Fig. 2. (We emphasize that these vertices will be used only when one line is closed in a loop. We will never consider a situation with *both* electric and magnetic charges appearing as external particles.) These lead to the one-loop diagrams shown in Figs. 1(a) and 1(b) for the B_μ and ϕ propagators. The crucial diagram for condensation is Fig. 1(b). By our earlier arguments, this is evaluated from Fig. 1(a) by the use of the original and the dual gauge symmetries which result in the replacement of e^2 by $-e^2$ to take care of the mixed mode nature of the vertex. Further the integrals are cut off at M. The mass correction δm^2 given by Fig. 1(b) is then

$$\delta m^2 = -be^2M^2 \ . \tag{5.2}$$

b is a positive constant. The $W_\mu^\pm$-ϕ interaction is effectively pure scalar electrodynamics with $e \to ie$. The self-energy graph due to B_μ loop [Fig. 3(b), first diagram] gives

$$\delta m^2 = \tilde{b} g^2 M^2 \ . \tag{5.3}$$

Thus we see a competition between e and g. For small e (large g) there is no confinement. Confinement is turned on at $e^2 \simeq g^2$.

We can actually compute the effective potential for ϕ by integrating out the vector bosons. Before we do this, we dispose of a question on the effect of Fig. 1(a). This is a contribution to magnetic charge renormalization. If the magnetic potential B_μ of the external legs were replaced by A_μ, it is well known that the contribution of diagram 1(a) is negative (corresponding to asymptotic freedom). With B_μ on the external lines the contribution is thus positive. This is consistent with the increase of g when e decreases. If the dual gauge invariance of B_μ is preserved Fig. 1(a) is only a charge-renormalization effect. But in the Higgs mode for B_μ (confinement), Fig. 1(a) also contributes to the mass of B_μ. A positive contribution thus means that the dual Higgs mode (or electric confinement phase) is stabilized, i.e., mass of B_μ is increased by radiative corrections of the type Fig. 1(a). This thus provides further support for our thesis that the duality induced sign change in e^2 is important for confinement.

Treating $W_\mu^\pm$-ϕ interaction as electrodynamics with $e \to ie$ replacement, the effective potential for ϕ can be computed. The two sets of diagrams corresponding to $W_\mu^\pm$ and B_μ loops are shown in Fig. 3. The integrals are cut off at $k^2 = M^2$. The result is

$$V_{\text{eff}}(\phi) = V(\psi_M) + V(\psi_E) \tag{5.4}$$

with

$$V(\psi) = \frac{3M^4}{64\pi^2}\left[\ln(1+\psi) + \psi - \psi^2 \ln\left[1+\frac{1}{\psi}\right]\right] \ , \tag{5.5}$$

where

$$\psi_M = \frac{g^2\phi^2}{M^2}\ , \quad \psi_E = \frac{M^2 - e^2\phi^2}{M^2} \tag{5.6}$$

corresponding to magnetic (B_μ) and electric (W_μ) loops. We also define

$$\tilde{V}(\phi) = V_{\text{eff}}(\phi) - V(1) \tag{5.7}$$

so that $\tilde{V}(0) = 0$.

In the limit $M \to \infty$, after subtraction of the divergent terms, $V(\psi)$ is the standard Coleman-Weinberg potential.[14] Now, however, we have a physical cutoff M and $V(\psi_m)$ is a monotonically increasing function of ψ_M (see Fig. 4) with a minimum at the origin. The simple Coleman-Weinberg mechanism for spontaneous symmetry breaking is not operative.

The electric piece $V(\psi_E)$ provides the source for spontaneous symmetry breaking of the dual gauge group. This is seen from Fig. 5 where $V(\psi_E)$ is plotted against ϕ^2. $V(\psi_E)$ becomes imaginary for $\psi_E < 0$ or $e^2\phi^2 > M^2$ but

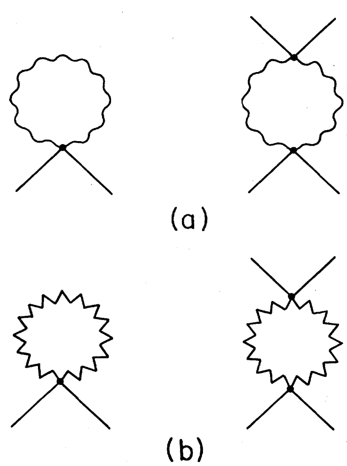

FIG. 3. The class of diagrams used to constrict the effective monopole potential. (a) are the internal gluon loops, while (b) are the internal magnetic-photon loops.

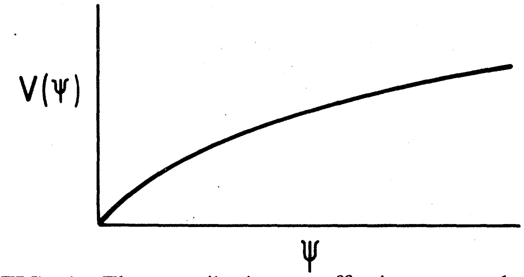

FIG. 4. The contribution to effective monopole potential realized by integrating out one-magnetic-photon loops as in Fig. 3(b).

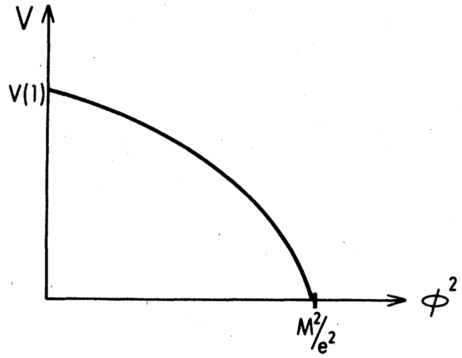

FIG. 5. The contribution to the effective potential of the gluon loops of Fig. 3(a).

this is the region where our effective theory breaks down (perturbative QCD takes over).

The occurrence of confinement depends on the competition between electric and magnetic contributions. Which component prevails can be seen by computing $\tilde{V}(\phi^2=M^2/e^2)$:

$$\tilde{V}\left[\phi^2=\frac{M^2}{e^2}\right]=V(g^2/e^2)-V(1)\ . \tag{5.8}$$

Since V is monotonic, whether a nontrivial minimum ($\phi^2\neq 0$) can develop or not depends on whether g^2/e^2 is less than or greater than 1 (see Fig. 6).

The emergence of a nonzero value of ϕ at the minimum is a generalized version of the Higgs mechanism which is itself the relativistic version of the Ginzburg-Landau theory of superconductivity. In our case it is the magnetic monopole field ϕ which develops nonzero vacuum value and condenses and so we have a magnetic superconductor in which the electric charges are confined.[15]

Returning to Fig. 6 we see that the confining phase is preferred whenever $g^2/e^2<1$ and that the confining phase transition is first order. By virtue of the Dirac quantization rule, $eg=2\pi$ the phase boundary

$$g^2=e^2 \tag{5.9a}$$

corresponds to

$$\alpha_e=\tfrac{1}{2}=\alpha_g\ . \tag{5.9b}$$

This equation determines the deconfinement point in terms of the scale parameter of QCD in some standard renormalization scheme.

That the point $e=g$ in coupling constant space is special is apparent in the chromomagnetic picture of confinement. In the perturbative phase only electric quantities are prominent. Confinement requires the magnetic variables to play a pivotal role. $e\simeq g$ is the transitional area where we go from one type of variable to the other. At just the confinement point we can neglect neither electric nor magnetic interactions. They will be on equal footing when (5.9) is satisfied.

If confinement sets in when $\alpha\approx\frac{1}{2}$ we can understand the precocity of scaling. Confinement is very close to the perturbative regime. In the early days of QCD the preferred value of $\Lambda_{\rm QCD}\approx 500$ MeV was intuitively very satisfying the because it seemed to set the confinement scale ($Q^2\approx\Lambda^2$, α very large) at a reasonable, typical hadronic mass scale. The most recent determinations[16] of Λ prefer values of 100—200 MeV which seem too small for the confinement scale. The original intuition is resurrected when we accept the above picture where confinement sets in at $\alpha=\frac{1}{2}$. The appropriate Q, for the new values of Λ, is 400—800 MeV right where it is expected. Just beyond this value of Q, α is small (less than $\frac{1}{2}$) and we expect perturbation theory to be valid.

It should be noted that our effective theory breaks down at $e^2\phi^2=M^2$ so that the nature of the minimum of V cannot really be described. However we can assert that $\phi=0$ loses its status as the global minimum once $g^2<e^2$. The preferred value of $\langle\phi^2\rangle$, it would seem, is M^2/e^2 so that the B_μ quanta acquire a mass M becoming degenerate with $W^\pm$.

The phase structure of QCD is now seen to be very simple. When α is very small perturbation theory holds. As

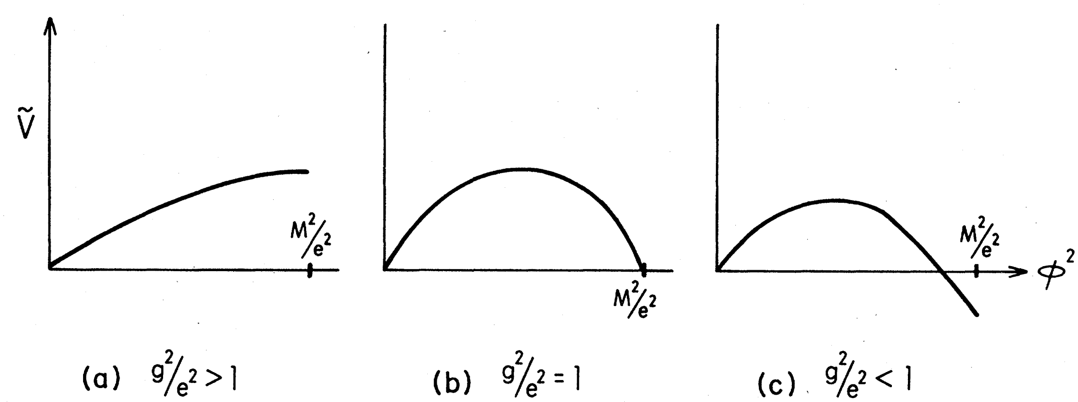

FIG. 6. The effective potential [Eq. (5.7)] for differing values of g^2/e^2. When $g^2/e^2<1$ the minimum at the origin is no longer the absolute minimum and a first-order, confining, phase transition occurs.

α increases we have a phase with monopoles and massive gluons. (These become dominant in the sense of being quasiparticles, i.e., they are collective states which are approximate normal modes of the complicated fully interacting Hamiltonian.) When α becomes $\frac{1}{2}$, the monopoles condense and we are in the confining phase. This is the Higgs mode in terms of the magnetic variables.

VI. CONCLUSIONS

QCD is an unbroken non-Abelian gauge theory whose most dramatic property is color confinement. In this paper we have attempted to demonstrate how confinement arises from just those deep properties (topological structure and monopoles, asymptotic freedom, and gauge symmetry) which are the essence, the heart, of non-Abelian gauge theories.

The topological structure of non-Abelian gauge theories provides for the existence of monopole-field configurations. The monopoles are important in determining the nonperturbative dynamics of the theory. In order to be able to study monopoles we have introduced a scalar field which serves as a regulator for the theory and makes the monopoles explicit. The scalar can be chosen to maintain the asymptotic freedom of the theory. The low-energy behavior of the theory is then dominated by almost pointlike monopoles of almost zero mass. These monopoles will couple to "electric" gluons and this "mixed mode" interaction is crucial for confinement. The monopole-gluon coupling is determined by duality arguments to be standard except for an extra, crucial, minus sign. This simple result is true only when all external particles are either all electric, or all magnetic.

An important sidelight which emerged from our discussion at this point was that a $1/k^4$ propagator for the gluon is dual equivalent to the statement that the gluon is propagating in a chromomagnetic superconductor.

We made our general discussion concrete by studying an SU(2) model. After regulating the theory we were left with a model of a scalar magnetic monopole, a "magnetic" photon (B_μ), and two massive charged gluons. An effective potential was generated by integrating out the magnetic photon and gluon loops. Because of the minus sign whose appearance was heralded by duality arguments the gluon loops tend to destabilize the potential and give rise to a minimum away from the origin. The transition from $\langle 0|\phi|0\rangle=0$ to $\langle 0|\phi|0\rangle\neq 0$ is first order and leads, in analogy with the Higgs-Ginzburg-Landau theory of superconductivity, to the vacuum becoming a chromomagnetic superconductor. The transition occurs at $e=g$, $\alpha_e=\frac{1}{2}$ which is obviously special when both electric and magnetic charges are present. The value $\alpha=\frac{1}{2}$ as the transition to confinement accords well with intuition and with the experimental fact of precocious scaling. This value emerged nontrivially from the dynamics.

The appearance of a minimum of V_{eff} away from the origin depended intimately on

(1) the existence of monopoles as a result of the topology of the gauge field,

(2) negative couplings involving the "non-Abelian" particles W which were derived on the basis of electric magnetic duality and gauge invariance, and

(3) the asymptotic freedom of the electric coupling constant which allowed the destabilizing contribution to become more important as e grew at low energies and eventually to overcome the magnetic contribution to V_{eff} at $e=g$.

The major theoretical problems which remain in this scenario are the introduction of the monopoles in a more natural way than by introducing a new Higgs scalar and the related question of the dynamics at $\phi^2=M^2/e^2$.

The physical implications of a chromomagnetic superconducting vacuum will be the subject of a sequel paper.[17] We have[18] already demonstrated that an attractive picture of chiral-symmetry breaking emerges and we will further explore this phenomena. We will also examine the connection of bag and string models of hadrons on one hand and type-I and type-II superconductors on the other. The insight gained from this analyses will be applied to the question of the phases of hadronic matter.

The superconducting model provides an attractive picture of quark confinement. We have seen how this model naturally comes forth from QCD. Further study of the model should prove fruitful.

ACKNOWLEDGMENTS

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